THE LONG TERM EFFECT OF MARGINS IN FUNDING ASSUMPTIONS

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1. Introduction

1.1 This paper is based on one particular section of a longer paper which we presented to the Institute of Actuaries in February 1992, entitled 'A Realistic Approach to Pension Funding'[1].

1.2 The paper reviewed recent developments affecting the financing of defined benefit pension schemes in the UK. These point to a need for bases to be realistic, and we described an approach to finding a "best estimate". We then described a number of areas in which we felt bases tended to contain unwanted margins of caution, and went on to show the long term effects of margins on the level of assets built up, using a simplified model fund, pointing out some problems which arise in the UK context. We next reviewed the various funding methods available, concluding that the projected unit and entry age methods should be used for normal purposes in conjunction with a discounted income approach to the valuation of assets. Finally we made some suggestions as to the degree of prudence appropriate for funding purposes.

1.3 In an international context the part of the paper of most universal application is the section dealing with the long term effect of margins in the assumptions, and this is the subject of the present paper.

2. Methodology

2.1 The effect of margins on valuation results can readily be investigated by substituting revised assumptions and comparing the results. We concentrated our investigations into one particular element namely the investment return, where, for two reasons, it may be appropriate consciously to adopt a margin. First, it is where the effect can most readily be demonstrated and hence understood. Secondly it is straightforward to make direct allowance for any deliberate margin of caution in the investment return assumed, in setting contribution rates which are designed to avoid unwanted surpluses arising.

2.2 As this paper is concerned with basic principles and concepts, we have chosen a very simple model of a stationary fund to investigate the effect, which relies solely on compound interest. A detailed description of the model is set out in the Appendix.

2.3 The calculations made in connection with the model demonstrate that the most important economic parameter in funding pensions is the rate of investment return which is achieved in excess of general pay increases, and we have designated this rate of interest by "i". The two other economic parameters required are the rate of investment return in excess of pension increases, and the rate of pay increases in excess of pension increases. The pension increases concerned are of two types: increases in deferment and increases in payment. For simplicity, we have only considered the case when the two are the same, and equal to the increase in cost of living (which is not an unrealistic assumption for many large defined benefit schemes in the UK).

2.4 If the rate of investment return over pension increases is denoted by "j" and the rate of increases in pay over pensions by "e", then the relationship between the three economic parameters is:

$$(1 + j) = (1 + i)(1 + e)$$

The rate of real increases in pay is regarded as a fairly stable parameter and for the purposes of the paper we have set "e" as .02. As already stated, and as we will show, "i" is the basic function in pension fund work and as our concentration will be on changes in "i" we have felt it natural to give examples which give integral values of "i". This means that the real investment return over prices will be non-integral. Thus, if "i" = .03, "j" = .0506. No doubt in this circumstance one might think of "j" as being 5%, but in our calculations we have rigorously used the true value.

2.5 For those who are interested, the detailed calculations under the model are given in a second Appendix to our UK paper. The results of the calculations by reference to values of i between 0% and 4% are set out in Table A1 in the Appendix to this paper. The table indicates that taking "i" = .03 a margin of 1% in the assumed rate of investment return leads to overstating the past service reserves needed by 14% and the contribution rate by 24%.

2.6 As a stationary fund, the conditions underlying the model fund are constant. In other words, the progress of the fund from one year to the next will simply depend on the rate of salary escalation suffered. The population of the scheme is stable and the characteristics of deferred pensioners and pensioners deriving from that membership remain static. Since all benefits are based on pay, it is not surprising that the answers come out to be constant multiples of payroll. The same will be true for different levels of pension increase, providing that the rate by which the pensions fail to keep pace with inflation (or rise faster than inflation) is constant.

2.7 If investment returns are constant, and each of the fund (F), the benefit outgo (B) and the contribution income (C) is a constant multiple of payroll, which they will be in a stationary fund, then the following relationship must apply:

$$B = C + F\delta$$

where δ is the force of interest earned in excess of pay increases. This relationship has been derived before in the actuarial literature, and is fundamental to the understanding of pension scheme finance. At a very simple level it makes the obvious statement that benefits must derive either from contributions or from the investment return achieved on assets. At another level it illustrates that if the population is stable and the economic conditions are stable, so that B is constant, then there are an infinite number of different stable contributions, each with its own unique stable fund.

