

# Assessing the Cost of Capital for Longevity Risk

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## Abstract

The cost of capital is a key element of the embedded value methodology for the valuation of a life business. Further, under some solvency approaches (in particular, the Swiss Solvency Test and possibly the developing Solvency 2 project) assessing the cost of capital constitutes a step in determining the required allocation.

Whilst the cost of capital is usually meant as a reward for the risks encumbering a given life portfolio, in actuarial practice the relevant parameter has been traditionally chosen, at least to some extent, inconsistently with such risks. The adoption of market-consistent valuations has then been advocated to reach a common standard.

A market-consistent value usually acknowledges a reward to shareholders' capital as long as the market does, namely if the risk is systematic or undiversifiable. When dealing with a life annuity portfolio (or a pension plan), an important example of systematic risk is provided by the longevity risk, i.e. the risk of systematic deviations from the forecasted mortality trend. Hence, a market-consistent approach should provide appropriate valuation tools.

In this paper we refer to a portfolio of immediate life annuities and we focus on longevity risk. Our purpose is to design a framework for a valuation of the portfolio which is market-consistent, and therefore based on a risk-neutral argument, while involving some of the basic items of a traditional valuation, viz best estimate future flows and allocated capital. This way, we try to reconcile the traditional with a market-consistent (or risk-neutral) approach. This allows us, in particular, to translate the results obtained under the risk-neutral approach in terms of the cost of capital.

**Keywords:** target capital, risk reward, embedded value, market-consistent valuation, risk-neutral setting, cost of capital, risk discount rate, longevity risk, life annuities.

# 1 Introduction

The lively debate about the valuation of the life business is still far to come to a proposal of a standard model. The main drawbacks of the classical Embedded Value (EV) framework are well-known; in particular, criticisms usually focus on the discount rate, given that it should include a reward for the risks inherent in the flows to be discounted, but its consistency with them is hard to disclose. On the other hand, Market-Consistent (MC) approaches, under which only undiversifiable risks are rewarded, while taking advantage of risk-adjustments assessed according to market experience, prove to be weak (when even not conceivable) when risks with poor or no market evidence are dealt with.

Among the risks affecting life annuity portfolios, special attention should be addressed to the “longevity” risk, i.e. the risk of unanticipated aggregate mortality, arising from mortality evolution at old ages. Unlike the risk of random fluctuations, which is diversifiable (or, better said, “pooling”) being due to deviations around expected mortality, the longevity risk is systematic, due to the fact that it concerns aggregate mortality (actually, it is realized when deviations from expected mortality are observed along time). So far, longevity risk has not been traded on the market; if one wants to assess its likely market cost through comparison with similar traded risks, a replicating strategy is hard to find. So, pricing longevity risk market-consistently keeps on being an open modeling problem.

The purpose of this paper is to design a setting for the valuation of a life annuity portfolio within which, using risk-neutral arguments, the cost of longevity risk is disclosed. We focus on immediate life annuities and we develop the valuation around some of the basic building-blocks of the traditional EV framework, viz the best estimate assessment of future flows and the so-called cost of capital (properly revised). This way, we try to reconcile the traditional with the market-consistent valuation. Rather than on the market pricing of longevity risk, we focus on developing a proper structure for the appraisal of a life annuity portfolio bearing longevity risk. Further work is then required to completely setting the valuation framework, in particular in regard of mortality modeling.

The paper is organized as follows. In Section 2 we recall the main features of the EV methodology and the purposes of a MC approach. In Section 3, referring to a portfolio of immediate life annuities, we describe a possible setting for a risk-neutral approach in respect of longevity risk; this involves a reinsurance arrangement and a portfolio of longevity bonds. In Section 4 we investigate bounds for the price of both the reinsurance arrangement and the bond portfolio. In Section 5 links between the traditional and a market-consistent assessment of the cost of capital are focussed. Finally, Section 6 concludes with some remarks on mortality modeling.

## 2 Life portfolio valuation: the traditional vs the market approach

The traditional approach to portfolio valuation is based on the so-called ‘‘Value of the In-Force business’’ (VIF), defined as the present value of future distributable earnings calculated with a given Risk Discount Rate (RDR), net of the amount of shareholders’ capital currently within portfolio assets. The distributable earning related to a given period, say a year, is defined as the flow from the portfolio assets to the residual assets of the insurance company (or vice versa) such that portfolio assets amount to a given level, viz the technical provision (or mathematical reserve) plus the target capital. According to this definition, the VIF at time  $t$ ,  $VIF_t^{[T]}$  (the  $[T]$  on the top refers to the underlying traditional approach), is given by

$$VIF_t^{[T]} = \sum_{h=t+1}^n K_h v_\rho(t, h) - M_t \quad (1)$$

where  $K_h$  denotes the distributable earning arising in year  $(h - 1, h)$ ,  $v_\rho(t, h)$  is the discount factor based on the annual RDR’s  $\rho_{t+1}, \rho_{t+2}, \dots, \rho_n$ ,  $M_t$  is the shareholders’ capital within portfolio assets at time  $t$  and  $n$  is the maximum residual duration of in-force policies.

