

Mortality in the Nordic countries – Sweden

IAA Life 2007, Stockholm

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Outline

- Background
- Methodology
- Data
- Results
- Conclusions

The need of a new common mortality investigation

- Legislation
 - IOPR-directive
 - Solvency II
 - IFRS Phase II
- Include mortality trends in life insurance reserves
- Latest common mortality investigation in 1989

M90

- Traditionally, the Makeham formula has been used

$$\mu(x) = a + b \cdot 10^{c(x-f)}$$

- Latest, public mortality investigation 1989-1990 resulted in the M90-parameters

$$a = 0.001$$

$$b = 0.000012$$

$$c = 0.044$$

$$f = \begin{cases} 6 & \text{females} \\ 0 & \text{males} \end{cases}$$

- "Flip" around age 60
 - Higher mortality in younger ages
 - Lower mortality in older ages

The Lee-Carter model

$$\mu_t(x) = e^{\alpha_x + \kappa(t) \cdot \beta_x}$$

- Developed by Lee and Carter in 1992
- The method is "extrapolative and makes no effort to incorporate knowledge about medical, behavioral, or social influences on mortality change"

The Lee-carter model - α_x

$$\mu_t(x) = e^{\alpha_x + \kappa(t) \cdot \beta_x}$$

- The average of the logarithm of the force of mortality over the observed period of time
- The general shape of the mortality

The Lee-carter model - $\kappa(t)$

$$\mu_t(x) = e^{\alpha_x + \kappa(t) \cdot \beta_x}$$

- The time trend

The Lee-carter model - β_x

$$\mu_t(x) = e^{a_x + \kappa(t) \cdot \beta_x}$$

- The age-specific patterns of the mortality change
- Shows the sensitivity of the logarithm of the force of mortality at age x to the time trend
- The higher value, the higher response of the time trend

Estimating the parameters – Basic concepts

- Exposure $R_x(t) = \frac{N_{x-1}(t-1) + N_x(t)}{2}$
- Number of deaths $D_x(t)$
- Force of mortality $\mu_x(t) = \frac{D_x(t)}{R_x(t)}$
- Assume that the number of deaths are approximately Poisson distributed, i.e.

$$D_x(t) \text{ is } Po(R_x(t) \cdot \mu_x(t)) \quad \rightarrow \quad E[D_x(t)] = R_x(t) \cdot \mu_x(t)$$

Estimating the parameters – Maximum Likelihood Method I

- Estimating the parameter vectors

$$\bar{\alpha} = (\alpha_x, \alpha_{x+1}, \alpha_{x+2}, \alpha_{x+3}, \dots)$$

$$\bar{\kappa} = (\kappa(t), \kappa(t+1), \kappa(t+2), \kappa(t+3), \dots)$$

$$\bar{\beta} = (\beta_x, \beta_{x+1}, \beta_{x+2}, \beta_{x+3}, \dots)$$

- Setting up and maximizing the likelihood sum:

$$L(\alpha, \kappa, \beta) = \prod_x \prod_t P(D_x(t) = d_x(t))$$

Estimating the parameters – Maximum Likelihood Method I I

- The maximization is done by the following three steps
 - Taking the logarithm of the likelihood sum
 - Taking the derivative of the logarithmed likelihood sum
 - Solving the equation by using a Newton-Raphson method, iterating one term at a time

Estimating the parameters – Iteration

$$a_x^{(v+1)} = a_x^{(v)} + h \cdot \frac{\sum_t (d_x(t) - \lambda_x^{(v)}(t))}{\sum_t \lambda_x^{(v)}(t)}$$

$$\kappa_t^{(v+1)}(t) = \kappa_t^{(v)}(t) + h \cdot \frac{\sum_t [(d_x(t) - \lambda_x^{(v)}(t)) \cdot \beta_x^{(v)}]}{\sum_t [\lambda_x^{(v)}(t) \cdot (\beta_x^{(v)})^2]}$$

$$\beta_x^{(v+1)} = \beta_x^{(v)} + h \cdot \frac{\sum_t [(d_x(t) - \lambda_x^{(v)}(t)) \cdot \kappa^{(v)}(t)]}{\sum_t [\lambda_x^{(v)}(t) \cdot (\kappa^{(v)}(t))^2]}$$

$$\lambda_x(t) = E[D_x(t) = d_x(t)] = R_x(t) \cdot \mu_x(t) = R_x(t) \cdot e^{\alpha_x + \kappa(t) \cdot \beta_x}$$

Estimating the parameters – Constraints

- The following constraints are used

$$\sum_t \kappa(t) = 1$$

$$\sum_x b_x = 1$$

- The constraints are used in each iteration

$$a'_x = a_x + \frac{1}{n} \sum_t \kappa(t)$$

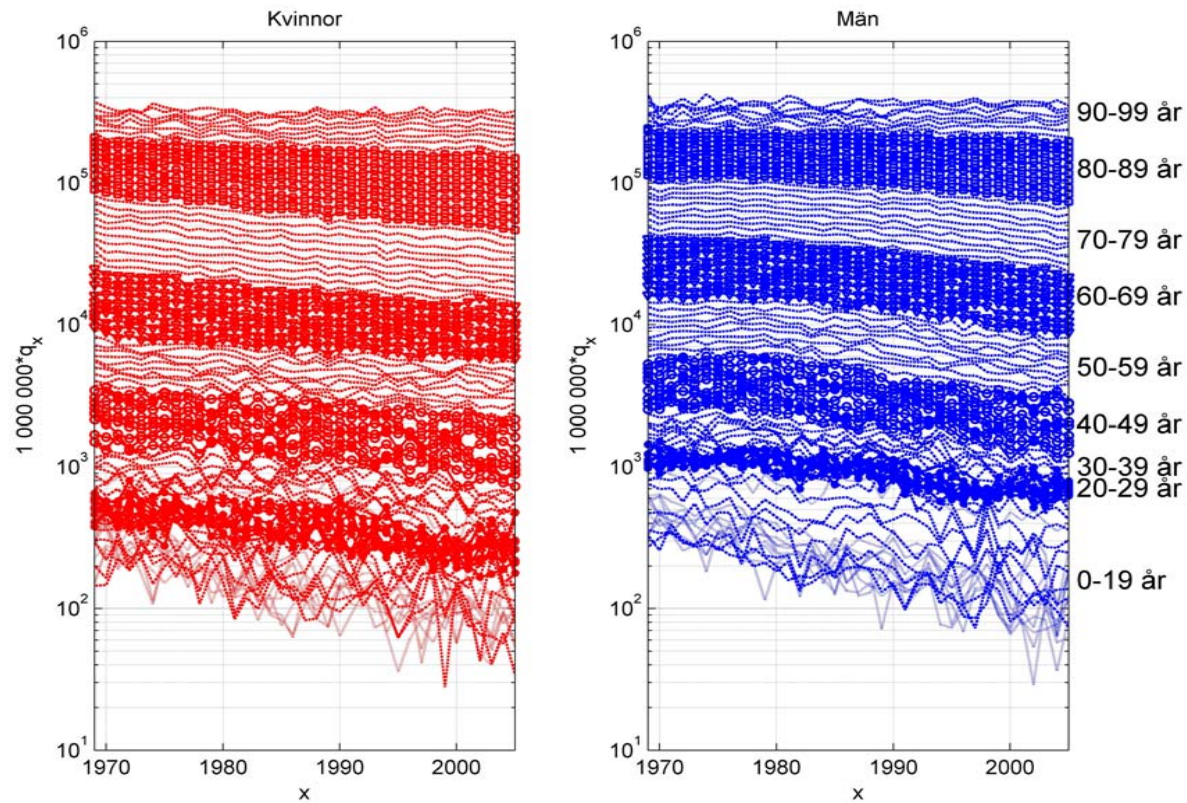
$$\kappa'(t) = \sum_x \beta_x \cdot \left(\kappa(t) - \frac{1}{n} \sum_t \kappa(t) \right)$$

