Managing risks for insurance companies and pension funds

IACA: Biennial Conference Amsterdam 24 June 2002

Risk measurement~ Characteristics of Random Variables

Characterisation of R.V.: old problem

Expectation, Variance, Skewness

Risk measurement:

New problem

ORM-TRM

Internal model versus risk measure

A premium measure (top-down)

A solvency measure (top-down)

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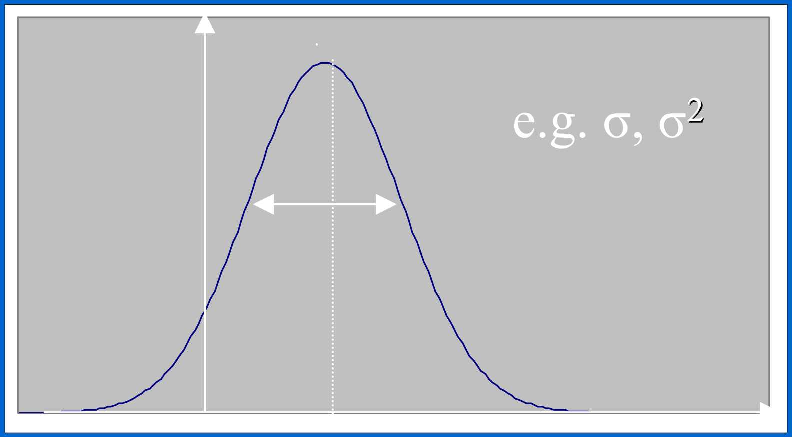
Premium-measure:

Compensation

Stability of the results

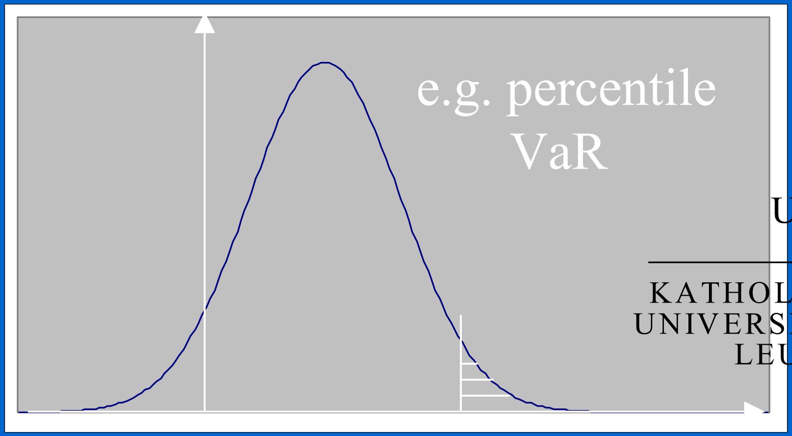


E(X)

Solvency-measure:

tail

failure



Properties

Prop1 (ordering of risks- R. Kaas et al)

$Y \geq_1 X \qquad \rho(Y) \geq \rho(X)$

(Variance is ruled out in case $E(X) \neq E(Y)$)

Prop2 (Subadditivity- M. Goovaerts et al 1984)

$\rho(X+Y) < \rho(Y) + \rho(X)$

(Motivation $\sigma(X+Y) < \sigma(X) + \sigma(Y)$)

Prop3 $\rho(X+a) < \rho(X) + a$

Prop4 $\rho(a X) < a\, \rho(X)$

Remark: Artzner coherence

Illustrations:

a) $Var(\Sigma X_i) = nVar(X_i)$ if $X_k \perp X_l$

$Var(\Sigma X_i^c) = n^2 \, Var(X_i)$

$\Sigma \, \sigma(X_i) \geq \sigma(\Sigma \, X_i^c) \geq \sigma(\Sigma \, X_i)$

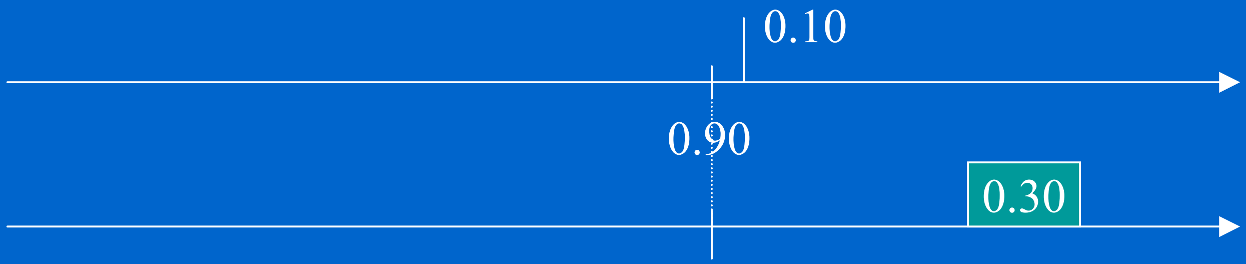
b) VaR does <u>not</u> satisfies the propertie

