On the regulator-insurer-interaction in a structural model

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Abstract: In this paper we provide a new insight of the previous work of Grosen and Jørgensen [2002]. More precisely, *firstly*, we investigate the impact of regulatory authorities' rules on the fair value of company's liabilities and assets. We study how to choose regulation intervention levels in order to control the shortfall probability of the issuing company. The fact that the regulation rule depends on the assets' volatility implies that "fixed volatility rule" becomes completely useless whenever the insurance company follows a dynamic investment strategy. Therefore, *secondly*, we study the interaction between the regulator who determines the regulation rule and the insurance company's risk management. Following the recent work of Ballotta, Haberman and Wang [2005] and the guidelines of the IASB, we develop an analysis of the model error when the insurance company is informed of the regulation rules and trades according to a certain discrete risk management hedging strategy instead of staying passive until the contract's maturity.

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Abstract: In this paper we provide a new insight of the previous work of Grosen and Jørgensen [2002]. More precisely, *firstly*, we investigate the impact of regulatory authorities' rules on the fair value of company's liabilities and assets. We study how to choose regulation intervention levels in order to control the shortfall probability of the issuing company. The fact that the regulation rule depends on the assets' volatility implies that "fixed volatility rule" becomes completely useless whenever the insurance company follows a dynamic investment strategy. Therefore, *secondly*, we study the interaction between the regulator who determines the regulation rule and the insurance company's risk management. Following the recent work of Ballotta, Haberman and Wang [2005] and the guidelines of the IASB, we develop an analysis of the model error when the insurance company is informed of the regulation rules and trades according to a certain discrete risk management hedging strategy instead of staying passive until the contract's maturity.

Introduction

In order to increase their competitiveness and to acquire as many customers as possible, new products with more and more complicated features are issued by life insurance companies (see the book of Hardy [2003] for a wide review of these products), whilst the economic context is difficult (longevity risk, changes in interest rates, hard competition between insurance companies). Nowadays the financial risks involved in life insurance products are a real challenge for actuaries. The ongoing issues are how to design and to apply the new accounting rules and solvency requirements, and capital adequacy to price the issued contracts and hedge the financial and mortality risk involved in these products. More details can be found in the recent work of Jørgensen [2004]. The impact of the methods is also described on a concrete example of a participating contract with a minimum guarantee by Ballotta, Esposito and Haberman [2006].

The IASB (International Accounting Standards Board) in Europe and the FASB (Financial Accounting Standards Board) in the United States have been working on new accounting standards. The "Fair Value" principle is more and more adopted. According to the FASB (and similar to the definition proposed by the IASB), the "fair value is the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date". Academicians are already working on the fair value of policies for more than thirty years. Indeed, let us recall the fundamental work of Merton [1974] on the valuation of the debt as an option written on the firm's assets. Indeed he modeled risky debt as riskless debt with a short position on a put option and equity as a call option on the firm assets. His closed-form formulae are directly obtained from the well known framework developed by Black-Scholes-Merton [1973]. Brennan and Schwartz [1976] and Boyle and Schwartz [1977] then study the fair valuation of unit-linked policies. They are at the origin of numerous works tackling with the present value of insurance policies. In more recent years this subject has been extended by Briys and de Varenne [1994, 1997a, 2001], Nielsen and Sandmann [1995], Bacinello [2001], Grosen and Jørgensen [2000, 2002], Miltersen and Persson [2003], Tanskanen and Lukkarinen [2003], Ballotta [2005], to quote only a few. In addition, a new valuation method, i.e. so called market-consistent valuation has attracted a lot of attention recently in order to aim

at valuing assets and liabilities in a consistent way. Models taking account of different financial risks and using new market tools are advocated and the methods need to be updated (see for example Bühlmann [2002, 2004]). I.e., this new valuation method intends to work on criticism like the absence of uncertainty, deterministic models and the stationary feature of the market. Nowadays lots of European countries including the UK, the Netherlands, the Switzerland, have already introduced accounting rules based on a market valuation of assets and liabilities. A detailed discussion can be found e.g. in Sheldon and Smith [2004].

With the increasing number of the insurance companies since the 1980s, the issue of solvency requirements is more and more concerned. Many studies (see for example Ballotta, Haberman and Wang [2005]) are dealing with the interpretation and the valuation of the possible default of life insurance companies. First the default at maturity has been modeled in insurance by Briys and de Varenne [1994, 1997a, 2001] (like Merton [1974, 1989]) did in corporate finance). The insurance literature on the default modeling has indeed followed step by step the finance works. The Merton's approach has been then widely extended in finance by Black and Cox [1976] who consider that ruin is possible at any instant. Stochastic interest rates have been then introduced in the previous models by Briys and de Varenne [1997b], Longstaff and Schwartz [1995] and Collin-Dufresne and Goldstein [2001] who provide a correction of the latter work. Modeling the default of insurance companies in a Black and Cox framework has been first done by Grosen and Jørgensen [2000, 2002], and then extended by Bernard, Le Courtois and Quittard-Pinon [2005b, 2006b]. Finally the recent works of Bernard et al. [2005a] and Chen and Suchanecki [2007] show how to apply the realistic procedure Chapter 11 of the US Bankruptcy code in the insurance field and directly extend the above literature.[1] The former one deals with the insurance of bank deposits, and the latter one with the modeling of default of a life insurance company issuing participating policies. These last contributions also follow some main financial studies, amongst others, Morellec [2001], François and Morellec [2004], Moraux [2004], Broadie, Chernov and Sundaresan [2004], Galai, Raviv and Wiener [2003]. As far as we know, the model of Leland [1998] is not yet interpreted for an insurance company, and all the above works assume that default is triggered by crossing a barrier which is exogenously given.

The default risk contained in the life insurance contracts lead to a consequence that customers of these contracts might even not get back their initial investment upon default/liquidation of the issuing insurance company. Therefore, in addition to the initial price of the contracts, the customers might be more concerned with the issue like with what probability the insurance company will become bankrupt and which amount they can expect to obtain after taking account of the default risk of the insurer. Whilst the insurance company tries to maximize returns for its equity holder, the regulatory authorities are responsible for protecting the policyholders. The regulators aim at avoiding subsidies that could distort the insurance system, therefore, they have supervised life insurance companies very strongly in order to avoid bankruptcy.

In this study we first focus on the impact of regulatory authorities rules on assets' and liabilities' market values and the shortfall probability of the issuing company. We set ourselves in the classical frameworks proposed in the literature on model bankruptcy and aim at interpreting optimal levels of regulators' intervention for a given shortfall probability constraint. We show how

[1] US bankruptcy procedure distinguishes between Chapter 7 and Chapter 11 bankruptcy code. According to Chapter 7 bankruptcy code, default and liquidation date coincide. While Chapter 11 bankruptcy code describes a more realistic procedure, i.e., default and liquidation are distinguishable events. Similar bankruptcy procedures can be found also in France, Germany and Japan etc.

to set the level of the barrier taking into account the risk, either in the volatility of the assets, in the debt ratio or in the default probability. From the first part of the analysis, we are very aware of the fact that there exists an interaction of the regulation rule and the volatility of the assets. This implies that "fixed volatility rule" becomes completely useless whenever the insurance company follows a dynamic investment strategy. Therefore, the second focus of this paper is on the interaction of the regulator's regulation rule and the insurer's risk management strategy. I.e. following the recent work of Ballotta, Haberman and Wang [2005] and the guidelines of the IASB, we develop an analysis of the model error when the insurance company is informed of the regulation rules and trades according to a certain discrete risk management hedging strategy instead of staying passive until the contract's maturity. By getting rid of this "static feature" and introducing some simple strategies, noticeable effects of this dynamic hedging are observed.

In the present work, we ignore lapses and any feature linked to the mortality risk such as early death benefit and focus on the financial risk. The solvency requirements are also based on financial considerations and aspects. By assuming independence between mortality and financial market risk, the model can easily be enhanced to take account of the mortality risk. This work is very general because the participating contracts studied here are only an example. The study could of course be extended to the equity-linked structured products sold by banks or the general equity-indexed annuities marketed by most of American and Canadian insurance companies.

The first section is devoted to the formulation of the problem in the context of solvency requirements. The following one section demonstrates two different ways to model default risk and calculate default probability in each case in a static framework. We then extend it to a dynamic framework in which the company is allowed to have a strategy linked to the regulatory rules. The last section discusses the results and adds some comments following the guidelines of the IASB.