$$\beta'_x = \frac{\beta_x}{\sum_x \beta_x}$$

Trend estimation data

- Long, homogeneous time series need to estimate trends in mortality
- Changes in mortality among insured individuals due to
 - The mortality has actually changed?
 - The characteristics of the insured has changed and therefor also the mortality?
- Only homogeneous time serie available for trend estimation is the mortality of the entire Swedish population

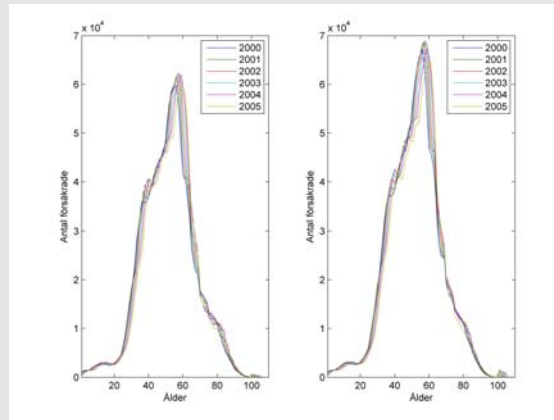
Swedish population data



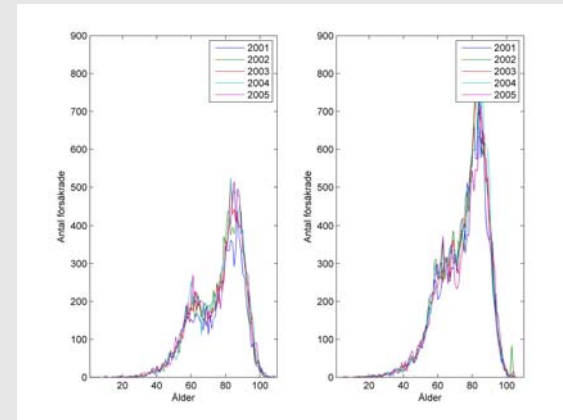
Insurance data

Voluntarily
insured

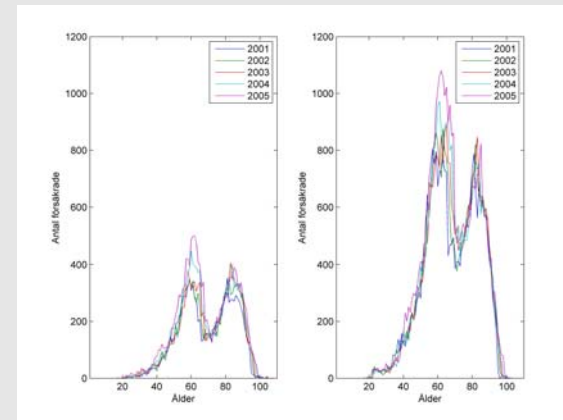
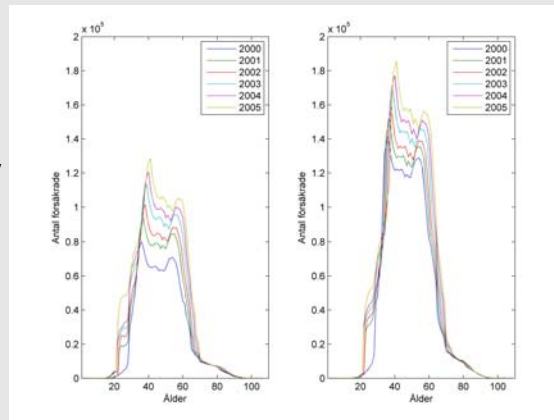
Number of insured



