Real Rates of Return
by John A. Jolliffe (United Kingdom)

(1) Most pension fund managers would now expect to be asked to submit their pension fund to the scrutiny of investment performance, so that the Company paymasters can determine whether they are securing a reasonable rate of return compared with the investment returns secured by other pension funds. The reason for the growth in the number of funds submitted to investment performance survey is not hard to find, since the very size of such pension funds and the important part which they play in the economy, in respect of both the nation as a whole and the Company which runs the fund, are sufficient justification for participation in investment performance measurement. Bad investment results mean that a company would have to pay more into its pension fund than would otherwise be the case to ensure its solvency by the actuarial standards adopted or in the absence of additional payment have to reduce its level of benefit, or pay lower pension increases.

(2) Company paymasters are also seeking the answer to an additional question. They want to know that their fund has secured a "real rate of return", when compared with the rise in the cost of living or pay rates, even if their investment performance has been good relative to the investment performance of other pension funds. There is often disbelief that a pension fund can have secured a real rate of return during a period of high inflation because of the tendency to compare the running yield secured from the fund with the rate of pay increase, forgetting that the capital value of the investments themselves may have changed over the period in question.

(3) We first need therefore to identify what we mean by "real rate of return". There are two main definitions. The first definition is that the real rate of return is equal to the difference between the rate of investment return secured by the fund, having regard both to its investment income and changes in its capital values, and the rate at which wages have increased over the same period. A second definition is that the real rate of return is equal to the difference between the rate of investment return secured by the fund and the rise in the cost of living over the same period. In my view, it is more important for a pension fund to aim to secure a real rate of return, where that return is measured by reference to rises in the wage rates paid to its employees, than where that return is measured by reference to the rise in the cost of living, since the growth in the liabilities of the pension fund to its existing employees are dependent upon the growth in its wage levels. Furthermore, investment managers of pension schemes, which have contracted out of the State Earnings Related pension Scheme will want to know that they have secured a greater real rate of return upon the investment of the contracted-out national insurance contribution abatement than that assumed by the Government Actuary in calculating the financial terms of contracting-out, since he will need to satisfy the Company paymasters that the decision taken to contract out of the State Earnings Related Pension Scheme, has been justified by the investment performance of the Company's pension fund.

(4) The contributions which a company pays to its pension fund are normally determined as a percentage of its payroll, and the amount of pension ultimately payable is also normally dependent upon an individual's rates of pay in his final year or years of employment. An investment manager might want to convince his Company paymaster that, if the company makes a contribution equal to, say, 10 per cent. of payroll in 1973, then the value of those contributions, together with interest and capital appreciation resulting therefrom accumulated to, say, 1983, would result in a value at least equal to 10 per cent. of the then payroll assuming a constant membership, or alternatively that if the Company pays a contribution equal to 10 per cent. of Mr. Blogg's salary, then the accumulated value of the contributions paid during the course of his employment should be equal to 10 per cent. of Mr. Blogg's final salary multiplied by the length of his contributory service. This notion would assume that Mr. Blogg follows an average wage career during his employment.

(5) It is possible to determine the real rate of return which a superannuation fund should have achieved by the use of a model fund. This model fund is assumed to be invested in a mixture of equities as measured by the Financial Times Actuaries All-Share Index and fixed interest investments as measured by a suitable government gilt-edged security.

Different returns may be obtained from the model by assuming different mixtures of equities and fixed interest securities, for example, a mixture of one-half equities, one-half fixed interest, and so on. It is then assumed that the model fund has received contributions, which are a fixed percentage of payroll during the period of years over which it is wished to assess the real rate of return, that no benefits have been paid and that payroll has increased in line with national average earnings. It is assumed that re-investment of investment income is made at the end of the month following its receipt, and that the expenses of re-investment amont to 4 per cent. in the case of equities and about ½ per cent. in the case of fixed interest securities. It is also assumed that each month is equal to one-twelfth of a year.

(6) The formula used to assess the rate of return secured by the model fund is determined on the assumption that contributions are paid at the beginning of each month on the basis of wages then being paid and invested immediately in the index. It is also assumed that investment income is re-invested in the same securities from which the income was derived. The formula therefore produces a cumulative model fund which you can start and stop at any time to cover any period over which you wish to measure the rate of return.

(7) Since the notional contributions which are invested in the model fund follow national average earnings, it becomes a relatively easy compound interest exercise to determine the average annual rate of interest secured on the invested contributions over the period in question, and then to compare this annual rate with the average annual rate which would have been required from the investment of the contributions in order for their cumulative value to equal exactly the rise in national average wages.

(8) The results may be derived from the following formula:—

Let, P_t = All-Share Actuaries Index at time t

D_t = All-Share Dividend Yield at time t

Q_t = Price of Treasury 8¾% 1997 at time t

E_t = Running Yield on Treasury 8¾% 1997 at time t

I_t = Wage Index at time t

Fund at time x invested in All-Share Index (A say)

$$= \frac{\sum_{t=0}^{t=x-1} \frac{P_x}{P_t} I_t(1+.96D_{t+1})(1+.96D_{t+2}) \ldots\ldots (1+.96D_x)}{}$$

Fund at time x invested in Fixed-Interest Index (B say)

$$= \frac{\sum_{t=0}^{t=x-1} \frac{Q_x}{Q_t} I_t(1+.995E_{t+1})(1+.995E_{t+2}) \ldots\ldots (1+.995E_x)}{}$$

Fund at time x required to match inflation (C say)

$$= x\, I_x$$

Equivalent Rate of Return

$$= i$$

Accumulated Contributions (D say)

$$= \sum_{t=0}^{t=x-1} I_t (1+i)^{x-t}$$

So we require the rates of interest i, which give D = A, D = B and D = C.

It may be seen from the above formulae that:—

$$A_X = (A_{X-1} + I_{X-1}).\frac{P_X}{P_{X-1}}(1 + .96D_X),$$

and likewise,

$$B_X = (B_{X-1} + I_{X-1}).\frac{Q_X}{Q_{X-1}}(1 + .995E_X),$$

and likewise,

$$D_X = (D_{X-1} + I_{X-1})(1 + i)$$

(9) The appropriate factors from which the calculated real rates of return may be derived can be built up in columnar form in the manner set out in Appendices A and B. The method also has the advantage that it can be continuously updated whenever new indices are published. In the case of the fixed interest index, it will be noted that interest is paid half-yearly on 1st March and 1st September on the stock taken, namely Treasury 8¾ 1997 Loan Stock, and due allowance has therefore been made for these dates in calculating interest paid, making allowance also for the time when the Stock goes ex-dividend on the market.