2.8 However, this stability is only reached at the expected values if the value of δ is actually achieved in practice. If, instead, a return of δ' is achieved then a variation must arise equal to $F(\delta' - \delta)$. The relationship between B, C and F will however still hold on the valuation rate of interest and the variation will need to be dealt with. If it is not removed immediately, the fund will grow or diminish, until a new stable level is eventually reached.

3. The Long-term Effect of Margins

3.1 To investigate the consequences in the long term, consider a pension scheme which has reached a stable position, with stable benefit outgo, B, fund, F, and contribution, C, on a valuation basis which assumes a force of interest in excess of pay escalation of δ. Suppose that instead a higher rate of return δ' is achieved; the fund will no longer be stable. Surplus will accrue, and will need to be removed over a period. Whilst that surplus is running off, more interest will be earned, and more surplus will arise.

3.2 Stability will only be resumed when two equations of value are obeyed. First, the equation between benefits, contributions and fund must work by reference to the rate of return actually achieved. That is, $B = C' + F'\delta'$. Secondly, the surplus in the fund $(F' - F)$ when removed over the amortisation period (say with annuity value, a) must result in the reduction in contribution, $(C - C')$, experienced.

We therefore have three identities:

$$B = C + F\delta$$

$$B = C' + F'\delta'$$

$$C - C' = (F' - F) \div a$$

From these formulae we can deduce that:

$$F' \div F = (1 - a\delta)/(1 - a\delta')$$

$$C' = C - F(\delta' - \delta)/(1 - a\delta')$$

3.3 Tables 3.1 and 3.2 show the results of applying these parameters to two valuation bases, using i (equivalent to δ) of 0.03 and 0.02.

3.4 The tables show a number of interesting features. If all surplus is removed instantaneously, the contribution rate is dependent on the valuation basis used, as well as the actual rate of interest earned. If a rate of return at the top end of our range is obtained, members contribute 5% of pay, and the investment return assumed in the valuation basis is only 2% above pay (a basis which would not be uncommon in the UK at the present time), then the ongoing contribution which will be required by the employers is very small or nil.

3.5 If the amortisation period is greater than zero, then as the investment return achieved increases above the return assumed in the basis, so the eventual contribution required falls quickly. Once again, if the returns achieved were in the range of 3% to 4% above pay increases (which we have suggested in our UK paper as a reasonable range for a best estimate), the use of an investment return of only 2% per annum above pay in the valuation basis is likely to mean, if members contribute 5% of pay, that the employers will never need to contribute again. Indeed, with longer periods of amortisation, no contributions at all would be required from either members or employers, and unless capital sums were removed, the fund could be expected to grow indefinitely.

3.6 At this stage it is worth drawing attention to the work which was done by Dufresne[(2)] in the mid 1980s. He showed that if the investment returns averaged the funding assumption in the long term, then there was an optimum period of amortisation. If the amortisation period was too short, excessive instability in contribution rates would result. If the amortisation period was too long, then the effect of random fluctuations could cause a random walk which would result in a drift in contribution rates. He concluded that, using rates of return consistent with typical valuation bases, the optimum period of amortisation was in the range 10 to 20 years. What the above analysis would show is that amortisation periods at the higher end of this range could be dangerous.

Table 3.1: Effect on Pension Funds of investment returns deviating from assumptions.

