

INCOME PROTECTION IN CASE OF DISABILITY
-
FRENCH CONTEXT AND TARIFF METHODOLOGY

BY

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ABSTRACT

This article describes the three pillars of French income protection and provides the reader with a straightforward method for claims experience rating of Short-Term Disability (STD) within the scope of self-employed income protection in France (Third Pillar).

Insurance products with short and medium deferred periods currently sold in the French market encourage stochastic modeling of STD based on a simple state model (active or disabled live) since high incidence and reactivation rates considerably reduce the impact on premium of other causes generally involved in a multi state model (occurrence of death, Long-Term Disability or retirement).

Although the rapid recovery development of STD actually calls for a continuous-time actuarial model, the present paper proposes an alternative in discrete-time for a spreadsheet calculation of premiums and mathematical reserve.

KEYWORDS

Sécurité Sociale Française, Convention Collective, Salaried Employees, Self-employed, Income Protection, Long-Term Disability (LTD), Short-Term Disability (STD), Accident, Sickness, Hospitalization, Deferred Period, GLM for Incidence and Reactivation Rates, Negative Binomial, Log-Logistic, Model Design, Data Formatting, Operative Mode, Occupational Classes, Risk Single and Level Premium, Mathematical Reserve.

1. INTRODUCTION

Technical notes of individual Short-Term and Long-Term Disability tariff sold in the French market usually show a gap between actuarial theory and practice, but also between underwriting guidelines and claims experience. The major role played by the French Social Security system in the income protection of salaried employees and self-employed largely contributes to this gap since the first and second pillars of the French Social Security have enforced part of the disability definitions used in individual insurance sold in the third pillar. The present paper addresses the question of Short-Term Disability tariff (STD) through the following sections:

- Section 2 discusses the three pillars of French income protection;
- Section 3 proposes a stochastic model of the French STD incidence and reactivation rates;
- Section 4 describes the necessary steps for analysing and modeling STD claims experience;
- Section 5 presents a STD premium and reserve model in discrete-time;
- A Conclusion is given in Section 6.

2. THREE PILLARS OF FRENCH INCOME PROTECTION

Section 2 of this article gives a broad outline of the three pillars of French income protection.

2.1. First Pillar – "Régime Général" of the French Social Security System

The French Social Security System (created in 1945) provides a compulsory disability cover for salaried employees and for the self-employed that is interdependent through generations and professions. The purpose of this cover is to compensate for loss of income resulting from a reduction in capacity to work and the inability to engage in gainful employment due to medical causes. It is financed two-thirds by the employer and one-third by the employee, if salaried, or totally by the self-employed. In the following discussion, accidents at work and diseases acquired while performing the normal duties of one's profession are excluded.

2.1.1. Short-Term Disability – Incapacité de Travail

Entitlement to a disability benefit is evaluated on an "own occupation" basis and requires approval from the family doctor. Social Security's medical advisors conduct random queries to confirm the disability status of the claimant. The benefit is paid after a deferred period of three days for salaried employees and after seven days for the self-employed (reduced to three days in case of hospitalization).

For salaried employees, the *monthly* benefit is equal to 50% of the average *last three months* gross salary limited to 50% of the "PMSS" (Plafond Mensuel de la Sécurité Sociale; in 2006, 1 PMSS = €2,589/month). The benefit is paid on a daily basis and limited to 360 days over one or several incapacities within each period of three consecutive years.

- For the self-employed the same rule applies to the *yearly* benefit by replacing "*last three months*" by "last three years" and "PMSS" by "PASS" (=12 PMSS) in the above-mentioned definition, and with a guaranteed minimum benefit since there is no minimum salary.
- For salaried employees, an additional protection paid by the employer ("Mensualisation" law, 1978) completes the above-mentioned benefit to a maximum of 90% of gross salary from the 1st to the 90th day of disability, which is reduced to 66% from the 91st to the 180th day of disability.
- For both the self-employed and salaried employees, the benefit is indexed every three months to the company's wages for salaried employees, and to PASS for the self-employed.

2.1.2. Long-Term Disability – Invalidité Permanente

Entitlement to a benefit is made by agreement between the Social Security's medical advisor and the claimant's family doctor after medical "consolidation" of disability at a permanent level or after three consecutive years in Short-Term Disability. The parties should agree on the following levels by order:

- *Functional disability* ($0 \leq F_n \leq 1$) – Pure physiologic and mental scale ("*Dammum emergens*"); and

- *Occupational disability* ($0 \leq Oc \leq 1$) – Professional scale based on an "any suitable occupation" disability definition and taking into account the nature of the functional disability, but also the educational level of the claimant (Articles L341.1-3 and R341.2 of Social Security Code).

