A simple Actuarial DFA Model Applicable On A Saudi Pension Experience

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Abstract

In this article, we review and demonstrate the DFA as a tool to assist in quantifying the financial strength of pension funds; also we adopted and applied a simple actuarial funding model to investigate the Saudi mortality experience and some actuarial assumptions. Two suitable methods are re-presented to evaluate and predict the contribution rate which will be needed to make the plan stable over time.

Key Words: Amortization, Asset/Liability Management, Dynamic Financial Analysis, Risk Management, Simulation Models.

Introduction:

Actuaries are using models and modeling to evaluate and analyze systems and programs which can be represented probabilistically, most of these models are projecting the expected time and amount of contributions and payments. Four major frameworks have been used in developing these models:

- The macroeconomic or macro time series approach.
- The interindustry or input-output approach.
- The transition matrix or markov approach.
- The micro analytic simulation approach.

However, some models combine more than one approach. A basic question which must be answered is should we use a deterministic or a stochastic model. Deterministic models are adequate for cash flow projections and valuations but little else. Stochastic models, on the other hand, allow us to investigate fully the dynamics of the fund through time and using different control strategies [5]. The separate question is whether models should be kept simple or be made very realistic. The answer here depends on the reasons for modeling.

A simulation model is a mathematical specification of a system that depicts quantitatively how the system behaves. It is usually a set of equations, with estimated coefficients and parameters, which depict the relationships among the variables represented in the model (the endogenous variables) given the values of other variables determined outside the model (the exogenous variables) and the parameters. Such a simulation model is usually based upon several conceptual models [11].

There are two methods in use to analyze the financial status of the pension and insurance systems over a specific time horizon:

- The Scenario testing method which is projecting business results under selected deterministic scenarios into the future; its results are valid only for this specific scenario and useful only if the scenario was correct and risks associated can roughly be quantified.
- The stochastic simulation method which managed to overcome this flaw by taking a random draw from the probability distributions of each component factor and combines them into a realization, it has provided us with a mathematical concept known as Dynamic Financial Analysis (DFA).

In this article, we review and demonstrate the (DFA) as a tool to assist in quantifying the financial strength of pension funds; also we adopted and applied a simple actuarial funding model to investigate the Saudi actuarial assumptions and mortality experience. Two suitable methods are presented and evaluated to see, if the pension liability exceeds the plan assets, also to predict the contribution rate which will be needed to make the plan stable over time.

The Dynamic Financial Analysis Model:

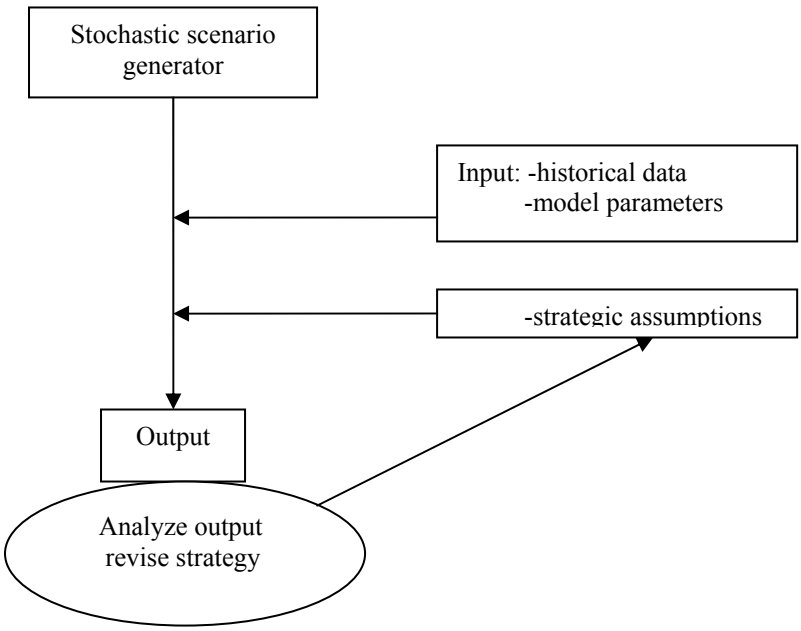
A DFA model is a stochastic model, it borrows many concepts and methods from statistics and economics to maintain two main tasks, the first is to maximize investment return, the second serving to maintain customer value through funding for pension benefits involves the substitution of contribution income(from plan participants and sponsor) by investment income from accumulated assets [3], to do these, DFA goes beyond designing an asset allocation strategy to performance measurement, and it should at least address the following risks:

- Pricing risk.
- Reserving risk.
- Investment risk.