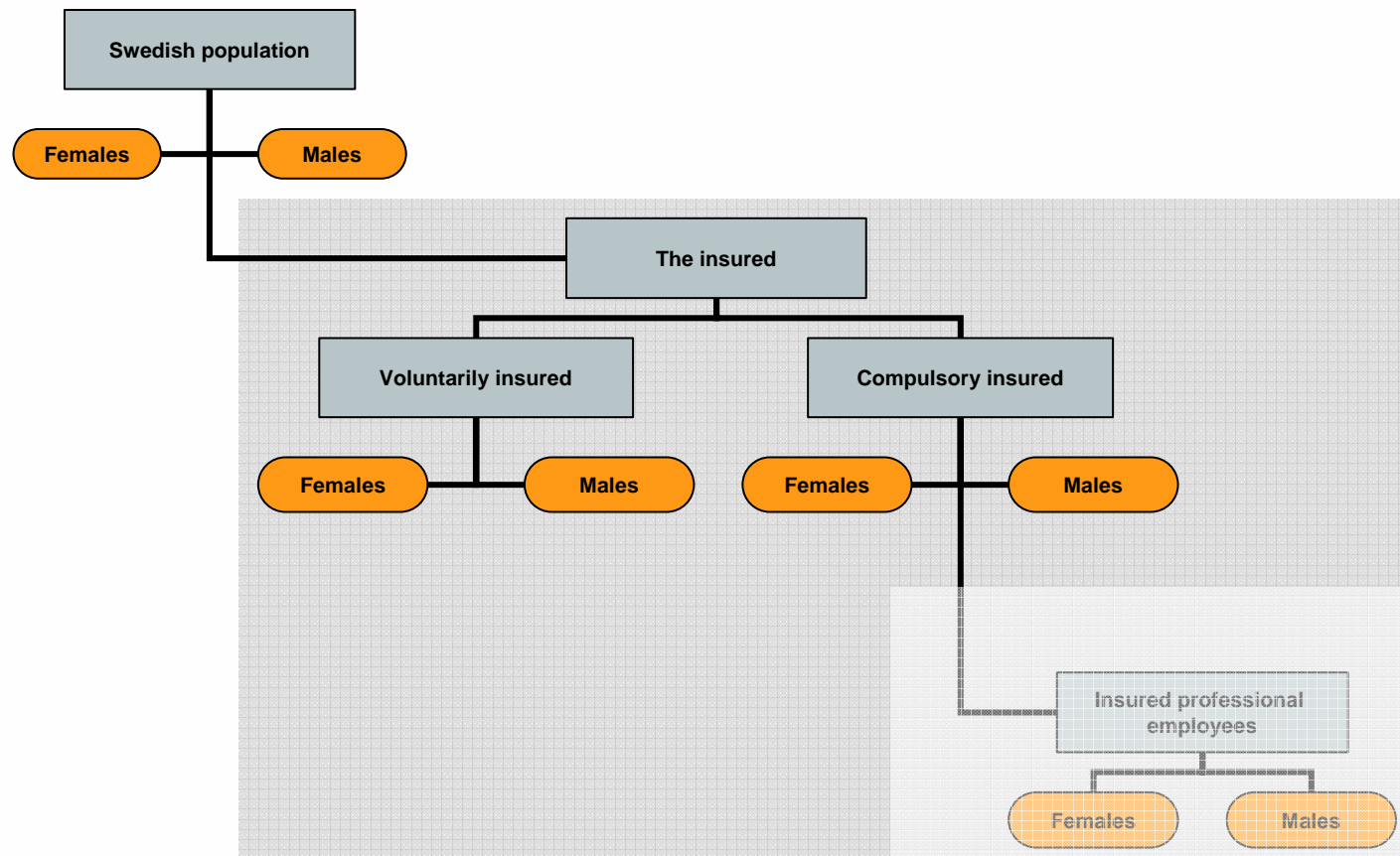
Number of deceased



Compulsory
insured

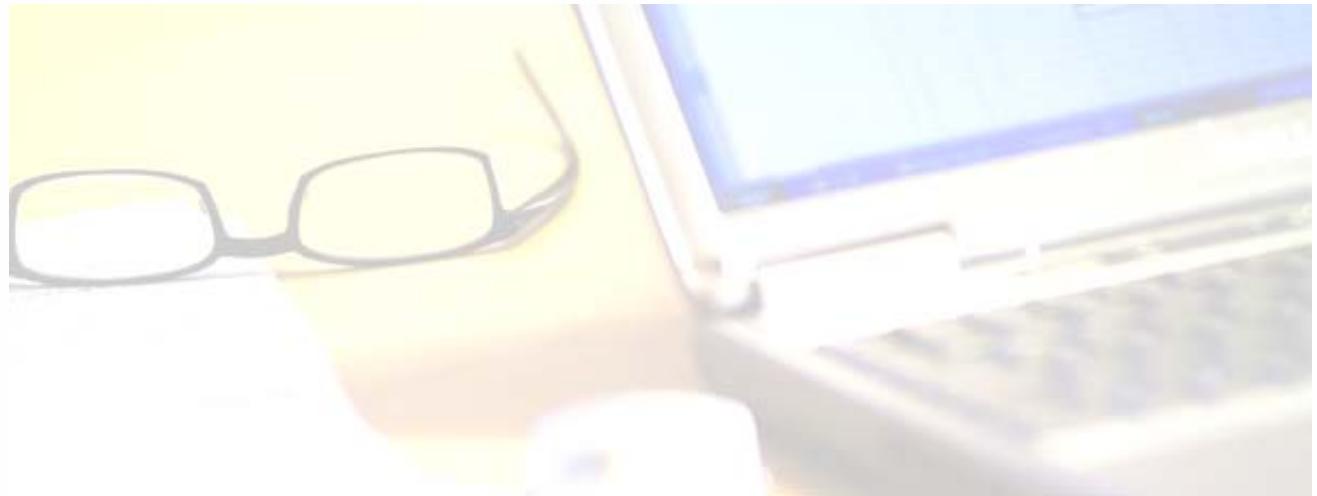


Classification of insurance data

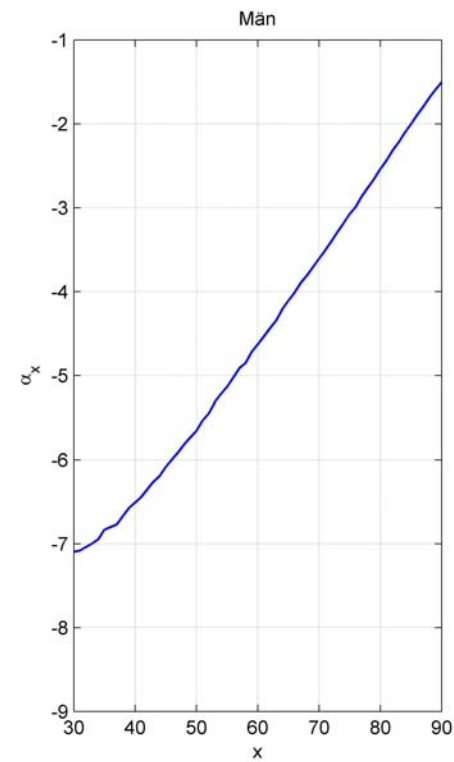
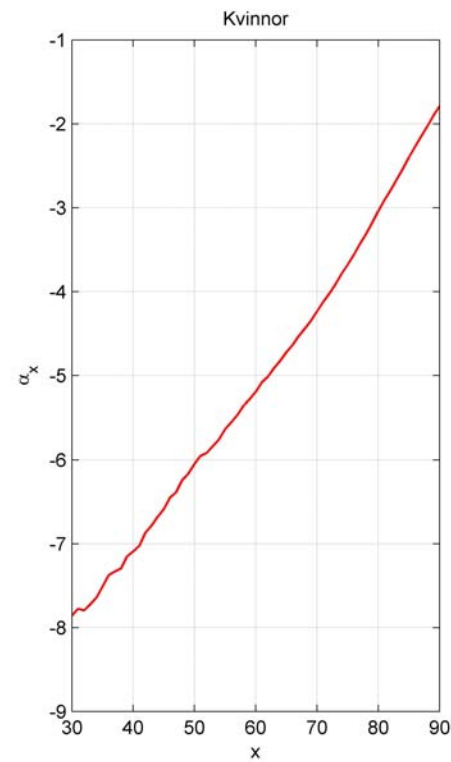


