Fair valuation of with profit life insurance policies: the Italian Case

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Abstract This work outlines a model used to estimate the fair value of Italian with profit policies (named "polizze vita rivalutabili"). The model considers that the rate of return on which depends the policyholder participation right is based on the book value of the assets in the reference portfolio. In addition, since reference portfolios are made up almost exclusively of high grade bonds, the valuation model also considers stochastic interest rates and bond investments.

Present version: January 29, 2007
First version (in Italian): October 26, 2006

Key words: With profit life insurance contract. International accounting standard (IAS) / International Financial Reporting Standard (IFRS). Fair value.

§ 1. Introduction

The with profit policies currently widespread in Italy, named "polizza vita rivalutabile", are part of a wider category of participating life insurance policies.

It is presumed that as from the year 2010 the insurance companies that adopt international accounting principles will have to evaluate all the insurance contracts issued at fair value, or, more precisely, according to the current exit approach, as tentatively outlined by the IASB *(International Accounting Standard Board)* during phase II of the *insurance* project (cf. for example, IASB update 1/2003, 12/2005 and 2/2006).

It is not by chance therefore, that literature on the fair valuation of with profit policies is being developed. The with profit policies category is however highly extensive and difficult to trace back to standardized types. This type of product, which has been widespread for decades, may, due to the diversity of its technical characteristics, be considered as the precursor of the structured securities created with modern financial engineering techniques.

In this work we have developed a model for the fair valuation of with profit policies used in Italy. This model may be used as a reference for the financial statement valuation according to the IFRS which is currently being developed.

Section 2 provides a summary of how, in accordance with the tentative conclusions of the IASB, participating life insurance contracts, including the Italian woth profit policies, should be evaluated. Section 3 reviews the literature on the fair valuation of with profit policies related to this paper. Section 4 shows the valuation model for the Italian with profit policies. More precisely, section 4.4.1 considers the non-stochastic interest rates case and a stock reference portfolio, while section 4.4.2 considers stochastic interest rates and a bond reference portfolio. Since the model does not allow for simple analytical solutions, section 5 analyses the main implications using numerical examples obtained via Monte Carlo simulations. Section 6 concludes with a brief discussion on the opportunity of introducing fair valuation of Italian with profit policies in the financial statement.

An extended version of this work (Floreani, 2006) discusses the main extensions for making the model effectively applicable and in particular it considers the mortality risk, the case of a policy portfolio that refers back to a single reference portfolio and the surrender option.

§ 2. Pricing and presentation in Financial Statements of participating life insurance policies according to the IASB tentative conclusions

In 2004, the IASB launched the second phase of the project for establishing a complete and structured set of accounting principles for insurance contracts (hereon called *insurance project)*. At present, the second phase of the insurance project is still in progress and should be completed by 2009. The resulting accounting standard should introduce uniform principles for the valuation of all insurance contracts (life and non-life) based on fair value or, more precisely, on a forward-looking, unbiased and current market based estimate (this approach is called current exit approach).

The work of the second phase of the insurance project refers to the class of with profit policies as a whole and does not specifically refer to the Italian policies. The valuation of this class of policy is one of the fundamental difficulties that need to be resolved before finalizing a complete and satisfactory standard .

According to the IASB's tentative conclusions, all policies must therefore be evaluated according to the current exit approach. This is "*the amount the insurer would expect to have to pay today if it transferred all its remaining contractual rights and obligations immediately to another entity*". Even if this definition is not identical to fair value, "*the amount for which an asset could be exchanged, or a liability settled, between knowledgeable, willing parties in an arm's length transaction*", for our purposes we will consider it the same and so from now on we can use the latter, more widespread term.

The fair value of the Italian with profit policies depends on the fair value of the assets in the reference portfolio (named "gestione separata"). In the unit and index linked products the fair value of the assets in the reference portfolio almost exactly match the fair value of the liabilities. In (Italian) with profit policies the link between the fair value of assets in the reference portfolio and the fair value of the policy liabilities is less direct, due to the minimum return guarantee, the discretionary participation and the surrender option .

In order to solve the problem, it could be useful to define a simplified context without taxes, future premiums, mortality risk and the surrender option. In this situation the balance sheet of a generic participating policy using fair valuation is given in Table 1.

TABLE # 1

The depiction in Table 1 will be used for the rest of the work. The items in bold print show a set of future stochastic cash flows. Specifically, ***A*** represents the stochastic cash flows originating from the reference portfolio. *G*, ***PhPR*** and ***ShPR*** represent, respectively:

- The contractually guaranteed benefits owed to the policyholder, *G* (*G* is not stochastic, having excluded the insurance risk components and the surrender option).
- The policyholder participation rights (***PhPR***).
- The shareholders' participation rights (***ShPR***); they receive all residual cash flows not paid to the policyholder .

Therefore: $\boldsymbol{A} = G + \boldsymbol{PhPR} + \boldsymbol{ShPR}$.

In some circumstances the assets of the reference portfolio may not be sufficient to guarantee the benefits promised the policyholder. The accounting standards requires that value of the future stochastic flows given by the shareholders to the policyholders in case of unsatisfactory performance of assets should be reported in the balance sheet. Total stochastic flows ***P*** is the payoff of an exotic put option in favour of the policyholder with underlying assets ***A***. Finally, operator *V(.)* measures the fair value of stochastic flows at the valuation date. The following equation is therefore applied also for the fair value:

$$V(\boldsymbol{A}) = V(G) + V(\boldsymbol{PhPR}) + V(\boldsymbol{ShPR})$$

It must be stressed that, in this context, the balance sheet in Table 1 is suitable for all with profit policies. The various types of policies differ exclusively in the contractual specification of *G*, ***PhPR*** and ***P***.

The valuation and financial reporting problems connected with the items presented in Table 1 are diverse and may be significant. The main problem encountered in financial reporting is the classification of the participation rights ***V(PhPR)*** and ***V(ShPR)*** between liabilities and equity. In particular, according to the IASB's preliminary views (cf. IASB update 3/2006), the policyholders' participation rights are a liability only if there is an "*enforceable constructive obligation*", i.e., only if "*it legally or equivalently compels potential outflows of cash or other economic resources*". In the case of Italian with profit policies, there seems to be no doubt of the existence of such an obligation, and consequently the policyholders' participation rights will be booked under liabilities and the participation rights of the shareholders under equity.

§ 3. Related literature

In this section we will attempt to give a brief overview of growing literature on fair valuation of with profit life policies.