- Catastrophes risk.
- Credit risk related to default.
- Currency risk.

Most DFA models consist of three major parts as shown in the following figure (Figure (1)): [6]. The stochastic scenario generator produces realizations of random variables representing the most important drivers of business results; the second data source consists of the organization specific input, assumptions regarding model parameters and strategic assumptions (investment strategy). The last part, the output provided by the DFA model, can then be analyzed by management in order to improve the strategy. This can be repeated until management is convinced by the superiority of certain strategy.

Figure (1)
Main structure of a DFA model

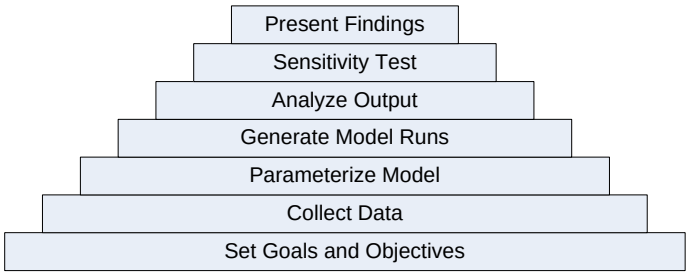


The Dynamic Financial Analysis Process:

DFA is more than a model, it is a way of thinking that weaves through the entire operations of an insurance organization, the central idea is to quantify in probabilistic

terms whether the organization will be able to meet its commitments in the future or not, while a modeling tool is essential for implementing dynamic financial analysis, it is just one element of a much grander picture (Figure (2)):[9]

Figure (2)
The Dynamic Financial Analysis Process



The DFA process starts with discussion and understanding of the goals, objectives, constraints and risk tolerance of the organization. This step determines the matrices that will be most important in evaluating alternative strategic initiatives. Steps 2 through 4 depend on the specifics of the DFA modeling system which has been used for the analysis. Whereas a common DFA process allows for effective and efficient sharing of concept and ideas to address the potential problem of model bias (model risk) and assumption bias (parameter risk). Step 5 and 6 of the DFA process relate to analysis and sensitivity testing [10]. Finally the presentation of the study (step 7) should do more than show the numbers and present the conclusions, it should review the highlights of each step of the DFA process and layout the logic that went into the analysis in such a way that the conclusions become evident before they are revealed.

Stochastically Modeled Variables:

A very important step in the process of building an appropriate model is to identify the key random variables affecting asset and liability cash flows. A key module of a DFA model is an interest rate generator. There are many different

interest rate models, the final choice of a specific interest rate model is not straightforward, and it might be helpful to post some general features of interest rate movements: [6]

1. Volatility of yields at different maturities varies.
2. Interest rates are mean-reverting.
3. Rates at different maturities are positively correlated.
4. Interest rates should not be allowed to become negative.
5. The volatility of interest rates should be proportional to the level of the rate.

In addition to these characteristics, there are some practical issues the interest rate model should be satisfied:

- simple enough that one can compute answered in reasonable time,
- flexible enough to cover most situations arising in practice,
- well- specific, in that required inputs can be observed or estimated,
- realistic, in that the model will not do silly things.

Finally, the model should pass the solvency testing process and it has to bridge the gap between stochastic simulation of cash flows and financial statements.

The Actuarial Valuation Basis:

The basic actuarial equation for pension fund is,

$$\text{Fund} + \text{Expected Contributions} + \text{Expected Investment Income} = \text{Expected Benefit Payment} + \text{Expected Expenses}$$

In pension actuarial calculations the usual objective is to determine the amount of current year contributions. The equation is first transformed to eliminate the investment income item by discounting the benefit payments, expenses and contributions to present value. The transformed equation is,

Fund	+	Discounted Present Value of Expected Contributions	=	Discounted Present Value of Expected Benefits and Expenses

One of the fundamental variables is surplus, defined as the difference between the market value of assets and the market value of liabilities. The amount of available surplus reflects the financial strength of the pension fund.