(10) A number of basic criticisms may be levelled at the use of the statistics derived by these formulae if the results are to be used generally. Firstly, market values vary considerably from time to time and there are points of time when market values are low, which would result in the fund appearing to secure a negative real rate of return whereas the following month a recovery of market values could indicate that a positive real rate of return might have been secured upon the money invested. Is there, therefore, a better yardstick to use than market values? Secondly, salaries of an individual company do not actually grow in line with national average wages but by reference to their own index of wages which is of course dependent upon the growth (or decline) of the Company in question. Is it still a relevant question to ask whether their fund had secured a real rate of return by reference to national average wages, when that rate of return should be measured by reference to their own wage index? Thirdly, contribution rates vary and pension funds do pay benefits to beneficiaries so that the incidence of contributions net of expenses received for pension funds are not in practice as smoothly arranged as in the model fund. Finally, investments are not actually made in the proportions which have to be assumed in the formula not in the exact proportions specified in the indices. The fund manager will want to know therefore how their actual performance has compared with the theoretical model assumed.

(11) The first criticism can be answered in two parts. It is important within either formula to retain an element of market values since purchases of securities are made at the market values which are relevant at the time of purchase. However, it is not so necessary to retain market values when we look at the ultimate value which a particular contribution has reached. It can be argued that in these circumstances it is better to adopt a common yardstick of measurement of the value of investments such as the concept of their "actuarial value", namely the discounted value of the income assumed to be derived from the particular set of investments allowing appropriately for the redemption monies and for anticipated rises in dividend levels. It will be noted that immediately an investment is made by the payment of a contribution, its actuarial value becomes the discounted value of the investments purchased, which is different from the amount of the contribution invested in the first place. A specific concept which can be used is to determine the actuarial value of the model fund at the end of the period over which you wish to measure the real rate of return, and compare this value with the cumulative value of the contributions paid, after they have been adjusted by their actuarial value.

(12) The market value of a particular set of securities at any point of time may be converted into their actuarial value by application of the following adjustment:

Equities

$$\text{Adjustment} = \frac{\text{Dividend Level at time t}}{\text{Valuation Rate of Interest} - \text{Dividend Increase Rate}}$$

Fixed Interest

$$\text{Adjustment} = \frac{8.75\, a^{(2)}_{97-t} + 100\, v^{97-t}}{\text{Mid-Market Price}}$$

In the case of the fixed interest securities, due allowance is made for the timing of the receipt of dividend income and redemption monies, as well as the time when the stock goes ex-dividend on the market.

(13) Variable results arise from the use of these actuarial adjustment formulae depending upon the underlying rate of interest and dividend increases used in their calculation.

For the purpose of this exercise, it is assumed that the underlying valuation rate of interest is 10 per cent. per annum, and that the underlying growth rate in dividend levels assumed is 5 per cent. per annum with annual stops or a continuous growth in dividend levels at the rate of 4.7619 per cent. per annum.

(14) Examples of the effect in this change in the formula may be seen in Appendix B, from which it appears that the real rate of return emerging from the use of this extension to the formula is much more even from year to year.

(15) It is quite clear that the pay rates of a particular company do not increase at the same rate as increases in national average earnings, as implied by the statistics which have been used for the purpose of this particular exercise. In the long-term, however, it is probable that the rate of growth in the earnings of the average employee in a particular company will be correlated to the growth in national average earnings when averaged over a long period, since otherwise the company's pay levels would progressively get out of step. However, an individual's pay would expect to grow during his career with the company on account of increasing age and responsibility, and this growth would be additional to the growth arising in respect of general pay levels.

(16) It would be possible to compare the actual growth rate in the average pay applicable to a particular company or group of employees with the increase applicable over the same period of time in respect of national average earnings. This adjustment on account of actual pay levels can then be applied to the rate of return required in order that the model fund should exactly equal that required over the period under measurement. The adjustment factor may be determined thus:—

$$\text{the square root of } \frac{\text{Salary of group at time t}}{\text{Salary of group at time o}} \times \frac{\text{National average wage at time o}}{\text{National average wage at time t}}$$

(17) This adjustment must be regarded as crude, since it would assume that the different growth rate in the salary levels has been evenly distributed over the period and that market values also have changed evenly over the period; the latter is most certainly untrue, and the adjustment will therefore be regarded in the light of all its deficiencies. Unless one can substitute an actual pay growth rate for the wage index, there is no alternative to the crude adjustment suggested above.

(18) The third criticism referred to in paragraph (8) relates to the fact that the amounts invested by pension funds are the aggregate of contributions and investment income net of expenditure. However, expenditure normally arises because benefits have to be paid. In the case of pensioners, the expenditure could be regarded as taking place out of his own individual fund which has been built up during the course of his career with the company and out of which his benefits are now being paid. We might be concerned to determine whether the fund that was needed to be set up to secure his benefits obtained a real rate of return during the course of its formation. In the case of benefits paid out to persons not retiring, for example, persons who change their employment or who die in service, then the fund is making an actuarial gain or loss, and this should not be regarded as effecting the calculation to determine

whether the fund secured a real rate of return over the period in question.

(19) One item of expenditure does, however, have an effect upon the results of this exercise, namely the cost of actually making the investments in the first place. In the case of equities, these expenses are stamp duty, commission and the jobber's turn, which, as in the case of the re-investment of investment income, we can take as 4 per cent. of the initial amount invested. In the case of fixed interest investments, however, the expenses of the initial investment will probably be somewhat less than the expenses arising in respect of the re-investment of the investment income; the allowance for expenses taken in the case of fixed interest investments is therefore 0.3 per cent. only. These adjustments are used in the examples set out in the Appendices.

(20) The fourth criticism referred to in paragraph (8) relates to the fact that the actual investment performance of a particular pension fund will not correspond exactly to that of the model fund. However, there are a number of investment performance measurement services available at present and in most cases the comparison of the actual performance of a specific pension fund is made with the performance of an index, normally the All-Share Index, so that a direct comparison can then be made with the return which would have been secured from an investment in the All-Share Index.

(21) The investment performance results produced by my own Firm show the following returns for the average pension fund participating in the service for the years 1971 to 1980:—

Calendar year to 31st December	Rate of return secured by the median fund relative to the All-Share Index
	%
1971	—6.6
1972	—3.7
1973	8.4
1974	45.8
1975	—32.7
1976	2.4
1977	—2.5
1978	—2.9
1979	—2.5
1980	—4.8

(22) It should be noted that the rates of return set out in the previous paragraph mean that, for example in the case of 1971, the market value of the average fund at 31st December 1971 was 93.4 per cent. of what it would have been had the fund been entirely invested at the All-Share Index. If the rates of return are accumulated over the ten years specified, it will be noted that the average fund would have a market value of some 86.1 per cent. of the market value which would have been applicable had the investments made been entirely in the All-Share Index.

(23) The investment performance results set out in paragraph (21) suffer from the disadvantage that only the performance of the UK Stock Exchange securities held by the pension funds included have been measured against the All-Share Index, overseas securities and property investments have been excluded. No doubt, it is possible to make some further adjustments to the examples set out in the Appendices to allow for this deficiency, but such adjustments would be open to criticism because in the first place, the performance of overseas securities has varied considerably depending upon their location, and also the operation of the "dollar premium", which until recently had to be paid under Exchange Control regulations on any UK investments being made overseas. In the second place, the performance of property investments depends very much upon the value which the valuer feels disposed to place upon the investments and therefore values are not subject to the disciplines of a market place.