Finally, the resultant disability level $R_s = \sqrt[3]{Fn^2 \cdot Oc}$ determines the disability category and thus the yearly benefit equal to x percent of the average last 10 years' gross salary but limited to x percent of the PASS plus 5% per dependent child with a maximum of 10% more:

- 1st Category – Partial and Permanent Disability: $33\% \leq R_s < 66\%$ and $x = 30\%$
- 2nd Category – Total and Permanent Disability: $R_s \geq 66\%$ and $x = 50\%$
- 3rd Category – Total and Irreversible Loss of Autonomy: $R_s \geq 66\%$ plus the claimant requires assistance of a third person in order to perform the activities of daily living. $x = 70\%$.

The Long-Term Disability (LTD) benefit, paid on a monthly basis, is PASS indexed and ends at age 60. It can be reviewed if the disability level R_s moves to another category (reductions are rare). After the age of 60, the pension system takes over.

In addition:

- The Social Security System requires that the claimant must have completed some minimum amount of time at work.
- Income taxes and limited social contributions are applicable to the benefit.
- The first pillar includes other protections, such as death, health, pension, accident at work, and diseases acquired while performing the normal duties of one's profession. These are regulated by other definitions.

2.2. Second Pillar – Group Income Protection on Top of the First Pillar for Salaried Employees

The second pillar consists of a Collective Agreement ("Convention Collective") between an employer and its salaried employees. It is negotiated by company or line of industry and regulated by the Labor Code (see also "Evin" Law 1989). In 2005, 128 Agreements were in force in France.

This compulsory cover, without medical underwriting, completes the first pillar to between 70% and 90% of gross salary with a limit of four or eight PASS. Disability definitions are generally in line with those of the first pillar.

Products of the market often propose:

- a Short-Term Disability (STD) benefit generally replacing the "Mensualisation" law, i.e., from the 90th day of disability or later; and
- a LTD benefit equal to $\text{Min}(3 \cdot R_s / 2; 100\%)$ for $R_s \geq 33\%$ (no benefit for $R_s < 33\%$). The benefit is paid until age 60 or 65.

The tariff – either standardized for small companies or tailored by age, gender and occupation distributions for larger ones – is expressed in a flat rate for all wages and partially financed by the employer (between 50% and 70%).

Exclusions are minimal, mostly for suicide and self-inflicted injury, automobile accidents caused by alcohol or drug use, and injuries incurred during riots, motor racing, gambling, or professional competition.

With €3.694 billion in group disability benefits paid in 2005, three groups of players are competing.

- *Provident Institutions* governed by the French Social Security Code – 45.3%;
- *Insurance Companies* governed by the French Insurance Code – 33.3% (including reinsurance of Mutual Societies and Provident institutions); and
- *Mutual Societies* governed by the French Mutuality Code – 21.3%.

Mutual Societies and Provident Institutions profit from their historical position in the second pillar (sometimes directly designated in the Collective Agreement) when insurance companies propose innovative products and efficient policy administration.

In addition:

- The premium is tax-deductible from employee's income / employer's result, but the benefit is taxable.
- The cover is not transferable if the employee leaves the company.
- In order to complete the first pillar, most benefits of the second pillar generally include other protections such as death, health, pension cover, and premium waiver.

2.3. Third Pillar: Individual Income Protection for Salaried Employees and the Self-Employed

For salaried employees, the third pillar completes the second one or replaces it if the Collective Agreement fails to propose a group plan. Characteristics are:

