THE INFLUENCE OF EXPENSE LOADINGS ON THE FAIRNESS OF A TARIFF

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ABSTRACT

In non-life insurance, it is nearly always assumed that the expense loading is a fraction of the risk premium. This may deeply affect the fairness of a tariff, as illustrated in the case of the Belgian bonus-malus system.

1. INTRODUCTION—SUMMARY

The exponential growth of the number of papers dealing with the theory of premium calculation principles is one of the significant trends of the actuarial science during the last decade. Also noteworthy is the fact that all of those papers concentrate on the risk premium (pure premium and safety loading) and deliberately leave aside the determination of the loading for expenses, commissions, taxes, profit, ... We shall attempt to show in this paper that this neglect has some severe consequences, that it is futile to try to assess the risk premium with great precision if the expense loading is only grossly estimated, that risk premiums with desirable characteristics in terms of the principles of risk classification are distorted through the loading process (this should be obvious since in many cases the expense loading is greater than the risk premium). Note that the same remark was made by JEWELL (1980): "The next step in premium setting is to determine the additional 50–200% increase which determines the commercial premium by adding expense and profit loadings. Except in life insurance where there are specific cost models for sales commissions (in many cases of regulated form), there seems to be no further modelling principles used, except [multiplying the risk premium by a factor $1 + \alpha$]. This lacuna in the literature is all the more surprising, as it is in sharp contrast to the fields of engineering and business management, where extensive and sophisticated cost allocation and modelling are the order of the day. Are these activities outside the realm of the actuary, ... ?"

2. APPARENT AND REAL RISK PREMIUMS

In all lines of insurance the policy-holders are partitioned according to some criteria that significantly affect the risk (like use and power of the car, age and occupation of the driver in motorcar insurance). Let s be the number of cells,

and $\{b_i; i = 1, \ldots, s\}$ the commercial tariff: b_i is the premium to be paid by a policy-holder that belongs to cell i. b_i is the sum of two components: the risk premium r_i and the expense loading e_i, that contains the company's general expenses g_i, the commissions c_i, the taxes t_i, and, in some cases, a profit loading p_i:

$$b_i = r_i + e_i, \qquad i = 1, \ldots, s$$

where $e_i = g_i + c_i + t_i + p_i$.

In non-life insurance, it is nearly always assumed* that the expense loading is a fraction of the risk premium:

$$b_i = r_i(1 + \alpha), \qquad \alpha > 0, \qquad i = 1, \ldots, s.$$

The loading coefficient $\alpha = \alpha_g + \alpha_c + \alpha_t + \alpha_p$, where

α_g = loading coefficient for general expenses

α_c = loading coefficient for commissions

α_t = loading coefficient for taxes

α_p = loading coefficient for profits.

This proportional approach is certainly open to some criticisms. Why should the salesmen of the company (brokers, agents, ...) be paid more for bad risks than for good risks (on the contrary we feel that they should be rewarded for bringing good risks to the company)? Is it fair that young drivers pay more taxes than older policy-holders? Is there any reason for the fact that drivers living in big cities contribute more to the profit of the company than inhabitants of small communities? If a proportional loading is applied, the high risk cells certainly pay a disproportionate share of the expenses. This means that the "real" risk premium they pay is not r_i, but $r_i' = r_i + (EX)_i$, where $(EX)_i$ is the excess charge of expenses (considered here as the "hidden" part of the risk premium)

(a) A Special Case: Flat Expense Loading

Suppose that there is no reason whatsoever that the high risk cells contribute more towards the expenses than the low risks, and denote by n_i the population of cell i. Instead of paying $b_i = r_i(1 + \alpha)$, a risk that belongs to cell i should pay

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* Among the few exceptions we found in the literature were

• the proposed new motor rating structure in the Netherlands, the authors' recommended rates are applicable for 90% of the premium income (including the part of the component for expenses contained therein), while the remaining 10% is considered to relate to expenses which should be appropriated towards each policy as a fixed amount (GREGORIUS (1982))

• a recommendation of the Massachusetts Insurance Service Office (ROY (1980)) The proposition is to allocate 75% of the operating costs as a fixed amount, and the remaining 25% as a proportional loading