(24) The tables which provide the statistics from which the real rates of return can be determined are set out in Appendices A and B following this paper. In addition, a number of examples are also set out which demonstrate how the tables can be used for this purpose.

(25) The tables may be retained for permanent use merely by adding on the new statistics for the All-Share Index, Fixed Interest Index and National Wage increases, whenever the relevant statistics are published, so that continued updating may be easily achieved. For convenience, the tables start as at 1st January 1971, but prior statistics can be included if desired, although this will alter the cumulative statistics shown in the tables. Some complications would arise if it is desired to measure a period beginning before the availability of the Financial Times Actuaries All-Share Index or before the issue of the relevant Treasury Loan used, although providing that care is taken to ensure that an appropriate conversion rate is introduced at the point of time at which the index measurement used is changed, then the whole process can revert to the continuous process as shown in the Appendices.

(26) A further refinement which can be allowed for relates to the matter referred to in paragraph (3), namely the rate of return relative to the rise in the cost of living, rather than rises in national average earnings. The adjustment required in this case is equal to:—

$$\text{the } t^{th} \text{ root of } \frac{\text{Cost of living index at time t}}{\text{Cost of living index at time o}} \times \frac{\text{Wage index at time o}}{\text{Wage index at time t}}$$

The adjustment may then be applied to the rate of return required to provide the fund which exactly matches the growth in wage levels, and the difference between the adjusted figure and the rate of return actually achieved by the model fund can then be regarded as the real rate of return secured by reference to the rise in the cost of living.

(27) The formulae may be easily adapted for use with other statistics relating to countries other than the UK. The results set out in the Appendices should be regarded solely as of interest and do not purport to throw any light on the likely trends of future real rates of return, inflation, pay increase rates, nor any correlation which these factors are assumed to have with increases in dividend levels or returns from fixed interest investments.

(28) The calculations are easily adapted for the use of computers, and indeed are more suited to computerisation than other means of calculations. For the sake of this paper, however, it has been thought appropriate to set out statistics and examples which can be followed by the use of a desk or pocket calculator.

Appendix A

Date	Wage Index	All Share Index	Dividend Yield	Fixed Interest Index	Equity Model Fund	Adjustment A	Fixed Interest Model Fund	Adjustment B	Contributions Accumulated at 1½% per month	Contributions Accumulated at 1¼% per month
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1.1.1971	114.2	136.26	4.39	93.00	—	1.000	—	—	—	—
1.2.	114.6	137.03	4.34	91.00	115.2	1.009	111.7	0.978	115.9	115.6
1.3.	115.8	133.68	4.48	93.50	225.0	0.988	237.9	1.052	234.0	233.1
1.4.	116.0	142.68	4.20	95.50	365.0	1.058	361.3	1.075	355.0	353.3
1.5.	117.6	159.17	3.79	96.00	538.2	1.184	479.8	1.080	478.1	475.1
1.6.	117.8	164.82	3.69	95.00	681.1	1.230	591.1	1.069	604.6	600.1
1.7.	119.4	167.95	3.64	97.50	816.4	1.257	727.6	1.097	733.3	726.9
1.8.	120.7	180.40	3.41	92.50	1008.0	1.353	803.6	1.041	865.4	856.9
1.9.	121.1	181.01	3.40	95.00	1135.6	1.362	987.1	1.118	1000.9	989.8
1.10.	122.0	181.25	3.40	101.00	1261.7	1.367	1178.2	1.189	1138.9	1124.8
1.11.	122.2	177.75	3.48	101.50	1360.8	1.345	1306.6	1.195	1279.8	1262.4
1.12.	123.3	182.53	3.40	105.25	1527.0	1.384	1481.6	1.239	1423.0	1401.9
