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Mortality in the Nordic countries – Sweden

IAA Life 2007, Stockholm

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Outline

- Background
- Methodology
- Data
- Results
- Conclusions

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The need of a new common mortality investigation

- Lagislation
  - IOPR-directive
  - Solvency II
  - IFRS Phase II
- Include mortality trends in life insurance reserves
- Latest common mortality investigation in 1989

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M90

- Traditionally, the Makeham formula has been used

$$\mu(x) = a + b \cdot 10^{c(\text{x-f})}$$

- Latest, public mortality investigation 1989-1990 resulted in the M90-parameters

$$a = 0.001$$
$$b = 0.000012$$
$$c = 0.044$$
$$f = \begin{cases} 6 & females \\ 0 & males \end{cases}$$

- ”Flip” around age 60
  - Higher mortality in younger ages
  - Lower mortality in older ages

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The Lee-Carter model

$$\mu_t(x) = e^{\alpha_x + \kappa(t) \cdot \beta_x}$$

- Developed by Lee and Carter in 1992
- The method is ”extrapolative and makes no effort to incorporate knowledge about medical, behavioral, or social influences on mortality change”

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The Lee-carter model - α_x

$$\mu_t(x) = e^{\alpha_x + \kappa(t)\cdot\beta_x}$$

- The average of the logarithm of the force of mortality over the observed period of time
- The general shape of the mortality

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The Lee-carter model - $\kappa(t)$

$$\mu_t(x) = e^{\alpha_x + \kappa(t) \cdot \beta_x}$$

- The time trend

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The Lee-carter model - β_x

$$\mu_t(x) = e^{\alpha_x + \kappa(t) \cdot \beta_x}$$

- The age-specific patterns of the mortality change
- Shows the sensitivity of the logarithm of the force of mortality at age x to the time trend
- The higher value, the higher response of the time trend

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Estimating the parameters – Basic concepts

- Exposure $$R_x(t) = \frac{N_{x-1}(t-1) + N_x(t)}{2}$$
- Number of deaths $$D_x(t)$$
- Force of mortality $$\mu_x(t) = \frac{D_x(t)}{R_x(t)}$$
- Assume that the number of deaths are approximately Poission distributed, i.e.

$$D_x(t) \text{ is } Po(R_x(t) \cdot \mu_x(t)) \Rightarrow E[D_x(t)] = R_x(t) \cdot \mu_x(t)$$

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Estimating the parameters – Maximum Likelihood Method I

- Estimating the parameter vectors

$$\overline{\alpha} = (\alpha_x, \alpha_{x+1}, \alpha_{x+2}, \alpha_{x+3}, \ldots)$$
$$\overline{\kappa} = (\kappa(t), \kappa(t+1), \kappa(t+2), \kappa(t+3), \ldots)$$
$$\overline{\beta} = (\beta_x, \beta_{x+1}, \beta_{x+2}, \beta_{x+3}, \ldots)$$

- Setting up and maximizing the likelihood sum:

$$L(\alpha, \kappa, \beta) = \prod_x \prod_t P(D_x(t) = d_x(t))$$

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Estimating the parameters – Maximum Likelihood Method II

- The maximization is done by the following three steps
  - Taking the logarithm of the likelihood sum
  - Taking the derivative of the logarithmed likelihood sum
  - Solving the equation by using a Newton-Raphson method, iterating one term at a time

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Estimating the parameters – Iteration

$$a_x^{(v+1)} = a_x^{(v)} + h \cdot \frac{\sum_t \left(d_x(t) - \lambda_x^{(v)}(t)\right)}{\sum_t \lambda_x^{(v)}(t)}$$

$$\kappa_t^{(v+1)}(t) = \kappa_t^{(v)}(t) + h \cdot \frac{\sum_t \left[\left(d_x(t) - \lambda_x^{(v)}(t)\right) \cdot \beta_x^{(v)}\right]}{\sum_t \left[\lambda_x^{(v)}(t) \cdot \left(\beta_x^{(v)}\right)^2\right]}$$

$$\beta_x^{(v+1)} = \beta_x^{(v)} + h \cdot \frac{\sum_t \left[\left(d_x(t) - \lambda_x^{(v)}(t)\right) \cdot \kappa^{(v)}(t)\right]}{\sum_t \left[\lambda_x^{(v)}(t) \cdot \left(\kappa^{(v)}(t)\right)^2\right]}$$

$$\lambda_x(t) = E\left[D_x(t) = d_x(t)\right] = R_x(t) \cdot \mu_x(t) = R_x(t) \cdot e^{\alpha_x + \kappa(t) \cdot \beta_x}$$

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Estimating the parameters – Containts

- The following constraints are used

$$\sum_t \kappa(t) = 1$$

$$\sum_x b_x = 1$$

- The constraints are used in each iteration

$$a'_x = a_x + \frac{1}{n}\sum_t \kappa(t)$$

$$\kappa'(t) = \sum_x \beta_x \cdot \left(\kappa(t) - \frac{1}{n}\sum_t \kappa(t) \right)$$

$$\beta'_x = \frac{\beta_x}{\sum_x \beta_x}$$

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Trend estimation data

- Long, homogeneous time series need to estimate trends in mortality
- Changes in mortality among insured individuals due to
  - The mortality has actually changed?
  - The characteristics of the insured has changed and therefor also the mortality?
- Only homogeneous time serie available for trend estmation is the mortality of the entire Swedish population

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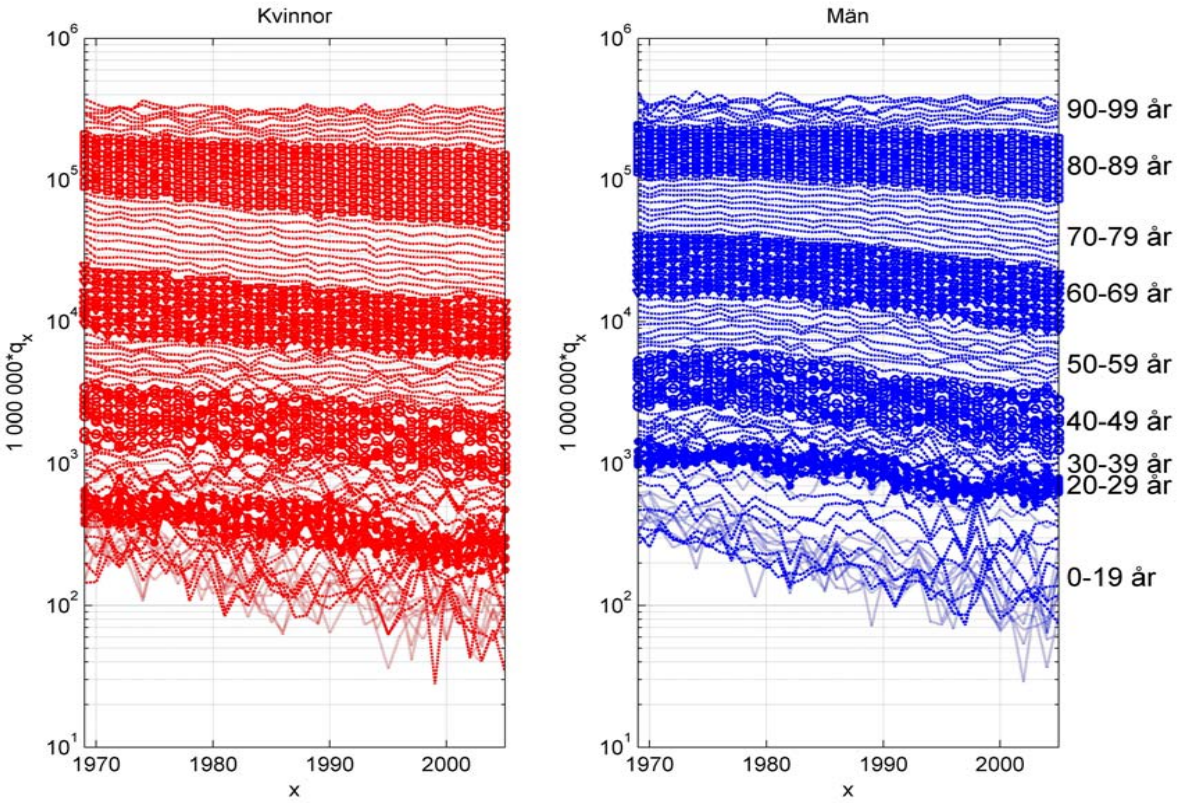
Swedish population data