1 Problem

We study the regulation of a company that would sell only one type of contract. These products ensure their holders a minimum guaranteed amount at maturity and a participation in the benefits of the company, if any. They are thus equity-linked since they are indexed on the assets of the issuing company.

1.1 The solvency issue

There are two viewpoints we will consider in the following of this paper.

- From the regulators' viewpoint. The regulators want to determine the optimal level of intervention in order to protect the policyholders. They could look at the default's probability for example or at the present value of the policyholders in case of default, and will choose the level in order to keep some fixed limit, e.g. to have a probability of default less than 0.5%.

- From the company's viewpoint. Assume regulators inform the company of their rule. How does the company modify its activity in order to have a probability of intervention of the regulators as low as possible? The company may find an optimal volatility level, i.e. may reduce its risk in order to minimize the probability of a regulatory's intervention but also

wants to maximize its value (due to its shareholders). It can also choose its volatility but ask for more capital in order to keep the probability of an intervention low.

We will examine two different situations. As a starting point, we consider the underlying contract presented in Grosen and Jørgensen [2002]. Monitoring is performed in continuous time: as soon as the level of the assets is not sufficient to fulfill its commitments the regulator liquidates the company. The continuous surveillance makes sense because a company has to be solvent at any time. It would not be interesting to consider only the Merton's case where solvency is required at maturity only. One reason could also be that most of the time such policies include some surrender options, meaning that the policyholder can claim at any time for his investment. The company should then be able to give back the promised amount. Then, we will have a brief view at how the model proposed by Chen *et al.* [2007] where the company is not immediately liquidated when its assets hit the fixed level but has a given period of time to recover before its liquidation by the regulators. The level should be higher than in the first section because hitting this level does not involve immediate liquidation.

The main drawback of these two models is the "static feature". Whatever are the regulatory constraints, the evolution of its assets level, the insurer does nothing. I.e., it is assumed that a constant fixed volatility is used by the insurance company. However, whenever the insurance company follows a dynamic investment strategy, such a "fixed volatility rule" becomes completely useless. Once and for all we cannot imagine that a static model to study a 10-year-term contract will give any realistic results. With this purpose, we introduce some simple dynamic strategies and see how for example it noticeably increases the final wealth of the policyholder for a given shortfall probability.

1.2 The valuation principle

We examine in our study a policy that provides a minimum guarantee and a participation clause (that could be a with-profit contract as in Bacinello [2001] or Ballotta [2005] among others, a French participating contract as in Briys et al. [1994], Equity-linked products...). The payoff can often be decomposed into three parts:

$$ZCBond + Call - Put,$$

where one has to read it as a long position on a zero-coupon bond (corresponding to the minimum guarantee classically embedded in such contracts), a long position on a call option (the bonus option given to the policyholders as the participation in equity) and a short position on a put (the default put). According to recent studies, criticisms come from Ballotta, Haberman and Wang [2005] or Ballotta, Esposito and Haberman [2006]. Indeed they show safety loadings should be added, otherwise the default probabilities are not realistic and the obtained market values are too low. Extra premiums need to be charged as a safety loading. As already noticed by Ballotta [2005], the put corresponding to the default option can be interpreted directly as a safety loading. But selling back the put to the policyholder, the solvency risk attached to the contract would disappear in a complete market where options can perfectly be hedged. Therefore, Bernard *et al.* [2006a] propose to sell back a part of the default part and not totally, arguing the safety loading would be too important and also that life insurance companies may default.

In a recent work, Boyle and Tian [2006] study the design of equity-linked contracts. In the first part of their study, they focus on a point-to-point classical Equity Indexed Annuity (EIA).

This is a product that ensure the consumer to get the maximum between a guaranteed amount and an index traded on a market at a fixed maturity. They propose a way to take into account the profit margin and the safety loading in the pricing of Equity Indexed Annuities. The premium paid by the investor is never equal to the market value of the contract because of the safety loading and the margin profit of the company. They propose to use for the pricing of such contracts a minimum guaranteed rate and a participation coefficient lower than the ones leading to a fair contract. The safety loading is here completely linked to the particular feature of the contract and is far away from the constant proportion of the actuarial value of the policy.

Following the ideas of the latter authors, we assume the company issues a product that provides a given minimum guaranteed rate g and a participation coefficient δ and sells it to a consumer at y_0. But y_0 corresponds in fact to a different fair contract. Indeed if the fair minimum guaranteed rate corresponding to the market value y_0 is equal to $\hat{g} = g$ then the fair participation coefficient is $\hat{\delta} > \delta$. The market value of the contract with (g, δ) is equal to x_0 and satisfies $x_0 < y_0$. The difference $y_0 - x_0$ corresponds to the safety loading. In the following, we assume

$$\delta = 90\%\hat{\delta}.$$

Then 10% of the fair participation represents the safety loading.

1.3 Adopted framework

In so called "structural models" which go back to Merton [1974] and Black and Cox [1976] and are applied in insurance by Briys and de Varenne [1994, 1997a] (no early default possible) and Grosen and Jørgensen [2002] (premature default possible), a representative liability holder pays an upfront premium which corresponds to an α-fraction of the entire company's initial assets. I.e., the equity-holder's contribution corresponds to $(1-\alpha)$-fraction of the initial assets' amount. The policyholders all invest in the same contract maturing at time T, guaranteeing a minimum interest rate g and a participation rate δ.

We make the standard assumptions of the Black and Scholes framework. First there exists a risk free asset with a continuous constant interest rate r. Trading takes place continuously. There are no tax, no transaction costs and no agency costs. Moreover there is no dividend. Throughout this paper, we use the following notations:

r	:	constant risk free interest rate.
σ	:	constant assets' volatility.
T	:	the contract's maturity date
$L_T = L_0 e^{gT}$	:	the guaranteed payment to the policy holder at maturity, where g could be interpreted as the minimum guaranteed interest rate
L_t	:	the minimum guarantee of the insured's investment at time $t \in [0, T]$
A_t	:	the value of the firm's assets at time $t \in [0, T]$
B_t	:	ηL_t, the barrier level, where η is the regulation parameter.

As compensations to their initial investments in the company, equity holders and policyholders both acquire a claim on the firm's assets for a payoff at maturity (or before maturity). The total payoff to the holder of such an insurance contract at maturity, $\psi_L(A_T)$, is given by:

$$\psi_L(A_T) = L_T + \delta[\alpha A_T - L_T]^+ - [L_T - A_T]^+.$$

This payment consists of three parts: the guaranteed amount at maturity (a guaranteed fixed payment which is the initial premium payment compounded by the interest rate guarantee), a bonus (call option) paying to the policy holder a fraction δ of the positive difference of the actual performance of his share in the insurance company's assets, and a short put option resulting from the fact that the equity holder has a limited liability.

It is noticed that the incentives for customers to buy this kind of contracts are not very high due to two reasons: first, the guaranteed interest rate is usually smaller than the market interest rate; and second, probably the customers cannot obtain the guaranteed amount which is against the nature of an insurance contract. Therefore, it seems more interesting to analyze the risk management of these contracts than to price them. In other words, it makes more sense to analyze different risk measures under the real market measure instead of under the equivalent martingale measure.

2 Optimal barrier under continuous surveillance

2.1 Framework

Grosen and Jørgensen [2002] model their regulatory intervention rule in the form of a boundary, i.e., an *exponential* barrier $B_t = \eta L_0 e^{gt}$ is imposed on the underlying assets value process, where η is a regulation parameter. When the asset price reaches this boundary, namely, $A_\tau = B_\tau$ with $\tau \in [0, T]$, the company defaults and is liquidated immediately, i.e., default and liquidation coincide. In addition, in Grosen and Jørgensen [2002], a rebate payment,

$$\Theta_L(\tau) = \min\{L_0 e^{g\tau}, B_\tau\} = \min\{1, \eta\} L_0 e^{g\tau},$$

is offered to the liability holder in the case of a premature closure of the firm, where τ denotes the liquidation date.

Moreover, in the above formulation, the regulator does not play a very important role. In reality, the regulator takes some measures in order not to let the insurer go bust immediately instead of claiming the company bankrupt after observing the barrier level is hit. In the following, we assume that by setting the regulation parameter, the regulator aims at ensuring a certain percent of guaranteed payment in case of an early default given that the insurance company is allowed to default with a constrained shortfall probability (premature+mature) (see Gatzert and Kling [2006]).

