SOCIETY OF ACTUARIES

A World of Mortality Issues and Insights Seminar
May 23, 2012

Session 12 – Mortality Uncertainty

Presenter

Henk van Broekhoven (IAA MWG)

Mortality Uncertainty

How to get a distribution around the Best Estimate Mortality

Henk van Broekhoven

May 23, 2012

What are we going to do?

- This presentations contains
  - Definition of mortality risk
  - What are the parts of the distribution around the BE
  - Example

Definition Life Risk

- Life Risk relates to the deviations in timing and amount of cash flows due to incidence or non-incidence of death
  - Deviations relative to the Best Estimate Assumptions (US: Best = Current)
- Overall mortality can be described by a probability function in which the Expected Value is the Best Estimate.

Definition Life Risk

- This probability function is not just a single distribution
- Following the IAA Book: "A Global Framework for Insurer Solvency Assessment" (also called "the Blue Book") EVERY risk type should be analysed in 3 parts:
  - Volatility
  - Uncertainty (model + parameter)
  - Extreme events

Distribution around the BE mortality

- Best Estimate Mortality rates can be analysed into two parts:
  - Level
  - Trend
- The distribution is defined by the following sub-risks**:**
  - **Volatility**
  - **Uncertainty Level**
  - **Uncertainty Trend**
  - **Extreme event risk (Calamity)**

Question

Stochastic?

OR

Analytic?

- Volatility
- Extreme Event (calamity)
- Uncertainty Level
- Uncertainty Trend

Volatility

- Risk of random fluctuations
  - frequency
  - important: variation of sum-at-risk across policies
  - also additional volatility because of external causes
    - like cold winters
    - influenza epidemics
    - severe accidents

<Mortality is not a full independent process>

Volatility

- Because of the (small) dependency between the several lifes the Compound Poisson distribution (instead of Compound Binomial) is used.

Volatility

- Via an analytical way this CP distribution can be estimated using the Normal Power Approach:

$$P\{[S - E(S)]/\sigma_s \leq s + \tfrac{1}{6}\gamma(s^2 - 1)\} \approx \Phi(s) = \alpha\%$$

- Or with x # standard deviations needed:

$$x = s + \frac{\gamma}{6}(s^2 - 1)$$

- The NP approach is based on the Cornish Fisher expansion only using the first 3 (or 4) moments.

Volatility

Standard deviation total claim level (c(j)) follows (for Compound Poisson):

- ***X is capital at risk*** *(=face value – reserve)*

$$\sigma = \sqrt{\sum_i q_i(x) X_i^2}$$

- And Skewness: $\gamma = \dfrac{\sum_i q_i(x) X_i^3}{\sigma^3}$

γ can be fine-tuned based on observed volatility in the past (Maximum likelihood)

Example Normal Power

Portfolio 1: Typical distribution of sum assured, typical age distribution
Portfolio 2: Distribution skewed to high sums assured.
Portfolio 3: Typical distribution, but compared to 1 a rather small portfolio

Portfolio	Number insured	Max insured / average	Skewness
1	125,970	11.6	0.13
2	60,777	40.3	0.77
3	24,570	14.7	0.38

Results (% RP):

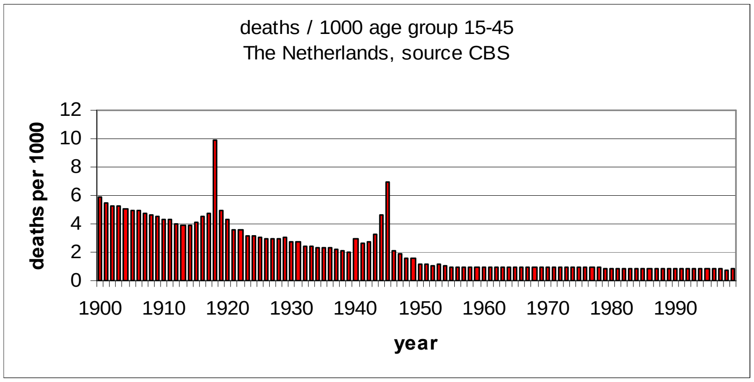
Portfolio	Simulation	Normal Power
1	22.7%	22.8%
2	69.9%	68.1%
3	57.2%	57.4%

- Volatility
- Extreme Event (calamity)
- Uncertainty Level
- Uncertainty Trend

Extreme event risk

- Worldwide the worst thing related to mortality that can happen is a Pandemic.
  - Some events are simple too extreme to model like an impact of a large asteroid.
- Still every insurance company should analyse concentration risk within their portfolio's. An incident can, in case of high concentrations, have a larger impact than a pandemic.