Firstly, we can distinguish between those models where the policyholders' participation rights depend directly and functionally on the fair value of the reference portfolio, and those models where the participation rights may also depend on the book value of the reference portfolio itself. This is the most important category for our purpose since, as already mentioned, models from the second category are required for fair valuation of Italian with profit policies. A large part of the literature however, refers to the first category (cf. Briys and De Varenne, 1997, Miltersen and Persson, 2000 and 2003 , Hansen and Miltersen, 2002, Bacinello, 2001, 2003a and 2003b).

The models of the second category (cf. Grosen and Jorgensen, 2000, Jensen et al, 2001, Taskanen and Lukkarinen, 2003 and Bauer et al, 2006) are fewer but more recent. Bauer et al (2006), in particular, examines the types of contracts used in Germany and has been taken as reference for developing the pricing model in this paper.

Furthermore, we can distinguish between specific models, i.e. those referring to contracts used in specific institutional settings, and those used in general settings and relative to the entire range of with profit policies or to a wide sub-range. Most literature may be referable to specific models. An example of the second category is the Tanskanen and Lukkarinen (2003) model. Obviously, the model developed in this paper is specific, since it has been expressly tailored to the pricing of Italian with profit policies.

A third type of category distinguishes between the basic and the more complex models. The basic, more essential models, including the one illustrated in this work, consider with profit policies as a financial contract with an established expiry date and ignore the insurance elements, including the surrender option and the mortality risk.

Grosen and Jorgensen (2000) and Bacinello (2003b), among others, examined the problem of valuation of the surrender option, even though this is de facto evaluated as a classic American option and does not consider that the surrender of a life policy does not depend exclusively on financial consideration.

Bacinello (2001) introduced the mortality risk. This is the only model explicitly developed for Italian with profit policies, even though the reference portfolio return is hypothesized as market based. Ballotta and Haberman (2006) built a model with stochastic mortality. Among the most recent extensions, Ballotta et al (2006) considered the risk of insolvency of the insurance company. In effect, it is probable that the IASB will establish that the valuation of the insurance liabilities must consider the credit risk of the insurer.

The last category type distinguishes between the stochastic characteristics of the assets related to the reference portfolio. The model in section 4.4.1, as per the majority of the models developed in literature, considers non-stochastic interest rates while the fair value of the reference portfolio evolves according to the traditional geometric Brownian motion and therefore rate of return is distributed lognormally (Ballotta, 2005, instead considers a jump-diffusion process). This dynamics is acceptable if the reference portfolio is made up mainly of stock. The reference portfolios of the Italian with profit policies are however made up mostly of low risk securities (government bond or high grade corporate bond). A realistic representation must therefore introduce stochastic interest rates and the more coherent structure of the reference portfolio. This is the case of the models developed by Bernard et al (2005), Jorgensen (2001) and the model in §4.4.2.

§ 4. Fair valuation of Italian with profit policies: the model

The functioning mechanism of the Italian with profit policies is particularly complex. Following is outlined a revaluation method that can considered closely similar to the one currently used for Italian with profit policies.

§ 4.1 The policyholder benefit

We consider a contract that expires at time $t=T$. L_0 indicates the benefit accrued in time $t=0$ (the valuation time). The capital accrued in time t ($\boldsymbol{L_t}$) is equal to the capital accrued in time $t-1$ (L_{t-1}) revaluated at a rate equal to the greater value between the minimum return guaranteed (rm) and the product between the rate of return of the reference portfolio ($\boldsymbol{rgs_t}$) and the participation rate[1], δ: :

$$\boldsymbol{L_t} = L_{t-1} \cdot \left[1 + \max\left(rm; \delta \cdot \boldsymbol{rgs_t}\right)\right] \qquad t = 1, \ldots, T \qquad [1]$$

Note that L_0 can represent the accrued value of a previously underwritten policy or the initial value of a policy subscribed in time $t=0$. The benefit is always paid at maturity date $t=T$ because the insurance elements of the problems (essentially mortality and surrender risks) are ignored.

Observe also that, as usually occurs in Italian with profit policies, the minimum rate of return is effective during every period, and not only at maturity. The effective contractual characteristics of Italian with profit policies may however be different from [1]. In any case, as opposed to what happens in other institutional contexts, the calculation criteria of policyholder benefits at maturity date are always contractually established without any discretionary features. In this situation, an adjustment of the functional relation [1.] it is possible without changing the rest of the model. The discretionary element of the Italian with profit policies is linked to the calculation of the rate of return of the reference portfolio ($\boldsymbol{rgs_t}$), which is uniform since it is established in detail by the supervisory authority (cf. "circolare ISVAP n.71/1986") and is not market-based.

The performance of the contract depends on the performance of the reference portfolio (named "gestione separata"). The assets in the reference portfolio are

[1] Note that L_{t-1} is not stochastic for t>1 if evaluated in time $t-l$, while it is stochastic if it is applied at the time of valuation ($t=0$). Only those variables whose stochastic nature depends on the return of the reference portfolio during the period considered are indicated in bold herebelow.

evaluated in the balance sheet in accordance with IAS/IFRS and so mainly at the current market value. The market value of the reference portfolio is given at time t with $\boldsymbol{A_t}$. However, in order to compute the reference portfolio rate of return, a different valuation criteria based essentially on the historical cost is applied. The asset value that results from the application of this criterion is called the book value and is indicated as $\boldsymbol{B_t}$. The difference $\boldsymbol{A_t - B_t}$ expresses the value of the capital gains and/or losses resulting from the reference portfolio valuations and not yet realized. This difference may be either positive or negative and is hereafter referred to as unrealized value or hidden reserves.

As will be illustrated shortly, at any moment $t = 1, \ldots, T\text{-}1$, the shareholders can receive their quota of the reference portfolio return and must integrate any deficiency deriving from the existence of the policyholder put option. For this reason, the time that precedes the operations with the shareholders at time t should be indicated with t^-, and the time following these operations with t^+. In generic time t^+ the reference portfolio assets are reinvested in a risky portfolio, details of which are illustrated in section 4.4 below.

§ 4.2. The Shareholder accounts

The resources invested in the reference portfolio must be strictly correlated to policyholder benefits. In particular the book value of the assets (B^+_t) must be at least equal to the value of the accrued benefits (L_t) at time t^+.

In our model any surplus should be credited to the shareholders, while any deficiency should be debited to them. It may be considered as a dividend paid to the shareholders, or more realistically, a withdrawal or contribution made by the insurance company with respect to the reference portfolio. In actual fact, the book value of the reference portfolio assets may exceed the benefits accrued. In this case, it will be necessary to allow for the portion of reference portfolio to attribute to the policyholders, and the portion to attribute to the shareholders. We did not follow this course as it would complicate the model without providing any substantial difference of valuation.