1.1.1972	124.4	193.39	3.25	106.00	1753.1	1.471	1616.3	1.247	1569.5	1544.3
1.2.	126.3	201.87	3.13	106.00	1964.7	1.539	1740.7	1.247	1719.3	1689.5
1.3.	128.3	213.93	2.99	105.50	2221.2	1.635	1929.7	1.293	1873.3	1838.5
1.4.	129.4	217.69	2.96	101.50	2396.5	1.667	1980.0	1.244	2031.6	1991.4
1.5.	130.5	226.50	2.88	104.00	2634.1	1.739	2161.3	1.274	2193.4	2147.3
1.6.	132.1	217.39	3.04	96.50	2659.9	1.673	2126.5	1.183	2358.8	2306.3
1.7.	132.8	206.42	3.22	92.50	2657.9	1.593	2165.0	1.134	2528.3	2468.9
1.8.	134.1	218.40	3.07	92.00	2960.0	1.689	2285.4	1.127	2701.0	2634.2
1.9.	137.8	222.68	3.03	91.50	3162.3	1.727	2514.5	1.175	2877.6	2802.9
1.10.	140.2	199.00	3.40	93.00	2957.2	1.547	2695.8	1.194	3060.6	2977.4
1.11.	141.7	206.24	3.33	93.50	3218.7	1.608	2851.2	1.200	3248.8	3156.6
1.12.	142.5	222.31	3.09	93.50	3631.2	1.737	2992.9	1.200	3441.4	3339.5
1.1.1973	143.1	218.18	3.15	93.00	3712.9	1.709	3118.7	1.194	3637.7	3525.6
1.2.	144.4	196.48	3.52	90.00	3482.2	1.544	3156.5	1.155	3837.5	3714.5
1.3.	145.9	188.98	3.69	90.00	3498.5	1.489	3453.6	1.211	4041.6	3907.2
1.4.	148.3	189.56	3.69	88.50	3666.4	1.498	3539.5	1.191	4250.3	4103.7
1.5.	149.5	191.50	3.65	89.00	3865.0	1.518	3708.6	1.198	4464.6	4305.2
1.6.	152.8	192.57	3.63	89.50	4048.6	1.531	3879.8	1.204	4683.3	4510.4
1.7.	153.4	191.95	3.67	89.00	4200.2	1.530	4010.1	1.198	4908.6	4721.5
1.8.	154.2	181.86	3.89	80.00	4137.6	1.454	3742.5	1.077	5138.0	4935.8
1.9.	155.8	175.57	4.04	78.50	4156.7	1.409	4027.3	1.115	5371.6	5153.6
1.10.	157.8	181.43	3.93	78.50	4470.5	1.460	4183.1	1.115	5610.3	5375.8
1.11.	158.8	186.83	3.82	80.50	4780.6	1.508	4451.4	1.143	5854.6	5602.7
1.12.	160.9	162.52	4.41	75.50	4311.9	1.317	4323.9	1.072	6103.6	5833.6
1.1.1974	154.0	149.76	4.77	75.50	4137.3	1.218	4484.8	1.072	6358.5	6069.4
1.2.	156.8	142.96	5.02	67.00	4112.9	1.167	4116.5	0.952	6610.1	6301.2
1.3.	166.6	149.27	4.82	67.00	4475.4	1.223	4540.8	1.014	6868.4	6538.7
1.4.	165.2	118.31	6.17	62.50	3697.3	0.974	4391.2	0.945	7140.6	6789.1
1.5.	174.9	128.30	6.00	63.50	4208.8	1.062	4629.3	0.961	7415.4	7041.3
1.6.	177.5	117.54	6.53	67.00	4037.0	0.978	5069.0	1.014	7704.1	7306.4
1.7.	181.0	105.35	7.31	62.00	3799.5	0.882	4855.0	0.938	7999.8	7577.4
1.8.	185.7	100.43	7.64	59.50	3817.8	0.846	4832.9	0.900	8303.6	7855.4
1.9.	188.8	88.15	8.71	58.00	3538.5	0.747	5245.7	0.943	8616.6	8141.6
1.10.	191.9	76.91	10.15	60.50	3278.4	0.657	5668.7	0.984	8937.5	8434.5
1.11.	199.2	78.95	9.97	55.50	3590.8	0.680	5376.3	0.903	9266.3	8734.3
1.12.	207.7	66.66	11.79	56.00	3230.2	0.580	5625.7	0.911	9607.5	9045.1
1.1.1975	205.6	66.89	11.71	54.00	3482.1	0.587	5625.1	0.878	9962.4	9368.5
1.2.	210.1	102.13	7.66	59.00	5665.0	0.902	6370.6	0.959	10320.5	9693.8
1.3.	212.7	126.44	6.21	62.50	7309.6	1.122	7441.1	1.807	10688.6	10027.7
1.4.	216.2	118.32	6.70	67.50	7077.0	1.056	8266.1	1.174	11064.8	10368.4
1.5.	220.8	139.59	5.93	61.00	8645.1	1.251	7665.5	1.061	11450.2	10716.9
1.6.	223.4	145.31	5.71	64.00	9271.3	1.309	8274.1	1.113	11846.1	11074.4
1.7.	230.9	128.21	6.51	64.50	8421.0	1.161	8563.9	1.122	12250.5	11439.0
1.8.	233.4	124.03	6.76	64.00	8415.1	1.129	8726.6	1.113	12668.7	11815.8
1.9.	237.6	140.92	5.97	65.50	9873.2	1.289	9673.6	1.215	13095.6	12199.8
1.10.	239.8	144.66	5.91	65.00	10428.2	1.329	9924.8	1.206	13533.2	12592.9
1.11.	241.1	150.22	5.70	62.50	11128.5	1.387	9773.7	1.159	13979.6	12993.1
1.12.	247.2	152.38	5.65	63.00	11585.2	1.413	10094.9	1.169	14434.0	13399.6
1.1.1976	248.1	158.08	5.47	64.50	12328.8	1.472	10588.4	1.197	14901.4	13817.4
1.2.	250.1	172.64	5.03	68.50	13790.5	1.614	11508.5	1.271	15376.8	14241.3
1.3.	253.7	167.12	5.22	67.00	13648.5	1.569	12232.5	1.324	15861.3	14672.6
1.4.	254.5	164.61	5.35	66.50	13752.0	1.552	12393.0	1.314	16356.7	15112.8
1.5.	258.7	169.39	5.26	67.50	14473.8	1.604	12837.7	1.334	16860.4	15559.4
1.6.	261.1	157.82	5.77	67.00	13789.6	1.501	12999.4	1.324	17375.8	16015.9
1.7.	263.1	155.41	5.91	68.50	13901.6	1.485	13557.3	1.353	17901.5	16480.4