Data specifics

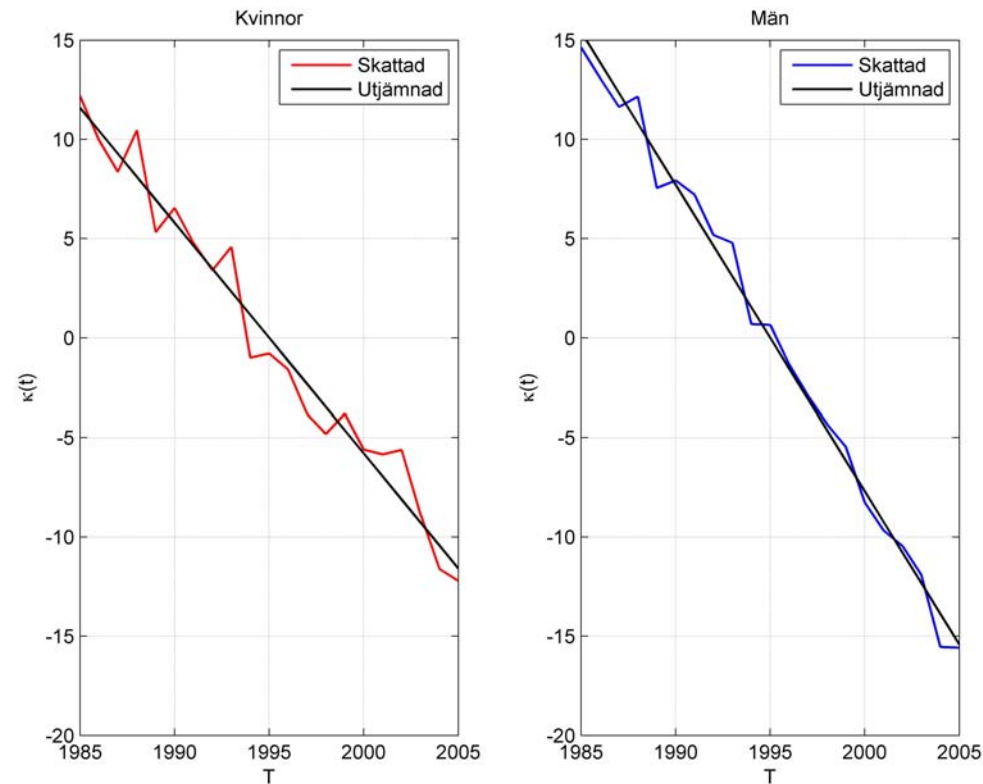
- Both mortality and longevity risks
- Health examination/able-bodied
- Insurance based and not individual based, i. e. individuals can be found in more than one group