In order to provide an independent valuation of the policyholder put option and participation rights, two distinct accounts registered in the names of the shareholders (or insurance company) will be considered. The first aims at measuring the policyholder put option and debits the shareholders with the difference of the revaluation made at time t and revaluation without option. More specifically, at each time t, we have a new debit equal to:

$$\boldsymbol{Q_t} = L_{t\text{-}1} \cdot \left[1 + \max\left(rm; \delta \cdot \boldsymbol{rgs_t}\right)\right] - L_{t\text{-}1} \cdot \left[1 + \delta \cdot \boldsymbol{rgs_t}\right] = L_{t\text{-}1} \cdot \left[\max\left(rm; \delta \cdot \boldsymbol{rgs_t}\right) - \delta \cdot \boldsymbol{rgs_t}\right] =$$
$$= L_{t\text{-}1} \cdot \left[\max\left(rm - \delta \cdot \boldsymbol{rgs_t}; 0\right)\right]$$

The remaining part of the difference between the value of the accrued capital and the reference portfolio book value, given by:

$$\boldsymbol{D_t} = \boldsymbol{B^-_t} - \boldsymbol{L_t} + \boldsymbol{Q_t}$$

is credited (if positive) and debited (if negative) to the shareholders on another account in their names. With simple algebra we can demonstrate that this totals:

$$\boldsymbol{D_t} = L_{t\text{-}1} \cdot (1\text{-}\delta) \cdot \boldsymbol{rgs_t}$$

making it possible to interpret this as the shareholder participation rights.

Bearing in mind that the funds debited or credited to the shareholders come from the reference portfolio, we obtain:

$$\boldsymbol{B}_t^+ = \boldsymbol{B}_t^- - \boldsymbol{D}_t + \boldsymbol{Q}_t$$
$$\boldsymbol{A}_t^+ = \boldsymbol{A}_t^- - \boldsymbol{D}_t + \boldsymbol{Q}_t$$

The amounts accrued in time t^+ in each of the two shareholder accounts are invested in a one-year zero-coupon bond (ZCB). This hypothesis is not strictly necessary for the valuation without stochastic interest rates, but it is particularly useful if we want to extend the model to the case of stochastic interest rates. We therefore obtain the sums accrued at time *t* in the two accounts registered in the shareholders' (or insurer' s) name as:

$$\boldsymbol{P}_t = \frac{P_{t-1}}{Z(t-1;t)} + \boldsymbol{Q}_t \qquad t = 1\ldots..T$$
$$\boldsymbol{ShPR}_t = \frac{ShPR_{t-1}}{Z(t-1,t)} + \boldsymbol{D}_t \qquad t = 1\ldots..T\text{-}1$$

with *Z(t-1,t)* as the price at time t-1 of a unit nominal value zero-coupon bond that matures at time t (residual time 1 year). It is obvious that, with non-stochastic interest rates, $Z(t\text{-}1,t)=e^{-r}$ (*r* is the instantaneous interest rate). The shareholders are also credited or debited in time T^+ with the investments remaining after the policyholders' policies have been liquidated. We therefore obtain:

$$\boldsymbol{ShPR}_T = \frac{ShPR_{T-1}}{Z(T-1,T)} + \boldsymbol{D}_T + \boldsymbol{A}_T^+ - \boldsymbol{L}_T$$

It is useful to note that the valuation of the amounts credited/debited to the shareholders has the exclusive purpose of achieving a breakdown by component of the value of the with profit policies, but it is not necessary for the valuation of the value of the contract as a whole.

Furthermore, the division into two accounts, one representing the amount covering the policyholder put option and the other the amount of the policyholder participation rights, is rather arbitrary. For example, to evaluate the put option we could compare the actual accrued benefit and the benefit that would have accrued without the put option, i.e.:

$$\boldsymbol{L}_t^* = \boldsymbol{L}_{t\text{-}1}^* \cdot \left[1 + \delta \cdot \boldsymbol{rgs}_t\right] \qquad \text{t=1, ….., T}$$

In this case, if we debited the effect of the put option period by period, we would obtain:

$$\boldsymbol{Q}_t^* = \left(\boldsymbol{L}_t - L_{t\text{-}1}\right) - \left(\boldsymbol{L}_t^* - L_{t-1}^*\right) = L_{t\text{-}1} \cdot \max\left(rm; \delta \cdot \boldsymbol{rgs}_t\right) - L_{t-1}^* \cdot \delta \cdot \boldsymbol{rgs}_t$$

A slightly different value would be obtained from this second classification of the policyholder put option and consequently a different value of the shareholders' (and policyholders') participation rights, while in no way affecting the value of the policy, whose benefits are univocally determined by equation *[1]*.

These considerations are not superfluous if we analyze the problem of the presentation of the with profit policies in the balance sheet and, in particular, the problem of unbundling, i.e. the breaking down of the various value components of a policy. In light of what has been said, it is clear that the unbundling of the various value components of a with profit policy is not so simple.

§ 4.3. The reference portfolio rate of return

The element that characterises the model is given by the calculation method of the reference portfolio rate of return. This is equal to the ratio between the reference portfolio's financial net income and the average book value of the period. Observe that the financial net income does not consider any capital gains or losses not yet realized.

In turn, the financial net income depends upon the level of the market interest rates, the quota of previously unrealized capital gains or losses that transform automatically into greater or lesser financial income with respect to the current interest rate (because, for example, the portfolio may contain high-yield bonds) and to the previously unrealized capital gains or losses that that company decides to negotiate (cf. Floreani and Rigamonti, 1999). In this work, the reference portfolio rate of return is:

$$\boldsymbol{rgs_t} = i_t + \gamma_t \cdot \frac{\left[\boldsymbol{A_t^-} - (1+i_t)\cdot B_{t-1}^+\right]}{B_{t-1}^+} \qquad [2]$$

Finally, we define the dynamics of the reference portfolio book value which is in line with the previously calculated reference portfolio rate of return:

$$\boldsymbol{B_t^-} = (1+\boldsymbol{rgs_t})\cdot B_{t-1}^+$$

Parameter γ_t of [2] may be interpreted as the portion of the hidden reserves realized during the period. It is influenced by the reference portfolio manager and this qualifies Italian with profit policies as (financial or) insurance contracts with discretionary participating features.

The precise significance of parameter γ_t depends upon how parameter i_t is defined.In an application context, it could be useful to define i_t as the rate of return of the reference portfolio in the case of a buy and hold investment strategy. In this case, γ_t would be completely discretionary, since it results exclusively from realized capital gains or losses. However, this representation requires a precise definition of the characteristics of the investments in the reference portfolio: if we consider stock portfolios, i_t depends on the dividend yield calculated on the initial book value. If we consider bond portfolios, i_t depends on many parameters (e.g. the interest rates, the maturity dates, the rates of return, duration and convexity). In this case, in the presence of decreasing interest rates, i_t will be higher than the current rates, while in the presence of increasing interest rates i_t will be lower than the current rates.