2.2 Optimal regulation parameter?

In contrast to the insurance company which tries to maximize the returns of its shareholders, the regulator wants to protect policyholders. As a starting point, we assume that the regulator plays a multiple role, i.e., he strives not only for a low shortfall probability of the company but also ensuring the policyholder a certain amount in case of default. Mathematically, the two goals of the regulator are given by:

$$\min\{\eta > 0 \ / \ P(\tau \leq T) \ \leq \ \varepsilon \ \} \tag{1}$$

$$\max\left\{\eta > 0 \ / \ E\left[\min\{\eta, 1\} L_\tau \, e^{r(T-\tau)} \mathbb{1}_{\{\tau<T\}} | \mathbb{1}_{\{\tau<T\}}\right] \geq \gamma L_T\right\}, \tag{2}$$

The goal of the regulator is to find a regulation parameter which first gives an acceptable level of default and second protects the policyholders by maximizing their expected conditional

σ	10%	15%	20%
$P(\tau \leqslant T)$	0.00257218	0.07269	0.239842

Table 1: Cumulative shortfall probabilities with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\eta = 0.5$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$.

cash flows given default. Some comments should be made concerning the second aim: (a) Given default, the policyholder obtains the rebate term, which corresponds to the term $\min\{\eta, 1\}L_\tau$; (b) Due to consistency reasons and in order to make it comparable with the final payment, the rebate payment is accumulated to the maturity date with the risk free asset r; (c) $\gamma \in [0, 1]$ implies that the regulator sets the regulation rule to ensure that γ percents of the final guaranteed payment will be paid to the policyholder in the expectation.

2.2.1 Aim 1: Minimizing the shortfall probability

In order to compute the shortfall probability and the expected value, the firm's assets value is assumed to follow a geometric Brownian motion under the historical measure P

$$dA_t = A_t\,(\mu\,dt + \sigma\,dW_t)$$

where μ and $\sigma > 0$ are respectively the instantaneous return rate and the volatility of the assets. W_t is assumed to be a standard Brownian motion under the historical measure P.

We begin with the first goal given in Equation (1), i.e. to compute the probability that an early default occurs: $P(\tau \leq T)$. According to the derivation of Appendix A,

$$P(\tau \leqslant T) \;= N\left(\frac{\ln(\frac{\eta L_0}{A_0}) - \hat{\mu}T}{\sigma\sqrt{T}}\right) + \left(\frac{A_0}{\eta L_0}\right)^{\frac{-2\hat{\mu}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) + \hat{\mu}T}{\sigma\sqrt{T}}\right)$$

with $\hat{\mu} = \mu - g - \frac{\sigma^2}{2}$.

Table 1 demonstrates several cumulative shortfall probabilities for a time horizon of 20 years. According to e.g. Moody's credit rating, a small volatility of 10% leads to a very small shortfall probability and this leads to rate Aaa of the company, a volatility value of 15% results in Baa[2].

Thanks to its definition, one can easily notice the probability of default is a strictly non-decreasing function with respect to the variable η. The higher the η, the higher the shortfall probability. Therefore η^ε is the unique solution satisfying the equation:

$$N\left(\frac{\ln(\frac{\eta^\varepsilon L_0}{A_0}) - \hat{\mu}T}{\sigma\sqrt{T}}\right) + \left(\frac{A_0}{\eta^\varepsilon L_0}\right)^{\frac{-2\hat{\mu}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta^\varepsilon L_0}{A_0}) + \hat{\mu}T}{\sigma\sqrt{T}}\right) = \varepsilon,$$

When the regulator sets a regulation parameter smaller than this critical value, i.e.,

$$\eta \;\leq\; \eta^\varepsilon, \tag{3}$$

a shortfall probability smaller than ε can be achieved. Figure 1 demonstrates how the optimal regulation parameter depends on the constrained shortfall probability ε for different volatility

[2]According to Moody's rating, for a 20-year horizon, the ratings are given as follows: Aaa, 1.55%; Aa, 2.70%, A, 5.24% and Baa, 12.59%.

values. The higher the ε-value, the higher the resulting optimal regulation parameter. Furthermore, the higher the volatility, for a given ε-value, the lower the optimal regulation parameter. In addition, it is observed that a quite low regulation parameter η which results in a quite low barrier level should be chosen in order to keep the insurance company to stay in a reasonable shortfall probability. Another important observation from Figure 1 is that the optimal regulation rule depends on the level of the volatility of the assets. I.e., there exists an interplay between the insurer and the regulator. For instance, if the insurer trades in a dynamic strategy with different volatilities, the regulator sets a regulation rule according this, then reacting to the regulator's new rule, the insurer adjust his strategy again... This perspective will be considered in section 3.

Optimal Regulation Parameter in Case of Grosen and Jørgensen

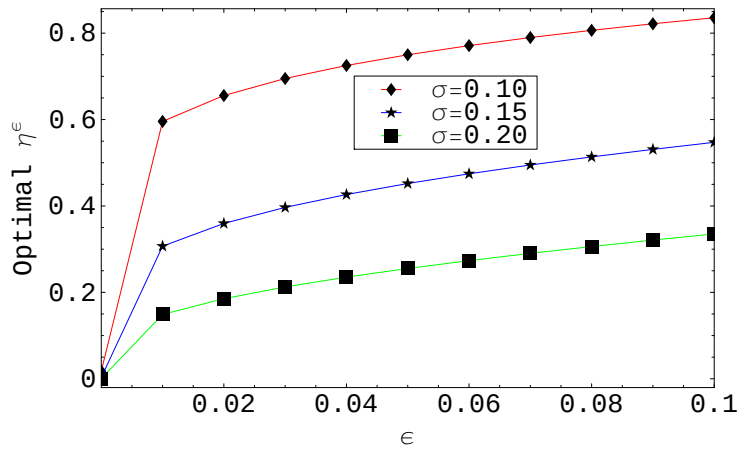

Figure 1: Trade-off between η^{ε} and ε when parameters are set to: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$.

ε	η^{ε}; $\sigma = 0.10$	η^{ε}; $\sigma = 0.15$	η^{ε}; $\sigma = 0.20$
0	0.0176057	0.002988	0.0005152
0.01	0.595660	0.306855	0.148879
0.02	0.655581	0.359548	0.185358
0.03	0.694975	0.396648	0.212528
0.04	0.725144	0.426470	0.235245
0.05	0.749929	0.451935	0.255261
0.06	0.77114	0.474452	0.273434
0.07	0.789786	0.494819	0.290258
0.08	0.806489	0.513537	0.306044
0.09	0.821664	0.530945	0.321006
0.10	0.835603	0.547280	0.335295

Table 2: Optimal regulation parameters η^{ε} for given shortfall probability constraint ε for diverse σ-values with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$.

γ	η^γ; $\sigma = 0.10$	η^γ; $\sigma = 0.15$	η^γ; $\sigma = 0.20$
0.70	0.926911	0.712758	0.575404
0.75	0.948932	0.744161	0.609739
0.80	0.969571	0.774388	0.64344
0.85	0.989005	0.803537	0.676536
0.90	1.01086	0.831693	0.709055
0.95	1.03638	0.858933	0.741021
1.00	1.06052	0.885323	0.772407

Table 3: Optimal regulation parameters η^γ for diverse γ and σ-values with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$.

2.2.2 Aim 2: maximize the expected payout of the contract given liquidation

We proceed with the second goal. The regulator wants to maximize the expected conditional cash-flows of the insured with respect to η. In Grosen and Jørgensen [2002], the rebate payment is paid out immediately at the premature liquidation time. For compatibility reasons, it is assumed now that the rebate payment will be accumulated with a risk free market interest rate and paid out at the maturity.According to the derivation in Appendix B, the expected conditional payoff is given by

$$\begin{aligned}
&\frac{E\left[(\eta \wedge 1)\, L_0 e^{g\tau} e^{r(T-\tau)} \mathbf{1}_{\{\tau<T\}}\right]}{P\{\tau < T\}} \\
= \;& (\eta \wedge 1)\, L_0 e^{rT} \left(\frac{\eta L_0}{A_0}\right)^{\frac{\hat{\mu}}{\sigma^2}} \left\{ \left(\frac{A_0}{\eta L_0}\right)^{\frac{\sqrt{(\hat{\mu})^2+2(r-g)\sigma^2}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) - \sqrt{(\hat{\mu})^2+2(r-g)\sigma^2}T}{\sigma\sqrt{T}}\right) \right. \\
&\left. + \left(\frac{A_0}{\eta L_0}\right)^{\frac{-2}{\sigma^2}\sqrt{(\hat{\mu})^2+2(r-g)\sigma^2}+1} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) + \sqrt{(\hat{\mu})^2+2(r-g)\sigma^2}T}{\sigma\sqrt{T}}\right) \right\} \Big/ P\{\tau < T\}
\end{aligned}$$

with the denominator corresponding to the result in the first aim.