Method: Projected Unit

Key parameters: Investment return over pay, i = .03
Standard Fund F = 3.80 times payroll
Standard Contribution C = 11.08% of payroll

Ultimate Fund (F') or Contribution (C') Amortisation Value (a)		Investment return over pay actually achieved, i': .02	.025	.03	.035	.04	.045
0	F'	3.80	3.80	3.80	3.80	3.80	3.80
	C'	14.8	12.9	11.1	9.2	7.4	5.6
5	F'	3.59	3.69	3.80	3.91	4.03	4.15
	C'	15.2	13.2	11.1	8.9	6.5	4.0
10	F'	3.34	3.55	3.80	4.08	4.40	4.78
	C'	15.7	13.5	11.1	8.3	5.0	1.3
15	F'	3.01	3.36	3.80	4.37	5.14	6.23
	C'	16.4	14.0	11.1	7.3	2.2	-5.1
20	F'	2.57	3.07	3.80	4.98	7.21	12.99
	C'	17.2	14.7	11.1	5.2	-6.0	-34.8

Table 3.2: Effect on Pension Funds of investment returns deviating from assumptions.

Method: Projected Unit

Key parameters: Investment return over pay, i = .02
Standard Fund F = 4.33 times payroll
Standard Contribution C = 13.75% of payroll

Ultimate Fund (F') or Contribution (C') Amortisation Value (a)		Investment return over pay actually achieved, i': .02	.025	.03	.035	.04	.045
0	F'	4.33	4.33	4.33	4.33	4.33	4.33
	C'	13.75	11.6	9.5	7.4	5.3	3.3
5	F'	4.33	4.45	4.58	4.71	4.85	5.00
	C'	13.75	11.3	8.8	6.1	3.3	0.3
10	F'	4.33	4.61	4.93	5.29	5.71	6.20
	C'	13.75	10.9	7.8	4.1	-0.1	-5.0
15	F'	4.33	4.83	5.47	6.29	7.39	8.96
	C'	13.75	10.4	6.2	0.7	-6.7	-17.1
20	F'	4.33	5.17	6.40	8.38	12.13	21.85
	C'	13.75	9.6	3.4	-6.5	-25.2	-73.9

3.7 The figures in Tables 3.1 and 3.2 show clearly the danger in setting too low a rate of investment return in the valuation basis, unless surpluses or deficits are amortised over a very short period.

3.8 The Tables also show that, where a low valuation rate of interest is used, the greater the value of the annuity value for amortisation, the greater the size of fund which would be built up. Even with relatively short amortisation periods, and relatively small margins between the valuation rate of interest and the rate of return actually achieved, substantial excess funds build up. Thus, with an annuity value of 10, a margin in the rate of interest of 1% leads to extra funds of the order of 60% of payroll. The position is exacerbated if the valuation basis used is even more conservative. For example, suppose the investment return actually achieved was at the favourable end of our forecast, namely 4% per annum in excess of pay, whilst only 2% was used for valuation purposes. In this circumstance, the extra funds built up with a 10 year amortisation period amount to 140% of payroll - coupled with the extra assets retained compared with a fund valued on a 3% basis (4.33 less 3.80), the extra funds built up are nearly 200% of payroll.

4 UK Surplus Restrictions

4.1 The discussion on the size of funds which will be built up led directly in our UK paper to the question of what maximum would be imposed by the surplus requirements contained in the UK tax legislation. Although the particular form of legislation is peculiar to the UK the principles involved may be of wider interest. In the UK, the Income and Corporation Taxes Act 1988 (ICTA 1988) imposes certain restrictions on the level of funding which can be built up without loss of tax relief on part of the investment income. An actuarial method and set of assumptions have been prescribed by the Government Actuary for the purposes of the Act, which would generally be regarded as being at the extreme conservative end of the range of assumptions which might be adopted for funding purposes. For a fund to enjoy the full benefit of tax relief on investment income, the assets must not exceed 105% of the value of the past service liabilities on the prescribed basis. If they are found to exceed this level at a formal valuation, and action is not taken to reduce the funding level to below 105% over a five year period, part of the investment income will become taxable. The action to be taken can include a combination of improvements in benefits, reduction in contributions, or a repayment of surplus to the employer.

4.2 Under the model fund, assuming pension increases in line with cost of living, but not guaranteed, the reserves under the surplus basis would be 4.96 x payroll. Adding the 5% margin gives a maximum fund of 5.21 x payroll before any allowance for reductions in contribution is taken into account. The contribution rate under the ICTA 1988 basis would be 16.76%, so that the value of five years employer's contributions would be some 57% of payroll if the members contributed at the rate of 5% or 81% of payroll if the scheme were non-contributory. In consequence, the maximum fund which could be held would be 6.02 x payroll if the scheme were non-contributory, or 5.78 x payroll if the employees contributed at the rate of 5% of pay. If pension increases were guaranteed to match increases in the cost of living the ICTA 1988 basis would permit a maximum fund even greater, as the Government Actuary prescribes an even stronger basis for such cases.