$b'_i = r_i + \beta$, where

$$\beta = \frac{1}{n}\left(\alpha \sum_{i=1}^{s} n_i r_i\right)$$

$$= \frac{\alpha}{1+\alpha}\frac{\sum_{i=1}^{s} n_i b_i}{n}$$

is computed in such a way that the total income $\sum_i n_i b_i$ of the company is not modified. As this risk effectively pays b_i, he is charged a (positive or negative) excess premium of

$$(EX)_i = \alpha r_i - \beta$$

$$= \frac{\alpha}{1+\alpha}\left[b_i - \frac{\sum_i b_i n_i}{n}\right].$$

The real risk premium paid is thus

$$r'_i = r_i + (EX)_i = b_i - \beta.$$

(b) *A More General Case: Linear Loading*

Suppose now that the expense loading should be partly proportional to the risk premium, partly uniform. Instead of being charged $b_i = r_i(1+\alpha)$, a risk of cell i should contribute

$$b'_i = r_i(1+\gamma) + \beta,$$

where $\gamma = \gamma_g + \gamma_c + \gamma_t + \gamma_p$, and $\beta = \beta_g + \beta_c + \beta_t + \beta_p$.

The total proportional part of the company's income is

$$(1+\gamma)\sum_i n_i r_i = \frac{1+\gamma}{1+\alpha}\sum_i n_i b_i.$$

In order to keep the same total income, β should then be equal to

$$\beta = \frac{1}{n}\left(\sum_i n_i b_i - \frac{1+\gamma}{1+\alpha}\sum_i n_i b_i\right)$$

$$= \frac{\alpha - \gamma}{1+\alpha}\frac{\sum_i n_i b_i}{n}.$$

The excess premium for cell i

$$(EX)_i = \alpha r_i - (\gamma r_i + \beta)$$

$$= \frac{\alpha - \gamma}{1+\alpha}\left[b_i - \frac{\sum_i n_i b_i}{n}\right].$$

Then the real risk premium is

$$r'_i = r_i + (EX)_i = \frac{1}{1+\alpha}\left[b_i(1+\alpha-\gamma) - (\alpha-\gamma)\frac{\sum_i n_i b_i}{n} \right].$$

Of course other expense allocation models are conceivable (commissions designed in such a way that the broker has an incentive to sign up good risks, for instance), but the model considered in (b) is more likely to be selected in practice, due to its simplicity.

3. AN APPLICATION

Since 1971, all Belgian companies are compelled to use an 18-class merit-rating system in motorcar third-party insurance. The premium levels $\{b_i; i = 1, \ldots, 18\}$ are presented in Table 1 along with the populations n_i observed in a company (columns 1 and 2). The expense loading is purely proportional, with the following coefficients

Company expenses		$\alpha_g = 0.5901$
Commissions		$\alpha_c = 0.3257$
Taxes $\begin{cases} \text{for the social security system} \\ \text{for the fund of the handicapped} \\ \text{for the Red Cross} \\ \text{tax} \end{cases}$	$\begin{matrix} \alpha = 0.1916 \\ \alpha = 0.1149 \\ \alpha = 0.0048 \\ \alpha = 0.1722 \end{matrix}$	$\alpha_t = 0.4885$
	Total loading	$\alpha = 1.4043$

The expense loading thus multiplies the risk premium by 2.4!

(a) *Flat Expense Loading*

Let us assume that the fair way to allocate expenses is that each policy-holder pays a fixed amount. In our example we obtain

$$\beta = \frac{\alpha}{1+\alpha}\frac{\sum_i n_i b_i}{n} = 39.9308.$$

We then compute the excess premium, and express it as a percentage of the commercial premium b_i (see Table 1, columns 3 and 4). For instance a policy-holder of class 18 can claim that he is being overcharged by 76.88, or 38.44%! Then, we substract β from b_i in order to obtain the real risk premium (column 5). By multiplying the figures of this column by 1.6647 (in order to bring back the premium of the initial class 10 to 100), we obtain the "real" merit-rating system applied by the Belgian companies. It differs markedly from the "alleged" one: for instance the ratio between the largest and smallest premiums is 8, instead of the apparent 3.33!