The recent experience of some actuaries in dealing with pension plan assets and liabilities together is suggesting that the discounting rate should be directly connected to the expected rate of expected return. If the pension liability exceeds the plan assets, then an unfunded liability exists. The unfunded liability varies over time and some suitable methods could generate a schedule of contributions that satisfies two objectives:

- Unfunded liabilities must be paid off and there must be enough funds to pay benefits when they are due.
- The contributions that are required from the sponsor and members of the plan must be stable over time.

The Model:

A stationary pension plan is assumed to simplify the model, with a fixed mortality and withdrawal rates at different ages [4]. The only benefit that is provided by the plan is a final salary pension paid at retirement. Also, there is no inflation on salaries or change in actuarial valuation assumptions over time. The equation of equilibrium holds when: [8]

$$AL = (1 + i_L)(AL + NC - B) \tag{1}$$

Where; AL: The actuarial liability.

i_L: The actuarial assumption for rate to discount pension liabilities (i.e. the interest rate used to discount pension liabilities cash flows).

NC: Normal cost or normal contribution rate.

B: Benefit paid every year.

Assuming that contributions C_t and benefits are paid at the start of year t, hence

$$F_{t+1} = (1+i)(F_t + C_t - B) \tag{2}$$

Where; F_t: The market value of the pension plan assets at time t.

i: The actual rate of return on plan assets.

C_t: The pension contribution paid at the beginning of year t.

The unfunded liability can be defined as,

$$UL_t = AL - F_t \tag{3}$$

It is assumed that, demographic and economic experience unfold in accordance with actuarial valuation assumptions, except that the actual investment rate of return i may differ from the assumed investment rate of return i_A.

If we define the gain as a negative loss,

$$C_t = NC + S_t \tag{4}$$

Where; S_t: The supplementary contribution paid at the start of year t.

If we let $v_L = (1+i_L)^{-1}$, it follows from equations (1) to (4) that,

$$UL_{t+1} = AL + (1+i)(UL_t - S_t - v_L AL) \tag{5}$$

Since the actual experience does not deviate from actuarial assumptions except possibly in investment returns, the unfunded liability at the end of year t would be,

$$UL_{t+1}^A = AL + (1+i_A)(UL_t - S_t - v_L AL) \tag{6}$$

Therefore the intervaluation in year t is

$$L_{t+1} = UL_{t+1} - UL_{t+1}^A = (i - i_A)(UL_t - S_t - v_L AL) \tag{7}$$

Equation (7) may be rewritten as,

$$UL_{t+1} - u_A UL_t = L_{t+1} - u_A [S_t - (v_A - v_L) AL] \tag{8}$$

Where; $u_A = 1 + i_A$ and $v_A = (1+i_A)^{-1}$

If the supplementary contribution S_t in equation (4) pays off over time past intervaluation losses as well as any initial unfunded liability at time 0, which can be arised for many reasons as, changing in the valuation basis or an amendment to benefit rules.

Assume that $L_t = 0$ *for* $t \leq 0$,

Also $UL_t = 0$ *for* $t < 0$

And the initial unfunded liability UL_0 is amortized over a finite period of n years at rate i_A by means of payments.

$$P_t = \begin{cases} UL_0 / \ddot{a}_{\overline{n}|} & 0 \leq t \leq n-1 \\ 0 & t \geq n \end{cases} \tag{9}$$

Where; $\ddot{a}_{\overline{n}|}$: represents the present value of an annuity-certain of term n payable in advanced and calculated at rate i_A. The unamortized part of the initial unfunded liability at time t is

$$U_t = \begin{cases} UL_0 \ddot{a}_{\overline{n-t}|} / \ddot{a}_{\overline{n}|} & 0 \leq t \leq n-1 \\ 0 & t \geq n \end{cases} \tag{10}$$

Observe that:

$$u_A U_t - U_{t+1} = u_A P_t \tag{11}$$

If Dufresene analysis have been extended [8], and allowing for a distinction between the liability valuation rate (i_L) and the investment return assumption (i_A),

$$S_t = \sum_{j=0}^{m-1} \frac{L_{t-j}}{\ddot{a}_{\overline{m}|}} + (v_A - v_L) AL + P_t \tag{12}$$