Shareholders’ capital within portfolio assets originates, year by year, from undistributable earnings, as well as from returns on pertaining assets. The former may be meant as the share of industrial profit to be maintained within portfolio assets (of course, in case of either an industrial loss or in case of an industrial profit below the amount to be maintained within portfolio assets, it turns out that undistributable earnings are negative, so that some residual assets must be allocated to the portfolio). Therefore, we have

$$M_t = M_{t-1} (1 + i_t^*) + U_t^I - K_t$$

where  $i_t^*$  is the investment yield,  $U_t^I$  the industrial profit in year  $(t - 1, t)$  and  $U_t^I - K_t$  the undistributable earning (which may also be negative). Relation (1) may be then rewritten as

$$VIF_t^{[T]} = \sum_{h=t+1}^n U_h^I v_\rho(t, h) - \sum_{h=t+1}^n M_{h-1} (\rho_h - i_h^*) v_\rho(t, h) \quad (2)$$

The quantity  $\rho_h - i_h^*$  in (2) represents the risk adjustment in the discount rate meant to reward, on a yearly basis, one monetary unit of shareholders’ capital for the risks encumbering the life portfolio (all but the market risk, whose reward is already embedded in  $i_h^*$ ); such risk adjustment is usually assumed to be constant in time, i.e.  $\rho_h - i_h^* = r^{(a)}$  for all  $h$ . The first term in the right-hand side of equation (2) is usually called Present Value of Future Profits (PVFP), whereas the second term represents the Cost of Capital

(CC), i.e. the present value of risk rewards required on the capital allocated to the life portfolio. Hence, equation (2) can also be written as

$$VIF_t^{[T]} = PVFP_t^{[T]} - CC_t^{[T]}$$

In order to better understand our discussion, it is useful to stress (or recall) some of the assumptions usually underlying (1) or (2) in practical implementations.

1. Typically, a best-estimate scenario is referred to for the assessment of future flows, whence a deterministic valuation follows. In recent years, stochastic models have been introduced for the valuation of financial options and guarantees embedded in life insurance products. Anyhow, the approach can be looked at as a deterministic one.
2. Portfolio riskiness could be allowed for mainly through the RDR, which as mentioned above should account for various risks inherent in the portfolio itself (mortality, investment, and so on), but also for inefficiencies in managing the portfolio (agency costs). This way, the reward in the RDR is not purely a risk adjustment. To this regard, it is worth stressing that within the traditional approach risks are rewarded as they are perceived by the company; no care is taken with reference to the way other agents in the market do value risks.
3. The RDR is often chosen by looking at common practice in this respect. As a consequence, the risk adjustment could not be properly tailored to portfolio features; hence, a biased valuation may follow.

Many papers have been devoted to the EV modeling. However, possibly due to the fact that it is a methodology developed along practice, a comprehensive formal description is somewhat lacking in literature. The main aspects are dealt with, among the others, by Turner (1978), Burrows and Whitehead (1987), LAVMWP (2001), Olivieri and Pitacco (2005) (the latter does provide a comprehensive description, however in Italian). Discussions about the RDR can be found also in Pemberton *et al.* (2000); to this regard, Sherris (1987), Coleman *et al.* (1992) and Burrows and Lang (1997) discuss the use of the CAPM.

The market approach to portfolio valuations aims at overcoming some of the weak points of the traditional actuarial model. Typically, a risk-neutral valuation principle is adopted, according to which annual flows must be adjusted with a risk margin assessed consistently with the price of securities suitable to transfer to the market the risk itself; risk-adjusted flows are then discounted with a risk-free rate. It follows, in particular, that the CC is no longer accounted for, given that no interest margin is included in the discount rate. It is worth stressing that, according to this setting, only undiversifiable risks (in particular systematic risks common to any agent) are rewarded. In practical terms, the value of risks with a market evidence can be assessed by applying marked-to-market arguments. The value of the portfolio (in particular if one looks for the value to shareholders) is anyhow affected by:

- i. systematic risks with poor or no market evidence (to which longevity risk belongs);
- ii. inefficiencies in managing the portfolio (for example, pooling risks not fully diversified);
- iii. agency costs.

The development of a market-consistent framework for life insurance valuations has been widely attacked in latest years, also following the proposals of the International Accounting Standard Board for new accounting principles. Just mentioning contributions moving from the EV setting, see for example O’Keeffe *et al.* (2005), CFO Forum (2004, 2006), Tillinghast-Towers Perrins (2004, 2005), Sheldon and Smith (2004).

### 3 Setting a risk-neutral framework for longevity risk

Longevity risk arises because of unanticipated aggregate mortality at old ages. As it is well-known, observations of past mortality suggest to adopt projected mortality tables for the actuarial appraisal of annuities (and other living benefits), i.e. to use mortality assumptions including a forecast of future mortality. Notwithstanding, whatever hypothesis is assumed, the future mortality trend is random, whence an uncertainty risk arises. Unlike the mortality risk of random fluctuations, the longevity risk is a systematic risk, namely a risk of systematic deviations from expected mortality (see, for example, Olivieri, 2001). Hence, it cannot be diversified with the usual pooling insurance argument. Technical tools to manage it include reinsurance, longevity securities and, of course, capital allocation. However, neither reinsurance arrangements nor capital market solutions have yet been fully developed. For some suggestions about reinsurance arrangements, see Olivieri (2005); for a discussion of possible structures of longevity securities, see Blake *et al.* (2006). Internal models for allocating capital to portfolio of life annuities bearing longevity risk are discussed in Olivieri and Pitacco (2003). In any case, mortality modeling must allow for uncertainty risk; this point will be shortly addressed in Section 6. In the remainder of this Section we focus on a structure for accounting for the cost of longevity risk when performing a valuation of a portfolio of life annuities.