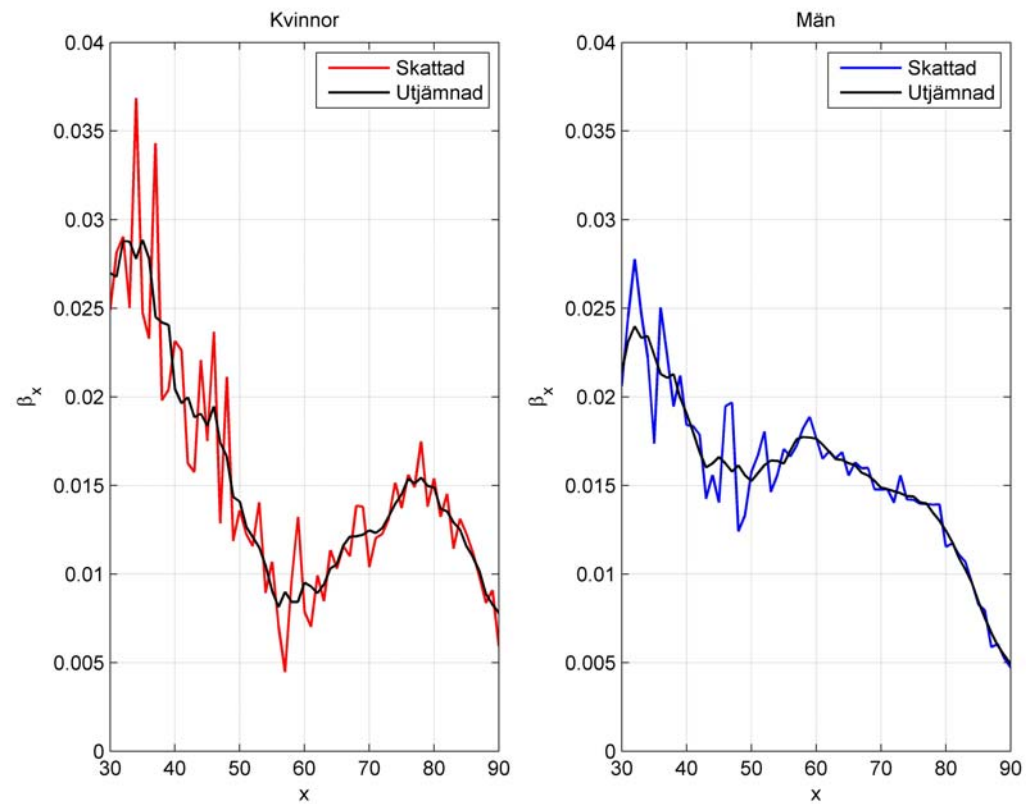
Lee-Carter parameters - α_x



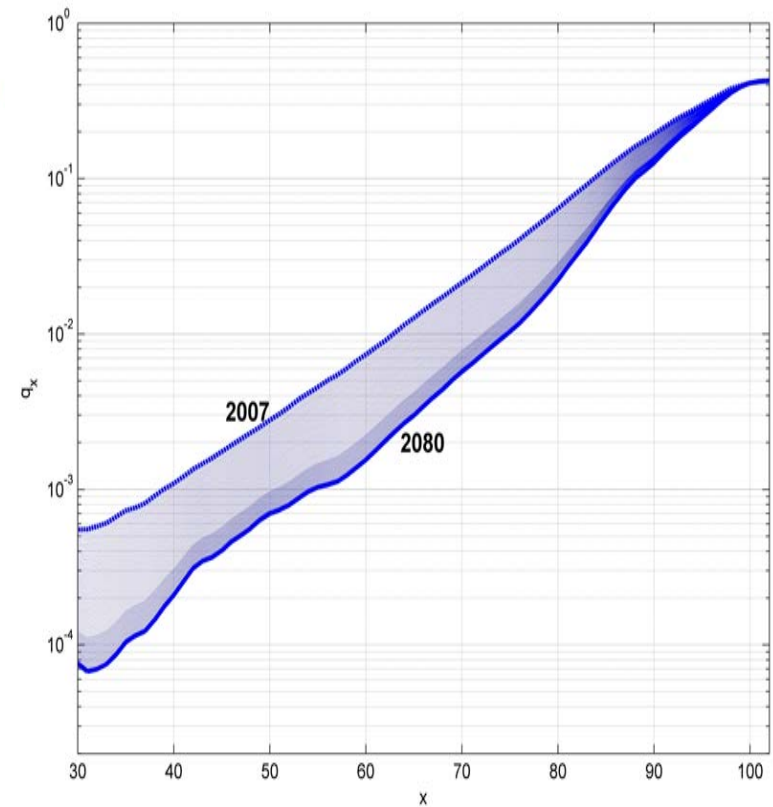
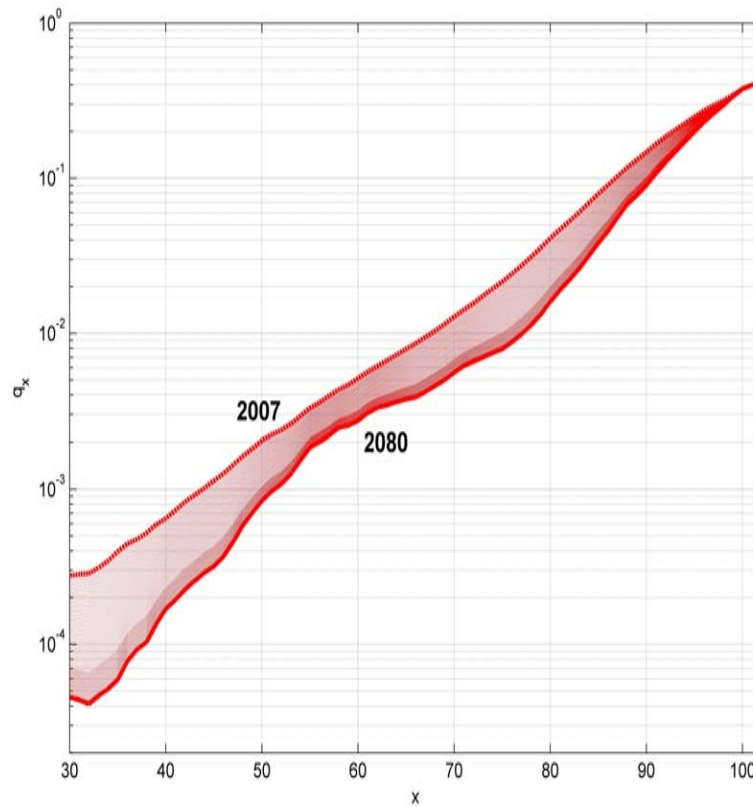
Lee-Carter parameters - $\kappa(t)$



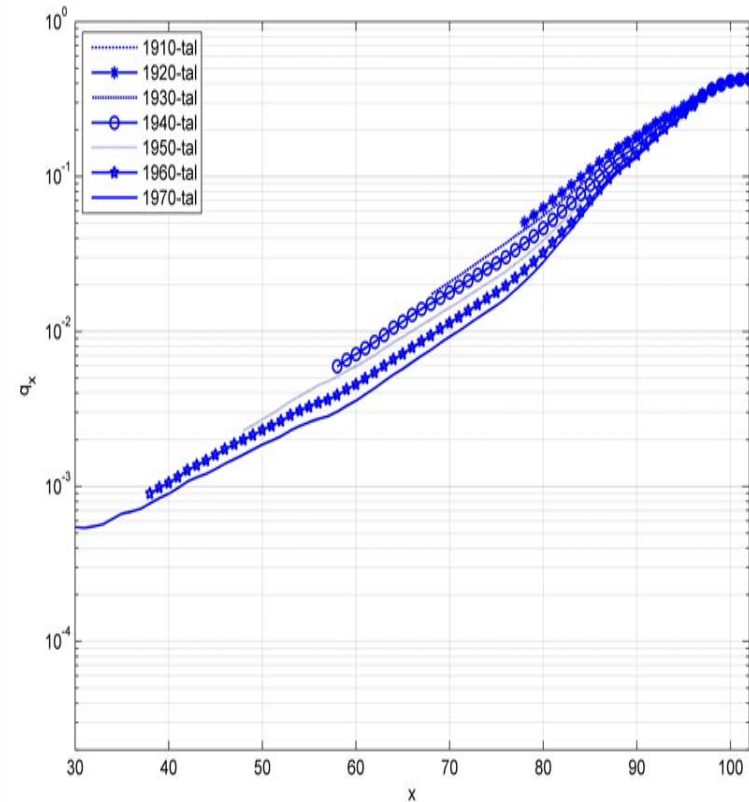
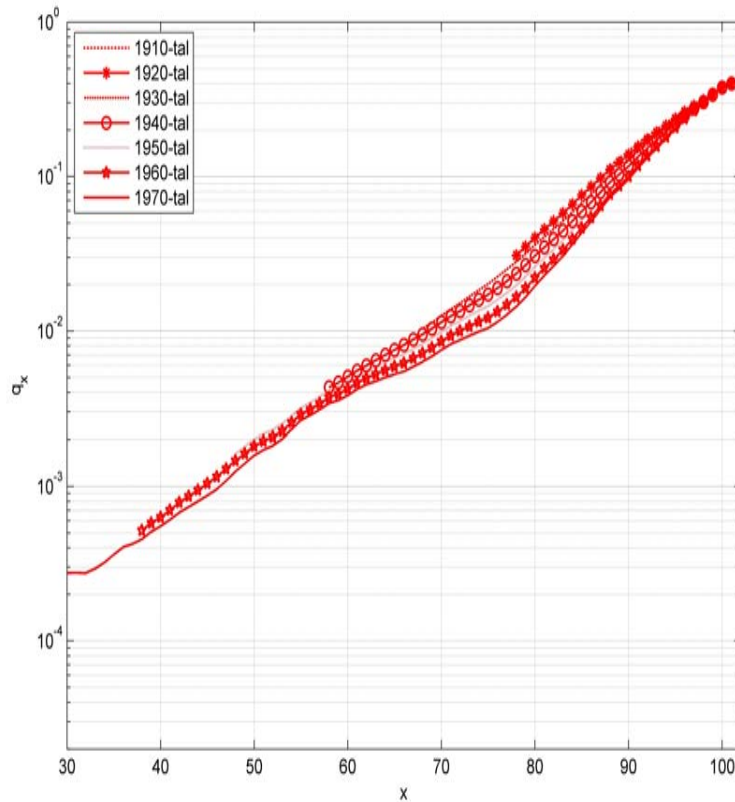
Lee-Carter parameters - β_x