Appendix A (cont'd)

Date	Wage Index	All Share Index	Dividend Yield	Fixed Interest Index	Equity Model Fund	Adjustment A	Fixed Interest Model Fund	Adjustment B	Contributions Accumulated at 1½% per month	Contributions Accumulated at 1¼% per month
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1.8.	267.1	151.37	6.21	64.00	13865.0	1.454	12912.5	1.264	18437.1	16952.8
1.9.	267.4	144.05	6.55	64.00	13519.1	1.391	14057.9	1.350	18984.7	17435.2
1.10.	269.8	135.00	7.12	61.00	12994.0	1.311	13653.8	1.287	19540.9	17923.8
1.11.	272.8	119.92	8.03	59.00	11857.9	1.172	13467.1	1.245	20107.9	18421.1
1.12.	275.3	128.99	7.53	61.50	13126.8	1.268	14322.1	1.298	20686.4	18927.5
1.1.1977	278.1	151.96	6.42	65.00	15869.7	1.502	15428.1	1.371	21276.1	19442.9
1.2.	279.1	166.52	5.86	67.00	17778.0	1.653	16189.5	1.414	21877.5	19967.5
1.3.	282.8	170.77	5.77	68.00	18603.4	1.703	17766.3	1.527	22489.0	20499.7
1.4.	282.1	176.51	5.69	76.00	19609.9	1.769	20172.5	1.706	23113.3	21042.3
1.5.	284.7	181.21	5.65	73.50	20514.0	1.824	19781.7	1.650	23746.4	21590.9
1.6.	285.6	186.36	5.57	73.50	21485.1	1.884	20066.4	1.650	24391.5	22149.1
1.7.	286.5	190.69	5.53	73.25	22375.1	1.936	20282.8	1.644	25047.3	22715.1
1.8.	288.8	185.71	5.76	69.25	22171.5	1.895	19446.1	1.555	25713.8	23289.1
1.9.	292.1	206.54	5.25	72.50	25084.4	2.116	21883.5	1.725	26392.6	23872.6
1.10.	295.7	224.45	4.89	89.50	27684.9	2.308	27375.3	2.130	27085.0	24466.8
1.11.	301.3	219.70	5.03	86.75	27498.7	2.269	26820.8	2.065	27791.4	25072.0
1.12.	304.1	208.45	5.37	82.00	26489.7	2.162	25637.0	1.951	28514.1	25690.5
1.1.1978	306.9	214.53	5.28	87.50	27691.8	2.234	27681.1	2.082	29250.5	26319.5
1.2.	311.7	203.94	5.57	81.00	26735.2	2.133	25908.9	1.928	30000.8	26959.3
1.3.	314.5	193.37	5.92	79.50	25766.6	2.032	27127.4	1.996	30767.1	27611.8
1.4.	323.7	205.27	5.64	79.00	27811.0	2.167	27269.3	1.983	31547.9	28275.4
1.5.	325.9	208.45	5.62	76.50	28699.0	2.211	26719.8	1.920	32349.6	28956.6
1.6.	332.5	216.74	5.46	76.00	30311.0	2.309	26869.0	1.908	33165.7	29648.5
1.7.	332.9	210.67	5.65	77.00	29920.0	2.254	27559.4	1.933	34000.6	30355.8
1.8.	334.9	224.85	5.39	74.75	32428.4	2.416	27077.2	1.876	34848.5	31072.3
1.9.	339.7	229.66	5.32	74.75	33606.6	2.478	28989.0	1.986	35711.2	31799.8
1.10.	344.6	228.35	5.43	75.25	33899.3	2.475	29524.9	1.999	36591.7	32541.2
1.11.	344.6	219.40	5.69	75.00	33051.5	2.389	29770.3	1.992	37490.3	33296.9
1.12.	349.8	222.56	5.66	74.50	34030.5	2.434	29914.1	1.979	38402.4	34062.0
1.1.1979	344.9	220.22	5.79	75.00	34176.4	2.420	30467.0	1.992	39333.5	34842.0
1.2.	355.9	223.62	5.71	68.00	35214.4	2.468	27936.1	1.806	40273.6	35626.7
1.3.	369.2	237.52	5.44	73.50	37945.7	2.633	32368.7	2.068	42232.3	37261.6
1.4.	367.5	266.28	4.94	81.50	43124.0	2.964	36301.2	2.293	41238.9	36432.4
1.5.	372.8	279.46	4.77	82.50	45818.4	3.122	37118.7	2.321	43238.7	38099.5
1.6.	385.9	261.27	5.16	79.75	43362.9	2.931	36241.7	2.244	44265.7	38953.2
1.7.	387.5	247.88	5.32	77.50	41683.3	2.793	35594.3	2.181	45321.4	39830.8
1.8.	385.2	237.14	6.00	76.25	40441.2	2.684	35401.4	2.145	46394.5	40721.1
1.9.	384.8	247.34	5.97	77.50	42785.8	2.813	38394.3	2.303	47481.4	41620.1
1.10.	401.6	254.73	5.96	77.25	44672.5	2.911	38654.1	2.296	48584.2	42529.9
1.11.	408.3	237.47	6.49	73.75	42238.1	2.728	37286.1	2.192	49720.6	43468.2
1.12.	417.0	231.28	6.76	69.00	41759.4	2.671	35266.7	2.050	50880.8	44424.9
1.1.1980	415.9	229.79	6.87	69.50	42135.0	2.669	35942.2	2.065	52067.3	45402.5
1.2.	424.2	251.70	6.33	69.00	46844.0	2.938	36096.6	2.050	53270.5	46391.1
1.3.	435.5	263.81	6.15	67.00	49786.2	3.094	37739.5	2.120	54500.1	47400.5
1.4.	439.9	240.38	6.91	67.25	46014.3	2.835	38317.4	2.128	55759.6	48433.9
1.5.	441.7	249.48	6.77	71.25	48473.9	2.958	41062.6	2.255	57042.5	49484.8
1.6.	458.9	243.60	7.00	72.00	48030.2	2.905	41941.2	2.279	58346.5	50550.5
1.7.	462.1	269.54	6.40	73.75	53927.2	3.231	43430.6	2.334	59687.4	51647.1
1.8.	465.4	282.29	6.18	74.25	57243.7	3.400	44190.3	2.350	61051.8	52760.5
1.9.	470.2	282.75	6.22	71.00	58090.7	3.423	45291.9	2.385	62439.9	53891.3
1.10.	469.3	290.26	6.12	74.25	60410.7	3.531	47856.8	2.494	63853.8	55041.0
1.11.	474.9	306.26	5.81	76.25	64534.7	3.743	49627.8	2.561	65287.9	56204.1
1.12.	482.4	307.33	5.78	76.50	65538.1	3.773	50267.0	2.569	66749.3	57387.5
1.1.1981	479.2	291.99	6.10	74.50	63031.3	3.602	49422.6	2.502	68240.2	58593.3
1.2.	483.1	288.97	6.19	71.00	63164.9	3.583	47557.4	2.385	69750.2	59810.9
1.3.	485.6	304.16	5.84	71.00	67306.7	3.789	50956.4	2.531	71286.8	61047.7
1.4.	489.5	309.73	5.78	74.50	69352.9	3.876	53977.8	2.656	72848.9	62302.5
1.5.	490.7	332.18	5.39	73.00	75227.8	4.175	53370.7	2.602	74438.5	63576.9
1.6.	500.3	315.58	5.70	72.00	72262.6	3.984	53123.6	2.567	76053.2	64868.4
1.7.	503.7	320.57	5.66	70.50	74248.2	4.065	52506.7	2.513	77701.8	66185.8
1.8.	518.7	319.42	5.67	65.50	74821.5	4.069	49250.8	2.335	79378.5	67523.1
1.9.	520.4	334.63	5.43	62.25	79270.6	4.282	50573.2	2.374	81095.7	68892.4

Notes on tables

(1) The Equity Model Fund has been determined using the formulae set out in paragraph (8).

(2) Adjustment A is the factor which enables the Equity Model Fund to be established for intermediate months which are not at the beginning or end of the table. The fomula is
$(6) = (6)_{-1} \times (2) \times (1 + .96(3))/(2)_{-1}$

(3) The fixed Interest Model Fund set out in column (7) has been calculated by the use of the following formula:—

$\left[(7)_{-1} + (1)_{-1}\right] (4)/(4)_{-1}$, with the addition every March or September as appropriate, of an amount equal to $(7)_{-1} \times 8.75 \times .995/2(4)_{-1}$.

Examples

(1) What is the likely real rate of return, allowing for expenses, which would have been secured over the period from 1st January 1971 to 1st September 1981 of a model fund invested two-thirds in equities and one-third in fixed interest, which is a period of 128 months?

Model Fund Invested in All-Share Index	$= 79270.6 \times .96$
Model Fund Invested in Fixed Interest Index	$= 50503.2 \times .997$
Market Value of Model Fund at 1.9.81	$= 67540.3$
Model Fund accumulated at 1½% per month	$= 81095.7$
Model Fund accumulated at 1¼% per month	$= 68892.4$
Fund Required at 1.9.81 to match Wage Increase	$= 128 \times 520.4$
	$= 66611.2$

To determine rate of return secured on model fund

let $(81095.7)^x (68892.4)^{1-x}$	$= 67540.3$
$x \log 81095.7 + (1-x) \log 68892.4$	$= \log 67540.3$
$4.908998x + 4.838171 (1-x)$	$= 4.829563$
	$x = .1215371$
Rate of Return	$= 1.25 - .1215371 \times .25$
	$= 1.219616\%$ per month
	or $= 15.66\%$ per annum

To determine rate required to match wage increases

$(81095.7)^x (68892.4)^{1-x}$	$= 66611.2$
$4.908998x + 4.838171 (1-x)$	$= 4.823547$
	$x = -.206475$
Rate of Return required	$= 1.25 - .206475 \times .25$
	$= 1.198381\%$ per month
	or $= 15.37\%$ per annum

The real rate of return secured by the model fund over the period $= 15.66\% - 15.37\%$
$= 0.29\%$

(2) What is the likely real rate of return secured over the period 1st October 1973 to 1st February 1980 of a model fund invested one-half in equities and one-half in fixed interest, which is a period of 76 months?