For the sake of brevity and for shortening notation, we adopt some simplifying assumptions.

1. The only risk perceived by the market is longevity risk. The risk is undiversifiable, but it can be transferred to other agents in the market, resorting to proper arrangements.
2. The risk of random fluctuations in mortality is fully diversified by the insurer, whence no reward is allowed for.
3. There is no financial risk, so that the annual yield from investments is the risk-free rate  $i$  (constant, for brevity).

4. The risk of reinsurers' default is negligible.
5. Expenses and transaction costs are disregarded.
6. All agents on the market (but the annuitants) hold the same information. Hence the same model for annuitants' mortality is shared by all agents.

It is reasonable to think that information asymmetries occur between the insurer on one hand and the annuitants on the other, concerning both information held by the annuitant only (whence the adverse selection risk arises) and information available to the insurer only (with regard to the description of the future mortality trend, and hence to the choice of the pricing mortality table). Such asymmetries lead to a safety loading embedded into the annuitisation rate chosen by the insurer.

The income at time 0 to the annuity provider is given by the single premium paid by annuitants. Referring to a cohort only, its total amount is  $N_0V_0$ , where  $N_0$  is the initial size of the portfolio (i.e. the cohort) and  $V_0$  is the single premium required to each annuitant;  $V_0$  represents the individual reserve at time 0 as well. With respect to a best estimate mortality table and for a given annual amount,  $V_0$  embeds some safety loading (i.e. expected profit). The individual reserve at time  $t$  will be denoted by  $V_t$ .

Assuming a proper cash unit, a unitary annual amount can be referred to. So the annual, random, outflows for the insurer are given by  $N_1, N_2, \dots$ , with  $N_t$  random number of annuitants alive after  $t$  years from issue.

Under the traditional valuation framework, the number of future lives is assessed through a best-estimate scenario; we denote such assessment as  $N_t^{[BE]}$ . The PVFP at time 0 is then

$$PVFP_0^{[T]} = \sum_{t=1}^n \left[ N_{t-1}^{[BE]} V_{t-1} (1+i) - N_t^{[BE]} - N_t^{[BE]} V_t \right] (1+\rho)^{-t} \quad (3)$$

where

$$U_t^I = N_{t-1}^{[BE]} V_{t-1} (1+i) - N_t^{[BE]} - N_t^{[BE]} V_t$$

is the best-estimate assessment of the industrial profit in year  $(t-1, t)$ , and the discount rate  $\rho$  (consistently with the risk-free rate) is assumed to be constant. Note that the adjustment  $r^{(a)} = \rho - i$  in the current setting should be meant as the reward for longevity risk only. The CC is

$$CC_0^{[T]} = \sum_{t=1}^n M_{t-1} (\rho - i) (1+\rho)^{-t} \quad (4)$$

whence we can get the appraisal of  $VIF_0^{[T]}$  as

$$VIF_0^{[T]} = PVFP_0^{[T]} - CC_0^{[T]} \quad (5)$$

In relation to the time horizon in (3) and (4) (and, hence, in (5)), note that it coincides with the maximum residual lifetime of annuitants, as it is assessed in the best-estimate scenario (in practice, it depends on the maximum attainable age assumed in the relevant mortality table).

Turning to the market approach, we aim to make use of risk-neutral arguments; hence, future outflows must be risk-adjusted and then discounted with the risk-free rate. However, we would like to design a framework where future flows can still be considered in terms of their best estimate appraisal; this will help us in reconciling the traditional with the risk-neutral approach (see Section 5).

Let us assume that the annuity provider transfers the longevity risk to a reinsurer through a swap-like arrangement. According to such arrangement, in each year the reinsurer pays to the cedant the random amount  $N_t - N_t^{[BE]}$  if positive; otherwise the amount  $N_t^{[BE]} - N_t$  is paid by the cedant to the reinsurer. It follows that the net annual outflow to the annuity provider is

$$N_t - (N_t - N_t^{[BE]}) = N_t^{[BE]} \quad (6)$$

which is certain.

As to the funding of this arrangement, the reinsurer may apply either a single or a periodic premium. The latter case should easily allow for possible changes of the pricing basis suggested by mortality evolution; it is reasonable to think that also in the former case the reinsurer may claim some revision of the initial premium, in particular in case of an adverse mortality evolution. However, this would imply that some longevity risk is still borne by the annuity provider or that it has not been neutralized. We will therefore address the case of a single reinsurance premium  $RP_0$ , to be paid at time 0 and subject to no future revision; this way, only the future, certain, outflows (6) are faced by the annuity provider.