The goal of the insurer here is to ensure the policyholder a certain γ-fraction of the guaranteed amount given liquidation. Hence, we use η^γ to denote the critical η-value which makes the above expected value equal to $\gamma\, L_T = \gamma\, L_0 e^{gT}$, then

$$\eta \;\geq\; \eta^\gamma \tag{4}$$

guarantees in the expectation that the payoff given liquidation is not smaller than γ-percents of the guaranteed payment. This is due to the fact that the expected rebate payment goes up with the regulation parameter η. This positive relation between γ and η^γ is observed in Figure 2.2.2.

Obviously, there is a trade-off between these two goals. A small shortfall probability requires a lower value of η, but at the same time leads to a lower expected rebate payment which is probably even lower than the γ-fraction of the final guaranteed payment. Therefore, a regulator has to set a regulation level in the area of

$$\eta^\gamma \;\leq\; \eta \;\leq\; \eta^\varepsilon \tag{5}$$

in order to achieve a plausible shortfall probability and to ensure a reasonable expected payoff to the policyholder given default. However, comparing the the optimal values of η^ϵ and η^γ given

in Tables 2 and 3, no intersection areas of η as in Equation (5) can be found. I.e., the regulator cannot aim at both of the goals. Hence, we focus on the first goal, i.e. constrain the shortfall probability of the insurance company as most regulators do in reality. In the rest of this section, we examine whether the result changes a lot when a more realistic bankruptcy procedure i.e. Chapter 11 comes into consideration.

Expected conditional payment given liquidation

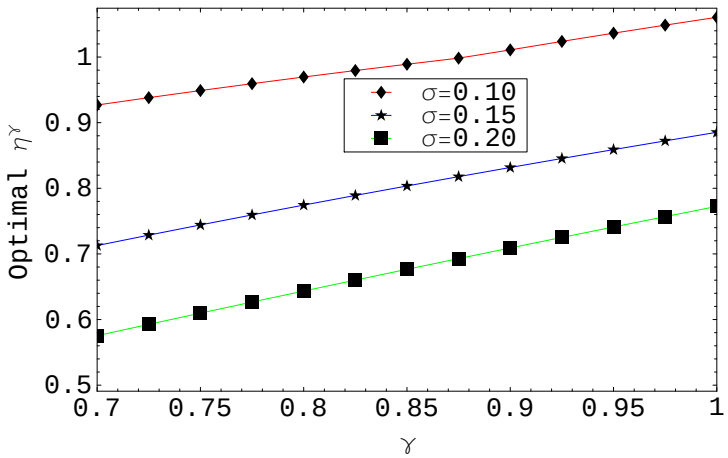

Figure 2: Expected conditional payment given liquidation with parameters $A_0 = 100$; $L_0 = 80; T = 20; r = 0.03;\ \mu = 0.04; g = 0.01$.

2.3 Under Chapter 11

In Chapter 11 bankruptcy procedure, default and liquidation are considered distinguishable events and mathematically this is realized by using Parisian option framework. Standard Parisian barrier feature corresponds to a procedure where the liquidation of the firm is declared when the financial distress has lasted successively at least a period of length d. Cumulative Parisian barrier feature corresponds to a procedure where the liquidation is declared when the financial distress has lasted in total at least a period of length d during the life of the contract. The economic idea behind these two extremes is the importance of the past of the company's assets. Indeed in the parisian case regulators completely forget the past. On the contrary the cumulative procedure corresponds to the extreme case when the regulators never forget the past. Alternative are discussed for example in Galai, Raviv and Wiener [2003].

The purpose of this subsection is to examine whether the realistic bankruptcy procedure Chapter 11 brings some new aspects to our analysis, therefore, we jump to the numerical results immediately. Those who are interested in the derivation of the shortfall probability can have a look at Appendix C.

Tables 4 and 5 demonstrate several optimal values of the regulation parameters for both the standard and cumulative Parisian option case. Above all, it is observed that the resulting optimal η^{ε}-values are higher than the results in Section 2.2. This is quite obvious, because

ε	η^ε; $\sigma = 0.10$	η^ε; $\sigma = 0.15$	η^ε; $\sigma = 0.20$
0.01	0.6536	0.35281	0.17954
0.02	0.7178	0.413186	0.223563
0.03	0.7603	0.45543	0.25576
0.04	0.7922	0.48964	0.28365
0.05	0.82015	0.51821	0.307534
0.06	0.8443	0.54312	0.32928
0.07	0.8659	0.56654	0.349880
0.08	0.8827	0.58754	0.36734
0.09	0.8991	0.6076	0.3855321
0.10	0.9156	0.62735	0.401856

Table 4: Optimal regulation parameters η^ε in case of standard Parisian option for given shortfall probability constraint ε for diverse σ-values with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$; $d = 0.5$.

default does not result in liquidation immediately by taking account of Chapter 11 bankruptcy procedure. Furthermore, the resulting optimal regulation parameters in the Standard Parisian framework are higher than those in the cumulative case. This is due to the fact that the knock-out condition in the standard case occurs with a lower probability than in the cumulative case if same parameters are taken. The knock-out condition for standard Parisian barrier options is that the underlying asset stays *consecutively* below barrier for a time longer than d before the maturity date, while the knock–out condition for cumulative Parisian barrier options is that the underlying asset value spends until the maturity *in total* d units of time below the barrier. In addition, the positive relation between the volatility and η^ε is still observed.

ε	η^ε; $\sigma = 0.10$	η^ε; $\sigma = 0.15$	η^ε; $\sigma = 0.20$
0.01	0.6332	0.33756	0.16965
0.02	0.69658	0.39485	0.210678
0.03	0.73819	0.43545	0.24126
0.04	0.77004	0.46778	0.266954
0.05	0.796205	0.485654	0.28935
0.06	0.81878	0.520094	0.30984
0.07	0.838412	0.54217	0.32865
0.08	0.855952	0.56254	0.34637
0.09	0.87200	0.581354	0.363189
0.10	0.88692	0.59997	0.3791764

Table 5: Optimal regulation parameters η^ε in case of cumulative Parisian option for given shortfall probability constraint ε for diverse σ-values with parameters: $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$; $d = 0.5$.

2.4 Comparison of the two frameworks

In section 1.1, we introduce two different viewpoints. In the two studied cases (subsection 2.2 and the last subsection) we only looked at the regulator's viewpoint. Now let us assume that

the regulation parameter η is given $\eta = 0.8$. The company is assumed to be perfectly aware of this coefficient, then it has an optimal volatility level or debt ratio level in order to avoid an intervention of the regulators with probability 99%. All parameters being fixed except the assets' volatility, we first give the two different risk levels in Table 6.

	Black and Cox	Standard Parisian	Cumulative Parisian
Volatility Level	0.0752	0.0817	0.07945

Table 6: Volatility Level $A_0 = 100$; $L_0 = 80$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$; $d = 0.5$; $\delta = 90\% \, \hat{\delta}$.

Table 6 shows that the company may choose a riskier portfolio in the case of a parisian surveillance for a given level of bankruptcy risk. The probability to have an intervention is indeed lower in the parisian setting than in a Black and Cox setting.

Table 7 gives results when all parameters are fixed except the debt ratio α, meaning that the company will ask for more capital to decrease its risk. In Table 7, we give the corresponding values of α.

	Black and Cox	Standard Parisian	Cumulative Parisian
Debt ratio α ($\sigma = 0.10$)	0.59566	0.65262	0.63329
Debt ratio α ($\sigma = 0.15$)	0.306855	0.35497	0.337397

Table 7: $A_0 = 100$; $T = 20$; $\mu = 0.04$; $r = 0.03$; $g = 0.01$; $d = 0.5$; $\delta = 90\% \, \hat{\delta}$; $L_0 = \alpha A_0$.