Examples

- Example1
  - Earthquake risk insurance

$$\Pi(X)+\Pi(Y) \;?\!\geq \Pi(X^c+Y^c) \geq \Pi(X+Y)$$

- Example2
  - VaR is on its own not a good risk measure



Examples

- Example 3
  - Conglomerate

$$\rho_{congl}(X_1 + \ldots + X_n - u)$$

$$\rho_1(X_1 - u_1) + \rho_2(X_2 - u_2) + \ldots \rho_n(X_n - u_n) =$$

$$\rho_1(X_1) + \rho_2(X_2) + \ldots \rho_n(X_n) - (u_1 + u_2 \ldots u_n)$$

- Example 4
  - Positive homogeneity conflicts with rational decision makers

Examples

- Example 5
  - Bankruptcy risk

$$\rho(X) + \rho(Y) + \rho(Z) \geq \rho(X+Y+Z)$$

(Sabena, Swiss air, …)

- Example 6
  - risk = X = $X_I + X_R$ (X_I and X_R are comonotonic)

$$\rho(X_I) + \rho(X_R) \geq \rho(X)$$

Examples

- <u>Example 7</u>

  Because of translation invariance

  a) $\rho(X) + \rho(Y) \geq \rho(X+Y) \Rightarrow \rho(X + (Y - \rho(Y)) < \rho(X)$

  $\text{Prob}(B=0) = 1-q = 1-\text{Prob}(B=1)$

  $S=\Sigma B_j^c$

  $\text{Prob}(\text{surplus}=u+n\,\rho(X)) = 1-q$

  $\text{Prob}(\text{surplus}=u+n\,\rho(X)-n) = q$

  b) $\rho(X - \rho(X)) = 0$!

Examples

- <u>Example 8</u>
  - E(X), Max(X) satisfy:

    $$\text{Prob}(Y \geq X) = 1 \implies \rho(Y) \geq \rho(X)$$

    $$\rho(aX+b) = a\,\rho(X) + b$$

    $$\rho(X) + \rho(Y) \geq \rho(X+Y)$$
  - Max(X) violates the no rip-off condition E(X) violates the mean preserving spread in the sense of Rothschild & Stiglitz.

Examples

- Example 9

	A	B
X	Euro 1000	Euro 5000
Surplus	Euro -10^6	Euro $+10^9$

Convexity order

$Y \geq_{cx} X$

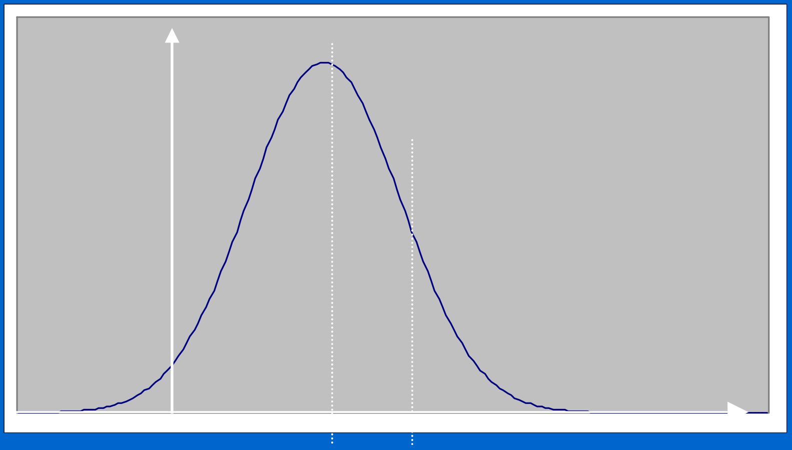
↓

$E((Y-t)_+) \geq E((X-t)_+)$ for all t

$E(Y) = E(X)$

Concept of RBC

- Π(X) - E(X) : loading *year t*
  - Premium risk measure



  - Compensation between the contracts in year t
  - Compensation between the global portfolio through the years 1,2,3,…,t