Population mortality - by calendar year



Population mortality - by birth cohort



Conversion from population to insurance mortality

- Ratio between population and insurance force of mortality 2001-2005

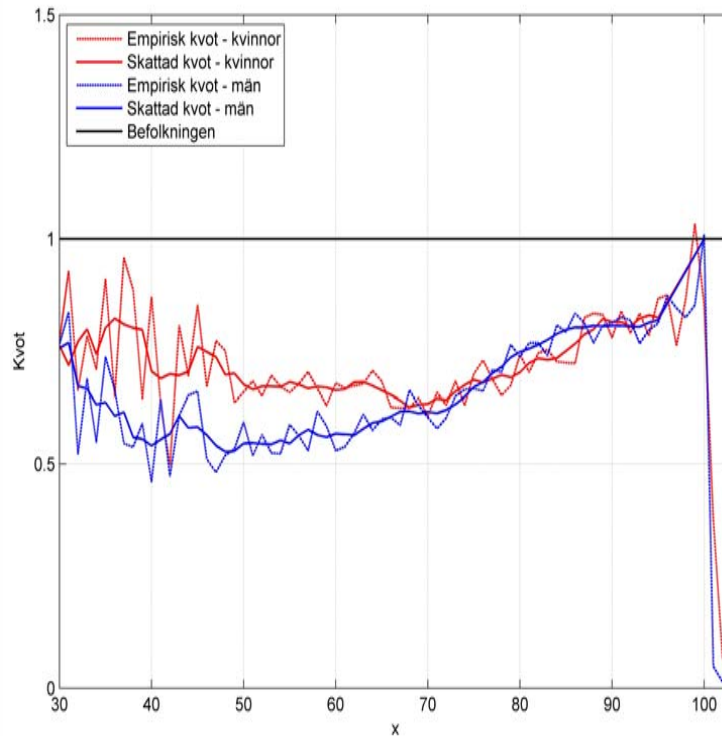
$$k_x^{i,P} = \frac{\mu_x^i}{\mu_x^P}$$

- Smoothing
- The smoothed ratio is applied to the projected population force of mortality 2007-2080

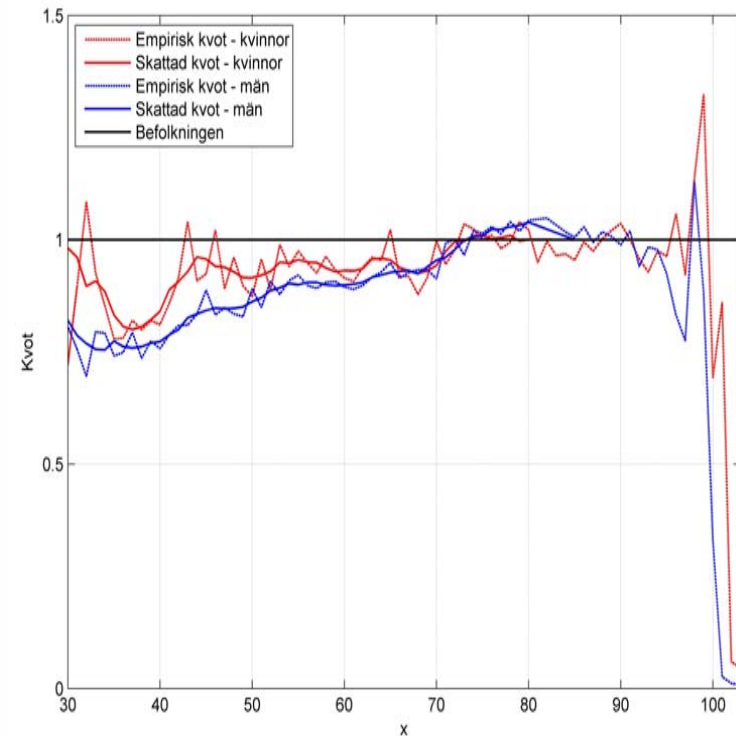
$$\mu_x^i(t) = k_x^{i,P} \cdot \mu_x(t)$$

Conversion from population to insurance mortality - ratios

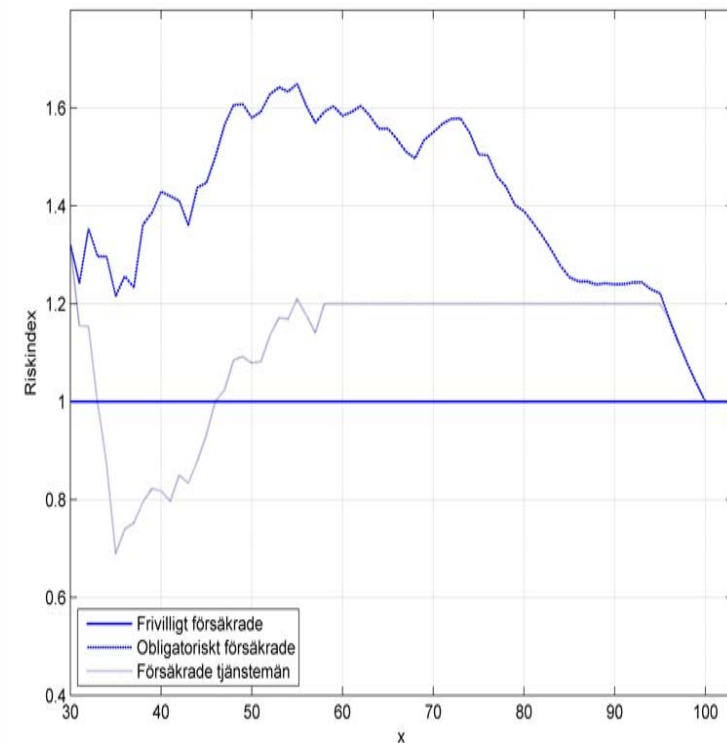
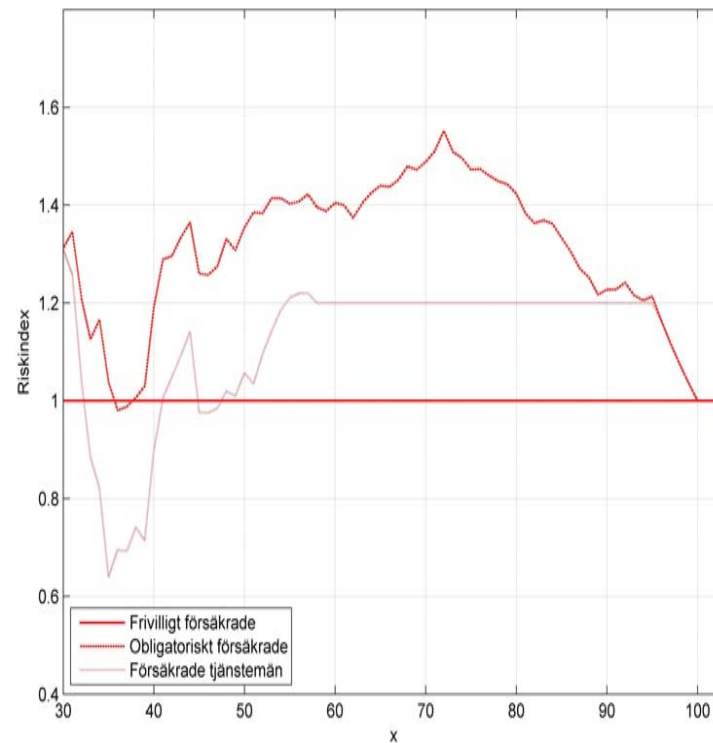
Voluntarily insured