To avoid complicating the model, in the examples i_t has been expressed at the current market rate referable to the period between *t-1* and *t*. In the model with stock investments and non-stochastic interest rates we therefore have $i_t = i = e^r - 1$, with *r* equal to the instantaneous interest rate. In the model with bond investments and stochastic interest rates, i_t varies in time ($i_t = 1/Z(t-1;t) - 1$, with $Z(t-1;t)$ the price at time *t-1* of a unit zero coupon bond that matures in time *t*). In both cases γ_t is partially determined by the spread between the coupon interest rate (or the dividend yield) and the market interest rate, and so it is not totally manoeuvrable by the manager.

The realized proportion of the hidden reserves depends upon the manager's strategy. In order to be able to determine the fair value of the contract, this strategy must be specified at the time of valuation (*t=0*). More precisely, to determine the fair value, we should not refer to the manager's specific strategy (entity-based estimate), but to the hypothetic "market" behaviour (market-based estimate). Two possible strategies have been considered in the examples:

- Maintenance of the realized proportion of the hidden reserves constant in time ($\gamma_t=\gamma$ for each t).
- Adjustment of the realized proportion of the hidden reserves in order to stabilise the rate of return on reference portfolio on a target level r*.

More precisely, if we indicate the target reference portfolio rate of return as r*, with $[\gamma_{min} ; \gamma_{max}]$, the discretionary interval of the realized proportion of the hidden reserves in order to achieve stabilization, and with:

$$\boldsymbol{rgs}_t(\gamma) = i_t + \gamma \cdot \frac{\left[\boldsymbol{A}_t^- - (1+i_t)\cdot B_{t-1}^+\right]}{B_{t-1}^+}$$

the reference portfolio return as a function of the realized proportion of the hidden reserved, $rgs_t(\gamma_{min})$ e $rgs_t(\gamma_{max})$ are the returns of the reference portfolio at the extreme points of the discretion interval.

Bearing in mind that function $rgs_t(\gamma)$ is monotone, we can represent the minimum between the two extreme rate of return as rgs^{min}_t and the maximum as rgs^{max}_t as follows:

$$rgs^{min}_t = MIN\ [rgs_t(\gamma_{min}),\ rgs_t(\gamma_{max})]$$
$$rgs^{max}_t = MAX\ [rgs_t(\gamma_{min}),\ rgs_t(\gamma_{max})]$$

In this case, the target return may be achieved if it lies within the discretionary interval $[rgs^{min}_t\ ,\ rgs^{max}_t]$. In this case, the realized proportion of the hidden reserves issue will be:

$$\gamma^* = (r^* - i_t)\cdot \frac{B_{t-1}^+}{\left[\boldsymbol{A}_t^- - (1+i_t)\cdot B_{t-1}^+\right]}$$

If on the other hand, the target return does not lie within the discretionary interval, the realized proportion will be placed at one of the two extremes. In brief, the reference portfolio return may be expressed at time *t* as follows:

$$\boldsymbol{rgs_t} = MAX\ [\ r^*\ ;\ \boldsymbol{rgs^{min}_t}\ ]\ +\ MIN\ [\ r^*\ ;\ \boldsymbol{rgs^{max}_t}]\ -\ r^*$$

§ 4.4. Dynamics of the interest rates, composition of the reference portfolio and resolution of the model

§ 4.4.1. Non-stochastic interest rates and stock investments

In the case of non-stochastic interest rates and reference portfolios made up of stock investments, the model is considerably simple. In this case, we can say that the market prices of the reference portfolio follow a geometric Brownian motion, i.e. they have the following risk neutral dynamics:

$$dA_t = r \cdot A_t \cdot dt + \sigma \cdot A_t \cdot dW_t$$

where r represents the instantaneous risk free interest rate and σ is the volatility of the assets that constitute the reference portfolio. In this situation, the reference portfolio values are distributed lognormally and the risk neutral distribution is:

$$\ln\left(\frac{\boldsymbol{A_t^-}}{A_{t-1}^+}\right) \sim \aleph\left[r - 0,5 \cdot \sigma^2 ; \sigma\right]$$

i.e. the log rate of return within each period have a normal risk neutral distribution with an expected value of $r - 0,5\ \sigma^2$ and a standard deviation of σ. In this situation, the composition of the reference portfolio investments influences the valuation of the with profit policies exclusively via parameter σ.

In the presence of a lognormal distribution of asset prices and non-stochastic interest rates, the fair value of any contingent claim on assets can be determined by calculating the present value at the risk-free rate of the expected payoff using the risk neutral probabilities. If $\mathbf{C_t}$ is a generic stochastic payoff of a contingent claim, we have:

$$V(\boldsymbol{C_t}) = E\left(\boldsymbol{C_t}\right) \cdot e^{-r \cdot t}$$

where *V(.)* is the fair value operator and *E(.)* is the expected value operator.

In some circumstances this value can be determined analytically. This is true when, for example, the policyholder put option is a European plain vanilla put option. In this case we would obtain Black and Scholes' classic formulae. In our model, the only component whose fair value can be determined analytically is the guaranteed component:

$$V(G) = V\left[L_0 \cdot (1 + rm)^T\right] = L_0 \cdot (1 + rm)^T \cdot e^{-r \cdot T}$$

It is not possible to identify analytical solutions for the other value components. In particular, the two key quantities for the valuation of the contract are the policyholder put option ***V(P)***, and the value of shareholders' participation rights, ***V(ShPR)***:

$$V(\boldsymbol{P}) = V(\boldsymbol{P_T}) = E(\boldsymbol{P_T}) \cdot e^{-r \cdot t} \qquad [3]$$

$$V(\boldsymbol{ShPR}) = V(\boldsymbol{ShPR_T}) = E(\boldsymbol{ShPR_T}) \cdot e^{-r \cdot t} \qquad [4]$$

The two expected values may be estimated numerically via the Monte Carlo technique. The values of the policyholder put option and the shareholders' participation rights are sufficient for determining the other values of the model. In particular, if we consider that the total value of the participation rights is:

$$V(PR) = A_0 - V(G)$$

the value of the policyholders' participation rights is:

$$V(\boldsymbol{PhPR}) = V(PR) - V(\boldsymbol{ShPR}) = A_0 - V(G) - V(\boldsymbol{ShPR})$$