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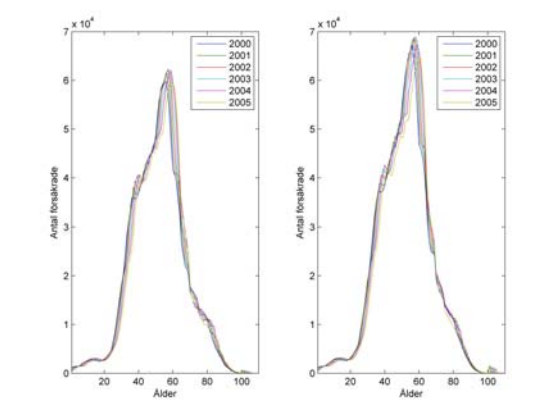
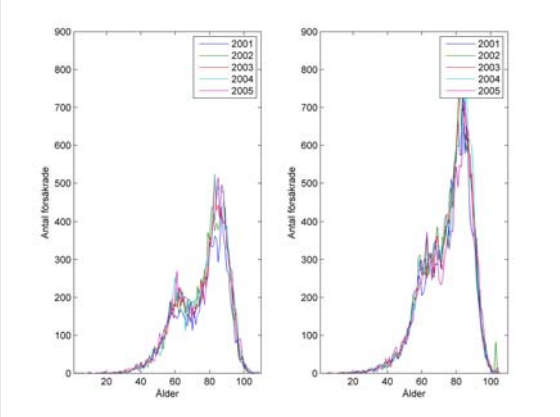
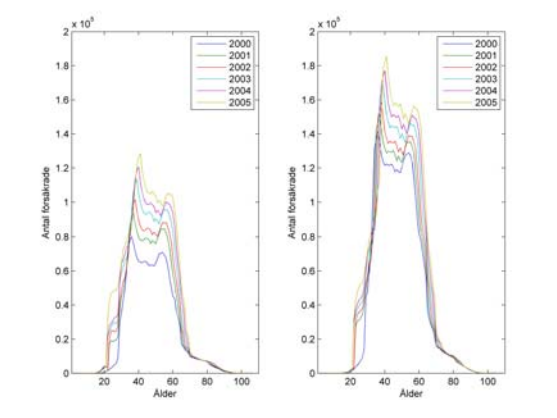
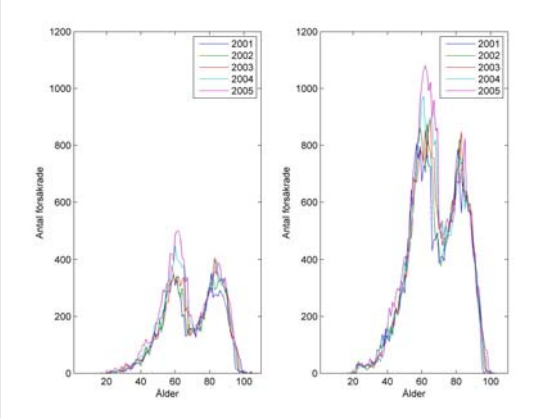
Insurance data

	Number of insured	Number of deceased
Voluntarily insured		
Compulsory insured		

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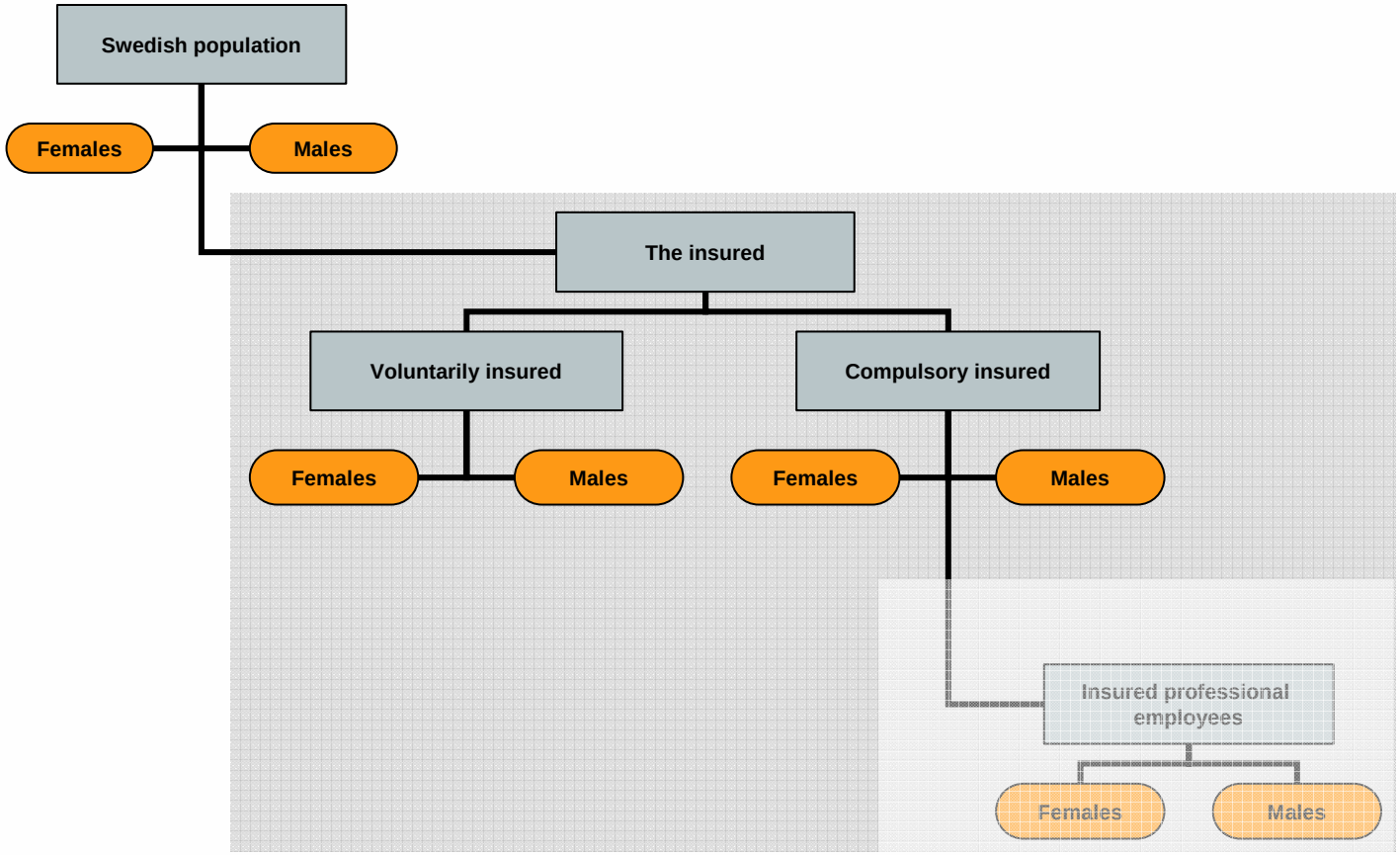
Classification of insurance data