TABLE 1

FLAT EXPENSE LOADING

b_i	n_i	$(EX)_i$	$\dfrac{(EX)_i \times 100}{b_i}$	r'_i	"Real" System
200	27	76 88	38 44	160 0692	266 47
160	28	53 52	33 45	120 0692	199 88
140	53	41 83	29 88	100 0692	166 59
130	81	36	27 69	90 0692	149 94
120	115	30 16	25 13	80 0692	133.29
115	201	27 24	23 69	75 0692	124 97
110	322	24 32	22 11	70 0692	116.65
105	507	21 40	20 38	65 0692	108 32
100	1 141	18 48	18.48	60.0692	100
100	1 429	18 48	18 48	60 0692	100
95	2 318	15 56	16 37	55 0692	91 68
90	3 385	12 64	14 04	50.0692	83 35
85	9 190	9 72	11 43	45 0692	75 03
80	9 791	6 79	8 49	40 0692	66 71
75	9 887	3 87	5 17	35 0692	58 38
70	12 231	0 95	1 36	30.0692	50 06
65	11 025	−1 95	−3 02	25.0692	41.73
60	70 962	−4.89	−8 14	20 0692	33 41
	132 693				

(b) Linear Loading

To be more realistic, let us compute the real "hidden" merit-rating system under the following assumptions.

(i) Commissions should be the same for every risk.

Indeed in Belgium a broker is nothing more than a salesman, and does not participate in the settlement of claims. He should not have any incentive to sign up customers that belong to the worst risk classes. So $\gamma_c = 0$ and

$$\beta_c = \alpha_c \frac{\sum_i n_i r_i}{n} = \frac{\alpha_c}{1 + \alpha} \frac{\sum_i n_i b_i}{n} = 9.2608.$$

(ii) The contributions to the social security system, the fund of the handicapped and the Red Cross should be proportional to the risk premium.

Bad risks have a higher propensity to cause claims with bodily injury, thereby adding their share to the deficits of the social security system and the fund of the handicapped. It is only fair that they should pay for it. So $\gamma_t = 0.3113$.

(iii) The tax should be the same for all policy-holders. So

$$\beta_t = \frac{\alpha_t - \gamma_t}{1 + \alpha} \frac{\sum_i n_i b_i}{n} = 5.0390.$$

(iv) The part of the general expenses related to the production and the administration of the policies should be uniformly distributed among the policyholders. The part related to the claims settlement should be proportional to the risk premium. In a large Belgian company, the former part accounts for 72.54% of the general expenses, the latter part for the remaining 27.46%. This leads to $\gamma_g = 0.1620$ and

$$\beta_g = \frac{\alpha_g - \gamma_g}{1 + \alpha} \frac{\sum n_i b_i}{n} = 12.1714.$$

Assembling the three components, we have

$$\gamma = \gamma_c + \gamma_t + \gamma_g = 0.4733$$

$$\beta = \beta_c + \beta_t + \beta_g = 26.4712$$

Altogether, around one third of the total expense loading is allocated proportionally, the remaining two thirds evenly.

The computations described in Section 2 enable us to compute the "real" merit-rating system applied by the Belgian companies; it is more severe than the "official" one, since for instance the ratio between the extreme premiums is 6.18, instead of the apparent 3.33.

It was stated over and over again [see for instance LEMAIRE (1982)] that the Belgian bonus-malus system is unefficient and unfair to the best drivers, since the penalizations for claims are much too small.

The preceding considerations show that the effect of a purely proportional loading is to attenuate this unfairness.

TABLE 2

LINEAR LOADING

b_i	$(EX)_i$	$\dfrac{100(EX)_i}{b_i}$	r'_i	"Real" System
200	50 97	25 48	134 15	249 16
160	35 48	22 18	102 03	189 50
140	27 74	19 81	85 97	159 67
130	23 86	18 36	77 94	144 75
120	19 99	16 66	69 90	129 83
115	18 06	15 70	65 89	122 37
110	16 12	14 65	61 87	114 92
105	14 18	13 51	57 86	107 46
100	12 25	12 25	53 84	100
100	12 25	12 25	53 84	100
95	10 31	10 86	49 83	92 54
90	8.38	9.31	45 81	85 08
85	6 44	7 58	41 79	77 63
80	4 50	5 63	37 78	70 17
75	2 57	3 42	33 76	62 71
70	0 63	0 90	29 75	55 26
65	−1 30	−2 01	25.73	47 79
60	−3 24	−5 40	21 72	40 33

REFERENCES

GREGORIUS, F (1982) Development of the Study In *New Motor Rating Structure in the Netherlands* ASTIN-Groep Nederland, pp 15–37

JEWELL, W (1980) Models in Insurance Paradigms, Puzzles, Communications and Revolutions *Proc of the 21st I C A Zurich-Lausanne*

LEMAIRE, J (1982) *L'assurance Automobile Modèles Mathématiques et Statistiques* Fernand Nathan-Editions Labor

ROY, J (1980) La Tarification en Assurance Automobile au Québec Rapport du Surintendant des Assurances