Finally, the total fair value of the contract is:

$$V(\boldsymbol{L_T}) = V(G) + V(\boldsymbol{PhPR}) + V(\boldsymbol{P}) = A_0 - V(\boldsymbol{ShPR}) + V(\boldsymbol{P}) \qquad [5]$$

while the value due to the shareholders (equity) is:

$$V(\boldsymbol{Sh}) = V(\boldsymbol{ShPR}) - V(\boldsymbol{P})$$

The liabilities value can also be estimated directly, i.e. evaluating via the Monte Carlo simulation the quantity:

$$V\left(\boldsymbol{L_T}\right) = L_0 \cdot E\left(\prod_{t=1}^{T}\left\{1 + \max\left[rm, \delta \cdot \boldsymbol{rgs_t}\right]\right\}\right) \cdot e^{-r \cdot T} \qquad [6]$$

We thus have a quantity that can be used to evaluate the reliability of the simulation. In particular, after having determined directly V($\mathbf{P_T}$), V($\mathbf{ShPR_T}$) e V($\mathbf{L_T}$) via [3], [4] e [6], and allowing for [5], it is possible to measure the simulation error by calculating the following indicator that for exact solutions is equal to zero:

$$\varepsilon = \frac{V(\boldsymbol{L_T}) - V(\boldsymbol{P_T}) + V(\boldsymbol{ShPR_T}) - A_0}{A_0}$$

The results of each of the numerical examples of the model (cf. §5) have been obtained via 10,000 simulated scenarios using the variance reduction technique of the antithetical variable. The values thus obtained have been deemed acceptable, considering that the absolute value of the simulation error is always lower than 0.1%. A simple method for reducing it even further without increasing the number of simulations is to proportionally distribute the simulation error between the various estimated components. The low number of simulated paths has made it possible to use a single worksheet for the entire Monte Carlo simulation and to limit computational time.

§ 4.4.2. Stochastic interest rates and bond investments

The model with a bond portfolio is a little more complex and requires calculations with stochastic interest rates.

Let us assume that the reference portfolio is made up exclusively of ZCBs (*zero-coupon-bond*) and that at each time t^+ all the reference portfolio resources are invested in ZCBs with time to maturity *D* and held until time $(t+1)^-$, when it will be possible to compute the performance for the period on the basis of the new interest rate structure. Dealings with shareholders are made at time *(t+1)* and at time $(t+1)^+$ the available resources are again reinvested in ZCBs with duration *D*[2]. Finally, all dividends credited or debited to the shareholders at time *t* are invested in annual ZCBs.

As a conseguence of this simple characterization, the investment strategy is univocally defined by the parameter *D*, the duration of the reference portfolio. Considering that the volatility of the bond prices is an increasing function of the duration, parameter *D* is at first assimilated to volatility σ.

In order to obtain plausible and coherent dynamics of the bonds prices, we have considered the following Q-risk neutral dynamic of the instantaneous interest rate:

$$dr_t = a \cdot [b - r_t] \cdot dt + s \cdot \sqrt{t} \cdot dW_t \qquad [7]$$

with r_0, *a*, *b*, *s* positive constants. Dynamic [7] was originally introduced by Cox, Ingersoll and Ross (1985) and the consequent valuation model is known as the CIR. Parameter *a* expresses the adjustment speed of the instantaneous interest rates towards long period value *b*. The stochastic term has a standard deviation proportional with the square root of the time. And, r_0 is the initial value of the instantaneous interest rate. The dynamic [7] has been chosen for its analytical tractability (e.g. the prices of bonds can be derived analytically, as can the prices of the plain vanilla options on zero coupon bonds). Furthermore, with respect to the more simple representations (e.g. with respect to Vasicek's model, 1977), this method does not allow for negative interest rates. Finally, the CIR examined herein that maintains parameters *a*, *b* and *s* fixed, may be generalized without too much difficulty in order to enable calibration with the market interest rate structure (Cf. Brigo and Mercurio, 2001, §§3.4 and 3.9).

As mentioned, it is possible to analytically determine the price of the risk-free zero coupon bonds. More precisely, if we let *Z(t,T)* the price at time t of a ZCB that pays €1 at maturity *T*, we have:

$$Z(t,T) = A(t,T) \cdot e^{-B(t,T) \cdot r_t}$$

$$A(t,T) = \left[\frac{2 \cdot h \cdot e^{0,5 \cdot (a+h) \cdot (T-t)}}{2 \cdot h + (a+h) \cdot (e^{(T-t) \cdot h} - 1)} \right]^{\frac{2 \cdot a \cdot b}{s^2}}$$

$$B(t,T) = \frac{2 \cdot (e^{h \cdot (T-t)} - 1)}{2 \cdot h + (a+h) \cdot (e^{(T-t) \cdot h} - 1)}$$

[2] Note that in order to effectively adopt such an investment strategy it would be necessary to liquidate the entire portfolio at each time *t* and this would determine the realized proportion of the hidden reserves as equal to 100%. More realistically, we could consider the existence of a compound bond portfolio whose duration is kept constantly between *D* and *D-1*.

$$h = \sqrt{a^2 + 2 \cdot s^2}$$

The model has been built in such a way as to report all the significant stochastic flows (shareholder dividends and policyholder benefits) at maturity time *T* of the policy. In this case, the most simple valuation approach is the adoption of the change of numeraire technique (Cf. Brigo and Mercurio, 2001, cap. 2). In this specific case, we need to determine the *T-forward measure*, Q^T. If we indicate a generic stochastic payoff at time *T* with $\boldsymbol{H_T}$, its value at time 0 may be determined as follows:

$$V(\boldsymbol{H_T}) = Z(0,T)\, E^T(\boldsymbol{H_T})$$

where *Z(0,T)* is the current ZCB price at 0 with maturity at time *T* and $E^T(\boldsymbol{H_T})$, is the expected value of $\boldsymbol{H_T}$ calculated on the basis of the *T-forward measure* Q^T. It can be demonstrated that the dynamics of the interest rates for the CIR model according to the *T-forward measure* Q^T is (cf. Brigo and Mercurio, 2001, p. 59):

$$dr_t = [a \cdot b - (k + B(t,T) \cdot s^2) \cdot r_t] \cdot dt + s \cdot \sqrt{t} \cdot dW_t^T$$

where

$$dW_t^T = dW_t + B(t,T) \cdot s \cdot \sqrt{r_t} \cdot dt$$

The only other calculation problem with respect to the section 4.4.1 model refers to the distribution of interest rates in the *T-forward measure* Q^T. It can be demonstrated that the distribution of the rates within this context is a non-central chi square. As opposed to the lognormal distribution, the non-central chi squared distribution is not tabulated in the electronic spreadsheets used for the numerical solution of the model. Consequently the series of interest rates according to the *T-forward measure* Q^T has been generated using the following Milstein scheme (cf. Brigo e Mercurio, p. 105):

$$r_{t+\Delta t} = r_t + \left[a \cdot b - (a + B(t,T) \cdot s^2) \cdot r_t \right] \cdot \Delta t - 0,25 \cdot s^2 \cdot \left[(W_{t+\Delta t}^T - W_t^T)^2 - \Delta t \right] + s \cdot \sqrt{r_t} \cdot (W_{t+\Delta t}^T - W_t^T)$$

T=10 years was set for all the simulations and the Milstein scheme was implemented considering the time intervals *Δt = 1/6*, i.e. 2 months time range, thereby generating a historical series of 60 interest rates for each simulated scenarios. Because of the computational effort deriving from the application of the Milstein scheme, the number of scenarios generated for each simulation is reduced (5,000 instead of 10,000). In spite of this, the simulation error was inferior with respect to that of the stock portfolio simulation.