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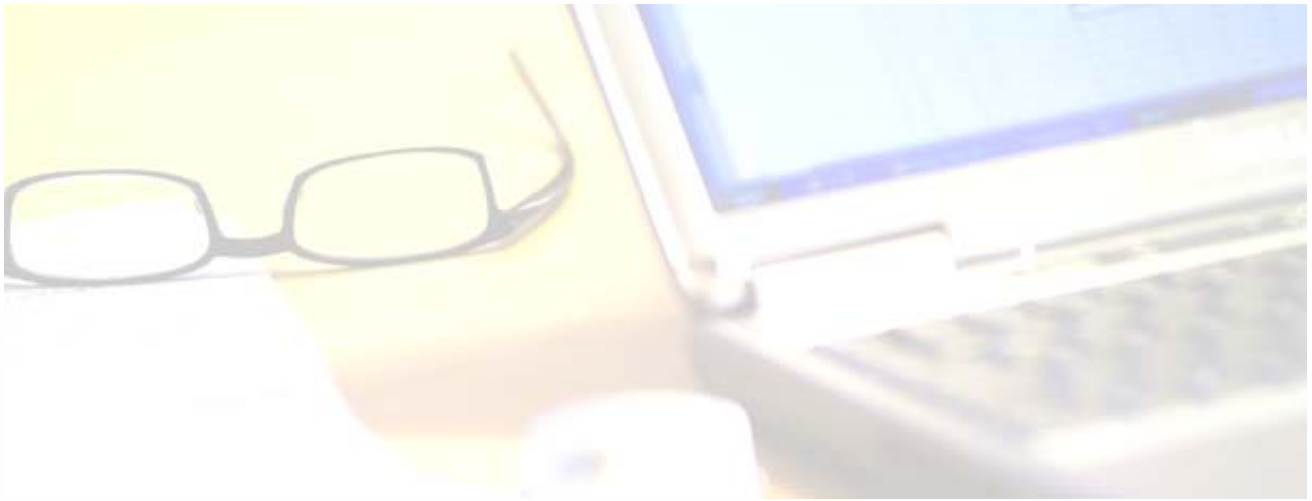
Data specifics

- Both mortality and longevity risks
- Health examination/able-bodied
- Insurance based and not individual based, i. e. individuals can be found in more than one group



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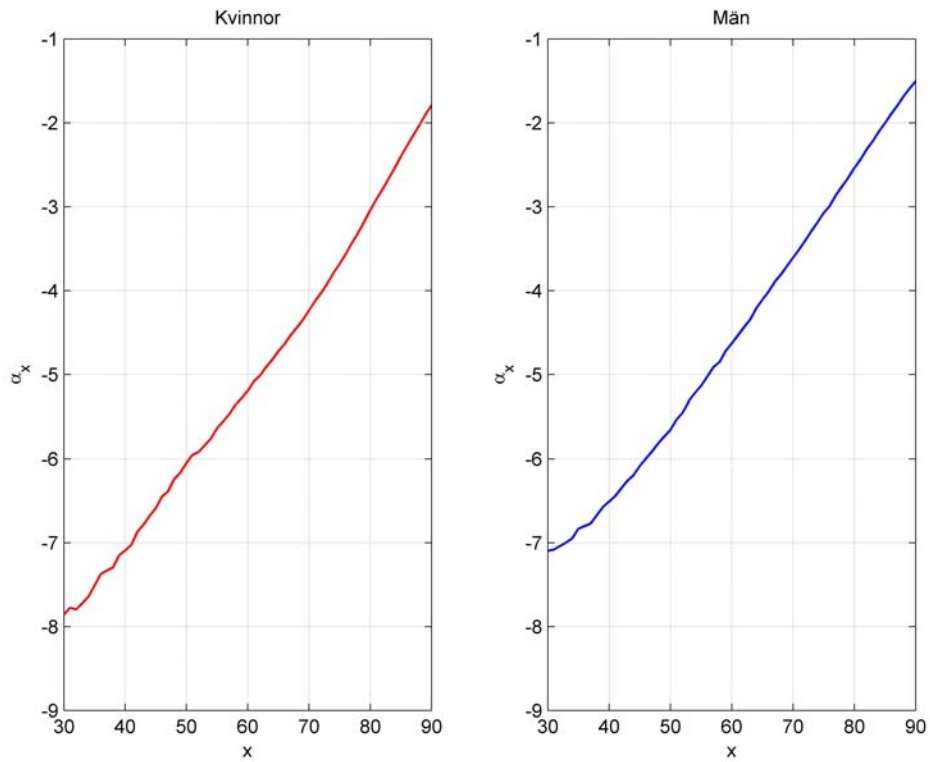
Lee-Carter parameters - α_x