For a given bankruptcy risk level, Table 7 illustrates the fact that the company needs more capital in a Black and Cox setting than in a parisian setting. For example when the volatility is set at 15%, the shareholders part should represent 69.3% in a Black and Cox setting instead of 64.5% in a Parisian setting to avoid a regulators' intervention with probability 99%.

If its initial volatility is set at 10% and initial debt ratio is $\alpha = 0.8$ then the probability of default is $7.6\% \gg 1\%$. A first solution is to decrease the portfolio's volatility level (Table 6), a second solution could be to decrease α, thus to ask for more capital (Table 7). These two tables summarize the impact of the regulators on the company. Regulatory constraints influence the market value of liabilities and assets by constraining either the portfolio management (volatility σ) and the capital structure (parameter α).

3 Moving to a dynamic approach

We did strong hypothesis in all the previous sections. We are valuing liabilities in the framework of Black and Scholes, assuming in particular that the assets of the company are lognormally distributed, the interest rate is constant, the volatility is constant and the company has no hedging strategy during the whole term of the contract. The IASB and the FASB insist on the fact that we should study the "model error". Following the guidelines of the IASB, the next step is now to ask ourselves wether the above results are consistent if the model is not well specified.

Carr et al. [2002] notice that the market prices reflect that the underlying can not be a simple diffusion process and thus that models are misspecified. Ballota et al. [2006] study the impact when the underlying is a Levy process instead of a diffusion process. The usual long term of the considered contracts contradict also the constant parameters used in the model. Many studies extend the previous models to a stochastic interest rate environment, see Nielsen et al. [1995], Tan [2003], Bernard et al. [2005b, 2006b] to quote only a few. Stochastic volatilities could also be introduced using for example the model of Bakshi et al. [1997].

In particular, in this section we concentrate ourselves on the model error due to the static assumption. We observe indeed that in the above case study analyzed in the paper the insurer stays passive until maturity. He has no hedging strategy. I.e. the insurer does not react to the regulator's regulation rule. Whatever are the regulatory rules, the evolution of its assets level, the insurer does nothing. In other words, it is assumed that the insurer trades just in an asst with a constant volatility. This might be reasonable for a short-term contract, but these considered life insurance contracts are often long-term contracts with a maturity T equal for example to 20 years. The long-term characteristic of the insurance contracts should incorporate the fact that the insurer will follow a dynamic strategy. According to the analysis in the last section, when the insurer decides to follow a dynamic instead of static hedging strategy, the optimal regulation rule cannot remain optimal, and the resulting shortfall probability might not be promising to the regulator any more. Consequently, this simple assumption that the insurance company is "passive" and just waits for default to occur or not is lifted in the following and we study the effects of a dynamic hedge of the insurer.

A general optimization problem of insurance company can be constructed as the following optimal investment problem:

$$\begin{aligned} \min_{\phi} \quad & E[l([H_T - V_T(\phi)]^+)] \\ \text{s.t.} \quad & \begin{cases} V_0(\phi) \leq E[H_T R_T] \\ P(\tau \leq T) \leq \epsilon \ . \end{cases} \end{aligned}$$

Here we denote by H_T the uncertain amount the policyholder is going to obtain at the maturity date. l is defined as a loss function which decides the attitude of the insurance company against loss. The first constraint is the budget restriction, where R_T can be considered as a deflator. The second condition is the shortfall probability constraint. The insurance company seeks to minimize its expected loss $E[l([H_T - V_T(\phi)]^+)]$ by choosing the optimal investment strategy ϕ subject to the budget restriction and an additional regulatory constraint (shortfall probability constraint). Or instead of controlling its loss, the insurance company might strive to maximize its shareholders' utility in a similar way, i.e., under the same restrictions just rephrase the objective function as

$$\max_{\phi} \quad E[U([V_T(\phi) - H_T]^+)],$$

where U stands for its shareholders' utility. However, these optimal investment problems are extremely difficult to solve. These problems are then reduced to a Monte Carlo approach because it seems not reasonable to obtain closed-form formulae, or quasi-closed form after introducing so many stochastic features. Indeed it is possible to have an internal model that takes into account the stochastic interest rates, the stochastic volatility, the periodic premiums, the surrender option (Bacinello [2003], Jørgensen [2001]), death benefits, guaranteed annuity options (Boyle and

Hardy [2003]), longevity risk...

In order to improve how noticeable effect a dynamic strategy might have, in the following we introduce a very simple strategy. More specifically, we examine how a great impact a simple dynamic hedge already has on the expected return and on the probability to have a regulators' intervention. As proposed by Dangl and Lehar [2004] for a bank, we assume there is two different portfolios with two asset risk levels. At the end of each year before the maturity of the contract, four different events might occur:

- the regulators look at the assets of the company and declare bankruptcy because they are too low.
- the company is solvent but too risky: regulators switch the level of asset risk to a lower level to satisfy the regulatory constraints.
- given the regulatory requirements, it is optimal for the managers to stick to the current risk level.
- given the regulatory constraints, it is optimal to switch the level of asset risk. In that case that means either company performs well and can take more risk or the company wants to avoid a future regulators' intervention.

3.1 Initial setting

We assume the parameters are set to $\mu_L = 4\%$, $r = 3\%$ and $\sigma_L = 10\%$ in the lower asset risk case. In the higher asset risk case, we assume $\sigma_H = 20\%$ and $\mu_H = 5\%$. At time 0, the volatility is set at $\sigma = \sigma_0$ and the ratio $\alpha = \alpha_0 = 0.4$, $A_0 = 100$, $L_0 = 40$, $T = 20$, $r = 0.03$, $g = 0.01$, $\delta = 90\% \hat{\delta}$.

We have closed-form expression of the probability of a regulators' closure decision before the maturity T in case of no monitoring and if the initial assets risk is set at σ_0. Surveillance is continuous in a Black and Cox framework (refer to section 2.2). We assume the level of bankruptcy at time t is given by ηB_t where $\eta = 0.4$. We define the expected return of the policyholders as:

$$\frac{\text{Expected Payoff at time } T \quad - \quad L_0}{L_0}$$

where the expected payoff of policyholders is given by

$$E[\left(\delta[\alpha A_T - L_T]^+ + L_T - [L_T - A_T]^+\right) \mathbb{1}_{\{\tau \geq T\}}] + E[e^{r(T-\tau)} \min\{A_\tau, L_\tau\} \mathbb{1}_{\{\tau < T\}}]$$

with $\delta = 90\% \hat{\delta}$. [3] The rebate payment is again accumulated with the risk free interest rate to the maturity and the expected final payment is calculated in Appendix D. In case of prior bankruptcy, we assume shareholders receive nothing, assets are used to pay bankruptcy costs and reimburse policyholders. The expected return of the shareholders is given by:

$$\frac{\text{Expected Payoff at time } T \quad - \quad (1-\alpha)A_0}{(1-\alpha)A_0}$$

[3] As mentioned, $\hat{\delta}$ represents the fair participation of a contract guaranteeing the minimum interest rate g. I.e. $\hat{\delta}$ results from the fair valuation principle, i.e.

$$E^*[e^{-rT} \left(\hat{\delta}[\alpha A_T - L_T]^+ + L_T - [L_T - A_T]^+\right) \mathbb{1}_{\{\tau \geq T\}}] = L_0,$$

where E^* represents the expectation taken under the equivalent martingale measure.

	$\sigma_0 = \sigma_L = 10\%$, $\mu_L = 4\%$	$\sigma_0 = \sigma_H = 20\%$, $\mu_H = 5\%$
Default Probability	3.8 $10^{-4}\%$	10.8%
Policyholders Expected Return	1.06	1.25
Shareholders Expected Return	1.34	2.00

Table 8: Shortfall probability and expected return in a static setting.

Hedging is always a trade-off between risk and return. We consider a very simple strategy. We fix a maximum level of risk (through a given early default probability) and use the volatility parameter to hedge our portfolio in order to maximize the expected payments to shareholders and policyholders keeping a shortfall probability prior maturity below a maximum level.