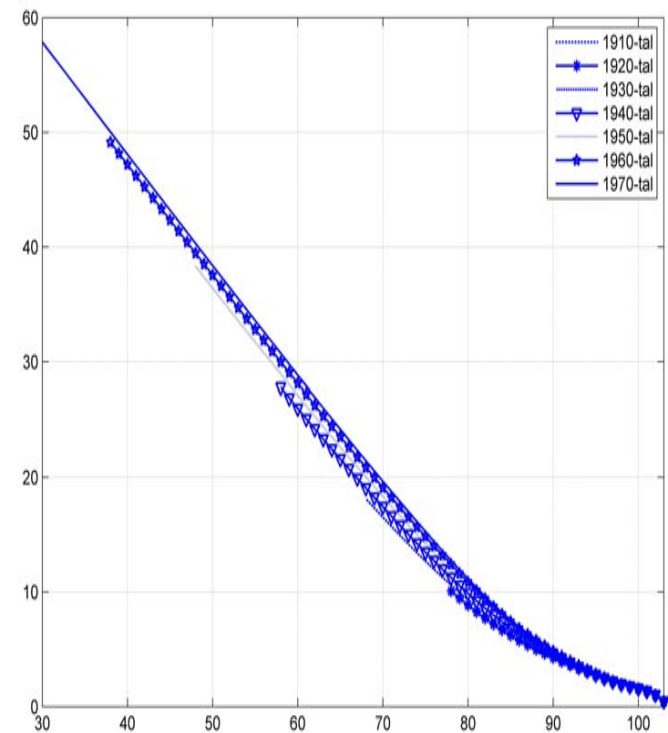
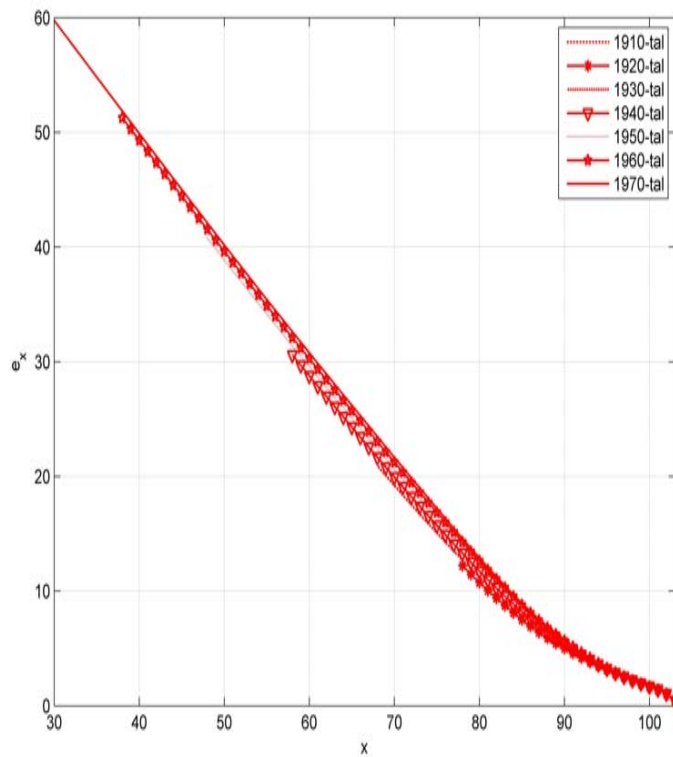
Compulsory insured



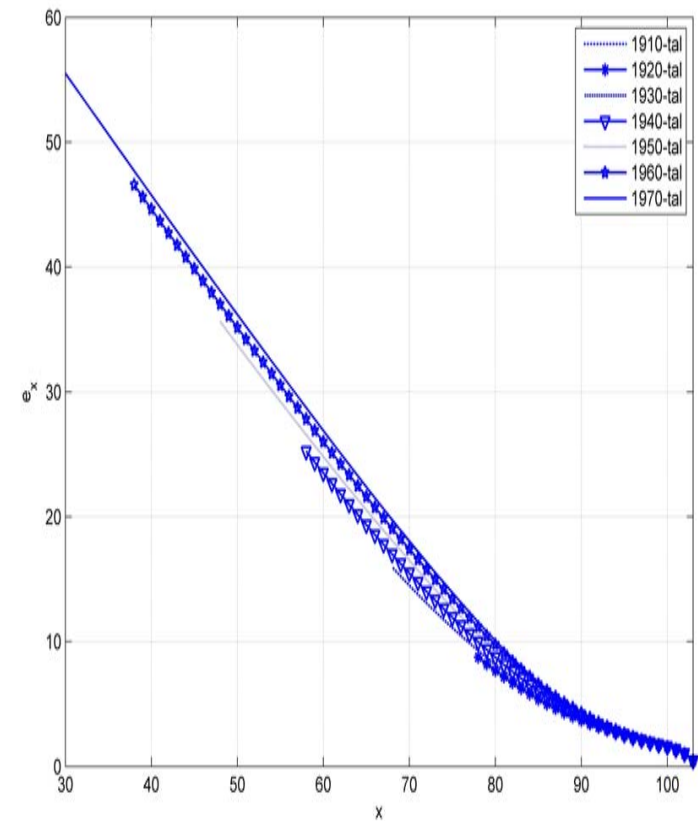
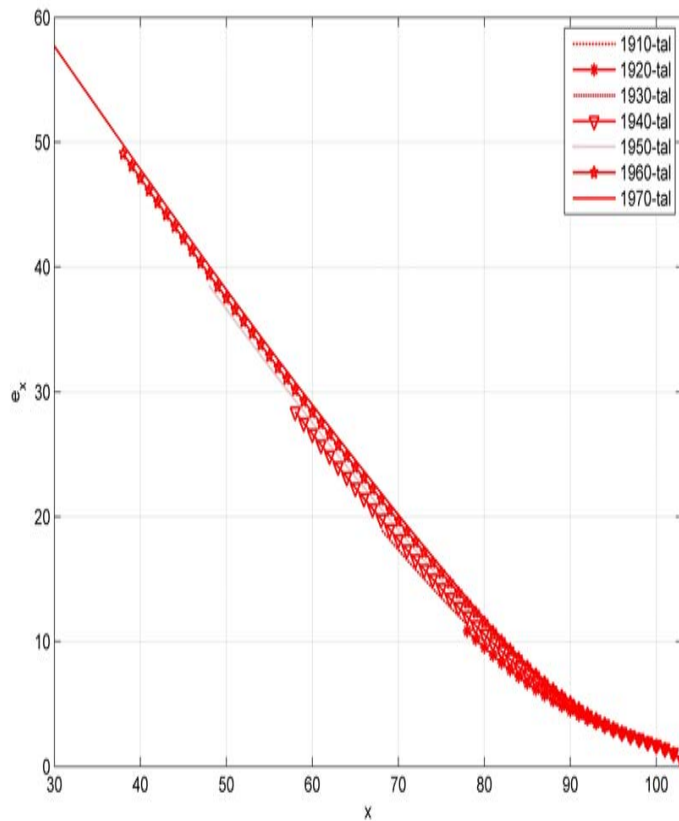
Riskindex