Model Fund Invested in All-Share Index
$= 46844.0 - 4470.5 \times \frac{2.938}{1.460} = 37847.9 \times .96$

Model Fund Invested in Fixed Interest Index
$= 36096.6 - 4183.1 \times \frac{2.050}{1.115} = 28405.7 \times .997$

Market Value of Model Fund at 1.2.80 $= 32327.2$

Model Fund accumulated at 1½% per month
$= 53270.5 - 5610.3 \times (1.015)^{76} = 35876.3$

Model Fund accumulated at 1¼% per month
$= 46391.1 - 5375.8 \times (1.0125)^{76} = 32572.5$

Fund required at 1.2.80 to match $= 76 \times 424.2 = 32239.2$

To determine rate of return secured on model fund

let $(35876.3)^x (32572.5)^{1-x}$	$= 32327.2$
$x \log 35876.3 + (1-x) \log 32572.5$	$= \log 32327.2$
$4.554808x + 4.512851 (1-x)$	$= 4.509568$
	$x = -.078247$
Rate of Return	$= 1.25 - .078247 \times .25$
	$= 1.230438\%$ per month
	or $= 15.81\%$ per annum

To determine rate required to match wage increases

$(35876.3)^x (32572.5)^{1-x}$	$= 32239.2$
$4.554808x + 4.512851(1-x)$	$= 4.508384$
	$x = -.106463$
Rate of Return required	$= 1.25 - .106463 \times .25$
	$= 1.223384\%$ per month
	or $= 15.71\%$ per annum

The real rate of return secured by the model fund over the period $= 15.81\% - 15.71\%$
$= 0.10\%$

(3)(a) What is the likely real rate of return secured over the period 1st January 1971 to 1st January 1981 of a fund whose investment performance closely resembles that of the average of pension funds over that period?

Model Fund Invested in All-Share Index	$= 63031.3 \times .96$
	$= 60510.0$
Model Fund accumulated at 1½% per month	$= 68240.2$
Model Fund accumulated at 1¼% per month	$= 58593.3$
Fund Required at 1.1.81 to match wage increases	$= 120 \times 479.2$
	$= 57504$

To determine rate of return secured on model fund

let $(68240.2)^x (58593.3)^{1-x}$	$= 60510.0$
$4.834040x + 4.767848(1-x)$	$= 4.781827$
$.066192x$	$= .013979$
	$x = .211189$
Rate of Return	$= 1.25 + .211189 \times .25$
	$= 1.302797\%$ per month
	$= 16.80\%$ per annum

Rate of Return secured by average pension fund over the ten year period by reference to the All-Share index, see paragraph (2), is 0.861. The Rate of Return secured is y, say, where,

$(1+y)^{10}$	$= (1.1680)^{10} \times .861$
$1+y$	$= 1.1506$
Rate of Return	$= 15.06\%$ per annum

To determine rate required to match wage increases

$(68240.2)^x (58593.3)^{1-x}$	$= 57504$
$4.834040x + 4.767848(1-x)$	$= 4.759698$
$.066192x$	$= -.008150$
	$x = -.123127$
Rate of Return	$= 1.25 - .123127 \times .25$
	$= 1.219218\%$ per month
	or $= 15.65\%$ per annum

A pension fund performing closely to the average of pension funds over the ten year period 1st January 1971 to 31st December 1980 will therefore have secured a negative rate of return relative to the wage index of 0.59% per annum.

(b) By how much over the same period has the average annual wage increase exceeded the average annual cost of living increase?

See Paragraph 26

$$10\sqrt{\frac{277.3}{76.6} \times \frac{114.2}{479.2}}$$

$= .985$ or $1/1.0149$

The average annual wage increase has exceeded the average annual cost of living increase by 1.49%, and therefore the real rate of return secured by the model fund over the ten year period is 0.9% per annum (i.e. 1.49% — 0.59%).

Appendix B

Date	Actuarial Adjustment to All-Share Index	Actuarial Adjustment to Fixed Interest Index	Contributions actuarially adjusted for All-Share Index and accumulated at 1½% per month	Contributions actuarially adjusted for All-Share Index and accumulated at 1¼% per month	Contributions actuarially adjusted for fixed interest index and accumulated at 1½% per month	Contributions actuarially adjusted for fixed interest index and accumulated at 1¼% per month	Cost of Living Index
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.1.1971	0.878	93.345	—	—	—	—	76.6
1.2.	0.868	89.749	101.8	101.5	116.3	116.1	77.1
1.3.	0.896	90.465	204.3	203.5	232.8	231.9	77.7
1.4.	0.840	91.186	312.6	311.1	350.0	348.3	79.4
1.5.	0.758	91.913	416.2	413.7	467.7	464.8	79.9
1.6.	0.738	92.646	513.0	509.1	589.0	584.6	80.4
1.7.	0.728	93.385	608.9	603.5	714.4	708.2	80.9
1.8.	0.682	89.789	706.2	699.0	841.2	832.9	81.0
1.9.	0.680	90.505	800.4	791.1	972.8	961.9	81.1
1.10.	0.680	91.227	896.0	884.4	1104.5	1090.7	81.5
1.11.	0.696	91.954	993.6	979.4	1232.9	1215.9	82.0
1.12.	0.680	92.688	1094.9	1077.8	1363.7	1343.2	82.4
1.1.1972	0.650	93.427	1196.4	1176.1	1494.4	1470.0	82.9
1.2.	0.626	89.831	1296.4	1272.7	1628.1	1599.4	83.3
1.3.	0.598	90.548	1396.1	1368.7	1761.2	1727.7	83.6
1.4.	0.592	91.270	1494.9	1463.5	1899.3	1860.8	84.4
1.5.	0.576	91.997	1595.1	1559.3	2045.9	2001.9	84.8
1.6.	0.608	92.731	1695.3	1654.9	2193.8	2143.8	85.3
1.7.	0.644	93.470	1802.3	1756.9	2355.6	2294.1	85.6
1.8.	0.614	89.875	1916.1	1865.5	2527.1	2463.7	86.3
1.9.	0.606	90.592	2028.4	1972.2	2698.0	2627.2	86.8
1.10.	0.680	91.314	2143.6	2081.4	2876.9	2798.1	88.0
1.11.	0.666	92.043	2272.5	2203.9	3059.8	2972.5	88.3
1.12.	0.618	92.777	2402.4	2327.0	3247.3	3150.9	88.7
1.1.1973	0.630	93.516	2527.8	2445.3	3439.5	3333.4	89.3
1.2.	0.704	89.922	2657.2	2567.1	3637.1	3520.8	89.9
1.3.	0.738	90.639	2800.3	2702.1	3838.1	3710.9	90.4
1.4.	0.738	91.362	2951.6	2844.9	4044.9	3906.0	92.1
1.5.	0.738	92.090	3106.9	2991.3	4260.9	4109.9	92.8
1.6.	0.726	92.824	3264.3	3139.2	4481.8	4317.9	93.3
1.7.	0.734	93.565	3425.9	3290.8	4709.9	4532.3	93.7
1.8.	0.778	89.970	3591.5	3445.9	4944.3	4752.2	94.0
1.9.	0.808	90.688	3767.2	3610.4	5194.4	4987.2	94.8
1.10.	0.786	91.411	3951.5	3783.0	5455.0	5231.8	96.7
1.11.	0.764	92.140	4136.6	3955.9	5723.4	5483.3	97.4
1.12.	0.882	92.875	4321.8	4128.2	5993.7	5735.8	98.1
1.1.1974	0.954	93.615	4530.7	4323.5	6284.5	6007.9	100.0
1.2.	1.004	90.021	4747.8	4526.3	6572.6	6276.4	101.7
1.3.	0.964	90.739	4978.8	4742.2	6885.0	6568.1	102.6
1.4.	1.234	91.463	5216.5	4964.1	7217.3	6878.7	106.1
1.5.	1.200	92.192	5501.6	5232.6	7571.0	7209.4	107.6
1.6.	1.306	92.927	5797.2	5510.5	7942.3	7556.7	108.7
1.7.	1.462	93.668	6119.4	5814.1	8311.3	7900.4	109.7
1.8.	1.528	90.075	6479.8	6154.7	5713.5	8276.0	109.8
1.9.	1.742	90.793	6865.0	6518.9	9129.5	8664.1	111.0
1.10.	2.030	91.517	7301.8	6933.4	9566.5	9071.6	113.2
1.11.	1.994	92.247	7806.8	7414.5	10004.6	9478.9	115.2
1.12.	2.358	92.982	8327.0	7909.4	10490.7	9932.7	116.9
1.1.1975	2.342	93.724	8949.0	8504.1	10998.1	10406.0	119.9
1.2.	1.532	90.131	9572.0	9097.9	11525.3	10897.4	121.9
1.3.	1.242	90.849	10042.3	9537.6	12023.9	11358.6	124.3
1.4.	1.340	91.574	10461.1	9924.2	12518.1	11813.6	129.1
1.5.	1.186	92.304	10912.0	10341.6	13003.6	12258.2	134.5
1.6.	1.142	93.040	11341.5	10736.0	13537.8	12749.7	137.1
1.7.	1.302	93.782	11770.6	11128.6	14070.5	13237.9	138.5
1.8.	1.352	90.190	12252.3	11572.1	14622.3	13743.3	139.3
1.9.	1.194	90.909	12756.3	12036.2	15175.5	14248.2	140.5
1.10.	1.182	91.634	13235.6	12473.9	15737.8	14760.1	142.5
1.11.	1.140	92.364	13721.9	12916.8	16317.0	15286.9	144.2
1.12.	1.130	93.101	14206.7	13356.6	16923.4	15838.8	146.0
1.1.1976	1.094	93.843	14703.3	13806.3	17548.1	16406.6	147.9
1.2.	1.006	90.251	15199.3	14253.7	18177.7	16977.2	149.8