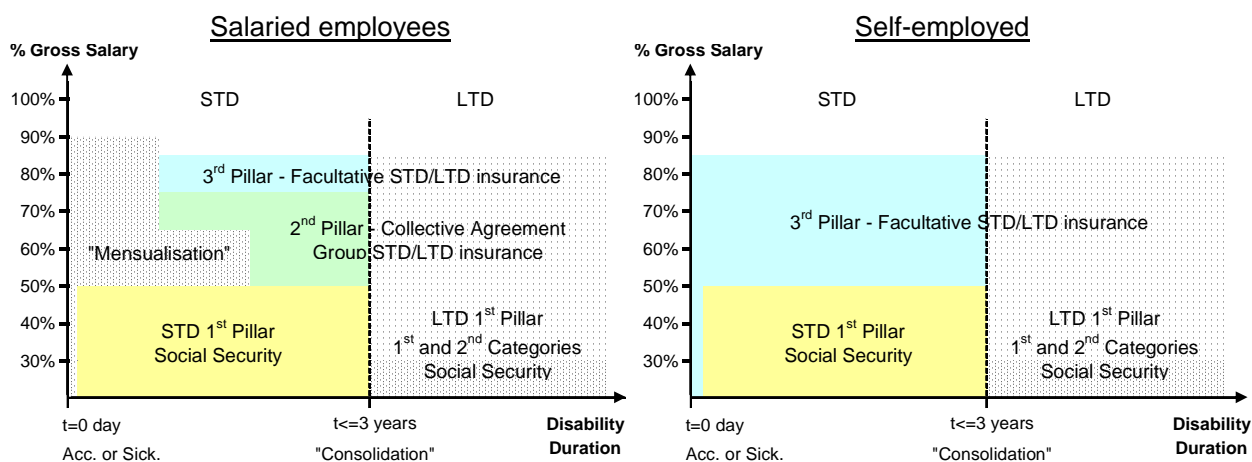
- benefit definition and deferred period often in line with second pillar products;
- age at entry and occupational differentiated level premium with medical underwriting;
- lump sum instead of annuity sometimes proposed for LTD; and
- plan not financed by the employer.

For the self-employed, individual income protection with medical underwriting acts as a second pillar since there is no Collective Agreement. It is often sold in addition to death cover (see also "Madelin Law" 1994). Characteristics are:

- short deferred period differentiated by cause, e.g., for sickness, accident, and hospitalization, the deferral period would be 30/0/3 days;
- cover and benefit until age 65, sometimes with a reduction from age 60 to 65;
- age at entry and occupational differentiated tariff with three or four classes mostly corresponding to retail trading, the craft industry and academic self-employed (sometimes with physicians and nurses as subgroups); and
- standard insurance exclusions, such as for pre-existing conditions; in some cases, exclusions for back disease or hospitalization for a mental disorder.

With €2.868 billion of gross premium for the individual disability market in 2005, insurance companies are the major players in this segment, which is more profitable than the second pillar.

The illustration below show the three pillars of French income protection for Salaried Employees and the Self-Employed.



3. A MODEL OF STD CLAIMS EXPERIENCE

The third section of this article is devoted to a statistical methodology that can be used to model French STD incidence and reactivation rates when analyzing claims experience. The intent is to provide the basis of an operative mode (Section 4) appropriate for a tariff.

3.1. Assumptions

In this section it is assumed that STD disability net single premium can be expressed as the expected value of a time-continuous stochastic process. For a healthy (active) insured of age x at policy issue:

$$\bar{a}_x^{ai} = \int_0^{65-x} {}_uP_x^{aa} \cdot \mu_{x+u}^{ai} \cdot v^u \cdot \left[\int_u^{u+\lambda} {}_{t-u}P_{x+u}^{ii} \cdot v^{t-u} \cdot dt \right] \cdot du \text{ where:}$$

- v is the annual financial discount factor;
- ${}_uP_x^{aa}$ is the probability per unit of time that an active insured of age x (policy issue) will remain active at age $x+u$;
- μ_{x+u}^{ai} is the probability per unit of time that an active insured of age x (policy issue) will become STD disabled at age $x+u$ (including deferred period); and
- ${}_{t-u}P_{x+u}^{ii}$ is the probability per unit of time that a STD disabled insured of age $x+u$ will remain STD disabled at age $x+u+t$. The integral in brackets is the actuarial value (present expected value) at age $x+u$ of a continuous annuity payable to a STD disabled insured until recovery, LTD or death, with a maximum of λ years.

Thus, the single premium estimation amounts to model μ_{x+u}^{ai} and ${}_{t-u}P_{x+u}^{ii}$. In the following section, an approach using Generalised Linear Models (GLMs) is given.

3.2. Stochastic Background

GLMs consist of a wide range of distributions in which the relationship (*link function*) between the random effect (expected value of the observations) and the systematic component (explanatory covariates) provides great advantages compared to simple distribution fitting (GLMs have more degrees of freedom) or to a classical linear regression (GLMs are not restricted to using only the Normal distribution).

Formally, the distribution of the *dependent* variables $(Y_i)_{i=1}^n$, independent, identically distributed (iid) and representing the observation to be predicted in each tariff cell $i(i=1, \dots, n)$, belongs to the two parameters exponential family defined by the density function:

$$f(y_i, \theta, \phi) = \exp \left\{ \frac{y_i \cdot \theta - b(\theta)}{a(\phi)} - c(y_i, \phi) \right\}$$

where $a(\phi)$, $b(\theta)$ and $c(y, \phi)$ are functions of the canonical parameter θ and the dispersion parameter ϕ .