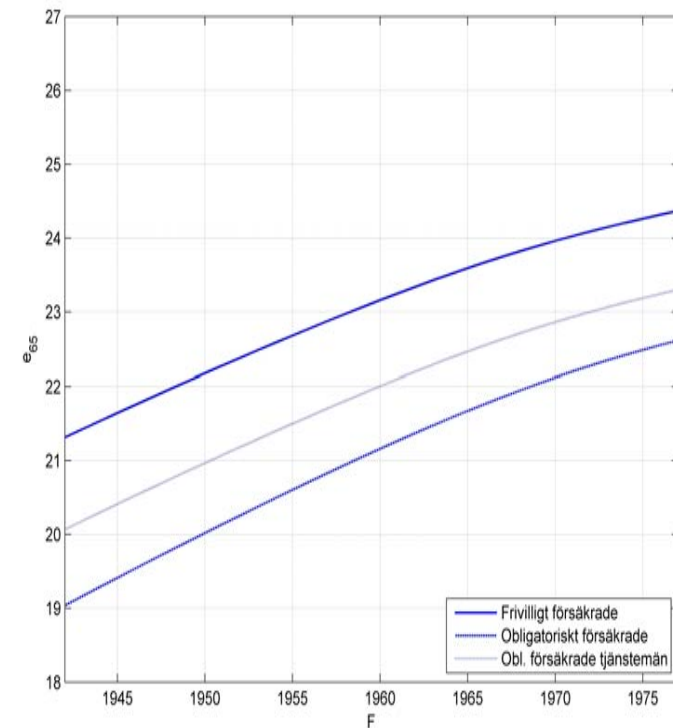
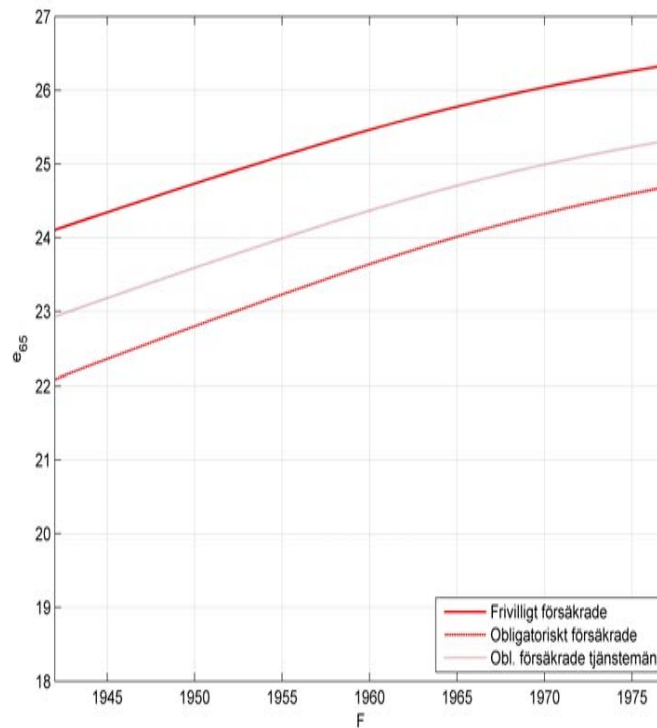
Expected remaining life time – all ages, voluntarily insured



Expected remaining life time – all ages, compulsory insured



Expected remaining life time – at age 65



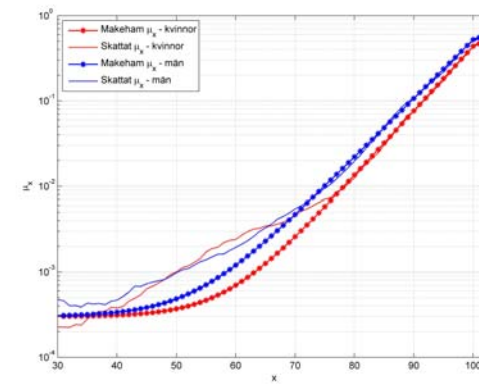
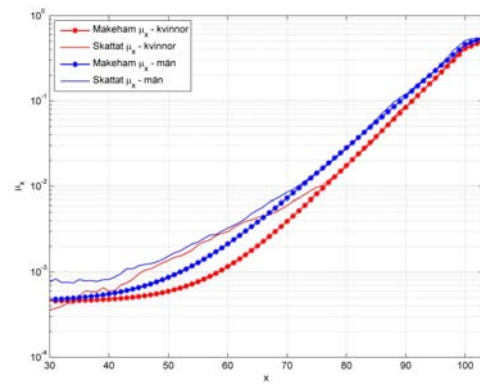
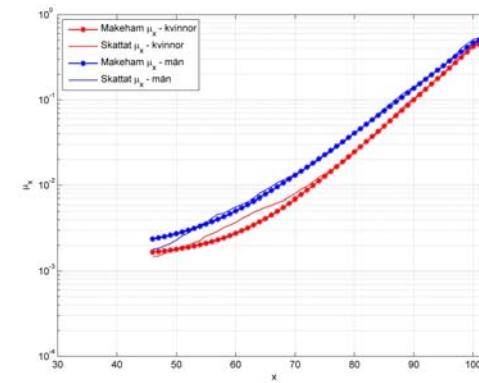
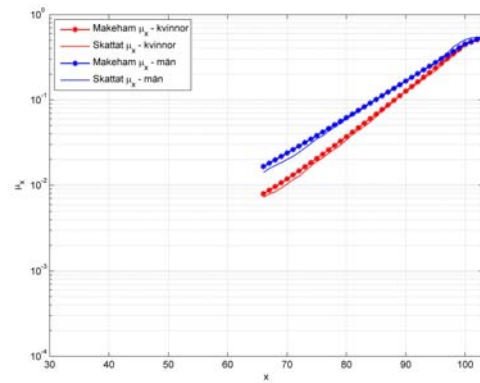
Makeham formula

- Makeham formula for each cohort

$$\mu_x^*(F) = \begin{cases} a + b \cdot 10^{c \cdot x}, & x \leq \omega \\ \mu_\omega + k \cdot (x - \omega), & x \geq \omega \end{cases}$$

- Manual adjustments for better fit in high and low ages

Cohort Makeham formulas – voluntarily insured



Conclusions

- Expected remaining life time depends on age, sex, year of birth and group of insured
- The later year of birth, the longer expected remaining life time
- Voluntarily insured individuals live longer than compulsory insured individuals born in the same in year
- Makeham is no good approximation at all ages at the same time