Appendix B (cont'd)

Date	Actuarial Adjustment to All-Share Index	Actuarial Adjustment to Fixed Interest Index	Contributions actuarially adjusted for All-Share Index and accumulated at 1½% per month	Contributions actuarially adjusted for All-Share Index and accumulated at 1¼% per month	Contributions actuarially adjusted for fixed interest index and accumulated at 1½% per month	Contributions actuarially adjusted for fixed interest index and accumulated at 1¼% per month	Cost of Living Index
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.3.	1.044	90.971	15682.7	14686.7	18784.8	17523.1	150.6
1.4.	1.070	91.696	16186.8	15138.4	19416.2	18090.9	153.5
1.5.	1.052	92.427	16706.0	15603.4	20063.6	18672.3	155.2
1.6.	1.154	93.165	17232.8	16074.0	20724.1	19264.4	156.0
1.7.	1.182	93.907	17797.1	16580.0	21403.5	19872.8	156.3
1.8.	1.242	90.316	18379.7	17102.1	22090.7	20486.4	158.5
1.9.	1.310	91.036	18992.1	17651.7	22804.6	21124.1	160.6
1.10.	1.424	91.762	19632.6	18227.1	23532.7	21773.3	163.5
1.11.	1.606	92.494	20317.0	18843.9	24297.7	22456.4	165.8
1.12.	1.506	93.231	21066.5	19523.0	25096.2	23170.1	168.0
1.1.1977	1.284	93.975	21803.3	20186.9	25896.3	23882.3	172.4
1.2.	1.172	90.384	22492.8	20800.7	26692.8	24587.9	174.1
1.3.	1.154	91.104	23162.2	21391.9	27475.4	25276.5	175.8
1.4.	1.138	91.830	23840.9	21989.8	28272.1	25976.0	180.3
1.5.	1.130	92.563	24524.3	22589.7	29042.1	26645.9	181.7
1.6.	1.114	93.301	25218.7	23197.8	29841.7	27342.0	183.6
1.7.	1.106	94.045	25919.9	23809.9	30657.3	28050.8	183.8
1.8.	1.152	90.455	26630.3	24428.3	31490.5	28773.9	184.7
1.9.	1.050	91.176	27367.5	25070.6	32345.7	29515.5	185.7
1.10.	0.978	91.903	28089.3	25694.5	33203.8	30256.4	186.5
1.11.	1.006	92.636	28804.2	26308.5	34010.0	30942.0	187.4
1.12.	1.074	93.375	29543.9	26944.2	34846.7	31654.6	188.4
1.1.1978	1.056	94.119	30318.6	27611.7	35720.9	32400.9	189.5
1.2.	1.114	90.529	31102.3	28285.0	36591.8	33140.1	190.6
1.3.	1.184	91.251	31921.3	28990.1	37494.3	33907.1	191.8
1.4.	1.128	91.979	32778.0	29729.5	38423.1	34696.4	194.6
1.5.	1.124	92.712	33640.3	30470.8	39382.0	35511.7	195.7
1.6.	1.092	93.452	34516.7	31222.6	40373.6	36355.5	197.2
1.7.	1.130	94.197	35403.0	31980.5	41394.2	37223.9	198.1
1.8.	1.078	90.608	36315.9	32761.2	42428.5	38101.6	199.4
1.9.	1.064	91.330	37227.1	33536.2	43476.9	38988.8	200.2
1.10.	1.086	92.058	38152.3	34321.4	44550.3	39896.4	201.1
1.11.	1.138	92.793	39104.5	35129.3	45646.5	40822.0	202.5
1.12.	1.132	93.532	40089.1	35965.5	46763.9	41763.9	204.2
1.1.1979	1.158	94.278	41092.3	36816.0	47911.1	42730.6	207.2
1.2.	1.142	90.690	42114.1	37680.6	49069.9	43703.8	208.9
1.3.	1.088	91.413	43158.3	38563.1	50287.7	44730.6	210.6
1.4.	0.988	92.142	44213.4	39451.8	51508.1	45754.7	214.2
1.5.	0.954	92.877	45245.1	40312.6	52702.4	46747.3	215.9
1.6.	1.032	93.617	46284.8	41176.6	53918.9	47756.6	219.6
1.7.	1.064	94.364	47383.3	42094.6	55187.5	48812.2	229.1
1.8.	1.200	90.776	48512.5	43038.2	56494.2	49900.1	230.9
1.9.	1.194	91.500	49709.4	44044.2	57807.1	50988.1	233.2
1.10.	1.192	92.229	50921.4	45059.9	59135.3	52085.5	235.6
1.11.	1.298	92.965	52171.1	46107.9	60509.0	53212.0	237.7
1.12.	1.352	93.706	53491.6	47220.8	61939.1	54408.4	239.4
1.1.1980	1.374	94.453	54866.2	48381.9	63443.0	55661.9	245.3
1.2.	1.266	90.866	56269.2	49565.3	64968.3	56930.0	248.8
1.3.	1.230	91.591	57658.3	50728.6	66509.8	58207.2	252.2
1.4.	1.382	92.321	59066.9	51905.1	68111.8	59537.6	260.8
1.5.	1.354	93.057	60570.0	53169.4	69746.4	60893.2	263.2
1.6.	1.400	93.799	62085.6	54439.6	71378.1	62238.5	265.7
1.7.	1.280	94.547	63668.9	55770.6	73055.6	63621.8	267.9
1.8.	1.236	90.961	65224.3	57066.6	74752.7	65016.9	268.5
1.9.	1.244	91.686	66786.6	58362.3	76452.7	66406.9	270.2
1.10.	1.224	92.417	68382.1	59684.1	78215.8	67851.7	271.9
1.11.	1.162	93.154	69990.8	61011.8	79981.9	69291.3	274.1
1.12.	1.156	93.897	71600.8	62333.1	81770.5	70744.9	275.6
1.1.1981	1.220	94.646	73240.8	63676.9	83598.1	72228.7	277.3
1.2.	1.238	91.060	74932.8	65064.8	85470.0	73747.9	279.8
1.3.	1.168	91.786	76663.9	66483.7	87380.9	75297.1	284.0
1.4.	1.156	92.518	78389.5	67889.0	89328.8	76874.0	292.2