When addressing profit assessment, some uncertainty does in any case lie on the annuity provider, not only because of the possible default of the reinsurer (which we are actually disregarding), but also in relation to the development throughout time of the technical provision facing portfolio liabilities. Actually, at time  $t$  the portfolio reserve is  $N_t V_t$ , i.e. a random quantity due to  $N_t$  as well as to possible revisions of the reserving basis. The latter aspect should be accounted for in the calculation of the target capital. Concerning  $N_t$ , thanks to the intervention of the reinsurer, which neutralizes risks in annual outflows, when assessing the value of the in-force business it is reasonable to make anyhow reference to the best-estimate value. In any case, a risk-adjustment in the discount rate is no longer acceptable. For the present value of future profits, the following (classical) results hold

$$\begin{aligned} & \sum_{t=1}^n \left[ N_{t-1}^{[BE]} V_{t-1} (1+i) - N_t^{[BE]} - N_t^{[BE]} V_t \right] (1+i)^{-t} \\ &= \sum_{t=1}^n \left[ N_{t-1} V_{t-1} (1+i) - N_t^{[BE]} - N_t V_t \right] (1+i)^{-t} \\ &= N_0 V_0 - \sum_{t=1}^n N_t^{[BE]} (1+i)^{-t} \end{aligned} \quad (7)$$

as it is well-known in actuarial mathematics. Note that also the maximum duration of future flows, assumed to be  $n$  in (7), is actually random. For what commented about the portfolio reserve, it is reasonable to refer to the maximum age in the best-estimate scenario.

The right-hand side of (7) is the so-called total cash flow, which will be denoted as

$$CF_0 = N_0V_0 - \sum_{t=1}^n N_t^{[BE]} (1+i)^{-t}$$

Under the assumptions above, the VIF can be assessed as follows

$$VIF_0^{[RN]} = CF_0 - RP_0 \quad (8)$$

( $[RN]$  on the top refers to the underlying risk-neutral setting), due to the following facts (compare with (5)): the PVFP, given now by (7), reduces to  $CF_0$ ; the CC is no longer considered, since no risk-adjustment is allowed for; this is counterbalanced by the outcome  $RP_0$  at time 0. In order to come to the valuation of the VIF, we need to address  $RP_0$ , which actually represents the adjustment for the longevity risk; to this aim, we have to investigate the reinsurer's position.

The reinsurer needs to hedge the accepted risk. To this purpose we assume that it takes a short position on a portfolio of Zero-Coupon bonds, one for each future time  $t$ ,  $t = 1, 2, \dots, n$ . The pay-off of the bond maturing at time  $t$ ,  $t = 1, 2, \dots, n$ , is  $N_0 - N_t$ , i.e. equal to the number of deaths among annuitants during  $(0, t)$ . Hence, the bonds can be looked at as longevity bonds. Note that the same sequence of cash flows could be realized by taking a short position on a longevity bond with principal 0 and (random) annual coupon  $N_0 - N_t$ ; so, the annual pay-off is increasing in time, but at an unknown pace, due to mortality.

Joint to the annual payment  $N_t - N_t^{[BE]}$  to/from the annuity provider, the reinsurer is now also liable for the outflows  $N_0 - N_t$  in respect of investors. Its net position is

$$\left(N_t - N_t^{[BE]}\right) + (N_0 - N_t) = N_0 - N_t^{[BE]}$$

which is certain. At time 0, the income  $RP_0$  by the insurer is increased by the income  $BP_0$  of the price of the bond portfolio. Note that the present value at time 0 of the overall position of the reinsurer,  $VR_0$ , is

$$VR_0 = RP_0 + BP_0 - \sum_{t=1}^n \left(N_0 - N_t^{[BE]}\right) (1+i)^{-t}$$

Actually, since future flows are certain, a risk-free discount rate must be used.

With regard to the mortality underlying the longevity bonds, a remark is required. Typically, for designing a bond whose pay-off (either the principal or the coupons) depends on mortality, reference must be made to a population whose mortality experience is recorded by some independent authority; so, typically the reference population of a

longevity bond should be the national population and not a population pertaining to the portfolio of an insurance company or a reinsurer. Anyhow, our assumption is backed by the initial hypothesis that all the relevant information about mortality are shared by all the agents (but the annuitants). At this stage we prefer to keep this simplifying hypothesis, which allows us to exclude basis risk, so to focus on the basic valuation problem; in order to implement the overall structure, further developments are clearly required (see also Section 6). Also in relation to the longest maturity of longevity bonds a remark is required. In our setting, reference is to the maximum residual lifetime of annuitants. It is reasonable that for a longevity security the longest maturity is chosen in some fixed way (given that the residual lifetime of the reference population is unknown at the time of bond issue). For what said above, we keep  $n$  as the maximum maturity.

Clearly, from the price of the portfolio bond some suggestions may be derived for the reinsurance premium.

## 4 Bounds for the reinsurance premium and the price of bonds

As regards the reinsurance premium and the price of the bond portfolio, a mortality model allowing for uncertainty in mortality evolution is required (see Section 6 for some remarks). In any case, some bounds may be obtained by investigating feasibility conditions concerning the annuity provider and the reinsurer.