In addition, exponential family allows expressing the variance as a function of the expected value:

$$\mu = E(Y) = b'(\theta) = g^{-1}(X\beta) \text{ and } \text{Var}(Y) = b''(\theta) \cdot a(\phi)$$

where:

- X is the *Model Design* (see below)
- $\beta = {}^t(\beta_0, \beta_1, \dots, \beta_k)$ is the $k+1$ - vector of the model parameters to be estimated; and

- g is a function *linking* the expected value $\mu = E(Y)$ to the linear predictor $X\beta$.

The purpose of modeling is to find β and ϕ , which maximize the Likelihood function:

$$L(y_1, y_2, \dots, y_n, \theta, \phi) = \prod_{i=1}^n f(y_i, \theta, \phi)$$

The Reweighted Least Square algorithm applied to the Log-Likelihood permits identification of the best estimates of β . This algorithm must be iteratively combined with the maximum likelihood equation with respect to any additional parameter representing ϕ (in the following α or σ).

In some cases, before modeling the process must "offset" the exposure (e.g., incidence rate model) and/or operate a transformation of the dependent variable (e.g., reactivation rate model).

Let's see an example of each model.

3.3. STD Incidence Rate: μ_{x+u}^{ai}

If the number of STD claims $(Y_i)_{i=1}^n$ observed in the tariff cell $i (i=1, \dots, n)$ is supposed to follow a Negative Binomial distribution with log-link function, then the Log-Likelihood is:

$$\begin{aligned} Ln\{L^{STD}(y, X, \beta, \alpha)\} = & \sum_{i=1}^n [y_i \cdot \ln\{\alpha \cdot e^{X_i \cdot \beta}\} \\ & - (y_i + \alpha^{-1}) \cdot \ln\{1 + \alpha \cdot e^{X_i \cdot \beta}\} + C_i] \end{aligned}$$

with C_i independent from β .

3.4. STD Reactivation Rate: $t-u P_{x+u}^{ii}$

If the reactivation time in days of STD claims $(T_i)_{i=1}^{n+m}$ observed in the tariff cell $i (i=1, \dots, n)$ is supposed to have a hazard rate $h^{STD}(t_i)$ following a Log-Logistic distribution with logit-link function:

$$h^{STD}(t_i) = \frac{1}{\sigma} \cdot t_i^{\frac{1}{\sigma}-1} \cdot \frac{e^{-\frac{X_i \cdot \beta}{\sigma}}}{1 + t_i^{\frac{1}{\sigma}} \cdot e^{-\frac{X_i \cdot \beta}{\sigma}}}$$

Then the Log-Likelihood is:

$$Ln\{L^{STD}(t, X, \beta, \sigma)\} = \sum_{i=1}^{n+m} w_i \cdot f^{STD}(t_i) + (1 - w_i) \cdot S^{STD}(t_i)$$

where:

- $f^{STD}(t_i)$ is the "STD reactivation" probability density on date t . It holds for the n "not censored" observations:

$$f^{STD}(t_i) = h^{STD}(t_i) \cdot \int_{-\infty}^{t_i} h(u) \cdot du;$$

- $S^{STD}(t_i)$ is the probability density of remaining STD disabled on date t . It holds for the m "right censored" observations. This function is also known as the "STD cumulative distribution":

$$S^{STD}(t_i) = 1 - F^{ITD}(t_i);$$

- w_i is the uncensored indicator of the observation i , which takes the value 0 if the observation is "right censored" and 1 if it is not. An observation is called "not censored" if STD terminates *within* the observation window and "right censored" if STD carries on *after the end* of the observation window.

3.5. Model Design and link function

The model design is expressed through a rectangular matrix $X = [X_i]_{i=1}^n$ called *Design Matrix*, where each selected covariate added to the model and each of its possible transformations (polynomial, log, crossed and nested effects, etc.) is a column of the *Design Matrix* making the linear regression form $X\beta$ an

integral part of the model. For example: $\text{Age} + (\text{Age})^2 + \text{Gender}(\text{Und. Year}) + \sum_{j=1}^4 1_{\{\text{Occup. Class } j\}}$

Two invertible forms of the *Design Matrix* are possible: "Over-Parameterized" or "Sigma-Restricted." Then the *link function* g insures the implementation of the regression in the model establishing a direct relation between the explanatory covariates and the expected value of the dependent variable.