3.2 Volatility Hedging

Given a maximum probability of bankruptcy before maturity T, denoted by $p_0 = P_0(T)$ (for example 4%) then the company wants to maximize the shareholders' value keeping the probability of an early closure under p_0. The insurance company trades (switch the portfolio) at the end of each year as long as no early default occurs. At the end of each year $t = t_i$, $i = 1..T$, managers face three different situations:
-Case 1: $A_t < \eta B_t$ Bankruptcy is declared, shareholders receive nothing and policyholders receive A_t.
-Case 2: $A_t \geq \eta B_t$ and $\sigma = \sigma_H$. We then compute at time t, the probability of bankruptcy before T when there is no monitoring until T (we use closed-form formulae provided in section 2.2). If this shortfall probability is above p_0, then regulators reduce the level of the volatility, otherwise they do not intervene.
-Case 3: $A_t \geq \eta B_t$ and $\sigma = \sigma_L$. The managers decide to switch to a higher risk level in order to increase their expected payment. Their decision should keep on satisfying that bankruptcy probability before maturity is below p_0.

We proceed by Monte Carlo methods assuming the initial volatility is either $\sigma_0 = 10\%$ or $\sigma_0 = 20\%$.

	$\sigma_0 = \sigma_L = 10\%$, $\mu_L = 4\%$	$\sigma_0 = \sigma_H = 20\%$, $\mu_H = 5\%$
Default Probability	0.62%	0.64%
Policyholders Expected Payment	1.40 (+32%)	1.15 (-8%)
Shareholders Expected Payment	1.69 (+26%)	1.86 (-7%)

Table 9: Shortfall probability and expected payments in case of dynamic hedging. In parenthesis, we give the percentage of increase or decrease compared to the situation with no monitoring.

Through results displayed in Table 8, we identify two different situations wether the company bears initially a high asset risk or a low asset risk. First if it has a low investment risk (initial volatility is set at 10%) then in the static framework, returns are rather small (1.06 for policyholders and 1.34 for shareholders). If it follows the above simple dynamic strategy, then the shortfall probability is higher but remains acceptable and expected returns are more interesting and realistic. Secondly, if the company is initially very risky, then the dynamic hedging strongly reduces its risk. Indeed if there is no monitoring the default probability is 10.8% which is not

realistic. This simple strategy decreases its shortfall probability to 0.64%. The expected returns are a bit lower but not significantly lower. A dynamic strategy can then have two interesting effects, it increases the expected payments without changing significantly the probability of default or it decreases significantly the shortfall probability keeping rather interesting expected returns.

We are aware that this strategy is an over simplified example but it already shows the possible impact of a dynamic setting. Indeed a dynamic strategy, even through a very simple example, changes strongly the results. Pricing and results given by closed-form formulae are of course useful to understand the influence of each parameter and to give us orders of magnitude but results need then to be discussed via Monte Carlo simulations to analyze the model error.

One drawback of our study is the constant participation coefficient. For the initially less risky company, the fair participation is about 90.1%, for the initially riskier company, it is 70.8%. Policyholders receive 90% of the fair participation, then hedging will be more interesting for the policyholders in the first case.

Conclusion

First, in the static framework of Black and Scholes, our study shows that regulatory rules and solvency requirements strongly influence the market values of company's liabilities and assets and thus the shortfall probability. Then through Monte Carlo simulations, we show that introducing a dynamic strategy can also significantly change the results. We aim at showing limits of the previous models giving the market value of long term liabilities under default risk.

Further studies may also include some realistic features such as dynamic distribution of benefits. Here the participation is fixed but practically it is often linked to yearly performances of the company and can also be a way to manage the financial risk of the contracts. We may also note that surveillance has a cost. The above study describes continuous scrutiny of the assets' process. Following Merton [1978] who extended his previous work by introducing random and costly audits, or the recent works of Battachaya *et al.* [2002] or Dangl and Lehar [2004] one may extend our framework to the case of random audits. The default barrier would not be continuous anymore and the company has to be solvent at any audits time. These works all assume that the audit's arrival process is independent to the assets' process and that the intensity is constant. It would be very interesting to make it depend on the previous information obtained by the regulators at the last audit. In addition, our work use the shortfall probability as a risk measure, but further investigations can extend it to more general risk measures. Furthermore, in this analysis, "non-market risks" are completely ignored in the geometric Brownian motion setting. Therefore, taking into account the non-hedgeable non-market risk into account is an important extension of the problem.

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Appendix

A Derivation of the shortfall probability

It is observed that

$$A_u \geq B_u \Leftrightarrow \left(\mu - \frac{1}{2}\sigma^2 - g\right) u + \sigma W_u + \ln\left(\frac{A_0}{B_0}\right) > 0$$

Hence, under the market measure P, passage of $A(.)$ through $B()$ is equivalent to the passage of the Brownian motion $Z_u = \left(\mu - \frac{1}{2}\sigma^2 - g\right) u + \sigma W_u + \ln\left(\frac{A_0}{B_0}\right)$ through zero. Now we assume that

$$\hat{\mu} = \mu - g - \frac{1}{2}\sigma^2 \quad ; \quad Z_0 = \ln\left(\frac{A_0}{B_0}\right)$$

consequently

$$\begin{aligned} A_T &= A_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T} = B_0 e^{Z_T + gT} \\ A_T &> B_T \Leftrightarrow Z_T > 0. \end{aligned}$$

Furthermore, it is known that the density of the first passage time is $g(\tau, Z_0, 0) = \frac{Z_0}{\sigma\tau^{3/2}} n\left(\frac{Z_0 + \hat{\mu}\tau}{\sigma\sqrt{\tau}}\right)$, with $n(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$. And the distribution function of τ is

$$G(\tau, Z_t, t) = N\left(\frac{-Z_t - \hat{\mu}(\tau - t)}{\sigma\sqrt{\tau - t}}\right) + e^{-\frac{2Z_t\hat{\mu}}{\sigma^2}} N\left(\frac{-Z_t + \hat{\mu}(\tau - t)}{\sigma\sqrt{\tau - t}}\right)$$

Hence, the shortfall probability we look for is given by:

$$\begin{aligned} P^{\varepsilon} &= N\left(\frac{-Z_0 - \hat{\mu}T}{\sigma\sqrt{T}}\right) + e^{-\frac{2Z_0\hat{\mu}}{\sigma^2}} N\left(\frac{-Z_0 + \hat{\mu}T}{\sigma\sqrt{T}}\right) \\ &= N\left(\frac{\ln\left(\frac{B_0^*}{A_0}\right) - (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) + \left(\frac{A_0}{B_0}\right)^{-\frac{2(\mu - g - \frac{1}{2}\sigma^2)}{\sigma^2}} N\left(\frac{\ln\left(\frac{B_0}{A_0}\right) + (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) \end{aligned}$$

B Derivation of the expected payoff given default

The expected payoff given default is given by

$$\frac{E\left[(\eta \wedge 1) L_0 e^{g\tau} e^{r(T-\tau)} \mathbf{1}_{\{\tau < T\}}\right]}{P\{\tau < T\}}$$

The denominator is already calculated in Appendix A, we just need to calculate the numerator. The numerator is given as follows:

$$\begin{aligned} & E\left[(\eta \wedge 1) L_0 e^{g\tau} e^{r(T-\tau)} \mathbf{1}_{\{\tau < T\}}\right] \\ = \; & (\eta \wedge 1) L_0 e^{rT} \int_0^T e^{-(r-g)\tau} g(\tau, Z_0, 0) d\tau \\ = \; & (\eta \wedge 1) L_0 e^{rT} \int_0^T e^{-(r-g)\tau} \frac{Z_0}{\sigma\tau^{3/2}} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{(Z_0 + \hat{\mu}\tau)^2}{2\sigma^2\tau}\right\} d\tau \\ = \; & (\eta \wedge 1) L_0 e^{rT} \left(\frac{\eta L_0}{A_0}\right)^{\frac{\hat{\mu}}{\sigma^2}} \left\{ \left(\frac{A_0}{\eta L_0}\right)^{\frac{\sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}}{\sigma^2}} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) - \sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}T}{\sigma\sqrt{T}}\right) \right. \\ & \left. + \left(\frac{A_0}{\eta L_0}\right)^{\frac{-2}{\sigma^2}\sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2} + 1} N\left(\frac{\ln(\frac{\eta L_0}{A_0}) + \sqrt{(\hat{\mu})^2 + 2(r-g)\sigma^2}T}{\sigma\sqrt{T}}\right) \right\} \end{aligned}$$

C Derivation of shortfall probability in the standard and cumulative Parisian framework

In the standard Parisian down–and–out option framework, an early default occurs only when the following technical condition is satisfied:

$$T_B^- = \inf\left\{t > 0 \middle| \left(t - g_{B,t}^A\right) \mathbb{1}_{\{A_t < B_t\}} > d\right\} \leq T$$

with

$$g_{B,t}^A = \sup\{s \leq t | A_s = B_s\},$$

where $g_{B,t}^A$ denotes the last time before t at which the value of the assets A hits the barrier B. T_B^- gives the first time at which an excursion below B lasts more than d units of time. In fact, T_B^- is the liquidation date of the company if $T_B^- < T$.