Appendix B (cont'd)

Date	Actuarial Adjustment to All-Share Index	Actuarial Adjustment to Fixed Interest Index	Contributions actuarially adjusted for All-Share Index and accumulated at 1½% per month	Contributions actuarially adjusted for All-Share Index and accumulated at 1¼% per month	Contributions actuarially adjusted for fixed interest index and accumulated at 1½% per month	Contributions actuarially adjusted for fixed interest index and accumulated at 1¼% per month	Cost of Living Index
1.5.	1.078	93.256	80139.7	69310.5	91285.7	78450.4	294.1
1.6.	1.140	93.999	81878.7	70712.5	93291.3	80065.7	295.8
1.7.	1.132	94.749	83685.8	72173.9	95353.6	81727.8	297.1
1.8.	1.134	91.164	85519.8	73653.4	97471.0	83434.9	299.3
1.9.	1.086	91.891	87399.7	75169.6	99665.9	85208.7	301.0

Notes on Tables — Appendix B

(1) The actuarial adjustments set out in columns (1) and (2) are based upon the formula referred to in paragraph (12).

(2) The accumulation of contributions set out in columns (3), (4), (5) and (6) follow exactly the same formula used to produce columns (9) and (10) in Appendix A, but allowing for the wage index applicable at the time the contribution is paid adjusted by reference to the actuarial adjustments in the manner indicated in paragraph (11).

Examples

(1) In the first example under Appendix A, what would the real rate of return have been relative to actuarial values?

Actuarial values of:—

Model Fund invested in All-Share Index $= 79270.6 \times .96 \times 1.086$

Model Fund invested in Fixed Interest Index $= 50573.2 \times .997 \times \frac{91.891}{65.25}$

Actuarial value of Model Fund at 1.9.81 $= 78765.6$

Actuarial Model Fund accumulated at 1½% per month $= 99665.9 \times \frac{1}{3} + 87399.7 \times \frac{2}{3}$
$= 91488.4$

Actuarial Model Fund accumulated at 1¼% per month $= 85208.7 \times \frac{1}{3} + 75169.6 \times \frac{2}{3}$
$= 78516.0$

Fund required to match wage increases (see example 1 — Appendix A)

$$= 66611.2 \times \frac{2}{3} \times 1.086 + 66611.2 \times \frac{1}{3} \times \frac{91.891}{62.25} = 81002.8$$

To determine rate of return secured on Model Fund

$(91488.4)^x (78516.0)^{1-2} = 78765.6$
$x \log 91488.4 + (1-x) \log 785160 = \log 78765.6$
$4.961366x + (1-x)\,4.894958 = 4.8963375$
$.066408x = +.001379$
$x = +0.020765$
Rate of Return $= 1.25 + .020765 \times .25$
$= 1.255191\%$ per month
or $= 16.15\%$ per annum

To determine rate required to match wage increases

$(91488.4)^x (78516.0)^{1-x} = 81002.8$
$4.961366x + (1-x)\,4.894958 = 4.908500$
$.066408x = .013542$
$x = .2039212$

Rates of return required $= 1.25 + .203921 \times .25$
$= 1.300980\%$ per month
or $= 16.78\%$ per annum

The real rates of return secured by the model fund adjusted for actuarial values $= 16.15 - 16.78$
$= 0.63\%$ per annum

(2) In the third example under Appendix A, what would the real rate of return have been in relation to actuarial values?

Actuarial value of:—

Model Fund invested in All-Share index $= 63031.3 \times .96 \times 1.220$
$= 73822.3$

Model Fund accumulated at 1½% per month $= 73240.8$

Model Fund accumulated at 1¼% per month $= 63676.9$

Fund required to match wage increases
$= 57504 \times 1.220 = 70154.9$

To determine the rate of return secured on the model fund

let $(73240.8)^x (63676.9)^{1-x} = 73822.3$
$4.864753x + 4.803982(1-x) = 4.868188$
$x = 1.056524$
Rate of return $= 1.25 + 1.056524 \times .25$
$= 1.514131$ per month
$= 19.76\%$ per annum

Rate of return for average pension fund (see example 3 — Appendix A) by reference to the All-Share index is 0.861.

The rates of Return secured is y where

$(1+y)^{10} = (1.1976)^{10}\,0.861$
$1+y = 1.1798$
Rate of Return $= 17.98\%$ per annum

To determine the rate required to match wage increases

$(73240.8)^x (63676.9)^{1-x} = 70154.9$
$4.864752x + 4.803982(1-x) = 4.846058$
$x = 0.692369$
Rate of Return $= 1.25 + 0.692369 \times .25$
$= 1.423092$ per month
or $= 18.48\%$ per annum

A pension fund performing closely to the average of pension funds over the ten year period 1st January 1971 to 30th December 1981 will therefore have secured a negative real annual rate of return relative to the wage index of 0.50%.

Printed in England by A G Bishop & Sons Ltd, Orpington, Kent