The feasibility of the net position of the annuity provider is

$$VIF_0^{[RN]} \geq 0$$

i.e.

$$RP_0 \leq CF_0$$

whence an upper bound for the reinsurance premium follows, viz

$$RP_0^{\max} = CF_0 \tag{9}$$

Conversely, the feasibility of the net position of the reinsurer is

$$VR_0 \geq 0$$

i.e.

$$RP_0 \geq \sum_{t=1}^n \left( N_0 - N_t^{[BE]} \right) (1+i)^{-t} - BP_0$$

whence we get a lower bound for the reinsurance premium

$$RP_0^{\min}(BP_0) = \sum_{t=1}^n \left( N_0 - N_t^{[BE]} \right) (1+i)^{-t} - BP_0 \tag{10}$$

which is dependent on the price of the bond portfolio,  $BP_0$ . Note that in order that bounds (10) and (9) are properly ordered, we must also require

$$\sum_{t=1}^n \left( N_0 - N_t^{[BE]} \right) (1+i)^{-t} - BP_0 \leq CF_0$$

whence a lower bound for the price of the bond portfolio follows

$$BP_0^{\min} = \sum_{t=1}^n N_0 (1+i)^{-t} - N_0 V_0 \quad (11)$$

An interesting interpretation can be given to bound (11). The quantity  $\sum_{t=1}^n N_0 (1+i)^{-t}$  is the price of  $N_0$  annuities certain (term  $n$ ), with a unitary annual amount and backed by a risk-free investment. The quantity  $N_0 V_0$  is instead the price of  $N_0$  life annuities. What is required, therefore, is that the price of the bond is at least equal (apart from a scale factor) to the price reduction achieved shifting from a life annuity (traded on the insurance market) to an annuity certain (traded on the capital market) with duration given by the assumed maximum residual lifetime of the reference population.

As far as an upper bound for the price of the bond portfolio is concerned, in principle we should require

$$BP_0 \leq \sum_{t=1}^n N_0 (1+i)^{-t} \quad (12)$$

where the right-hand side of (12) represents the price of a portfolio made of Zero-Coupon bond maturing at time  $t$ ,  $t = 1, 2, \dots, n$ , each one with face value  $N_0$ . However, if (irrespective of the fact that, under the reinsurance arrangement, flows may go also from the cedant to the reinsurer) one thinks reasonable that  $RP_0 \geq 0$ , then we must require

$$RP_0^{\min}(BP_0) \geq 0$$

whence

$$BP_0 \leq \sum_{t=1}^n \left( N_0 - N_t^{[BE]} \right) (1+i)^{-t} \quad (13)$$

Note that, in our description of the reinsurance market, it is unrealistic that in case the proper reinsurance premium turns out to be negative (before being forced to some other amount by the reinsurer) the arrangement is developed; actually, since the annuity provider and the reinsurer are holding the same information about mortality, the annuity provider would not seek for protection in this case. Aiming just at the valuation of the life annuity portfolio, however, a negative reinsurance premium would make sense. However, recent experience does not back such hypothesis. So, we will refer to the following upper bound for the price of the bond portfolio:

$$BP_0^{\max} = \sum_{t=1}^n \left( N_0 - N_t^{[BE]} \right) (1+i)^{-t}$$

In Table 1 a simple numerical illustration is given. We refer to a portfolio of annuitants initial age 65. The risk-free rate is set equal to 2.5% and this is used also for setting the premium of the annuity. The pricing mortality table is given by the IPS55 (which is a projected table built up with reference to Italian male annuitants). It is assumed that such table provides mortality rates which are 20% lower than the best-estimate ones. Table 1 quotes the bounds for the reinsurance premium and the bond portfolio price (calculated per policy issued, i.e. with  $N_0 = 1$ ).

Upper bound for the reinsurance premium ( $RP_0^{\max} = CF_0$ )	0.80047
Lower bound for the bond portfolio price ( $BP_0^{\min}$ )	12.74646
Lower bound for the reinsurance premium ( $RP_0^{\min}$ )	$13.54693 - BP_0$
Upper bound for the bond portfolio price ( $BP_0^{\max}$ )	13.54693

Table 1: Bounds for the reinsurance premium and the bond portfolio price

## 5 Reconciling the traditional with the risk-neutral approach

In order to reconcile the traditional with the risk-neutral approach, a comparison must be made between the VIF's calculated under the two settings. In particular, we can be interested in finding the "equivalent RDR", i.e. the RDR  $\rho$  such that

$$VIF_0^{[T]} = VIF_0^{[RN]} \quad (14)$$

From (5) and (8), we get that the RDR  $\rho$  must be chosen so that

$$RP_0 = (CF_0 - PVFP_0^{[T]}) + CC_0^{[T]} \quad (15)$$

where, in particular, the term in brackets represents the reduction of the PVFP with respect to the total cash flow due to discounting future profits with a rate higher than the investment yield. Actually, thanks to (7) we may write

$$CF_0 = \sum_{t=1}^n U_t^I (1+i)^{-t}$$

whence (15) can be rewritten as

$$RP_0 = \sum_{t=1}^n U_t^I [(1+i)^{-t} - (1+\rho)^{-t}] - \sum_{t=1}^n M_{t-1} (1+\rho)^{-t} \quad (16)$$

It is evident from (16) that it is not possible to find a closed formula for the equivalent RDR. However, from numerical investigations possibly some approximate short-cut formulae may be found through proper fitting.

The problem of reconciling the traditional with the risk-neutral approach can be attacked also in a different manner.