4.3 It will be seen from Tables 3.1 and 3.2 that if a value of "a" of more than 10 is used, there is a very real danger that the maximum fund will eventually be breached. Yet again, there seem to be good reasons for ensuring that the valuation basis does not contain too many margins, and that amortisation is over a reasonably short period.

4.4 Tables 3.1 and 3.2 also show that if the funds do get to the multiple of 5½ to 6 times payroll, there becomes a substantial probability that no further contributions will ever be required and that the over funding will increase. The circumstances for this to happen are easily described. If the scheme is non-contributory, one would simply require a force of return δ', to be achieved where: $\delta' = B/F'$. With a non-contributory scheme, and a maximum fund of 6.02, the implication would be that it would only require a rate of investment return of 3.78% per annum above pay to be earned for no contributions to be required. If the scheme is contributory, say with rate of contribution of 5%, the equation then becomes:

$$\delta' = (B - .05)/F'$$

With a maximum fund of 5.78, this implies that a rate of return of just 3.04% per annum in excess of pay is required before no further contributions are needed. Even with a fund of 5.21, the rate of return required is just 3.38%.

4.5 These figures apply to a fully mature fund. Clearly, if the scheme is not yet mature the effect of taking a contribution holiday will be more helpful, and a higher rate of return will be required before this zero position is reached. However, many schemes in the UK are now super-mature, in the sense that the reduction in workforce during the 1980s has been so severe that the rate of pension in payment is currently running at far more than would be sustained by a mature scheme. In such schemes, the rate of return required before one finds that, once the fund has reached the maximum on the ICTA 1988 basis, no further contribution will ever be required, is much lower than given above. The position is even worse for schemes where pension increases to match the cost of living are guaranteed.

4.6 It may be thought that this would not matter unduly, on the grounds that a capital repayment could always be made from the scheme to the employer. However, it is not usually a simple matter in the UK to arrange a repayment, and the employer may need to share the surplus with the members of the scheme. What our analysis is showing is that once a scheme gets to the position where a repayment does need to be made to the employer in order for the surplus requirements not to be breached, it is quite likely that that position will persist and further repayments be required, even if the employer never makes a further contribution.

5 The Effects of Prudence on Accounting Requirements

5.1 Another important aspect of margins in funding assumptions arises in connection with the calculation of pension cost for the sponsoring company's accounts. In the UK this has to conform to Statement of Standard Accounting Practice Number 24 (SSAP 24), but again the issues may be of wider interest. In the interests of space we do not set out here a description of the requirements of SSAP 24. For present purposes it is assumed that the pension costs for SSAP 24 purposes are calculated consistently with the funding calculations, with surpluses or deficiencies amortised over the average future working lifetime of the active members of the fund. For SSAP 24 purposes the assumptions used have to represent a "best estimate" basis, which need not be the same as the funding basis.

5.2 Where the trustees, and perhaps the company, have chosen to fund for pensions on a prudent basis, there is a natural desire to adopt the same basis for SSAP24 purposes, on the grounds that the company will prefer the pension costs shown in its accounts not to differ from the payments actually made to the pension scheme. In consequence, there are likely to be many examples where an actuary has used a basis which he considers is perhaps ½% or even 1% per annum on the prudent side in the investment return assumed, and this basis is then used for SSAP24 purposes. What are the consequences of this in the long term?

5.3 In this circumstance, no provision or prepayment will be built up in the company balance sheet.

However, as shown in paragraph 3.8, the effect will be to create permanent surpluses in the pension fund of the order of 30% or 60% of payroll if the margin were ½% or 1% respectively, and a permanent reduction in contribution below the standard contribution rate of the order of 3% or 6% of payroll respectively.