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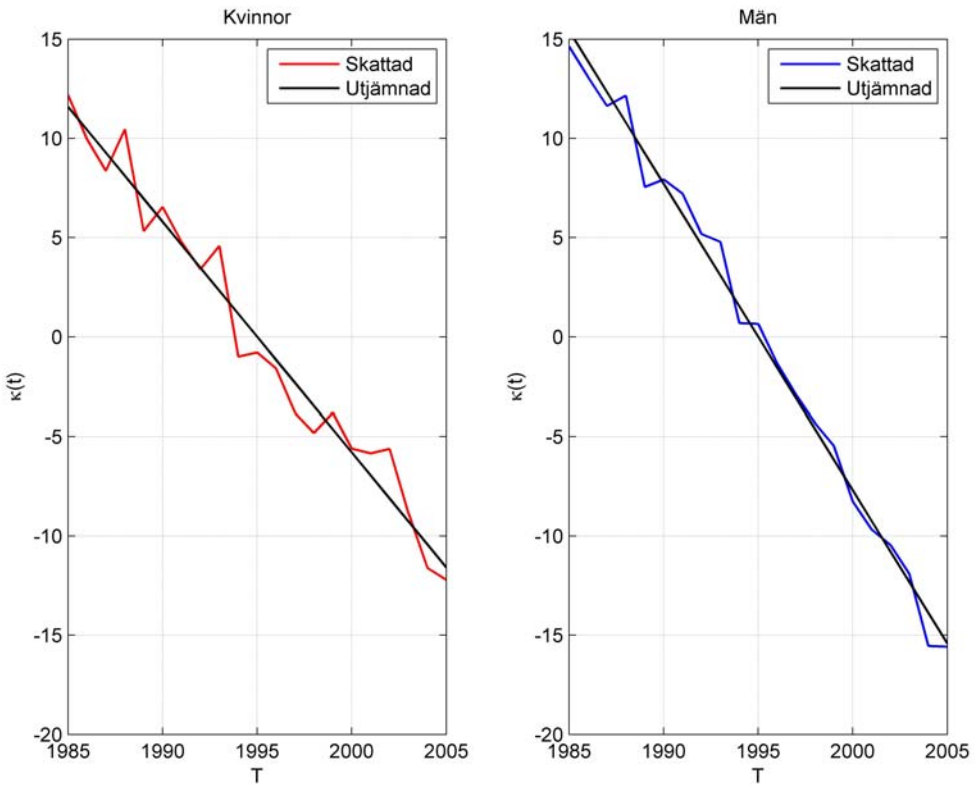
Lee-Carter parameters - $\kappa(t)$



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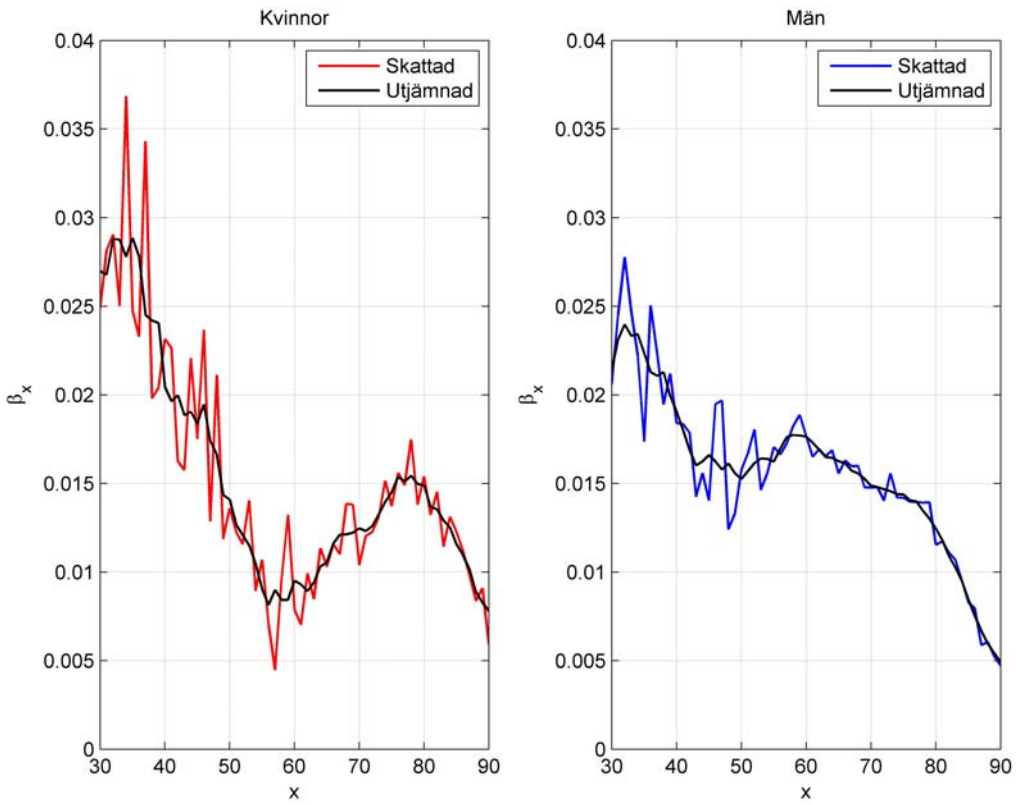
Lee-Carter parameters - β_x



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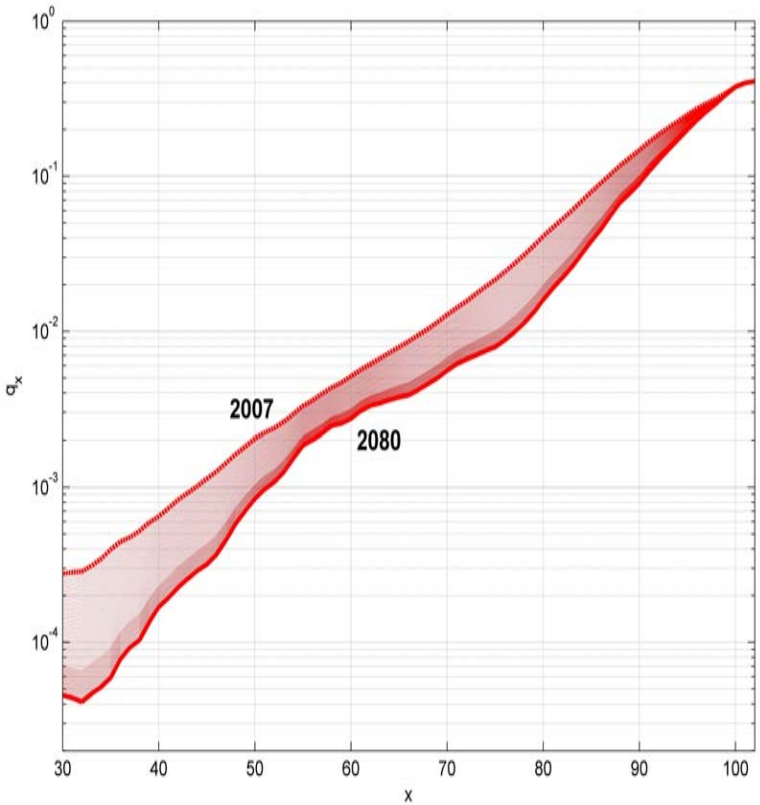
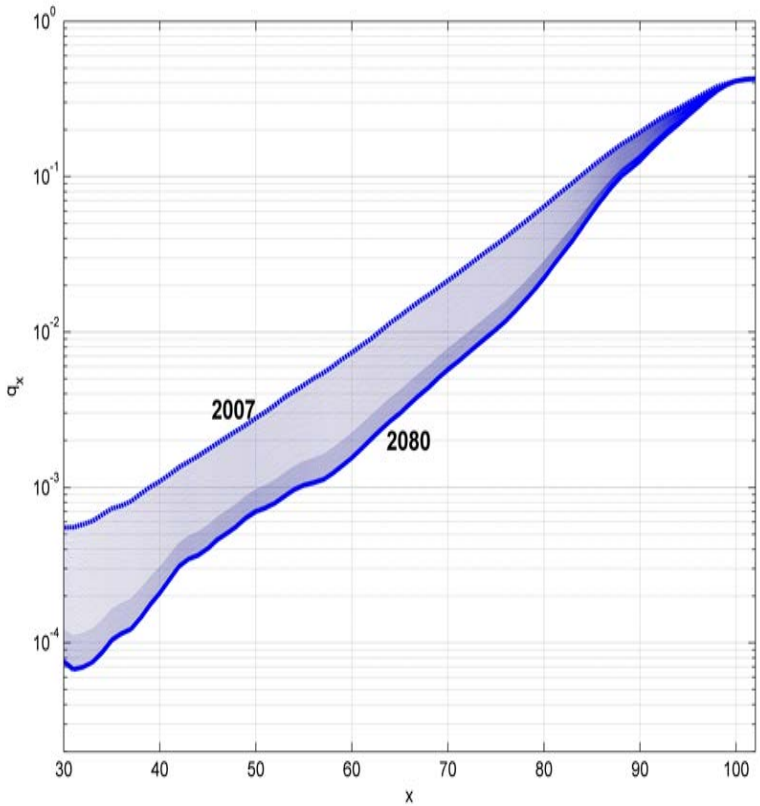
Population mortality - by calender year