3.6. Model Testing

Several methods allow checking the goodness of fit provided by the model as well as the adequacy of the covariates and of the residual distribution (= observed – predicted):

- Plots of predicted versus observed values (straight line "y=x" = good fit) or predicted versus residuals (symmetrical spread = good fit);
- Plots of the normal probability distribution versus scaled deviance residuals distribution (straight line "y=x" = good fit);
- Plots of Leverages, which identify tariff cells distant from the center of the observations (no Leverage outliers = good fit).
- Likelihood-Ratio tests 1 and 3: Contribution to the model of each covariate (1) taken individually and (3) by removing it from the full model (all covariates);
- Model robustness – Tests if the model holds (= robust) on subsamples

4. OPERATIVE MODE FOR MODELING CLAIMS EXPERIENCE

The fourth section of this article reviews the necessary steps for modeling STD claims experience from data extraction to the final model testing of μ_{x+u}^{ai} and ${}_{t-u}P_{x+u}^{ii}$.

4.1. Scope and Product Feature

The operative mode presented in this section stemmed from the following STD product design:

- Third pillar of the French income protection for self-employed – Individual business;
- Complementary to Death and LTD 1st and 2nd category of French Social Security;
- 0, 8, 15, 30, 90 days deferred period in case of Accident;
- 8, 15, 30, 90 days deferred period in case of Sickness;
- Reduction in Accident and Sickness deferred periods in case of hospitalization;
- Cover limited to age 65 and benefit limited to 1,095 days and to age 65.
- Exclusions as usual of Third Pillar policies (see section 2.2);
- Standard medical underwriting with three occupational classes and substandard risks;
- Level premium by age at policy inception.

4.2. Data formatting

Data formatting consists of drawing up two separate tables according to the analysis required.

4.2.1. Table for analysis of incidence rates

This table is the result of a breakdown of the history of each policy in risk status with the related claims experience. It contains the following information:

- Start and end dates as well as exposure by fraction of year of the risk status (anniversary date of birth or of policy inception, date of addendum);
- All variables remaining unchanged within each period defined by a risk status (age, gender, deferred period, insured daily benefits, occupational class etc);
- Any claim that may have affected the risk status.

4.2.2. Table for analysis of reactivation rates

This table of claims includes censoring indicators within the bounds of the observation window and contains the following information:

- Dates of STD occurrence (recognized by the medical advisor of the insurance company for entitlement to a benefit) and STD first day of benefit payment (after the deferred period);
- All variables remaining unchanged within the risk status affected by the claim (see related criteria of the table for incidence rates);
- Medical cause (Accident or Sickness) triggering the claim and also any degrees of functional as well as professional disability;
- Date of STD reactivation – if within the observation window – and its censored indicator.

The tables should be designed separately by causes triggering STD cover – Sickness and Accident, both derived in with or without hospitalization where Sickness is defined as the complementary to accident over all STD causes.

4.3. Sequence for STD Model Building

The following steps are required to find out the best model of μ_{x+u}^{ai} (and respectively ${}_{t-u}P_{x+u}^{ii}$). The sequence should apply to Sickness and Accident separately and be derived in with or without hospitalization since most policies of the French market propose a deferred period specifically for hospitalization.

- Distribution fitting on aggregated data.

The parametric distribution most appropriate for the nature of the data is selected. This step includes an analysis of the relation between expected value and function of variance (Quasi-likelihood). The Kaplan-Meyer product-limit method may help to select the best distribution for ${}_{t-u}P_{x+u}^{ii}$.

- Analysis of variance for simple effect to the model – ANOVA.

Each effect is tested alone in order to select the best subset of covariates discriminating μ_{x+u}^{ai} (and respectively ${}_{t-u}P_{x+u}^{ii}$). Cox models and Cox tests may help to select the best subset of covariates for ${}_{t-u}P_{x+u}^{ii}$. A first selection of outliers is made.

- Analysis of variance for multiple effects to the model – MANOVA

Cross effects are tested. They often act as a corrective term of the stand alone effects in the model. This step may be an alternative to covariates failing the ANOVA tests. A second selection of outliers is made.

- Parametric model of μ_{x+u}^{ai} and ${}_{t-u}P_{x+u}^{ii}$ – GLM

This step is discussed in Section 3. In addition it should include an analysis of the Deviance function.

Notes: The deferred period has a double functionality. In addition to discriminate μ_{x+u}^{ai} and ${}_{t-u}P_{x+u}^{ii}$, this covariate acts as a priority (retention).