5.4 An alternative might be to continue to use a prudent level of funding, but to extract surplus immediately even though the funding basis was regarded as satisfactory for SSAP 24 purposes. In that circumstance, no surplus would arise in the fund, but if the contribution paid were regarded as initially less than that required under SSAP 24, a provision would accumulate in the company accounts. In a similar manner to paragraph 3.2 above, it can be shown that that provision would eventually be the same amount as the surplus which would have accrued in the fund if the amortisation had been in accordance with SSAP 24, whilst the SSAP 24 cost would reduce accordingly - i.e. to the amounts shown in Tables 3.1 and 3.2. The SSAP24 cost would thus ultimately be below the funding contribution.

5.5 A further alternative would be to keep the prudent basis for funding purposes, but adopt a best estimate basis for SSAP 24 purposes, amortising over the SSAP 24 period on both the prudent funding basis and the SSAP24 basis. In this circumstance, the contribution rate paid would initially exceed the SSAP 24 cost, and a prepayment would be incurred in the company accounts. Again, it can be shown mathematically that the amount of that prepayment will eventually become the sum of the surplus in the pension fund on the valuation basis plus the excess of the standard fund on the valuation basis over the standard fund on the SSAP 24 basis.

5.6 The important conclusion which we reach from this analysis is that it is very difficult to prevent either large provisions or large prepayments being built up in company accounts, unless some means is found of reconciling both the needs and desires for a stronger fund than a 50-50 chance of solvency by trustees and company with the best estimate requirements of SSAP 24.

6 Reconciling Funding Strength with Accounting Requirement

6.1 It might be thought that our conclusion from the above analysis is that it must be right in all circumstances to fund on a best estimate basis. However, this is not the case. What we have been trying to show is that funding conservatively does involve the risk of over-funding, even against the levels desired, and that ways need to be found to avoid this happening. This is especially true for schemes which are already over-funded. It is highly desirable to seek methods which will give the company full credit for the amount of money in the fund, whilst at the same time meeting the natural desires of the trustees that, once a strong position has been built up, it should not be lightly taken away. Furthermore, there are a number of occasions when funding on a best estimate basis for past service could lead to insufficient margins on discontinuance if that should occur after one or two years of poor investment performance. In consequence, we are attracted to the idea of retaining a margin for caution in setting funding bases of ½% to 1% per annum on the investment returns. Indeed, because the range within which the best estimate might fall is so large, one can never be certain in advance that the best estimate has been correctly identified.

6.2 Our proposed method of reconciling these conflicting desires, which has been used in practice by a few actuaries, is to use a variation of the projected unit method whereby the standard fund is calculated by reference to the prudent funding basis, but the contribution rate set takes into account the additional interest which it is expected will be earned on the fund, using a best estimate basis. The intended result of this method is that surpluses should not build up, that a best estimate rate of interest is used for setting the contribution rate as required for SSAP24 purposes, whilst a strong fund is preserved which will satisfy trustees and members. The regular contribution under this approach would be the regular contribution rate by reference to the basis used for <u>funding</u>, less the additional interest which is expected to be earned by reference to best estimate assumptions on the standard fund held on the funding basis.

6.3 The approach becomes clear if one considers a new scheme just setting up. At the end of the first year, the reserves which will be required will be the reserves by reference to the basis used for past service. The contribution rate will therefore need to be set by reference to that basis. There would be no offsetting interest to be earned. In subsequent years, the contribution rate so set should be offset by the interest which is expected to be earned on the funds held. In practice, this would mean setting a contribution rate which one knew would gently fall over the years. For example, if the investment rate of return above pay was 2% for the purposes of funding past service, but 3% for the purposes of setting the contribution rate, the contribution rate in a new scheme with no past service benefits will initially be 13.75%. At the first valuation after three years, the fund built up would only be of the order of 40% of payroll, so the reduction in contribution at that time would be from 13.75% of pay to about 13.35% of pay. Eventually, over a large number of years, the contribution would fall to 9.42%. We would not regard such a pattern as being inconsistent with SSAP24, or the disclosure requirements, especially if the expected reduction was mentioned in the valuation report.