Let us redefine the traditional VIF as follows

$$VIF_0^{[TRN]} = CF_0 - \sum_{t=1}^n M_{t-1} (\rho - i) (1 + i)^{-t} \quad (17)$$

(the  $[TRN]$  on the top stands for “traditional risk-neutral”). The difference between (17) and (5) consists in the discount rate, which is risk-free in (17) and risk-adjusted in (5). In (17) the CC is defined as

$$CC_0^{[TRN]} = \sum_{t=1}^n M_{t-1} (\rho - i) (1 + i)^{-t} \quad (18)$$

where  $\rho - i = r^{(r)}$  is the (risk) reward which can be acknowledged on allocated capital. Note that this is in line with the Cost of Capital addressed in modern solvency frameworks. The “equivalent risk reward” is the value of  $r^{(r)}$  such that

$$VIF_0^{[TRN]} = VIF_0^{[RN]}$$

We easily find

$$r^{(r)} = \frac{RP_0}{\sum_{t=1}^n M_{t-1} (1 + i)^{-t}} \quad (19)$$

Note that for a given reinsurance premium  $RP_0$ , the higher is the allocated capital (expressed by the quantities  $M_{t-1}$ ), the lower is the risk reward. Indeed, the amount of the shareholders’ capital should be determined with the goal of a high probability of meeting the relevant obligations, namely aiming at solvency. The risk reward should therefore account for the residual risks, i.e. for the possible default of the annuity portfolio. Actually, the CC calculated with (18), thanks to (19) accounts for the neutralization of risks.

The main advantages of (17) in respect of (5) are that the discount rate suggested in a market-consistent framework is adopted; further, for the calculation of the CC, a closed formula is obtained for the relevant parameter, which is also easier to disclose. The advantage of (17) in respect of (8) is the possibility to refer to a methodological structure already adopted in practice (whence the relevant packages just require a resetting of the main parameters). Clearly, the open problem is still the appraisal of the reinsurance arrangement.

Before moving to conclusions and remarks about mortality modeling, let us quote a numerical example on the issues of this Section. We adopt the same parameters used in Section 4. As to the target capital, we assume two alternative rules:

- a “Solvency 1”-like rule, according to which the target capital at time  $t$  is 4% of the portfolio reserve;

- a “Solvency 2”-like rule, according to which the target capital at time  $t$  is the change in net asset value against a 25% permanent decrease of mortality rates.

In our example, the Solvency 2-like rule turns out to be a more severe requirement than the other one.

Tables 2 and 3 quote the equivalent RDR  $\rho$  obtained solving (14), having adopted the two alternative target capitals mentioned above. Just to give an example, in each row of the two tables, it is assumed that the reinsurance premium  $RP_0$  is such that it reduces the  $VIF_0^{[RN]}$  to a given percentage of  $CF_0$  (which is worth 0.80047; see Table 1). Although the behavior of the equivalent RDR  $\rho$  is increasing in respect of  $RP_0$  (i.e. in respect of a decreasing VIF), it is not possible to find a satisfactory approximate short-cut formula. Of course, more useful results in this regard could be found when addressing more than one cohort.

$RP_0$ s.t. $\frac{VIF_0^{[RN]}}{CF_0}$	$VIF_0^{[T]} = VIF_0^{[RN]}$	$\rho$	$r^{(a)} = \rho - i$	$PVFP_0^{[T]}$	$CC_0^{[T]}$
100%	0.80047	2.500%	0.00%	0.80047	0.00000
90%	0.72042	2.895%	0.40%	0.75392	0.03350
80%	0.64038	3.327%	0.83%	0.70725	0.06687
70%	0.56033	3.800%	1.30%	0.66047	0.10014
60%	0.48028	4.324%	1.82%	0.61360	0.13332
50%	0.40023	4.908%	2.41%	0.56667	0.16644
40%	0.32019	5.566%	3.07%	0.51971	0.19952
30%	0.24014	6.315%	3.82%	0.47276	0.23262
20%	0.16009	7.180%	4.68%	0.42587	0.26579
10%	0.08005	8.195%	5.70%	0.37913	0.29908
0%	0.00000	9.413%	6.91%	0.33262	0.33262

Table 2: Equivalent RDR  $\rho$ ; Solvency 1-like rule for target capital

Table 4 quotes the factor  $\frac{1}{\sum_{t=1}^n M_{t-1} (1+i)^{-t}}$  in the expression of the equivalent risk reward (see (19)). Once  $RP_0$  has been assigned, the equivalent risk reward can be assessed through (19), as it is suggested in Table 4.

Note, in all examples, that the equivalent RDR  $\rho$  and the equivalent risk reward depend on the amount of the target capital, as noted above. In particular, the equivalent risk reward (see Table 4) may help in disclosing the severity of the capital allocation policy adopted by the annuity provider.

$RP_0$ s.t. $\frac{VIF_0^{[RN]}}{CF_0}$	$VIF_0^{[T]} = VIF_0^{[RN]}$	$\rho$	$r^{(a)} = \rho - i$	$PVFP_0^{[T]}$	$CC_0^{[T]}$
100%	0.80047	2.500%	0.00%	0.80047	0.00000
90%	0.72042	2.845%	0.34%	0.75965	0.03923
80%	0.64038	3.225%	0.72%	0.71789	0.07751
70%	0.56033	3.647%	1.15%	0.67512	0.11479
60%	0.48028	4.120%	1.62%	0.63127	0.15099
50%	0.40023	4.657%	2.16%	0.58624	0.18601
40%	0.32019	5.273%	2.77%	0.53995	0.21976
30%	0.24014	5.991%	3.49%	0.49228	0.25214
20%	0.16009	6.846%	4.35%	0.44312	0.28303
10%	0.08005	7.891%	5.39%	0.39232	0.31227
0%	0.00000	9.210%	6.71%	0.33973	0.33973

Table 3: Equivalent RDR  $\rho$ ; Solvency 2-like rule for target capital

target capital	$r^{(r)}$
Solvency 1-like rule	$0.11292 \times RP_0$
Solvency 2-like rule	$0.08297 \times RP_0$

Table 4: Equivalent risk reward

## 6 Mortality models allowing for longevity uncertainty and final remarks

As it was mentioned earlier, longevity risk concerns aggregate mortality (and this originates its systematic nature), due to an unanticipated decrease in mortality rates at adult ages.