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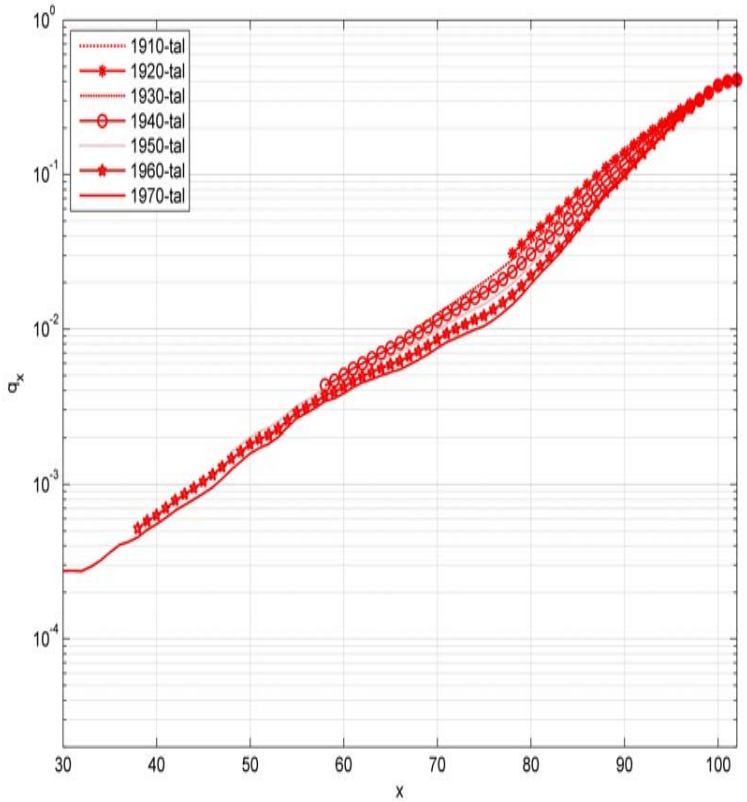
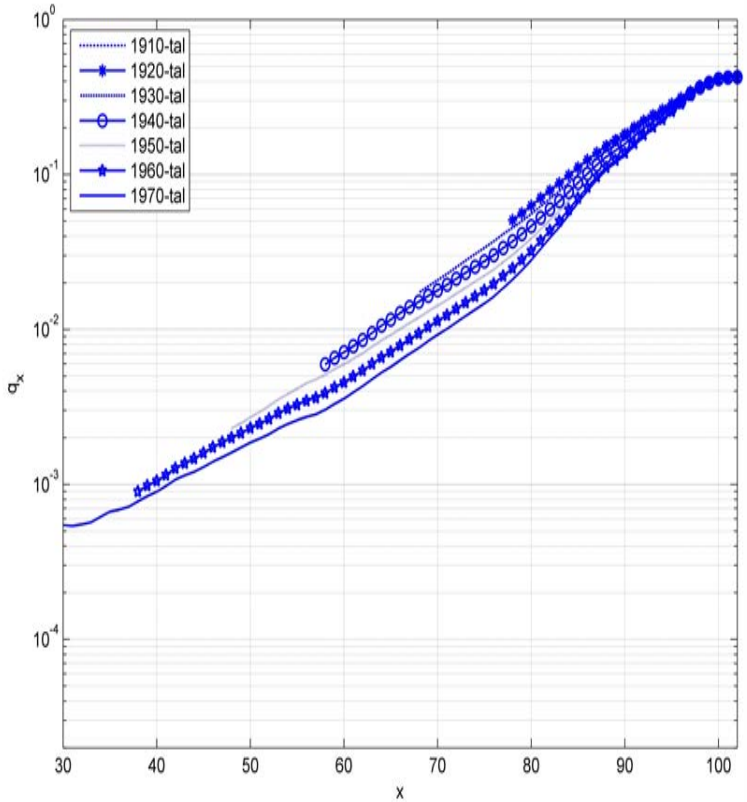
Population mortality - by birth cohort





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Convertion from population to insurance mortality

- Ration between population and insurance force of mortality 2001-2005

$$k_x^{i,P} = \frac{\mu_x^i}{\mu_x^P}$$

- Smoothing
- The smoothed ratio is applied to the projected population force of mortality 2007-2080