§ 5. Results

A systematic sensitivity analysis was made in the extended version of this work (Floreani, 2006) to investigate the reaction of the various components of the fair value of the Italian with profit policies to the numerous parameters of the model. Some relations are so evident that it is not worth reporting the results (e.g. the relation between the policyholder put option value and volatility σ in the case of a reference

portfolio comprising only stock investments). This work reports the results of the simulations deemed to be of greater significance.

To this end, we have considered the following specifications for the stock model.

- Initial fair value of the assets invested in the reference portfolio: A_0= 1.000
- Absence of initial hidden reserves: $L_0 = B_0 = 1.000$
- Instantaneous risk-free interest rate: $r=r_0$ = 4%
- Asset volatility: σ = 8%;
- Maturity of with profit policy: T=10 years
- Minimum guaranteed rate of return: rm= 2% per year
- Participation quota: δ = 85%
- Portion of hidden reserves realized in the period $\gamma_t = \gamma$ =25% p.a.

The basic situation should be considered quite realistic except for the volatility parameter that seems considerably high. The high volatility of the basic situation may be in part compensated by the non-excessive maturity of the policy under valuation. A high volatility has been maintained to prevent the value of policyholder put option from becoming completely negligible, as will become clear from the results.

§ 5.1. Policyholder put option value

A first and not surprising result is the fact that in the basic situation the put option value is rather low (3.8% of A_0) and negligible (0.2% of A_0) for more plausible volatility values (σ=3%); (cf. Table 2).

TABLE #2

By increasing the portion of the hidden reserves realized (γ), the value of the policyholder put option increases. A realized proportion of the hidden reserves of 100% gives a situation where the return of the reference portfolio is market value based. In this case, the put option value is extremely high (21.9% of assets against 3.8% in case of γ=25%) (cf. Figure 1).

FIGURA #1

The above results show that the stabilizing effect of the computational method of the reference portfolio rate of return is very high. This effect reduces the volatility of the reference portfolio returns and so reduces the value of the put option.

This stabilizing effect is further strengthened when we consider a portfolio of bond investments and an objective of stabilization of the reference portfolio returns.

In the presence of stochastic interest rates and with a plausible representation of the CIR model (a=0.08, b=0.04 and s=0.06), a reference portfolio duration of 18 (!) years is necessary to achieve a fair value compared to that of the basic situation shown in Table 2, while for more realistic durations the value of the put option tends to reduce considerably (cf. Figure 2). We can also see that the put option value is not an increasing monotone function of the duration. In effect, when the duration is “too low”, a decrease in market interest rates is rapidly reflected in the reference portfolio rate of return and, if the market rates fall below the minimum guaranteed, the guarantee ends *in-the-money*. With longer durations, on the other hand, a decrease in interest rates below the minimum guaranteed gives rise to capital gains that, if realized progressively,

make it possible to maintain the reference portfolio returns above the minimum guaranteed for a long period of time, even in cases of persistent low interest rates and so prevents the exercise of the option. If however, the duration is too high, the put option value tends to increase since the greater persistence in time of low interest rates is more than compensated by the greater fragility derived from the possible losses originated from an increase in interest rates.

FIGURE #2

If we could freely and jointly fix the realized proportion of the hidden reserves and the duration of the portfolio, the minimum put option value is reached via low realized proportions of hidden reserves and high duration values of the portfolio (cf. Figure 3 which shows the isocurves of the put option value).

This result is not insignificant and is due to the peculiar calculation method of the reference portfolio return (we have already seen that γ is not totally manoeuvrable by the management and, in particular, there is a value γ_{min} under which we cannot go. In addition, the manoeuvrability of γ increases – i.e. γ_{min} decreases – with the increase of the duration of the portfolio).

FIGURE #3

Finally, the put option value becomes completely negligible when we have the possibility of manoeuvring γ in function of an explicit policy for the stabilization of the reference portfolio returns (cf. Table 3).

TABLE #3

§ 5.2. Value of the policyholder participation rights

The model developed in this work breaks the policy's fair value down into its elementary components. We have so far concentrated on the parameters that significantly influence the put option value. We will now expound the significant relations that also involve the policyholder participation rights value.

Of special interest is the analysis of the trend of the various value components with unrealized values (cf. Figure 4). More precisely, the ratio B_0/A_0 is indicated on the x-axis. Toward the left of the graph we have values below 1. This means that the current assets value exceeds the book value and we have unrealized capital gains. Toward the right of the figure we have high ratio values and so unrealized capital losses. To better understand the figure we have also reported the value of accrued liabilities in time 0 L_0, that coincides with book value B_0 (straight brown line), while the current value of the assets is always 1,000.

FIGURE #4

The value of the guaranteed component increases in a linear manner in function of L_0. On the other hand, the value of the policyholders' participation rights is very high for the low levels of the B_0/A_0 ratio, and progressively reduce and become negative when the B_0/A_0 ratio is too high. In short, when there are many unrealized capital gains,

the value of the policyholders' participation rights is high and causes the fair value of the policy to rise beyond the initial accrued value (L_0).

This situation is due to the fact that past positive market-based performances have not yet had any effect on the reference portfolio book-based rate of return, and so on the capital accrued at time t=0. This does not seem to be cause for concern for the insurance companies. In effect, even the shareholders' participation rights are high and the put option value is low, since the hidden reserves guarantee higher returns for the coming period. However, as observed by Bauer et al (2006) for German with profit policies, this situation should be considered with caution when new policies at price L_0 are issued. In this case, the policy would be underpriced and the difference between the fair value and the underwriting price would erode the participation rights of the shareholders and of the pre-existing policyholders.