Extreme Events in mortality



Extreme event risk

- Although often a model for extreme events is connected to some confidence level, like 1 in 200 for Solvency II, in my opinion this is wrong.
- A pandemic is a conditional event.
  - The 1 in 200 connected to the 0.15% is valid in case we are in a WHO phase 1 situation.
    - We never were!

Pandemic phases

Inter-pandemic phase	Low risk of human cases	1
New virus in animals, no human cases	Higher risk of human cases	2
Pandemic alert	No or very limited human-to-human transmission	3
New virus causes human cases	Evidence of increased human-to-human transmission	4
	Evidence of significant human-to-human transmission	5
Pandemic	Efficient and sustained human-to-human transmission	6

Extreme event risk

- Better to talk about a scenario, like the Spanish Flu: combined with the 1 in 200 VAR of the other risks you should be able to survive a certain scenario.
  - Beside this "conditional" nature of a pandemic it is also a fact that nobody really knows the real pandemic distribution.
  - This is also the case for other types of extreme events

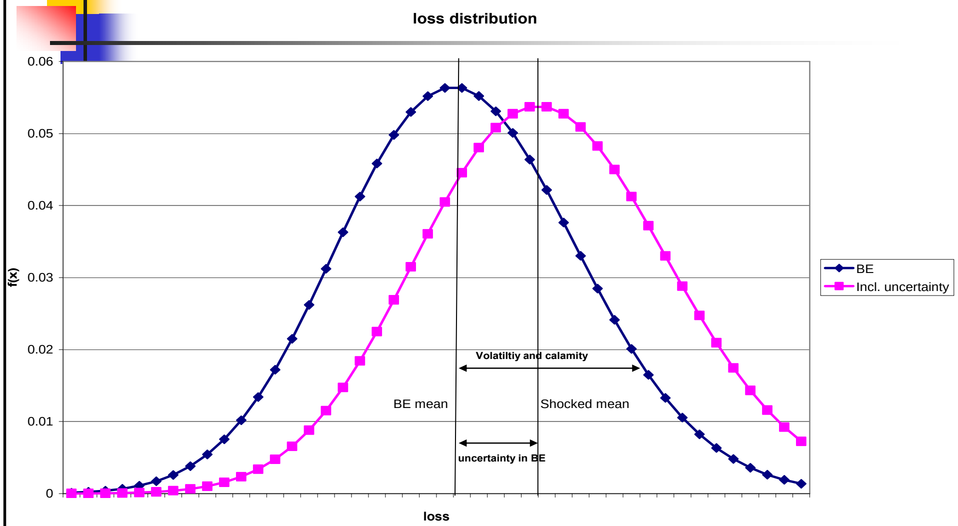
Question / Discussion

- Now we start calculating the uncertainty.
- It should at the end result in an uncertainty around the BE.
- Should it be a one-year approach or a multi-year approach?
  - Solvency II Standard Model is based on a one year approach…

Multi-year

- Long term guarantees are given
- So liabilities also contain uncertainty in the future (e.g. medical developments)
  - This uncertainty doesn't appear within one year
  - Random walk is no reason to adjust BE assumptions (it is modelled in volatility)
  - Uncertainty has to deal with the BE

Multi-year



Multi-year

- In the internal model I'm using the uncertainty is based on Multi-year
- Volatility and calamity on one-year.

- Volatility
- Extreme Event (calamity)
- Uncertainty Level
- Uncertainty Trend

Uncertainty Level

- The best estimate is based on a factor times the population mortality (Basis Best Estimate)
- The factor in formula:

$$factor = \frac{\mu_{obs}}{\mu_{pop}}$$

Uncertainty Level

- Because the μ_{obs} is the result of a random process (=volatility in the past!) We are not sure that this number represent the real expected claim level.
  - It could have been an observation in the (wrong) tail.
- To be sure for α% that the liabilities are sufficient we need to add a capital.

Uncertainty Level

- Recalculate the liabilities based on mortality rates on "α% level", this is

$$q_{EC}(x;t) = factor_{EC} \times q_{pop}(x;t)$$

$$factor_{EC} = \frac{\mu_{obs} + (-)unc_{\alpha\%}}{\mu_{pop}}$$

Uncertainty Level

- To find the uncertainty (unc) around the Best estimate we use like in Volatility the Compound Poisson distribution
  - Poisson instead of binomial because there is some dependency in mortality
  - Compound because of the spread of the insured sums
  - Like in volatiltiy a correction can be made to the skewness based on historical volatlity

Uncertainty Level

- The Compound Poisson distribution can be calculated in a rather simple way using the normal power approach similar to the volatility calculations
- The distribution is translated into a normal distribution using the first three moments
  - -> NP(3)