Replacing S_t from equation (12) into equation (8) and using equation (11) yields

$$(UL_{t+1} - U_{t+1}) - u_A (UL_t - U_t) = L_{t+1} - u_A \sum_{j=0}^{m-1} \frac{L_{t-j}}{\ddot{a}_{\overline{m}|}} \tag{13}$$

That means;

The unfunded liability at the end of the year	=	The accumulation of the unfunded liability at the start of the year	+	The loss that emerges during the year	−	The accumulated value of payments made in respect of past losses

It is easily verified that the solution of equation (13) is: [4]

$$UL_t - U_t = \sum_{j=0}^{m-1} \frac{\ddot{a}_{\overline{m-j}|}}{\ddot{a}_{\overline{m}|}} L_{t-j} \tag{14}$$

As in Dufresene (1989), replace S_t from equation (12) and UL_t from equation (14) into equation (7), and use equation (11), to obtain: [8]

$$L_{t+1} = (i - i_A) \left[\sum_{j=0}^{m-1} \frac{L_{t-j} (\ddot{a}_{\overline{m-j}|} - 1)}{\ddot{a}_{\overline{m}|}} - v_A (AL - U_{t-1}) \right] \tag{15}$$

Dufresene obtains a sufficient condition for the convergence of $\{L_t\}$, $\{UL_t\}$ and $\{S_t\}$ as $t \to \infty$. The following result is due to Dufresene (1989);

$$\lim_{t \to \infty} S_t = \frac{m}{\ddot{a}_{\overline{m}|}} \lim_{t \to \infty} L_t + (v_A - v_L) AL \tag{16}$$

Provided that $|i - i_A| \sum_{j=0}^{m-1} \frac{(\ddot{a}_{\overline{m-j}|} - 1)}{\ddot{a}_{\overline{m}|}} < 1$

Owadally obtains the following corollary [8]:

Corollary: Assume that $|i - i_A| \sum_{j=0}^{m-1} \frac{(\ddot{a}_{\overline{m-j}|} - 1)}{\ddot{a}_{\overline{m}|}} < 1$

If $i_A = i$, then $UL_t = 0$.

If $i_A > i$, then $limUL_t > 0$.

If $i_A < i$, then $limUL_t < 0$.

That is confirming, if the actuarial investment returns assumption is optimistic (that is $i_A > i$), then a persistent deficit occurs ($limUL_t > 0$); if the investment return assumption

is conservative (that is $i_A<i$), then a persistent surplus occurred ($limUL_t<0$). Note also that, if $i_A \neq i$, $limUL_t$, depends on the period m over which gains and losses are amortized.

The equations may be extended to allow for the distinction between the rates, as well as for the separate treatment of the initial unfunded liability.

One can represent the supplementary contribution paid in year $(t,t+1)$ by the following equation ,

$$S_t = \sum_{j=0}^{\infty}(1-k)k^j u_A^j L_{t-j} + (v_A - v_L)AL + p_t \tag{17}$$

Where $0 \leq k \leq v_A$, and any unit loss is paid off by means of a sequence of exponentially declining payments, the unit loss being paid of in perpetuity since,

$$\sum_{j=0}^{\infty}(1-k)k^j u_A^j v_A^j = 1 \tag{18}$$

The larger the parameter k, the slower the loss is paid off. The loss is never completely defrayed except in the limit as $t \rightarrow \infty$.

This is not a weakness [11], as intervaluation losses occur randomly in practice and never completely removed.

Replacing S_t from (17) into (8) and using equation (11) yields

$$(UL_{t+1} - U_{t+1}) - u_A(UL_t - U_t) = L_{t+1} - u_A \sum_{j=0}^{\infty}(1-k)k^j u_A^j L_{t-j} \tag{19}$$

It is easily verified from the previous equation that,

$$UL_t - u_t = \sum_{j=0}^{\infty} k^j u_A^j L_{t-j} \tag{20}$$

This equation is sensible and showing that, at any time t, the unfunded liability is the present value of payments yet to be made in respect of all past and present losses, together with the unamortized part of the initial unfunded liability.