$$\mu_x^i(t) = k_x^{i,P} \cdot \mu_x(t)$$

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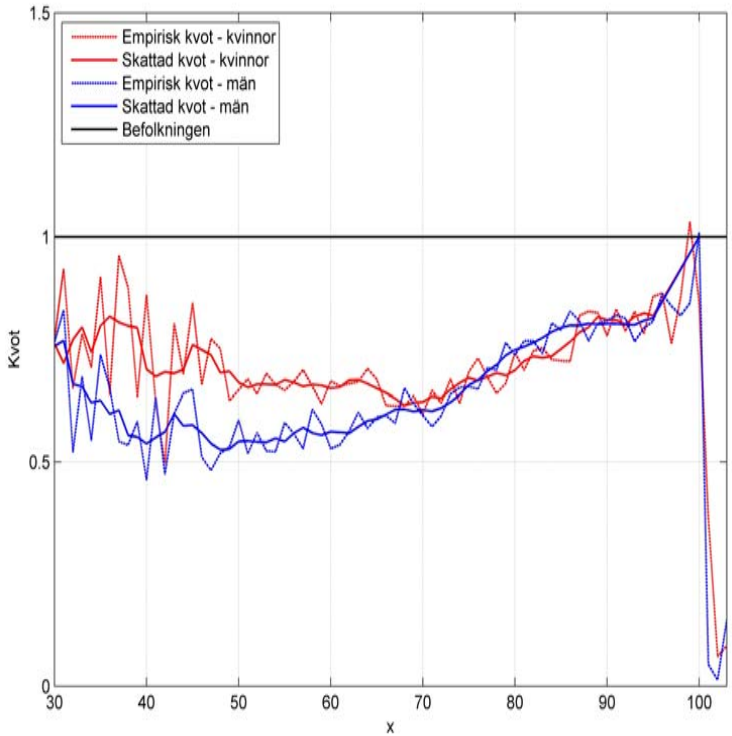
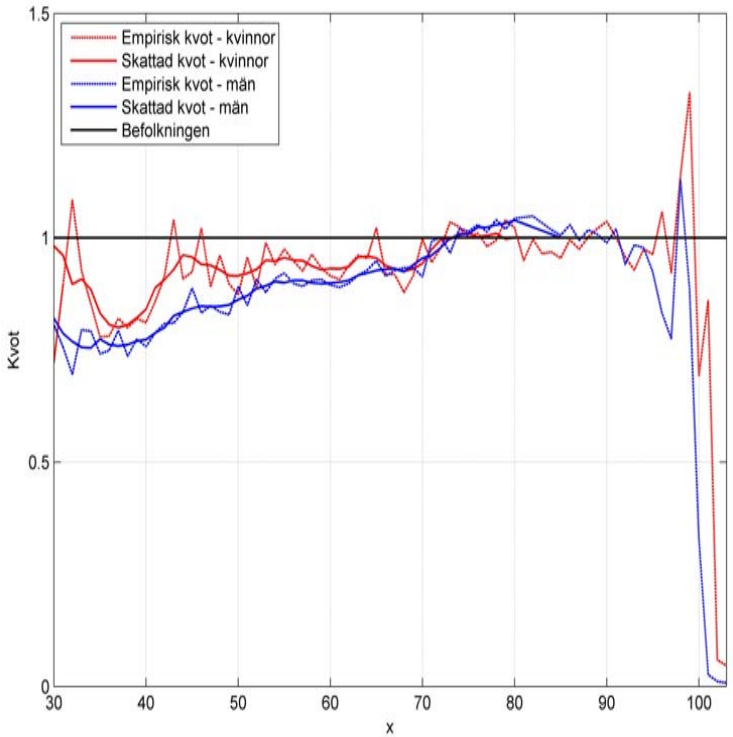
Convertion from population to insurance mortality - ratios

Voluntarily insured



Compulsory insured



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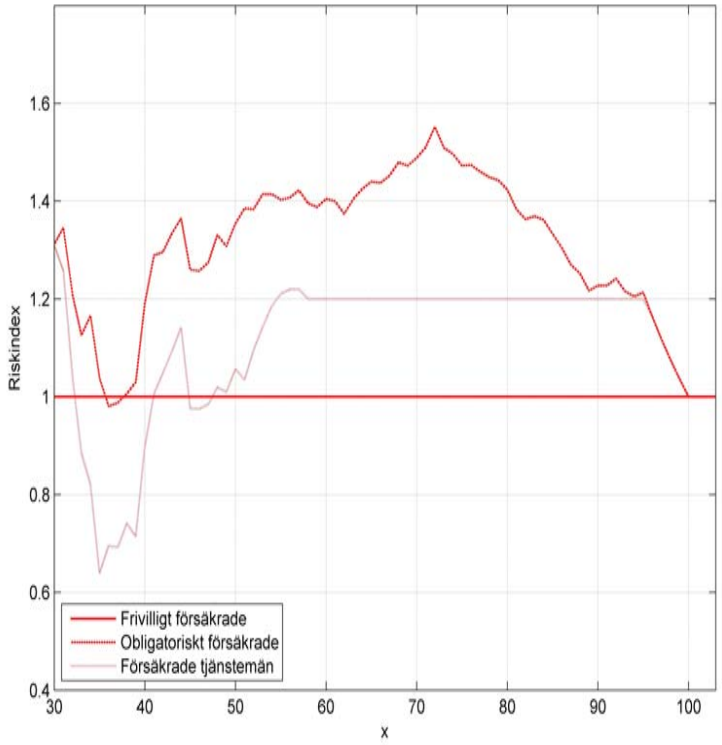
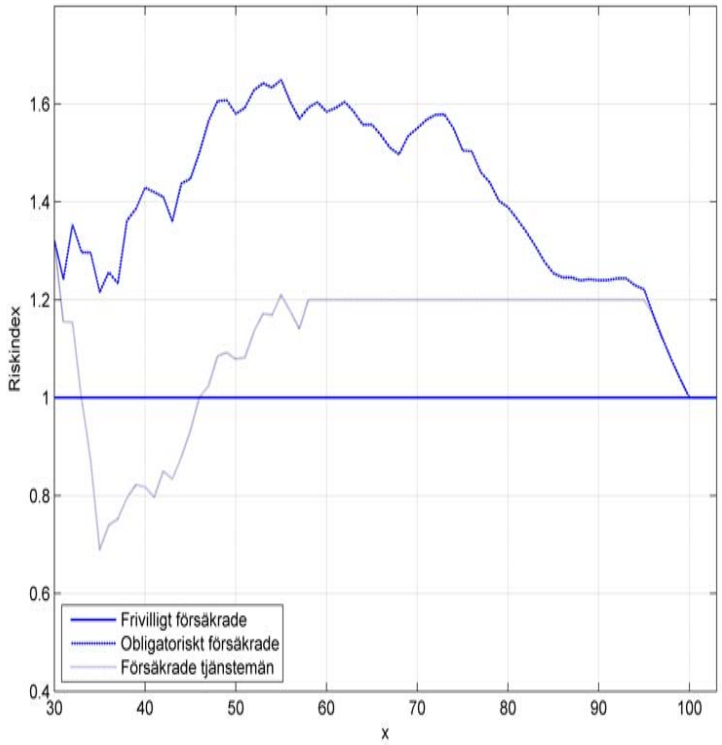
Riskindex