If we look toward the right of the graph, the valuation gains give way to the valuation losses that will be progressively realized in time. There will consequently be a progressive erosion of the policyholders' participation rights that, at a certain point onwards, will become negative. However, the put option comes to the aid of the policyholders. When the losses are so high as to have significant negative effects on the reference portfolio rate of return, the put option kicks in. In such situations, the fair value of the policy remains decidedly lower with respect to the initial accrued value (L_0), but it is nevertheless higher than the current value of the assets (A_0), thus resulting in a negative equity value.

Finally, we can see (cf. Table 3) that the move from a strategy with a constant realized proportion to a strategy that aims to stabilise the reference portfolio rate of return also has significant effects on the policyholders' participation rights.

In particular, the higher the target rate of return ***r****, the more capital gains the portfolio manager is willing to realize with declining interest rates and the fewer capital losses with rising interest rates, in order to achieve target ***r****. On the contrary, when ***r**** approaches the minimum guaranteed rate of return, the company adopts protective measures to stabilize the reference portfolio rates of return at a low level. The policyholders' participation rights value therefore grows with increase of ***r****.

§ 6. The fair value of with profit policies ?

The fair value of Italian with profit policies depends considerably on the management strategy used for the negotiation of the hidden reserves. This is a distinctive characteristic of Italian with profit policies and, in part, contributes to determining their success and longevity.

In function of the financial market trends, of the business or profitability objectives, it would seem normal and legitimate that the insurance company could continue to change not only its current realized proportion of the hidden reserves in the reference portfolio, thereby deciding to realize or not the accrued capital gains or losses, but also its expected behaviour for the future, and consequently modifying the policies' fair value. This ever-changing is normal at the management level, but creates considerable problems at the valuation level, where the future strategy of the realization of hidden reserves must be fixed for valuation purposes. Since it is not possible, for valuation purposes, to request a constant maintenance of the realized proportion of hidden reserves or the criteria for its calculation, the way is opened to obvious or less obvious financial reporting policies.

Of partial comfort is to remember, that the fair value should not be determined with reference to the specific business situation of the company that makes the valuation (entity based estimate) but should use the market as reference (market based estimate). In this case therefore, we should look not at the specific strategies, but at the expected future behaviour of the market operators.

The calculation of the fair value of Italian with profit policies is therefore useful for pricing the product, for valuating profitability and for modernizing the systems for valuating performance and measuring the creation of value within the insurance industry.

The discretionary features cause however considerable problems with regard to fair value estimation for financial accounting purposes, since they open the door to window dressing, whose containments, according to the IASB, represent one of the most convincing arguments in favour of fair value. Thus a new peculiar feature of the insurance business emerges: while in general the introduction of fair value makes the financial statements more transparent and less open to window dressing, the introduction of fair value in Italian with profit policies could decrease the transparency and increase the possibility of window dressing by the insurance companies.

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Table 1. Full fair value balance sheet of a with profit life insurance policy

Assets		**Equity and liabilities**	
Assets	*V(**A**)*	*Liabilities*	
		Guaranteed benefit	*V(G)*
		Policyholder Put Option (holder)	*V(**P**)*
		Participation Rights	
		Policyholder Participation Rights	*V(**PhPR**)*
		Shareholders Participation Rights	*V(**ShPR**)*
		Equity	
		Policyholder Put Option (writer)	*-V(**P**)*
Total Assets	*V(**A**)*	Total Liabilities	*V(**A**)*

Table 2. The fair value of Italian with profit life insurance policies with stock investments and non-stochastic interest rates

(A_0 = 1,000; B_0 = 1,000; L_0 = 1,000; r=4%; rm=2%; δ=85%; γ=25%; T=10)

Assets		**Liabilities and equity**		
			σ=8%	*σ=3%*
Assets	1,000	*Liabilities (Policyholder rights)*	980	945
		Guaranteed benefit	817	817
		Policyholder Participation Rights	125	126
		Policyholder Put Option (holder)	38	2
		Equity (Shareholders' Rights)	20	55
		Shareholders Participation Rights	58	57
		Policyholder Put Option (writer)	- 38	-2
	1,000		1,000	1,000

Figure 1. The fair value of Italian with profit policies in function of the portion of hidden reserves realized
(stock investments and non-stochastic interest rates)
(A_0 = 1,000; B_0 = 1,000; L_0 = 1,000; r=4%; rm=2%; σ=8%; δ=85%; T=10)

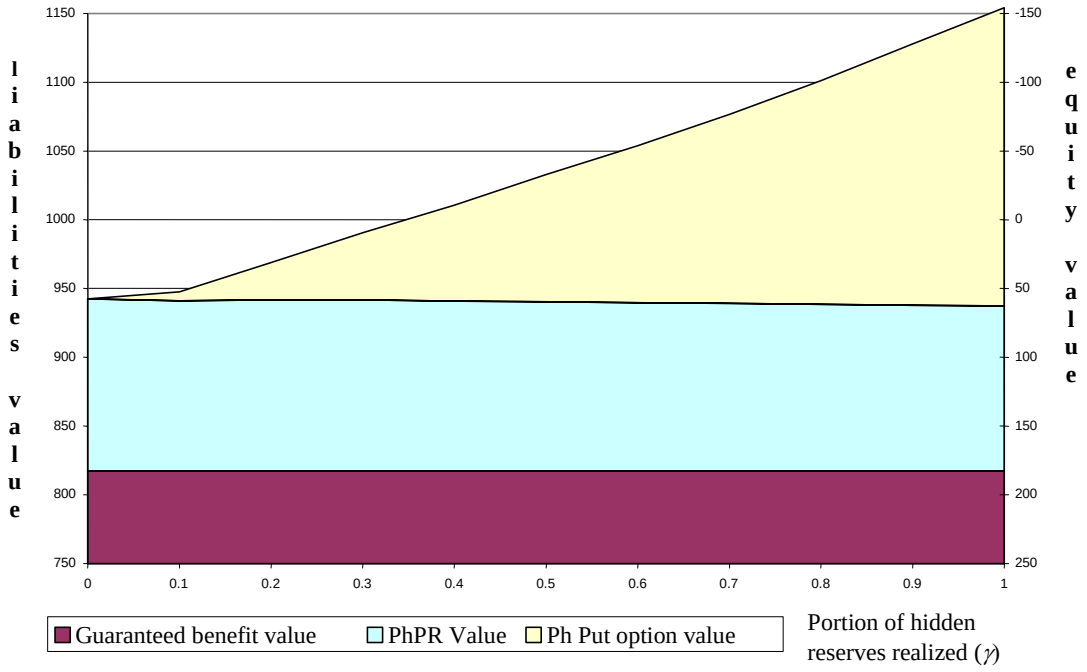

Figure 2. The fair value of Italian with profit policies in function of the duration of the reference portfolio investments (stochastic interest rates and bond investments)