One can calculate S_t directly as a portion (*1-k*) of the unfunded liability by comparing equations (17) and (20),

$$S_t = (1-k)(UL_t - U_t) + (v_A - v_L)AL + P_t \qquad (21)$$

For simplicity Dufresne disregards the separate treatment of initial unfunded liability and the distinction between i_A and i_L and considers only $S_t=(1-k)UL_t$. Also, Dufresne state that parameter k is calculated as $k = 1 - \frac{1}{\ddot{a}_{\overline{M}|}}$, where M is between 1 and 10 years.

Thus if $M = m$, the first payment made in respect of a unit loss is $\frac{1}{\ddot{a}_{\overline{m}|}}$.

<u>The Model in Action:</u>

An Application on the Saudi mortality rates is given that have been based on the following:

- Demographic projections:
  - Mortality: Saudi life table [1].
  - Plan population: Stationary with single entry age of 20 and single retirement age of 60.
  - Salary: Constant throughout working lifetime (by taking its average).
  - Benefit: A level pension at age 60 paying 75% of annual salary.
- Economic Projection:
  - No inflation.
  - Assets earn a constant rate of return of 5%.
  - Initial unfunded liability = 0.
- Actuarial Valuations:

- Frequency: Yearly.
- Actuarial cost method: Unit credit.

- Actuarial Assumptions (2):
  - Fixed with valuation assumptions $i_L = 5\%$, $i_A = 6\%$.
  - Other valuation assumptions are identical to projection assumptions.
- Valuation Data: have been calculated;
  - Actuarial liability : $AL = 14.8$
  - Normal Cost : $NC = 0.362$
  - Both expressed as a proportion of the yearly benefit outgo (B).
- Funding Method Parameters:
  - Amortization : $m = 5$
  - Spreading : $k = 1 - \frac{1}{\ddot{a}_{\overline{5}|}}$

Table (1)
Saudi Funded Value (% of *AL*) and Saudi Contribution (% of *NC*)

Time	Funded Value (%)		Contribution (%)	
	Amortization (A)	Spreading (S)	Amortization (A)	Spreading (S)
0	100.0	100.0	12.3	12.3
2	98.2	98.1	32.4	30.8
4	97.6	97.5	44.5	37.6
6	96.4	95.8	52.7	46.2
8	96.2	94.6	53.4	51.3
10	96.6	94.8	56.7	56.4
12	96.8	94.6	57.4	57.6
14	96.7	94.2	59.5	61.3
16	96.4	94.0	65.1	72.2
18	96.3	93.8	72.2	83.6
20	96.2	93.7	89.3	94.1
25	96.1	93.6	89.3	94.6
30	96.1	93.4	89.3	95.1
35	96.1	93.3	89.3	95.1
40	96.1	93.2	89.3	95.2
45	96.1	93.2	89.3	95.2
50	96.1	93.2	89.3	95.2

When i = 5% and i_A = 6%, the investment return assumption is optimistic. Fund values and contributions over time are presented in table (1), figures (3), (4), (5) and (6) respectively. A contribution equals 12.3% of normal cost is required initially under the two methods. Under amortization, the required contribution levels off at 89.3% of normal cost and an unfunded liability of 3.9%. Under spreading, the contribution rises steadily to 95.2% of normal cost and unfunded liability of 6.8% of actuarial liability. Note that the pension fund is finally in balance under the two methods. For example, under amortization, using units of yearly benefit outgo, a fund of 96.1 × 14.8 = 14.22 yields investment income of 14.22 × 5% = 0.711 at the end of the year. At the start of the year, the present value of this income is $\frac{0.711}{1.05} = 0.677$. Contribution income is 89.3% × 0.362 = 0.323. Total income is 0.677 + 0.323 = 1, that balances the benefit of 1 which is paid out.