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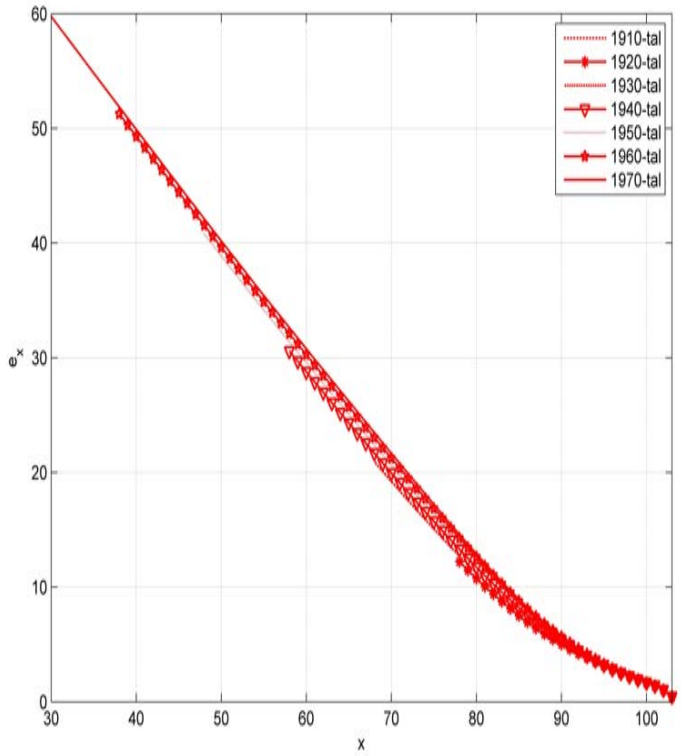
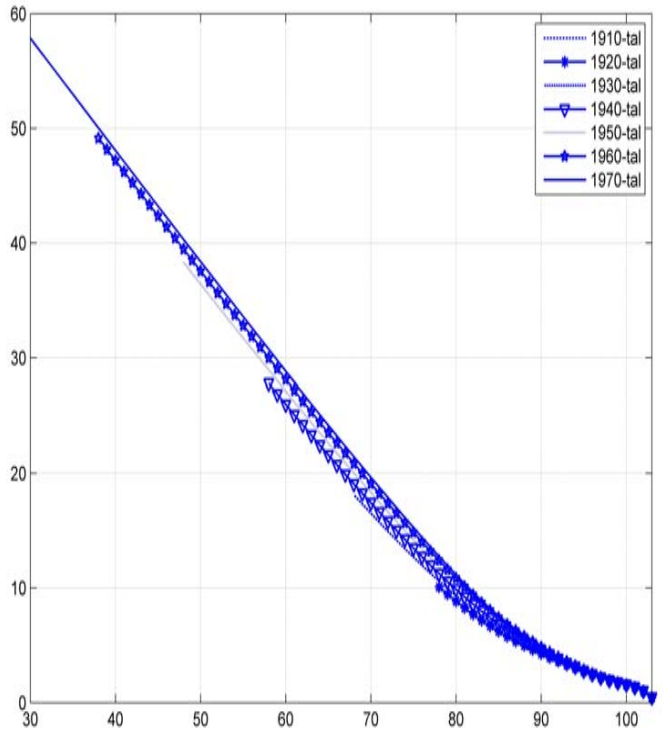
Expected remaining life time – all ages, voluntarily insured





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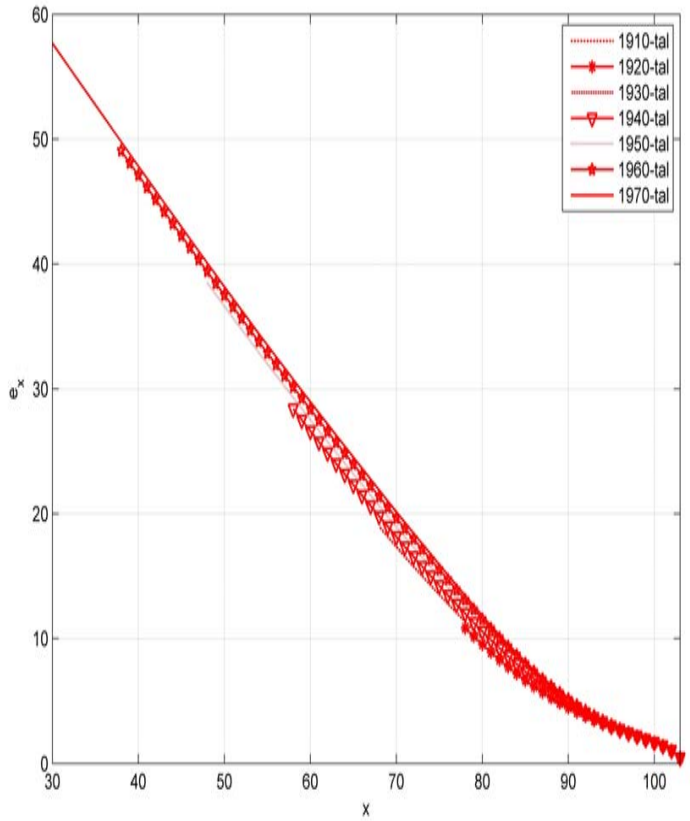
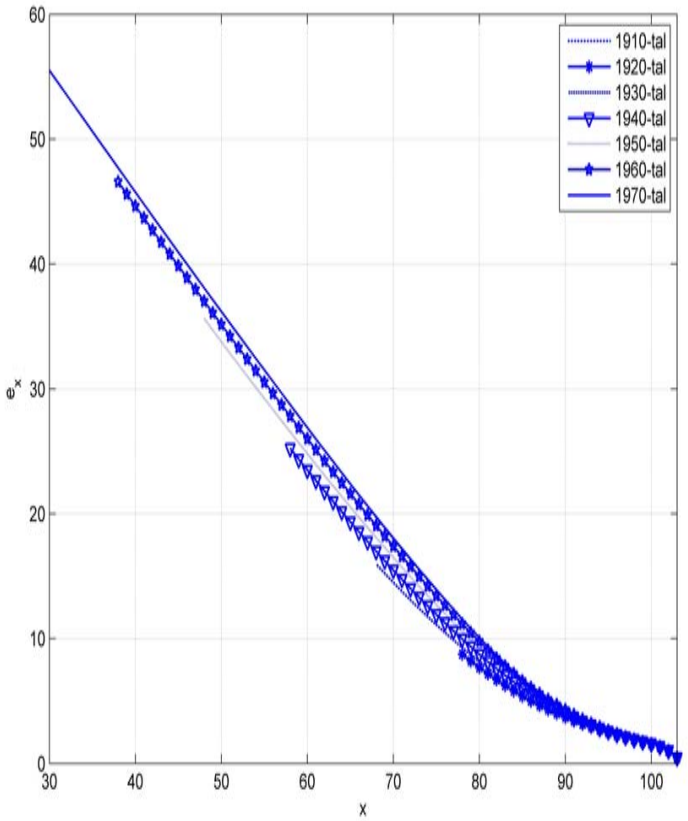
Expected remaining life time – all ages, compulsory insured