4.4. Examples of STD Model Design

- Negative Binomial with Log-Link for STD incidence rates by Accident with hospitalization

Model Design – Gender x Age Reached x Accident Deferred Period x Accident Occupational Class:

Coefficient	Estimate	Std Err.	p (Wald)	p (LR1)	p (LR3)
<i>Intercept</i>	-7.8338	0.438352	0.00%	--	--
<i>Gender = "F"</i>	-0.1156	0.029459	0.01%	0.01%	0.01%
<i>Gender = "M"</i>	0.1156	--	--	--	--
<i>Age reached</i>	-0.0588	0.021871	0.72%	0.00%	0.00%
<i>(Age reached)²</i>	0.0005	0.000265	4.06%	3.17%	1.24%
<i>Acc. Def. Per.</i>	-0.0208	0.002211	0.00%	0.01%	0.02%
<i>Occ. Class = 1</i>	-0.5735	0.049818	0.00%	0.07%	0.03%
<i>Occ. Class = 2</i>	-0.1826	0.047548	0.01%	0.02%	0.03%
<i>Occ. Class = 3</i>	0.0056	--	--	--	--
<i>Occ. Class = 4</i>	0.7504	0.035122	0.00%	0.92%	0.41%
α	0.7207	0.044793	1.38%	--	--

- Log-Logistic with Logit-Link for STD reactivation rate by Sickness without hospitalization

Model Design – Age at time of Claim x Sickness Occupational Class x Sickness Deferred Period:

Coefficient	Estimate	Std Err.	p (Wald)	p (LR1)	p (LR3)
<i>Intercept</i>	2.6705	0.273217	0.00%	--	--
<i>Age Claimed</i>	0.0260	0.005425	0.00%	0.00%	0.00%
<i>Occ. Class = 1</i>	-0.7169	0.136477	0.00%	0.00%	0.00%
<i>Occ. Class = 2</i>	-0.3721	0.121213	0.21%	0.52%	0.34%
<i>Occ. Class = 3</i>	-0.2733	0.129335	3.46%	1.03%	0.96%
<i>Occ. Class = 4</i>	0.0000	--	--	--	--
<i>Sick. Def. Per.</i>	0.0066	0.002423	0.62%	0.01%	0.02%
σ	0.9199	0.024459	0.06%	--	--

4.5. Occupational Classes

4.5.1. Limits of the classes

There are two possible ways of defining the limits of occupational classes.

- Strategic orientation – Usual classification

The limits of the classes are pre-defined by experience of underwriters in accordance with the strategic orientation of the product in the market. In addition to a misclassification, this method often leads to classes with heterogeneous size – a large class 1 for "preferred risks" and a small class 4 for substandard risks.

- Premium weighted orientation

Occupational classes are defined in accordance with the premium volume represented by each class of the portfolio. The classification is made by scoring the incidence rate of each occupation. The cumulative premium volume by order of occupation in the ranking defines the limits of each classe. This method should apply to mature and stable portfolios only.

4.5.2. Dual Classification

Most products sold in the French market operate with a single occupational classification, which is convenient for producing a paper tariff.

Nevertheless, in most of the cases it appears very clearly that occupational classes, when designed by underwriters, are often influenced by a risk perception of each occupation that is oriented towards incidence and/or accidental components. Unconsciously the reactivation and sickness components are playing a second role even when the classification is designed by experimented underwriters. The evidence based experience is often different, and a dual occupational classification appears to be necessary in order to avoid a bias.

The Model Designs of incidence and reactivation rates mentioned in section 4.4 are built on 2x2-occupational classes: (accident, sickness) x (incidence, reactivation). This means 3⁴=81 instead of 3 classes, which is too complicated to sell. In practice a final ranking and grouping of the 2x2-occupational classification must be operated. The ranking/grouping must alternate with the premium calculation in a recursive process.

5. ACTUARIAL METHODOLOGY FOR BUILDING A STD TARIFF

The fifth section of this article presents a STD premium model in discrete-time. The premium is drawn up using the models for STD incidence and reactivation rates as described in Section 3.

5.1. Definition of a Simplified Actuarial Model

The rapid recovery development of STD actually calls for a continuous-time actuarial model of premium (hazard rate on a daily basis), that is difficult to achieve with a spreadsheet calculation.