In case of the cumulative Parisian framework, the options are lost by their owners when the underlying asset has stayed below the barrier for at least d units of time during the entire duration of the contract. Therefore, an premature or mature default appears when the following condition holds:

$$\Gamma_T^{-,B} = \int_0^T \mathbb{1}_{\{A_t \leq B_t\}} dt \geq d,$$

where $\Gamma_T^{-,B}$ denotes the occupation time of the process describing the value of the assets $\{A_t\}_{t\in[0,T]}$ below the barrier B during $[0,T]$. Again, we denote τ as the premature liquidation date and it implies:

$$\Gamma_\tau^{-,b} := \int_0^\tau \mathbb{1}_{\{\tau \leq T\}} \mathbb{1}_{\{A_t \leq B_t\}} dt = d.$$

The shortfall probability in the case of Standard Parisian option is given by

$$\begin{aligned} P^\varepsilon &= P\left(T_B^- = \inf\left\{t > 0 \middle| \left(t - g_{B,t}^A\right) \mathbb{1}_{\{A_t < B_t\}} > d\right\} \leq T\right) \\ &= e^{-\frac{1}{2}m^2 T} \left(\int_{-\infty}^b h_2(T,y) e^{my}\, dy + \int_b^\infty h_1(T,y) e^{my}\, dy \right) \end{aligned}$$

with $m = \frac{1}{\sigma}\left(\mu - g - \frac{1}{2}\sigma^2\right)$. $h_1(T,y)$ and $h_2(T,y)$ are uniquely determined by inverting the corresponding Laplace transforms which are given by

$$\begin{aligned} \hat{h}_1(\lambda, y) &= \frac{e^{(2b-y)\sqrt{2\lambda}} \psi\left(-\sqrt{2\lambda d}\right)}{\sqrt{2\lambda}\psi\left(\sqrt{2\lambda d}\right)} \\ \hat{h}_2(\lambda, y) &= \frac{e^{y\sqrt{2\lambda}}}{\sqrt{2\lambda}\psi\left(\sqrt{2\lambda d}\right)} + \frac{\sqrt{2\lambda d} e^{\lambda d}}{\psi\left(\sqrt{2\lambda d}\right)} \left(e^{y\sqrt{2\lambda}} \left(N\left(-\sqrt{2\lambda d} - \frac{y-b}{\sqrt{d}}\right) - N\left(-\sqrt{2\lambda d}\right) \right) \right. \\ &\quad \left. - e^{(2b-y)\sqrt{2\lambda}} N\left(-\sqrt{2\lambda d} + \frac{y-b}{\sqrt{d}}\right) \right). \end{aligned}$$

with

$$\begin{aligned} b &= \frac{1}{\sigma} \ln\left(\frac{B_0}{A_0}\right) = \frac{1}{\sigma} \ln\left(\frac{\eta L_0}{A_0}\right) = \frac{1}{\sigma} \ln(\eta\alpha) < 0 \\ \psi(z) &= \int_0^\infty x \exp\left\{-\frac{x^2}{2} + zx\right\} dx = 1 + z\sqrt{2\pi} e^{\frac{z^2}{2}} N(z), \end{aligned}$$

and λ the parameter of Laplace transform. The shortfall probability in the case of cumulative Parisian option is determined by

$$\begin{aligned}
P^{\varepsilon} &= P(\tau \leq T) = P\left(\frac{1}{T}\int_0^T \mathbb{1}_{\{W_u + m\,u \leq b\}} du \geq \frac{d}{T}\right) \\
&= P\left(\frac{1}{T}\int_0^T \mathbb{1}_{\{W_u - m\,u \leq -b\}} du \leq 1 - \frac{d}{T}\right) \\
&= 2\int_0^{1-\frac{d}{T}} \left\{ \left[\frac{N\left(-m\sqrt{T}\sqrt{1-u}\right)}{\sqrt{1-u}} - m\sqrt{T}N\left(-m\sqrt{T}\sqrt{1-u}\right)\right] \right. \\
&\quad \left. \cdot \left[\frac{1}{\sqrt{u}} N\left(\frac{(-b)/\sqrt{T} + m\sqrt{T}u}{\sqrt{u}}\right) + m\sqrt{T}e^{2mb}N\left(\frac{b/\sqrt{T} + m\sqrt{T}u}{\sqrt{u}}\right)\right]\right\} du,
\end{aligned}$$

where $N(\cdot)$ is the density function of the standard normal distribution. In the above derivation, Equation (12) of Takács [1996] is applied.

D Derivation of the expected final value

The expected final payment is described as follows:

$$\begin{aligned}
& E[\left(\delta[\alpha A_T - L_T]^+ + L_T - [L_T - A_T]^+\right)\mathbb{1}_{\{\tau \geq T\}}] + E[e^{r(T-\tau)}\min\{A_\tau, L_\tau\}\mathbb{1}_{\{\tau < T\}}] \\
= \; & \delta E[(\alpha A_T - L_T)^+\mathbb{1}_{\{\tau \geq T\}}] + E[L_T\mathbb{1}_{\{\tau \geq T\}}] - E[(L_T - A_T)^+\mathbb{1}_{\{\tau \geq T\}}] \\
& + E\left[e^{r(T-\tau)}\left(\eta \wedge 1\right)L_0 e^{g\tau}\mathbb{1}_{\{\tau < T\}}\right] \\
= \; & \delta \cdot A + B - C + D
\end{aligned}$$

where E gives the expectation under the market measure. In the following, we begin with the first term, the expected value of the down-and-out call option.

$$\begin{aligned}
A &= E[(\alpha A_T - L_T)^+, \inf_{t\in[0,T]} A_t > B_t] \\
&= \int_{B_T}^{\infty} (\alpha A_T - L_T)^+ f(A_T, T; A_0, 0)\, dA_T \\
&= \int_{\left(B_T \vee \frac{L_T}{\alpha}\right) \equiv X}^{\infty} (\alpha A_T - L_T) f(A_T, T; A_0, 0)\, dA_T
\end{aligned}$$

where $f(A_T, T; A_0, 0)$ denotes the density of A_T with an absorbing barrier B_T. According to Appendix A, it holds

$$\begin{aligned}
A_T &> (B_T \vee \frac{L_T}{\alpha}) := X \Leftrightarrow Z_T > \ln\left(\frac{X}{A_0}\right) + Z_0 - gT = \ln\left(\frac{X}{B_0^*}\right) - gT := q \\
A_T &= A_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T} = B_0 e^{Z_T + gT}.
\end{aligned}$$