Uncertainty Level

- The economic capital "level" follows:

$$EC_{level} = LIAB_{EC} - LIAB_{BE}$$

Liabilities based on $q_{EC}(x;t)$

Liabilities based on $q_{BE}(x;t)$

- Volatility
- Extreme Event (calamity)
- Uncertainty Level
- Uncertainty Trend

Uncertainty Trend

- It is impossible to predict a future trend.
- Medical development, environment, new diseases and resistance agains a medical cure can change trend (drift).
- Also volatility in the observations will cause uncertainty (random walk).
- Are we using the right model?
- We must try to say something about the uncertainty

Uncertainty Trend

- In Lee Carter the stochastic part is based on
  - The random walk around the drift
  - Mistake in estimation of the drift because of volatility in used data
  - The drift is rather linear (in standard LC model)
  - This means that changes in drift are modeled as random walk
  - Often the Normal Distribution is used.

Uncertainty Trend

- The real uncertainty is the uncertainty in the drift. But exactly this part is hard to model.
- Other than assumed under Lee Carter is the drift not a straight line and for sure not Normal Distributed
- It is influenced by several factors like medical developments, resistance against medicines, climate change.
- Even extreme events can occur like a cure against important causes of death (cancer)

Uncertainty Trend

Lee Carter model

$$c = \frac{1}{T}\sum_{t=1}^{T}[k(t) - k(t-1)] = \frac{k(T) - k(0)}{T}$$

Standard error e(t)σ :

$$se(e) = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}[k(t) - k(t-1) - c]^2}$$

Error in c:

$$se(c) = \sqrt{\frac{\sigma^2}{T}} \approx \frac{se(e)}{\sqrt{T}}$$

Uncertainty Trend

- In my opinion this method is underestimating the risk around trends.
- The random walk part of a trend analysis is better separate modelled under volatility.
  - It contains for example impacts of cold winters, flu epidemics etc.
- Future uncertainty (e.g. medical developments) are not included

Uncertainty Trend

- The uncertainty is not only based on a statistical error but also:
  - Model choice
  - Used data
  - Unexpected developments (extreme events)
    - E.g. a medical cure against an important cause of death.

Uncertainty Trend

- Therefore uncertainly trend should be partly based on information and ideas out of the medical world to get an insight of the extreme events related to longevity
- Also historical changes in trend can help to get information about model choices and data used.

Uncertainty Trend

- The trend uncertainty is analysed in two part:
  - Statistical Uncertainty in used trend data
  - Uncertainty is long term projection, also includes Expert Judgement on
    - Medical developments

Uncertainty Trend

- For the uncertainty in the long term life expectancy the goal table we use a recent study of the Dutch CBS. In that study also several international conclusions are used
- It is stated that a 95% confidence interval means an increase of plus / minus 5 in the projected life expectancy in 2060.
  - This is based on both statistical and medical information and medical expert ideas
- Using this, by adjusting the mortality rates of the BE long term projection in such a way that the life expectancy is adjusted with 5 years a shocked projection table (both upper and lower) can be created

Playing with a goal table

- The uncertainty can be included in the projection using the following formula:

$$q(x; j+t) = q(x;t) \times f(x; j)^{t} \times e^{\frac{\alpha(x)t(t+1)}{2}}$$

Original trend

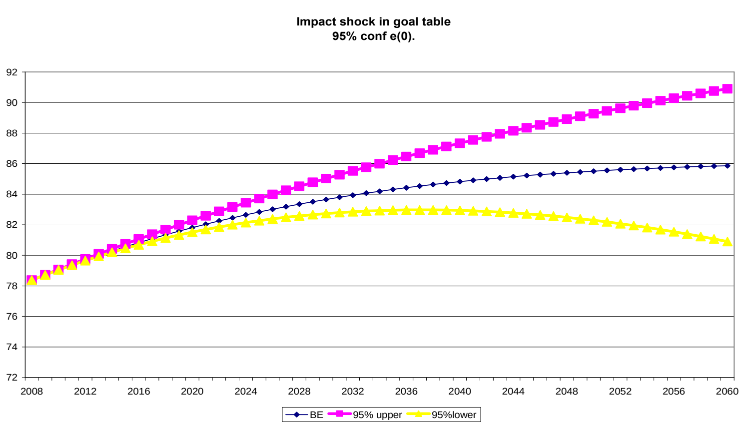
For j+t is final year of projection the adjusted goal table this formula can be solved

Playing with a goal table

- Solving $\alpha(x)$ <j+t is the last year of the projection:

$$\alpha(x) = \frac{\log q(x; j+t) - \log q(x;t) - t \times \log f(x; j)}{\frac{1}{2} \times t \times (t+1)}$$