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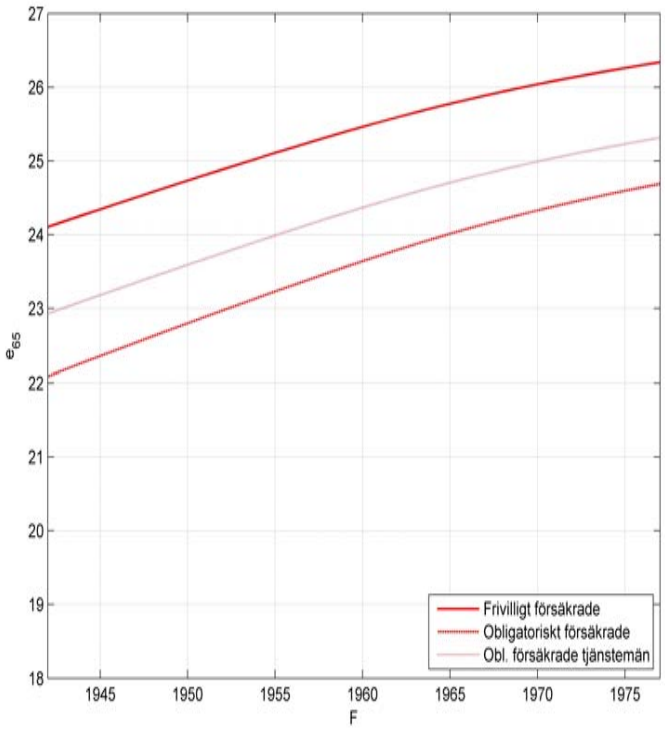
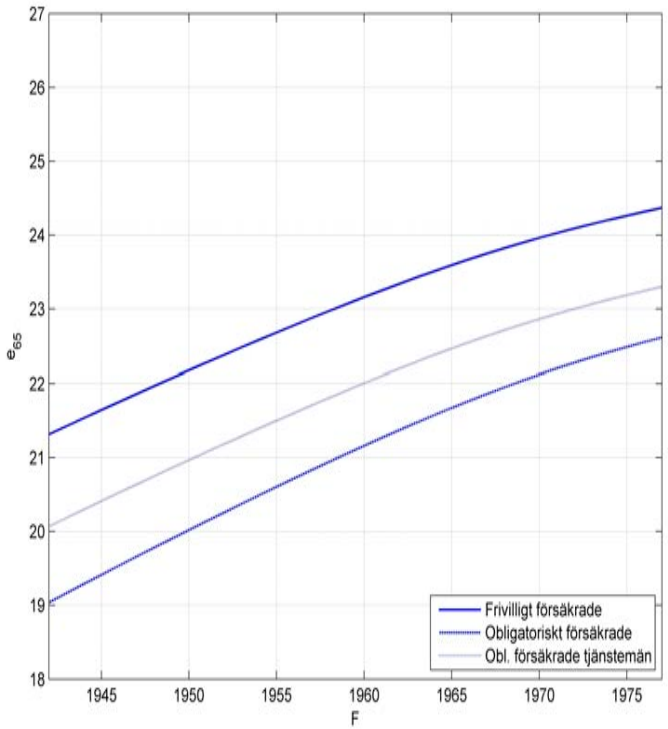
Expected remaining life time – at age 65





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Makeham formula

- Makeham formula for each cohort

$$\mu_x^*(F) = \begin{cases} a + b \cdot 10^{c \cdot x}, & x \le \omega \\ \mu_\omega + k \cdot (x - \omega), & x \ge \omega \end{cases}$$

- Manual adjustments for better fit in high and low ages

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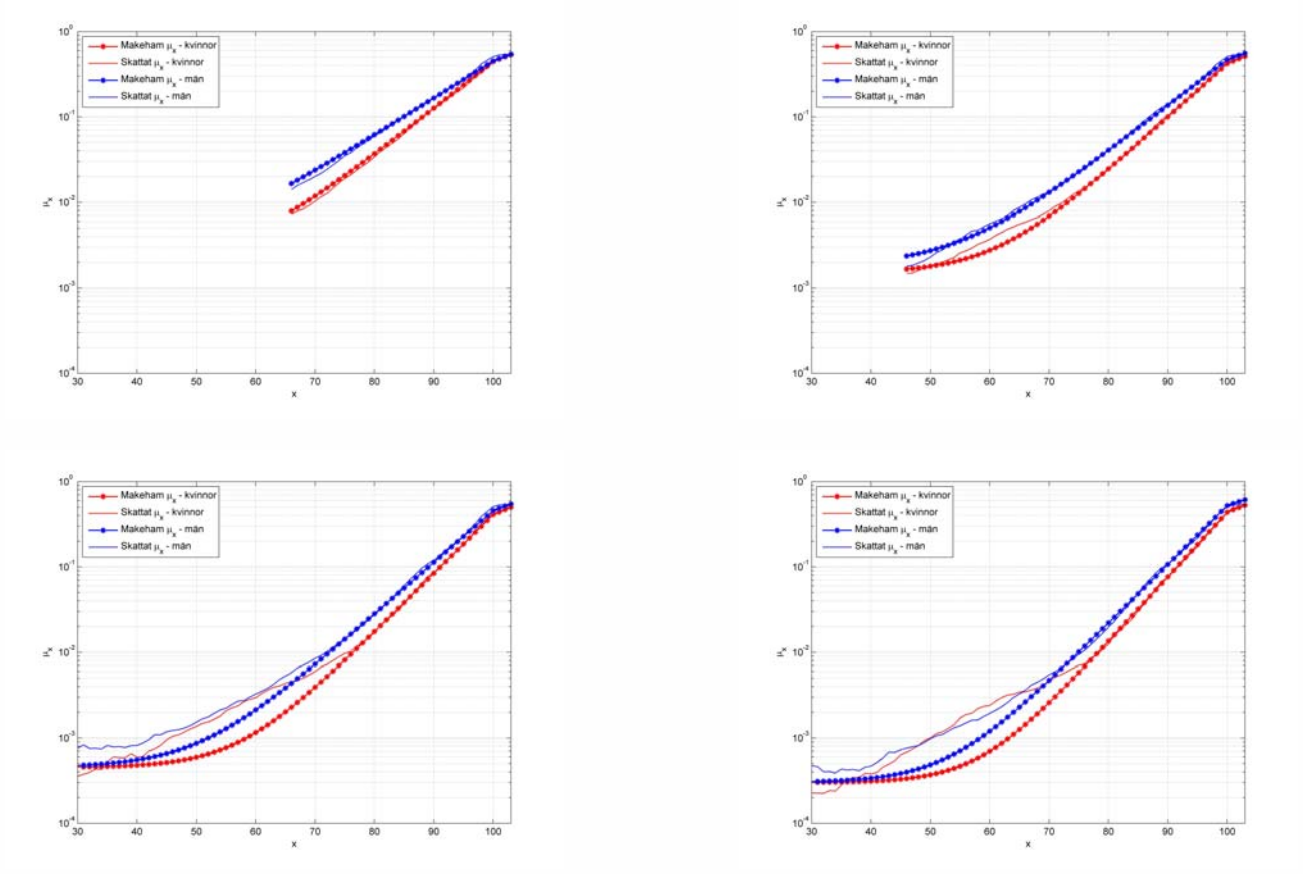
Cohort Makeham formulas – voluntarily insured



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Conclusions

- Expected remaining life time depends on age, sex, year of birth and group of insured
- The later year of birth, the longer expected remaining life time
- Volontarily insured individuals live longer than compulsory insured individuals born in the same in year
- Makeham is no good approximation at all ages at the same time