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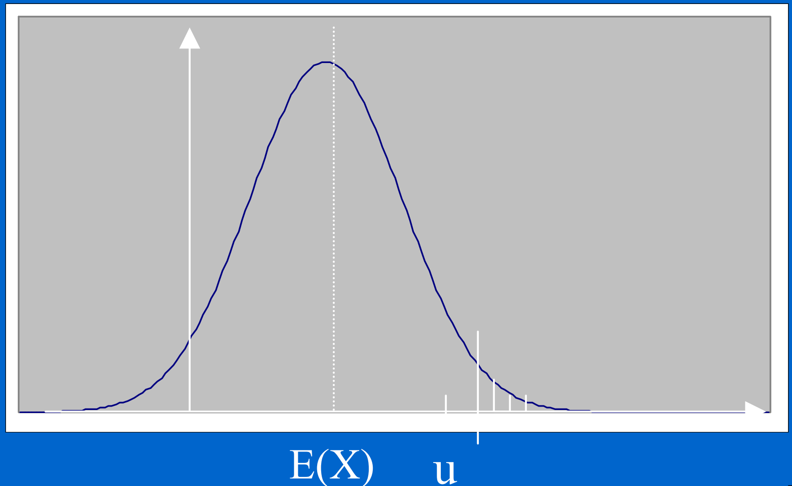
Aim of RBC

- Shareholder provides: u at a cost i u
- Remaining risk: $E(Y-u)_+$ where Y=X-E(X) or
  $$\int_u^\infty g(1-F_Y(y))dy$$
- Cost: $E((Y-u_+)+ i\, u$
- optimalize:
  $u=F_Y^{-1}(1-g(i))$



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A risk measure based on exponential premiums

$u_t = u_{t-1} + c - S_t$ $\qquad$ $S_t \sim$ i.i.d. $0 \geq E(S-c)$

$c = \Pi(S)$

$\varepsilon = \exp(-Ru) \geq$ Ruin probability

$\exp(Rc) = E(\exp(RS))$

choose $c = \ln E(e^{RS})/R$

$$\rho(S) = \frac{u}{|\ln \varepsilon|} \ln E\left(e^{\frac{u}{|\ln \varepsilon|} S} \right)$$

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Properties

- 1) S & T independent $\rho(S+T) = \rho(S) + \rho(T)$
- 2) $T \geq_{cx} S$ then $\rho(T) \geq \rho(S)$
- 3) ρ is invariant for a proportional change in monetary units
- 4) $\rho(S^{\circ}+T^{\circ}) \geq \rho(S+T)$ (S°,T°) more related

Remark: ρ is not subadditive

Economic context

$$\frac{u}{|\ln \varepsilon|} \ln E\left(e^{\frac{|\ln \varepsilon|}{u} \sum X_i} \right) \leq \sum_i \frac{u_i}{|\ln \varepsilon|} \ln E\left(e^{\frac{|\ln \varepsilon|}{u_i} X_i} \right)$$

It makes only sense to add measure for portfolios having the same ruin probabilities

Optimal capital allocation

$$\frac{u_j}{u} \approx \frac{Var(X_j)/2u_j}{\sum_i Var(X_i)/2u_i}$$

Remark: risk measure has the dimension of Euro:

$$\frac{Var(X_j)}{2u_j}$$

is in Euro

Risk measure based on convex order

$$\rho(X_1 + X_2 + \ldots + X_n) \leq \rho(X_1^c + X_n^c + \ldots + X_n^c)$$

$$(X_1 + X_2 + \ldots + X_n)_+ \leq \sum_{i=1}^{n} (X_i - u_i)_+$$

with

$$\sum u_i = u$$

Problem A

$$A = \underset{\substack{u_1,\ldots,u_n \\ \sum_i u_i = u}}{Min} \sum_{i=1}^{n} E\left(\left(X_i - u_i\right)_+\right)$$

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Problem B

$$B = Max\left(\left(X_1 + X_2 + \ldots + X_n - u\right)_+\right)$$

over the dependence structure

Solution A=B

$$E\left(\left(X_1^c + X_2^c + \ldots + X_n^c - u\right)_+\right) =$$

$$\sum_j E\left(\left(X_j - F_{X_j}^{-1}(F_W(u))\right)_+\right)$$

with

$$F_W^{-1}(u) = \sum_j F_{X_j}^{-1}(u)$$

Conclusions

- Internal model is and remains very important (work for consulting actuary)
- Replacing the notion of solvency by some measures is not yet for this decade

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