6.4 This method seems to us to have the merit of reconciling the natural desires of trustees, sponsors and auditors, without building up unnecessary provisions, prepayments or surpluses. It also allows all the detailed calculations to be made on just one basis. As the method involves using two different rates of interest, we propose calling the method the "Dual Interest Projected Unit Method".

7 CONCLUSION

7.1 As we have shown, problems can arise if the bases used for funding purposes contain too great a margin for caution compared with best estimate assumptions.

We believe that our suggested approach of removing year by year the excess of the expected investment return on a best estimate basis over the return assumed for funding purposes provides a sound method of addressing these problems.

REFERENCES

(1) "A Realistic Approach to Pension Funding" by P N Thornton MA FIA and A F Wilson BSc FIA, presented to the Institute of Actuaries 24 February 1992.

(2) "The Dynamics of Pension Funding". PhD Thesis by D Dufresne, 1986, City University, London.

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APPENDIX

The Model Fund

A1.1 The basic benefit of the model fund we have chosen is a pension payable from age 60 of one-sixtieth of final pay for each year of service. Age 60 has been chosen as representative of the average age at retirement, inclusive of early retirement at the request of the employer. It has been assumed that part of that pension would be commuted on the terms available under the 1989 Revenue code, that is to produce a lump sum of 2.25 per unit of pension before commutation at a rate of 12 for 1. (Allowance for commutation is essential if the size of fund which might be built up is to be studied.)

A1.2 It has been assumed that the pension, once it comes into payment, will be paid for 22 years certain. This period has been chosen to equate (broadly) the value of the annuity certain to the value of a pension payable from age 60 to an individual, for 5 years certain and thereafter for life, with a reversionary pension of one-half (before commutation) to a surviving spouse. Using PA(90) mortality rated down two years, 5% interest, and proportions married of .835 (men) and .6 (women), the values of the pension benefits are approximated by annuity certains of duration 21.5 (men) and 23.1 (women). Each deduction of a year in rating from the mortality table would increase the period of the annuity certain by 0.7, whilst each addition of 1% to the rate of interest would reduce the period of annuity certain to give equivalence by 0.4 years. The effect of changing the annuity certain by one year would be to change the figures quoted by about 3%, and would not invalidate the conclusions.

A1.3 No allowance has been made for mortality before retirement, and withdrawals have been allowed for in a stylised way. It has been assumed that members join at age 20, and leave at age 25, to be replaced by other members at age 25. In turn, those leave at age 30 and are replaced at the same age. Those members leave at age 40 and are replaced by other members of age 40, who survive to age 60. There are two basic reasons for choosing this pattern: first, it represents the career of a "typical" employee. Secondly, introducing other forms of exit will not change significantly the past service fund; the effect is mainly limited to the contribution rate.

A1.4 Confirmation that this pattern of employment is typical is drawn from two sources. First, various surveys which have been done in the past on the pattern of employment, including the General Household Survey. It has been assumed that each period of employment has been of sufficient duration to produce pension benefits. It is therefore appropriate to the position since 1988, when the vesting period was reduced to two years, and many companies introduced immediate entry to pension arrangements unless the joiner specifically requested otherwise. The fact that, before 1980 when the preservation requirements introduced by the Social Security Act 1973 first had full effect (or 1978 for contracted-out schemes), there was relatively little preservation, will mean that the model overstates the number of deferred pensions and amounts of deferred pension currently in existence, but it does give an indication of future trends.

A1.5 The idea that employees change jobs for the last time in their late thirties or early forties is borne out in practice. We have studied the past membership durations of members of a number of large pension schemes, and find that the past duration at age 55 is typically in the range of 15 to 20 years. Between age 55 and age 60 average past duration sometimes increases, but not as fast as a year for each year of age, and indeed sometimes decreases. The main reason for this would seem to be that early retirement, especially on redundancy, is weighted quite heavily towards those with long service, who will therefore not suffer such a large drop in income on retirement as others. This tendency is reinforced by the practice in some schemes for service-related supplementary credits to be given to encourage early retirement.