Modeling the longevity phenomenon and the consequent uncertainty is currently a core topic in actuarial literature. A proper modeling should first address the features of mortality evolution. Among these, rectangularization and expansion (see, for example, Olivieri, 2001), period and cohort effects (see Willets, 2004; Richards *et al.*, 2006) have been highlighted.

- Expansion: this aspect is related to the increasing average duration of life, joint to the increasing maximum age observed in cross-sectional observations. The risk of systematic deviations has turned out to be stronger than in the past.
- Rectangularization: cross-sectional observations lead to a survival function (at birth) with a more rectangular shape; this reduces the risk of random fluctuations. However, the survival function conditional on outliving an adult age (say, for

example, age 65) is less rectangular, which means that referring just at the adult ages, as in the case of immediate annuities, the dispersion of the residual lifetime has increased in time. This could be linked to the increased maximum age.

- Period and cohort effect: cross-sectional observations suggest that the general trend in the population consists in a decline in time of mortality rates. Due to particular events (such as an epidemic flu or bad weather conditions, also in terms of very high temperatures in summer time), it may happen that in some years the trend is reversed, especially in relation to some ages. Further, going deeper into the analysis of data, it may turn out that some cohorts are experiencing a specific improvement. From such considerations, the notions of cohort effect and period effect follow. The idea is that each cohort has its own mortality trend; however, some changes (possibly, temporary) are common to more than one cohort (or even to the overall population).

When addressing adult and old ages, a general model representing mortality risks should therefore account for:

- random fluctuations, whose impact can be faced by mutuality;
- systematic deviations due to the (unknown) trend specific of a cohort;
- period (unknown) deviations, common to more than one cohort in the population;

A number of stochastic models have been developed to represent mortality improvements. A very influential model is the Lee-Carter (see Lee and Carter, 1992; Lee, 2000), which accounts in particular for random fluctuations. The Lee-Carter model has been extended in many papers, in particular aiming at removing some simplifying hypotheses which are not satisfactory in actuarial applications. See, for example, Brouhns *et al.* (2002) and Renshaw and Haberman (2003, 2006). In order to account properly for cohort effects, CMI (2002) propose the adoption of the P-Spline methodology (see also CMI, 2006). A further class of proposals focus on the extension of credit risk and interest rate models; see, among the others, Biffis (2005), Biffis and Millosovich (2006), Cairns *et al.* (2006). A more naive approach consists in designing a finite set of alternative mortality scenarios. This could be useful for stress tests or for solvency investigations; see CMI (2002, 2006) and Olivieri and Pitacco (2003). Within the Solvency 2 project, a scenario-based approach should be also addressed for the capital requirement; see CEIOPS (2007).

When addressing the valuation of longevity bonds, the behavior of financial markets should be taken into consideration joint to mortality features. Most authors make use of the risk-neutral pricing framework; see, for example, Cairns *et al.*, (2006), Blake *et al.*, (2006b) and references therein. Other authors resort to some distortion of the relevant probability distribution, typically of the survivor index on which the longevity bond is based; see Lin and Cox (2005). However because of differences in mortality and

market behavior, merging properly the two aspects is a very difficult and controversial exercise. Further research is required before a reliable result can be reached.

Let us now move to some concluding remarks. We are well aware of the limits of the model we have described, due to the simplifying hypotheses which have been adopted. However, our aim was to design the basic valuation framework reconciling traditional tools with modern methodologies; proper adjustments should lead to a setting more suitable for practical implementations. First, the pricing of the longevity bonds has to be addressed. Some initial tests could be based on the models already discussed in literature (see above). In any case, this is the major problem, due to lacking experience in the capital market.

Also pricing the reinsurance arrangement is not an easy task, due to poor experience in reinsuring longevity risk. The starting point for the appraisal is the cost of the hedging strategy; however, consideration should be given to the basis risk due to the fact that the mortality underlying longevity bonds can be unlike the one in the reinsurer portfolio (see also Section 3). A further important aspect is the maturity of longevity bonds in comparison with the lifetime of the reinsured life annuity portfolio (see Section 3).

In order to develop a modeling framework which can be calibrated to the available market data, further policy conditions should be considered for the reinsurance arrangement, such as barriers in determining the size of flows to/from the cedants (so that some risk is left on the annuity provider). However, the valuation framework of the annuity portfolio, moving from a best-estimate assessment of future flows, forces to refer to the setting described in Section 3. When other policy conditions are considered, proper adjustments should be applied to the reinsurance premium.

Finally, among further extensions, more than one cohort as well as heterogeneity in respect of annual amounts should be investigated in order to improve the valuation model.