Furthermore, if we denote $f(Z_T, T, Z_0, 0)$ the density function of the Brownian motion with drift and with an absorbing barrier at the origin which has a form of

$$f(Z_T, T, Z_0, 0) = \frac{1}{\sigma\sqrt{T}} n\left(\frac{Z_T - Z_0 - \hat{\mu}T}{\sigma\sqrt{T}}\right) - e^{-\frac{2Z_0\hat{\mu}}{\sigma^2}} \frac{1}{\sigma\sqrt{T}} n\left(\frac{Z_T + Z_0 - \hat{\mu}T}{\sigma\sqrt{T}}\right),$$

we can continue with the calculation of the expected value as follows

$$\begin{aligned}
A &= \alpha \int_q^\infty \left(B_0 e^{Z_T + gT} - \frac{L_T}{\alpha} \right) f(Z_T, T, Z_0, 0) dZ_T \\
&= \alpha A_0 e^{\mu T} N \left(\frac{\ln(\frac{A_0}{X}) + (\mu + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) - L_T N \left(\frac{\ln(\frac{A_0}{X}) + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \\
&\quad -\alpha B_0 \left(\frac{A_0}{B_0} \right)^{-\frac{2(\mu - g - \frac{1}{2}\sigma^2)}{\sigma^2} - 1} e^{\mu T} N \left(\frac{\ln(\frac{(B_0)^2}{A_0 X}) + (\mu + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \\
&\quad +L_T \left(\frac{B_0}{A_0} \right)^{\frac{2(\mu - g - \frac{1}{2}\sigma^2)}{\sigma^2}} N \left(\frac{\ln(\frac{(B_0)^2}{A_0 X}) + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right)
\end{aligned}$$

$$\begin{aligned}
\delta A &= \delta \left(\alpha A_0 e^{\mu T} N \left(\frac{\ln\left(\frac{A_0}{X}\right) + (\mu + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \right) - L_T N \left(\frac{\ln\left(\frac{A_0}{X}\right) + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \right) - \alpha B_0 \left(\frac{A_0}{B_0} \right)^{-\frac{2\hat{\mu}}{\sigma^2} - 1} \cdot \right. \\
&\quad \left. e^{\mu T} N \left(\frac{\ln\left(\frac{B_0^2}{A_0 X}\right) + \left(\mu + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \right) + L_T \left(\frac{B_0}{A_0} \right)^{\frac{2\hat{\mu}}{\sigma^2}} N \left(\frac{\ln\left(\frac{B_0^2}{A_0 X}\right) + (\mu - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \right) \right)
\end{aligned}$$

with $X = \max\left\{B_T, \frac{L_T}{\alpha}\right\}$. Before we come to the calculation of the second part B, we observe that

$$A_T > B_T \Leftrightarrow A_0 e^{Z_T + gT - Z_0} > B_T \Leftrightarrow Z_T > 0.$$

This implies:

$$\begin{aligned}
B &= L_T \int_{B_T}^\infty f(A_T, T, A_0, 0) dA_T \\
&= L_T \int_0^\infty f(Z_T, T, Z_0, 0) dZ_T \\
&= L_T N \left(\frac{\ln(\frac{A_0}{B_0}) + (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) - L_T \left(\frac{A_0}{B_0} \right)^{\frac{-2(\mu - g - \frac{1}{2}\sigma^2)}{\sigma^2}} N \left(\frac{\ln(\frac{B_0}{A_0}) + (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right)
\end{aligned}$$

The expected fixed payment has a form of

$$B = L_T N \left(\frac{\ln\left(\frac{A_0}{B_0}\right) + \hat{\mu}T}{\sigma\sqrt{T}} \right) - L_T \left(\frac{A_0}{B_0} \right)^{\frac{-2\hat{\mu}}{\sigma^2}} N \left(\frac{\ln\left(\frac{B_0}{A_0}\right) + \hat{\mu}T}{\sigma\sqrt{T}} \right).$$

The third term is given as follows:

$$
\begin{aligned}
C &= \mathbb{1}_{\{\eta<1\}} \int_{B_T}^{L_T} (L_T - A_T) f(A_T, T, A_0, 0) dA_T \\
&= \mathbb{1}_{\{\eta<1\}} \int_0^{\frac{L_0}{A_0}} (L_T - B_0 e^{Z_T + gT}) f(Z_T, T, Z_0, 0) dZ_T \\
&= \mathbb{1}_{\{\eta<1\}} \Bigg\{ L_T \left(N \left(\frac{\ln(\frac{L_0}{B_0}) - (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) - N \left(\frac{\ln(\frac{B_0}{A_0}) - (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \right) \\
&\quad - L_T \left(\frac{A_0}{B_0} \right)^{-\frac{2(\mu - g - \frac{1}{2}\sigma^2)}{\sigma^2}} \left(N \left(\frac{\ln(\frac{A_0 L_0}{(B_0)^2}) - (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \right. \\
&\quad \left. - N \left(\frac{\ln(\frac{A_0}{B_0}) - (\mu - g - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \right) - A_0 e^{\mu T} \left(N \left(\frac{\ln(\frac{L_0}{A_0}) - (\mu - g + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \right. \\
&\quad \left. - N \left(\frac{\ln(\frac{B_0}{A_0}) - (\mu - g + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \right) + B_0 e^{\mu T} \left(\frac{A_0}{B_0} \right)^{-\frac{2(\mu - g - \frac{1}{2}\sigma^2)}{\sigma^2} - 1} \\
&\quad \left(N \left(\frac{\ln(\frac{A_0 L_0}{B_0}) - (\mu - g + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) - N \left(\frac{\ln(\frac{A_0}{B_0}) - (\mu - g + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \right) \Bigg\}
\end{aligned}
$$

The expected value of the shorted put option can be computed as follows:

$$
\begin{aligned}
C &= \mathbb{1}_{\{\eta<1\}} \Bigg\{ L_T \left(N \left(\frac{\ln\left(\frac{L_0}{B_0}\right) - \hat{\mu}T}{\sigma\sqrt{T}} \right) - N \left(\frac{\ln\left(\frac{B_0}{A_0}\right) - \hat{\mu}T}{\sigma\sqrt{T}} \right) \right) - L_T \left(\frac{A_0}{B_0} \right)^{-\frac{2\hat{\mu}}{\sigma^2}} \\
&\quad \left(N \left(\frac{\ln\left(\frac{A_0 L_0}{B_0^2}\right) - \hat{\mu}T}{\sigma\sqrt{T}} \right) - N \left(\frac{\ln\left(\frac{A_0}{B_0}\right) - \hat{\mu}T}{\sigma\sqrt{T}} \right) \right) - A_0 e^{\mu T} \left(N \left(\frac{\ln\left(\frac{L_0}{A_0}\right) - (\hat{\mu} + \sigma^2)T}{\sigma\sqrt{T}} \right) \right. \\
&\quad \left. - N \left(\frac{\ln\left(\frac{B_0}{A_0}\right) - (\hat{\mu} + \sigma^2)T}{\sigma\sqrt{T}} \right) \right) + B_0 e^{\mu T} \left(\frac{A_0}{B_0} \right)^{-\frac{2\hat{\mu}}{\sigma^2} - 1} \\
&\quad \left(N \left(\frac{\ln\left(\frac{A_0 L_0}{B_0}\right) - (\hat{\mu} + \sigma^2)T}{\sigma\sqrt{T}} \right) - N \left(\frac{\ln\left(\frac{A_0}{B_0}\right) - (\hat{\mu} + \sigma^2)T}{\sigma\sqrt{T}} \right) \right) \Bigg\}
\end{aligned}
$$

The expected rebate part can be derived as follows:

$$
\begin{aligned}
D &= E\left[(\eta \wedge 1) L_0 e^{g\tau} e^{r(T-\tau)} \mathbf{1}_{\{\tau<T\}} \right] \\
&= (\eta \wedge 1) L_0 e^{rT} \int_0^T e^{-(r-g)\tau} g(\tau, Z_0, 0) d\tau \\
&= (\eta \wedge 1) L_0 e^{rT} \int_0^T e^{-(r-g)\tau} \frac{Z_0}{\sigma \tau^{3/2}} \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{(Z_0 + \hat{\mu}\tau)^2}{2\sigma^2 \tau} \right\} d\tau
\end{aligned}
$$

The expected rebate part is given by

$$
\begin{aligned}
&(\eta \wedge 1) L_0 e^{rT} \left(\frac{\eta L_0}{A_0} \right)^{\frac{\hat{\mu}}{\sigma^2}} \Bigg\{ \left(\frac{A_0}{\eta L_0} \right)^{\frac{\sqrt{\hat{\mu}^2 + 2(r-g)\sigma^2}}{\sigma^2}} N \left(\frac{\ln\left(\frac{\eta L_0}{A_0}\right) - \sqrt{\hat{\mu}^2 + 2(r-g)\sigma^2} T}{\sigma\sqrt{T}} \right) \\
&+ \left(\frac{A_0}{\eta L_0} \right)^{1 - \frac{2}{\sigma^2}\sqrt{\hat{\mu}^2 + 2(r-g)\sigma^2}} N \left(\frac{\ln\left(\frac{\eta L_0}{A_0}\right) + \sqrt{\hat{\mu}^2 + 2(r-g)\sigma^2} T}{\sigma\sqrt{T}} \right) \Bigg\}
\end{aligned}
$$

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