A1.6 The pattern of withdrawals before age 40 is less crucial, providing that the periods in each job are sufficient to give rise to a deferred pension. The overall rate of benefit commencing at age 60 of 53.3% (= 32/60) calculated under the model implies that the 20 years service before age 40 is equivalent to 12 years service in the final employment after allowing for the difference between pension increases and pay increases. If instead the pattern assumed was for changes in job every two years until age 40, the overall rate of benefit commencing at age 60 would reduce to 52.2% (= 31/60), but such a rapid turnover in staff is not supported by the evidence. Overall, it seems unlikely that different patterns of withdrawals would change the level of benefits (and therefore funds, and costs) by more than ± 10%.

A1.7 Because the withdrawals are stylised, the reserves at each age will not be typical of a real pension fund, but the overall levels of, and relationships between, actives, deferreds and pensioners should be. Thus, for a total active population of 40 (i.e. one for each age between 20 and 59 inclusive) the model gives the number of deferred pensioners (assuming no-one transfers) as 85, and the number of pensioners as 88 (four at each age, emanating from those who left at ages 25, 30, 40 and 60). Thus, at this rate of turnover the number of pensioners and deferred pensioners is each more than twice the number of actives. Clearly, these ratios depend crucially on the pattern of withdrawals, and it is therefore difficult to gauge the maturity of a scheme from relative numbers: a much better guide is the level of benefit outgo as a multiple of payroll.

A1.8 To keep matters simple, we have assumed in the model that the number of members is such that we can assume a density of one per age. To introduce a number of members, say N per age, would simply be to introduce a further parameter in the calculations which would cancel out at all stages.

A1.9 The absence of deaths and ill health retirements from the model will only cause distortions to the extent that the reserves on death or ill health retirement differ from the reserves held under the model. In practice, the value of past service benefits payable on ill health retirement is generally found to be slightly more than the reserve before retirement, whilst the value of spouse's benefits on death in respect of past service will not be very different from the reserve held, at the ages at which deaths occur. In consequence, the inaccuracies in the size of the reserves by ignoring these exits will be small: their main importance lies in the effect on contribution rates of providing the risk benefits of lump sums on death and notional years of service on ill health retirement and death. These costs have been ignored in the results below, which therefore understate the total cost of providing benefits, possibly by 1% - 2% of pay.

A1.10 Another advantage of the model is the investigation of the comparability of money purchase benefits with final salary benefits. Money purchase benefits are usually accumulated on the assumption that the member concerned will survive to retirement, leaving the provision of enhancement of benefits on death in service and ill health retirement to be separately funded. It therefore makes sense to equate money purchase and final salary benefits on this footing.

A1.11 Finally, no salary scale has been assumed. As is shown in Section 5.3, the error which is involved by ignoring the salary scale is relatively small, and mainly affects only those members joining and leaving by age 30. Indeed, since pay, when expressed as a multiple of the national average, tends to decline with age after about age 45, the effect of assuming no salary scale is to overstate the past service liabilities for active members somewhat, but to understate the value of deferred pensions. Overall, the effect is small.

A1.12 The results of the calculations at various rates of interest are summarised in Table A1.

A1.13 It will be found that the figures in the table satisfy the identity $B = C + F\delta$.

TABLE A1: Valuation Characteristics of Stationary Funds and how they vary with Investment Return

Rate of return in excess of pay increases (%)	0	1	2	3	4
Past service liabilities as multiple of payroll					
Pensioners	1.971	1.839	1.722	1.617	1.523
Deferred Pensioners	2.127	1.713	1.399	1.157	0.968
Actives (Past Service)	1.721	1.431	1.205	1.028	0.887
Total Past Service	5.819	4.983	4.326	3.802	3.378
Standard Contribution Rates (% of payroll)					
Entry Age	22.31	17.12	13.25	10.34	8.13
Projected Unit	22.31	17.36	13.75	11.08	9.07
Attained Age	25.04	20.70	17.28	14.56	12.38
Future Service Reserve under Entry Age Method					
(a) as multiple of payroll	0.187	0.233	0.249	0.248	0.238
(b) as percentage of active past service	11	16	21	24	27