For this reason, it has been settled on the hypothesis of a discrete weekly time (\checkmark), which governs the following actuarial model:

- $\checkmark = 52 \cdot x + s$, age in weeks of the insured, with $1 \leq s \leq 52$ and $20 \leq x \leq 65 = \omega$;
- b , maximum duration benefit in weeks as per policy conditions $1 \leq b \leq 156$;
- $\checkmark i = (1+i)^{\frac{1}{52}} - 1$ weekly technical financial discount rate (with i , yearly rate);
- $\checkmark i_{idx} = (1+i_{idx})^{\frac{1}{52}} - 1$ weekly benefit indexation rate from the payment of the first weekly benefit after the deferred period (with i_{idx} , yearly rate);
- $\checkmark q_x^{aa} = 1 - (1 - q_x^{aa})^{\frac{1}{52}}$ probability of death within the week for an active insured aged $\checkmark x$ (with q_x^{aa} , yearly rate and x age in years);
- $\checkmark i_x = 1 - (1 - i_x)^{\frac{1}{52}}$ probability of becoming STD disabled within the week for an active insured aged $\checkmark x$ (with i_x , yearly rate and x age in years);
- $\checkmark q_x^i = 1 - (1 - q_x^i)^{\frac{1}{52}}$ probability of death within the year for a STD disabled insured aged $\checkmark x$ (with q_x^i , yearly rate and x age in years);
- $\checkmark r_{x+1}^{aa} = \checkmark l_x^{aa} \cdot (1 - \checkmark q_x^{aa}) \cdot (1 - \checkmark i_x) + \checkmark r_x^i$, weekly change in portfolio of active insured population where

$$\checkmark r_x^i = (1 - \checkmark q_x^i) \cdot \left\{ \sum_{k=1}^{b-1} \checkmark l_{\checkmark x-(k+1)}^{aa} \cdot \left(1 - \frac{1}{2} \cdot \checkmark q_{\checkmark x-(k+1)}^{aa} \right) \cdot \checkmark i_{\checkmark x-(k+1)} \cdot \prod_{j=1}^{k-1} (1 - \checkmark H_{\checkmark x-(k+1)+j,j}^i) \right. \\ \left. - \sum_{k=2}^b \checkmark l_{\checkmark x-k}^{aa} \cdot \left(1 - \frac{1}{2} \cdot \checkmark q_{\checkmark x-k}^{aa} \right) \cdot \checkmark i_{\checkmark x-k} \cdot \prod_{j=1}^{k-1} (1 - \checkmark H_{\checkmark x-k+j,j}^i) \right\}$$

represents returns to work (and therefore payment of premiums) with $\checkmark H_{\checkmark y,j}^i$ ($1 \leq j \leq b$), the weekly probability of STD reactivation for all causes (including death) at the end of the j^{th} week of STD for a claimant aged $\checkmark y$, provided that he was STD disabled at the end of the $j-1^{\text{th}}$ week.

The population of weekly STD reactivation $\checkmark r_x^i$ is defined as the difference of STD disabled portfolio observed between the beginning and the end of the period $[\checkmark x-1, x)$ of disabled insured whose STD disability occurred before age $\checkmark x-1$ and having survived to the period $[\checkmark x-1, x)$. It has been supposed

that STD disability occurred within the period $[\bar{x}-1, x)$ or claimant still STD disabled at week b did not return to active population.

5.2. Commutation Factors

- $\tilde{D}_{\bar{x}}^{aa} = \tilde{l}_{\bar{x}}^{aa} \cdot \left(\frac{1}{1+i} \right)^{\bar{x}}$, $\tilde{N}_{\bar{x}}^{aa} = \sum_{y=\bar{x}}^{52 \cdot \omega + 27} \tilde{D}_y^{aa}$ et $\tilde{a}_{\bar{x}}^{aa} = \frac{\tilde{N}_{\bar{x}}^{aa}}{\tilde{D}_{\bar{x}}^{aa}}$, present value commutation factors for active insured (Note: $N_x^{aa} = \sum_{y=x}^{\omega} \tilde{D}_{52 \cdot y + 27}^{aa}$);
- $\tilde{D}_{\bar{x}}^i = \tilde{l}_{\bar{x}-1}^{aa} \cdot \left(1 - \frac{1}{2} \cdot \tilde{q}_{\bar{x}}^{aa} \right) \cdot \tilde{i}_{\bar{x}} \cdot \left(\frac{1 + \tilde{i}_{idx}}{1 + \tilde{i}} \right)^{\bar{x}}$ and $\tilde{N}_{\bar{x}}^i = \tilde{D}_{\bar{x}}^i \cdot \sum_{k=1}^b \left(\frac{1 + \tilde{i}_{idx}}{1 + \tilde{i}} \right)^{(k-1)} \cdot \prod_{j=1}^{k-1} (1 - \tilde{H}_{\bar{x}-(k+1)+j,j})$, present value commutation factors for STD disabled insured;
- $\tilde{a}_{\bar{x}}^i = \frac{\tilde{N}_{\bar{x}}^i}{\tilde{D}_{\bar{x}}^i} = \sum_{k=1}^b \left(\frac{1 + \tilde{i}_{idx}}{1 + \tilde{i}} \right)^{(k-1)} \cdot \prod_{j=1}^{k-1} (1 - \tilde{H}_{\bar{x}-(k+1)+j,j})$, present value of a weekly benefit of EUR 1, paid in advance for up to b weeks of STD;
- $\tilde{a}_{\bar{x}}^i = \tilde{a}_{\bar{x}}^i - 1 \left(\neq \frac{\tilde{N}_{\bar{x}+1}^i}{\tilde{D}_{\bar{x}}^i} \right)$, present value of a weekly benefit of EUR 1, paid in arrears for up to b weeks of STD.