## References

- [1] Biffis E. (2005), Affine Processes for Dynamic Mortality and Actuarial Valuations, *Insurance: Mathematics & Economics*, **37**: 443–468.
- [2] Biffis E., Millosovich P. (2006), A Bidimensional Approach to Mortality Risk, *Decisions in Economics and Finance*, **29**: 71–94.
- [3] Blake D., Cairns A.J.G., Dowd K. (2006a), Living with mortality: longevity bonds and other mortality-linked securities, *British Actuarial Journal*, **12**: 153–228.
- [4] Blake D., Cairns A.J.G., Dowd K., MacMinn R. (2006b), Longevity bonds: financial engineering, valuation, and hedging, *The Journal of Risk and Insurance*, **73**: 647–672.
- [5] Brouhns N., Denuit M., Vermunt J.K. (2002), A Poisson log-bilinear regression approach to the construction of projected life tables, *Insurance: Mathematics & Economics*, **31**: 373–393.
- [6] Burrows R.P., Lang J. (1997), Risk discount rates for actuarial appraisal values of life insurance companies, *Proceedings of the 7th International AFIR Colloquium*, Cairns (Australia): 283–307.
- [7] Burrows R.P., Whitehead G.H. (1987), The determination of life office appraisal values, *Journal of the Institute of Actuaries*, **114**: 411–465.
- [8] Cairns A.J.G., Blake D., Dowd, K. (2006), A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration, *Journal of Risk and Insurance*, **73**: 687–718.
- [9] CEIOPS (2007), *QIS3 Technical Specifications*.
- [10] CFO Forum (2004), *European embedded value principles and basis for conclusions*.
- [11] CFO Forum (2006), *Elaborated principles for an IFRS phase II insurance accounting model. Elaborated principles and basis for conclusions*.
- [12] CMI (2002), *Working Paper 1 – An interim basis for adjusting the “92” Series mortality projections for cohort effects*, The Faculty of Actuaries and Institute of Actuaries.
- [13] CMI (2006), *Working Paper 20 – Stochastic projection methodologies: Further progress and P-Spline model features, example results and implications*, The Faculty of Actuaries and Institute of Actuaries.
- [14] Coleman A.M., Edwards B.A., Torrance D.M. (1992), *Maintainable earnings and actuarial methods for valuing life insurance companies*, The Institute of Actuaries of Australia, Sessional Meeting 1992.

- [15] LAVMWP (Life Assurance Value Measurement Working Party) (2001), *Summary and comparison of approaches used to measure life office values*, The Staple Inn Actuarial Society, London.
- [16] Lee R.D. (2000), The Lee-Carter method for forecasting mortality, with various extensions and applications, *North American Actuarial Journal*, **4**: 80–93.
- [17] Lee R.D., Carter L.R. (1992), Modeling and forecasting U.S. mortality, *Journal of the American Statistical Association*, **87**: 659–675.
- [18] Lin Y., Cox S. H. (2005), Securitization of Mortality Risks in Life Annuities, *Journal of Risk and Insurance*, **72**: 227–252.
- [19] O’Keeffe P.J.L., Desai A.J., Foroughi K., Hibbett G.J., Maxwell A.F., Sharp A.C., Taverner N.H., Ward M.B. and Willis F.J.P. (2005), Current developments in embedded value reporting, *British Actuarial Journal*, **11**: 407–496.
- [20] Olivieri A. (2001), Uncertainty in mortality projections: an actuarial perspective, *Insurance: Mathematics & Economics*, **29**: 231–245.
- [21] Olivieri A. (2005), Designing longevity risk transfers: the point of view of the cedant, *Giornale dell’Istituto Italiano degli Attuari*, **LXVIII**: 1–35. Reprinted on: *ICFAI Journal of Financial Risk Management*, March 2007.
- [22] Olivieri A., Pitacco E. (2003), Solvency requirements for pension annuities, *Journal of Pension Economics and Finance*, **2**: 127–157.
- [23] Olivieri A., Pitacco E. (2005), *La valutazione nelle assicurazioni vita. Profili attuariali*, EGEA, Milano.
- [24] Pemberton J., Allen I., Brown D. Hardwick S., Shah N., Stevens A., Wilson C. (2000), *Classifying DCF models to explicate EVA*, The Staple Inn Actuarial Society, London.
- [25] Renshaw A.E., Haberman S. (2003), Lee-Carter mortality forecasting with age-specific enhancement, *Insurance: Mathematics & Economics*, **33**: 255–272.
- [26] Renshaw A.E., Haberman S. (2006), A cohort-based extension to the Lee-Carter model for mortality reduction factors, *Insurance: Mathematics & Economics*, **38**: 556–570.
- [27] Richards S.J., Kirkby J.G., Currie I.D. (2006), The importance of year of birth in two-dimensional mortality data, *British Actuarial Journal*, **12**: 5–38.
- [28] Sheldon T.J., Smith A.D. (2004), Market consistent valuation of life assurance business, *British Actuarial Journal*, **10**: 543–626.

- [29] Sherris M. (1987), On the risk adjusted discount rate for determining life office appraisal values, *Journal of the Institute of Actuaries*, **114**: 581–589.
- [30] Tillinghast-Towers Perrins (2004), *Market-consistent embedded value*.
- [31] Tillinghast-Towers Perrins (2005), *Market-consistent embedded value. Dispelling the myths*.
- [32] Turner S. (1978), Actuarial appraisal valuations of life insurance companies, *Transactions of the Society of Actuaries*, **30**: 139–167.
- [33] Willets R.C. (2004), The cohort effect: insights and explanations, *British Actuarial Journal*, **10**: 833–877.