($A_0=B_0=L_0=1{,}000$; $r_0=4\%$; $rm=2\%$; $\delta=85\%$; $\gamma=25\%$; $T=10$; $a=0.08$; $b=0.04$; $s=0.06$)

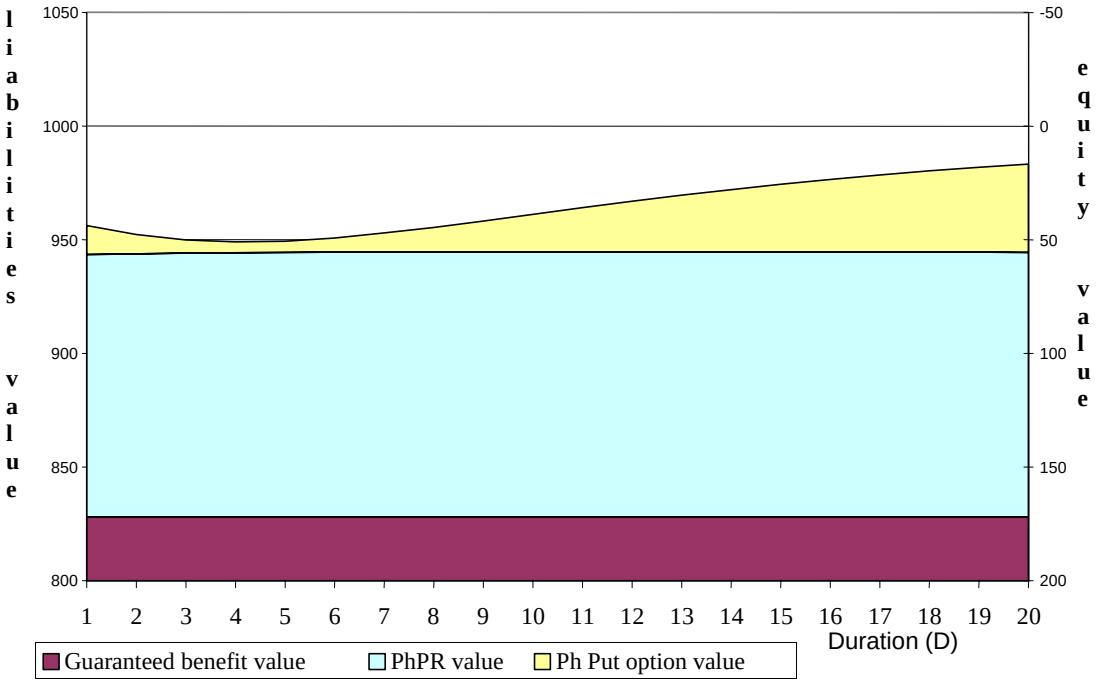

Figure 3. Isocurve of the policyholder put option value. (stochastic interest rates and securities investments)

($A_0=B_0=L_0=1{,}000$; $r_0=4\%$; $rm=2\%$; $\delta=85\%$; $T=10$; $a=0.08$; $b=0.04$; $s=0.06$)

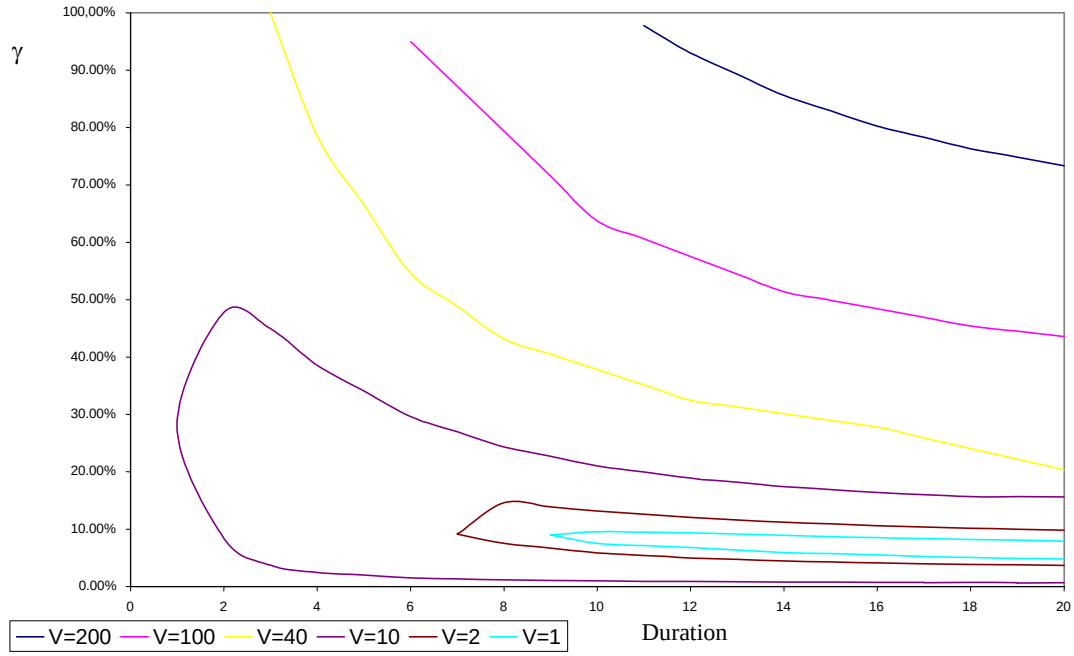

Table 3. The fair value of Italian with profit life policies in the presence of a target rate of return r* (Stochastic interest rates and bond investments)

(A_0 = 1,000; B_0 = 1,000; L_0 = 1,000; r_0=4%; rm=2%; δ=85%; T=10; a=0.08; b=0.04; σ=0.06; D=18)

	Basic case (Table 4)	**r*=7%**	**r*=5,5%**	**r*=4%**	**r*=2,5%**
Liabilities (policyholder rights)					
Guaranteed benefit	828	828	828	828	828
Policyholder participation rights	117	181	158	118	88
Policyholder put option (holder)	36	8	4	2	1
Total policyholder rights V(L)	981	1.017	990	948	917
Equity (shareholders rights)					
Shareholders' participation rights	55	-9	14	54	84
Policyholder put option (writer)	-36	-8	-4	-2	-1
Total shareholders' rights	19	-17	10	52	83
Total Liabilities	1,000	1,000	1,000	1,000	1,000

Figure 4. The fair value of Italian with profit policies in function of the ratio between the book and the market value of the reference portfolio at time t=0 (stock investments and non-stochastic interest rates)

(A_0 = 1,000; r=4%; rm=2%; σ=8%; δ=85%; γ=25% ; T=10)