5.3. Premium and Mathematical Reserve

The premium for an insured aged x (i.e. $\bar{x} = 52 \cdot x + s$ weeks), covering in case of STD a weekly allowance of EUR 1 paid after deferred period for up to b weeks of disability and up to the age of ω years, are as follows:

- Risk Premium:

$$RP_{\bar{x}} = \tilde{i}_x \cdot \left(\frac{\tilde{a}_{\bar{x}}^i + \tilde{a}_{\bar{x}}^i}{2} \right) \quad \text{and} \quad RP_x = \frac{\sum_{y=52 \cdot x + 27}^{52 \cdot (x+1) + 26} RP_y \cdot \tilde{D}_y^{aa}}{\tilde{D}_{52 \cdot x + 27}^{aa}} \cdot \left(\frac{1}{1+i} \right)^{\frac{Def. Per.}{365}}$$

- Single Premium:

$$SP_{\bar{x}} = \frac{\sum_{y=\bar{x}}^{52 \cdot \omega + 26} RP_y \cdot \tilde{D}_y^{aa}}{\tilde{D}_{\bar{x}}^{aa}} \quad \text{and} \quad SP_x = SP_{52 \cdot x + 27 + \frac{Def. Per.}{7}}$$

- Level Premium:

$$LP_{\bar{x}} = \left(\frac{SP_{\bar{x}}}{\tilde{N}_{\bar{x}}^{aa} - \tilde{N}_{52 \cdot \omega + 27}^{aa}} \right) \cdot \tilde{D}_{\bar{x}}^{aa} \quad \text{and} \quad LP_x = \frac{SP_{52 \cdot x + 27 + \frac{Def. Per.}{7}}}{\sum_{y=x}^{\omega-1} \tilde{D}_{52 \cdot y + 27}^{aa}} \cdot \tilde{D}_{52 \cdot x + 27}^{aa}$$

Finally, the mathematical reserve is expressed as follows:

$${}_tV_x = SP_{x+t} - LP_x \cdot \ddot{a}_{x+t, \omega-(x+t)}^{aa} = S\bar{P}_{52 \cdot (x+t) + 27 + \frac{Def. Per.}{7}} \left(1 - \frac{\sum_{y=x+t}^{\omega-1} \ddot{D}_{52 \cdot y + 27}^{aa}}{\sum_{y=x}^{\omega-1} \ddot{D}_{52 \cdot y + 27}^{aa}} \cdot \frac{\ddot{D}_{52 \cdot x + 27}^{aa}}{\ddot{D}_{52 \cdot (x+t) + 27}^{aa}} \right)$$

6. CONCLUSION

This article addresses most of the centers of competences involved in the French income protection in general and in Short-Term Disability of self-employed in particular: Medical and financial underwriting, exclusions, policy conditions, based experience evidence for modeling incidence and reactivation rates, actuarial methodology of premium and reserve calculation as well as legal and commercial environments. These key factors cannot be overlooked in a commercial success.

In practice, for modeling STD incidence and reactivation rates, the production and claims data have to be formatted in order to provide daily risk exposure, claim censoring indicators and time dependent covariates. Then, after selection of covariates discriminating the tariff (incidence and/or reactivation rates), a distribution and a link function appropriate for the nature of the observations must be chosen. Statistical packages provide algorithms for simple and multiple analyses of variance as well as for model fitting and testing. And finally, before implementation in the premium model, results must be adjusted by the experience of underwriters and claim managers since some effects cannot be predicted (e.g., social context, anti-discrimination and taxation laws, and sometimes exclusions).

Further estimations show that the level premium calculation, when evaluated on a simple state model in discrete-time (weekly basis with Excel) as presented in this article deviates from less than one percent from the multi state model in continuous-time (Hazard rate with Mapple). The simplified model has considerable advantage because it is more flexible to experience data provided by policy administration systems of companies operating in the market. Furthermore the simplified model has been positively tested in the French STD commercial environment. Examples of further actuarial methods are given by Haberman and Pitacco (Actuarial Models for Disability Insurance).

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