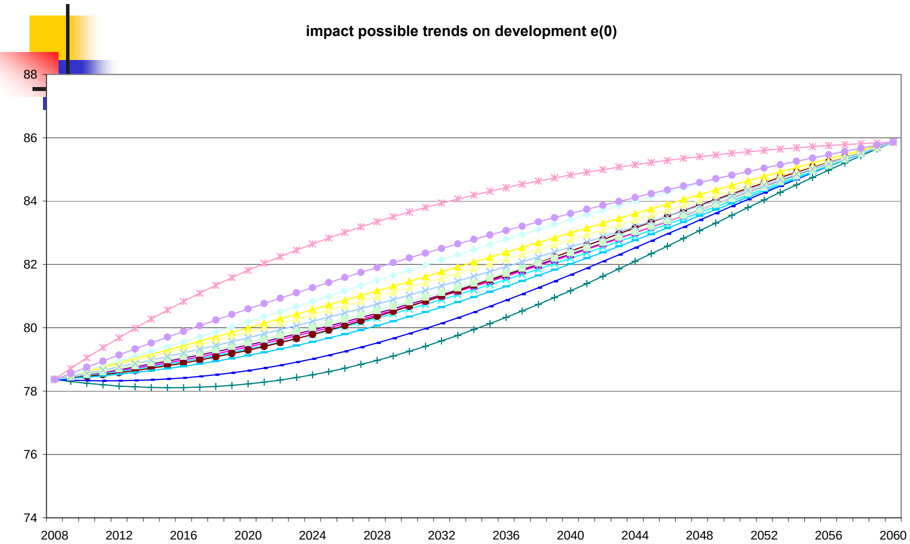
Uncertainty Trend (goal table)



Uncertainty Trend

- Based on the history it can be analysed how different 10 year trends look like
  - I used here Dutch figures
- This gives an idea how also in the future short term trend can change or what the impact can be for other assumptions
- By setting all those different trends at the beginning of the prognosis some conservatism will be introduced
  - With each trend observed in the past a separate generation table can be calculated
  - I used 12 trends
- See next graph

Uncertainty Trend



How to use this?

- An insurance company and pension funds need to know what the impact is on the Best Estimate liabilities of the uncertainty around the trend
- Because a complex dependency between the development for the several ages it is not advised to calculate liabilities directly with the at a certain confidence level shocked mortality rates
- Better is to use each of the generation tables (11 for start trend and 1 for the shocked goal table) plus the BE generation table to calculate liabilities.

Uncertainty capital starting trend

- With each of the 12 historical 10 year trends a generation table is derived and with each of them liabilities can be calculated:

1	13.09420843
2	13.33246787
3	13.066988
4	13.17006177
5	13.33750594
6	13.02018636
7	13.07364983
8	13.02874271
9	13.48099007
10	13.09073366
11	13.25972385
12	**14.0581669**

Standard deviation:0.293

Distribution: Student with 11 DoF

97.5% upper conf. level: 2.2*0.293

= 4.9% of the mean

Uncertainty trend

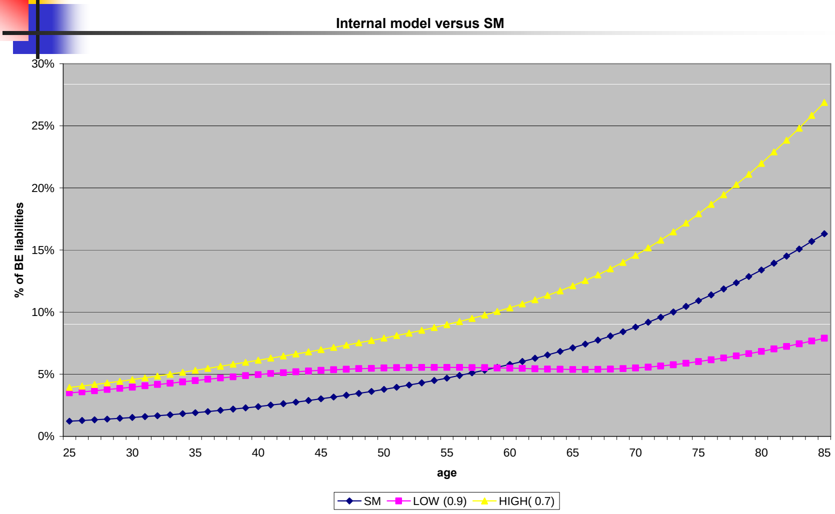
- Assuming 100% correlation between the uncertainty goal table and uncertainty start trend the total capital can be set at the sum.
- The results will be highly dependent on the discount rate!
- Most likely the ratio’s mentioned can be used for other countries with not enough data available to make own calculations.

Standard model Solvency II

Standard Model Solvency II

- For longevity just a simple shock of 20% on the mortality rates
  - Not duration dependent
  - One year time horizon

Comparison S II with internal model



Standard model

- IMO:
  - Standard model less suitable for risk management
  - Perhaps OK for SCR calculations for average portfolio's