Figure (3)

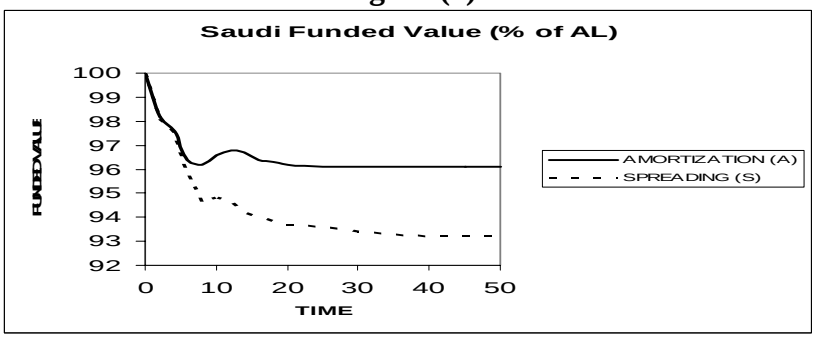


Figure (4)

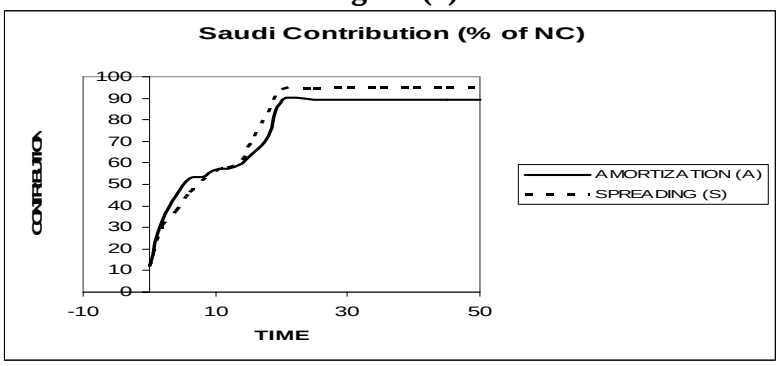


Figure (5)

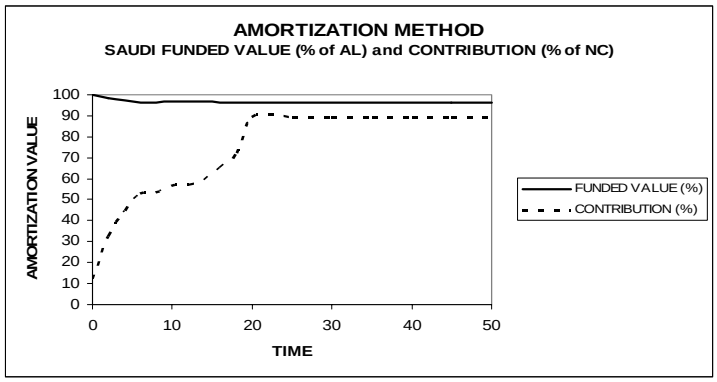
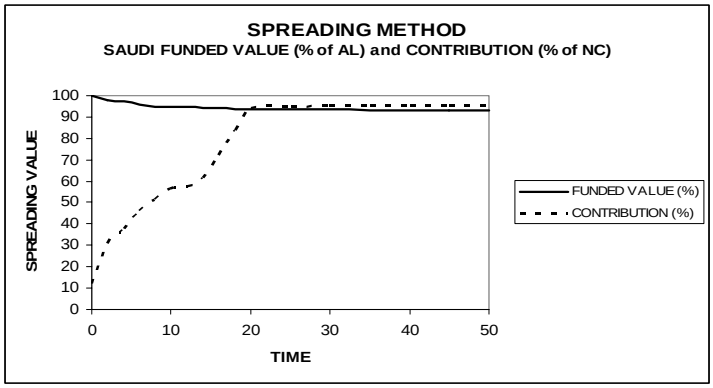


Figure (6)



Conclusions

Actuaries are using models and modeling to evaluate and analyze systems and programs which can be represented probabilistically, most of these models are projecting the expected time and amount of contributions and payments. In this article we review and demonstrate the DFA as a tool to assist in quantifying the financial strength of pension funds, the next logical step in promoting DFA is to show, how these models and process can be implemented to produce value and to provide a quantitative support to the Saudi management. To do that, a simple DFA model has been adopted and two funded methods have been presented and used to analyze the financial status of a Saudi pension fund, by predicting the contribution rate and testing the liability.

The results are presented, discussed and showed that, a contribution equals 12.3% of normal cost is required initially. The unfunded liability is 3.9% under the amortization method and 6.8% under the spreading method. The two methods are showing that, the pension fund is finally in balance.

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