MORTALITY AND DISABILITY RISKS IN LONG TERM CARE INSURANCE[•]

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Abstract
Demographical risks related to Long Term Care products are discussed. In particular, the risks deriving from mortality and disability trends are analysed, depicting possible scenarios in which insurance products will evolve and briefly illustrating how such risks can be faced.

1. Introduction

Long Term Care (LTC) is care required in relation to chronic (or long-lasting) bad health conditions. LTC insurance (LTCI) provides income support for the insured, who needs nursing and/or medical care, in the form either of a forfeiture annuity benefit or nursing and medical expense refunding. In some countries (e.g. Japan) LTC products are sold according to which the insured can choose, in case of a claim, between an annuity benefit or an appropriate care service provided by organizations offering nursing care services. Given the type of risks covered, LTC insurance has a lifetime duration.

The common approach to the mathematical representation of LTC covers is multistate modelling. Actuarial theory has developed models easy to handle from a formal viewpoint. The major task in implementing such models is due to the fact that LTC covers are recent products and experience data are still scanty. Moreover, given their lifetime duration, these covers are affected by demographical trends. Because of the uncertainty in estimating the future levels of mortality and morbidity, considerable difficulties are therefore encountered in pricing and reserving.

This paper is devoted to the analysis of the risks coming from uncertainty of future demographical trends. As it is well-known, such uncertainty suggests the adoption of projected tables. The use of mortality projections for pricing (and reserving) life covers sold to the elderly, such as annuities, is extensively widespread; on the contrary, such practice is not common for health insurance. Difficulties for these latter covers originate from the several causes of uncertainty by which they are affected, namely mortality levels, morbidity rates, amount of claims (when expense refunding covers are dealt with). In particular, the paucity of sickness data relating to very old ages increases the difficulty in estimating claim frequencies and amounts.

The main aim of this paper is to investigate the effects of uncertainty related to the future scenario in which a portfolio of LTC insurance will evolve, in terms of the risk borne by the insurer. For this purpose, projected mortality and disability tables for LTC covers are constructed, through which it is shown how the impact of demographical trends can be considerable on LTC portfolios. Some tools for facing such risks are then briefly discussed, in particular with regard to proper capital allocation strategies which can be adopted in conjunction with suitable pricing bases.

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For the sake of simplicity, the following hypotheses are adopted throughout the paper. Insurance covers with only one level of disability and involving forfeiture benefits are considered. In particular, the paucity of data concerning annual claim amounts and frequencies suggests to disregard expense refunding covers (for which premiums are usually assessed with reference to the maximum amount insured). Randomness other than the demographical one is disregarded; in particular, a deterministic financial structure is adopted. Expense loading, profit assessment and reinsurance are not dealt with. Further, for brevity the characteristics of LTCI are recalled just for the types of policy dealt with in this paper, i.e. Stand Alone and Enhanced Pension covers. For a deeper description see, for example, Haberman, Pitacco (1999). In this paper, only the viewpoint of the provider of LTCI is considered. The economics of LTC, whose discussion goes beyond the scope of this paper, is for example dealt with in Chen (1994), Whynes (1996) and Zweifel (1996).

The paper is organized as follows. In Section 2 the multistate approach is briefly recalled. In Section 3 demographical projections for LTCI are described. Section 4 and 5 deal with the analysis of mortality and disability risks for Stand Alone and Enhanced Pension covers, respectively. In Section 6 some tools to face LTC risks are discussed. Finally, Section 7 concludes.

2. Multistate model for LTC covers

LTC covers, similarly to other health insurance products, are usually represented in terms of a multistate model (see, for example, Haberman, Pitacco (1999)). In this setting, it is assumed that the evolution of an insured risk can be described in terms of the presence of the risk itself, at every point of time, in a certain state belonging to a specified set of states (the state space) and that the events which give rise to cash flows of premiums and benefits correspond to transitions from one state to another state.

In the case of LTC covers, the multistate model usually consists of the following states: state 1 = “healthy”, state 2 = “disabled at level I”, state 3 = “disabled at level II”, …, state N = “dead”. Considering one level of disability only, the multistate model depicted in Figure 2.1 is adopted where, according to the chronic character of LTC illness, it is assumed that no recovery (neither partial) is possible (i.e., disability is permanent).

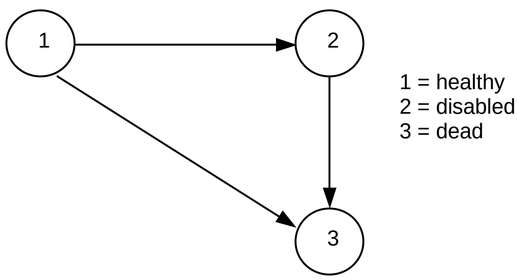

Figure 2.1 – A three-state model for LTCI

Upon entry into state 2, the LTC annuity is paid until death; in some covers (for example, Enhanced Pension) an annuity is paid also in state 1, but at a lower amount. The appraisal of insurer’s liabilities (for pricing, reserving or risk investigations) requires information about the rates of disability (transition $1 \rightarrow 2$) and the rates of death both for healthy people (transition $1 \rightarrow 3$) and disabled people (transition $2 \rightarrow 3$). This choice is discussed in Section 3.

The basic tool for risk investigations (but also for pricing and reserving) is the so-called loss function which is defined as follows

loss function at time t = random present value at time t of future benefits
- random present value at time t of future premiums

The loss function can be defined both at individual and portfolio level. We denote by L(t) the individual loss function and by $L^P(t)$ the portfolio loss function. Clearly, $L^P(t)$ can be obtained by summing up the individual loss functions with respect to the policies in-force at time t.
Premiums and reserves are assumed to be calculated according to the equivalence principle, as it is quite common for life covers. Given a conservative hypothesis, describing the future scenario, premiums must be determined so that the expected present value of the loss function at time 0 is equal to zero. The reserve is then defined as the expected present value of the loss function at time t, based either on conservative hypotheses, in case a reserve on the safe-side must be assessed, or on realistic hypotheses, in case of a fair valuation of the reserve itself. A specific reserve must be set up for each state in which the policy is still in force; according to Figure 2.1, a reserve for healthy people and a reserve for disabled people are defined.

3. Demographical scenarios

The choice of a proper description of the future scenario is particularly difficult when LTC covers are dealt with for two main reasons:
(i) LTC are recent products and insurance experience data are rather scanty in many countries;
(ii) recent trends in mortality and morbidity witness significant changes that contribute in defining a moving scenario in which LTC products will evolve.
Aspect (i) is usually overcome by resorting to population medical data, properly transformed to reflect selection of insured people with respect to the general population. Moreover, population medical data are usually in the form of prevalence rates, whereas inception rates are required for insurance pricing; hence, a transformation is necessary also in this regard. Such problems are not dealt with in this paper (see, for example, Gatenby (1991), Olivieri (1996) and Haberman and Pitacco (1999)).
Aspect (ii) has emerged in recent years. Mortality trends at adult ages reveal decreasing annual probabilities of death (see for example Benjamin and Soliman (1993), Macdonald (1997), Macdonald et al. (1998)). In particular, the following phenomena are observed in populations including both healthy and disabled lives:
(1) an increasing concentration of deaths around the mode of the density function of the future lifetime distribution (the so-called “curve of deaths”);
(2) a forward shift of the mode of the curve of deaths.
These changes clearly affect any cover involving lifetime benefits. In particular, whilst the former aspect reduces uncertainty since with higher probability the actual duration of life might coincide with its mode, the latter increases the risk inherent in the management of a policy, the magnitude of the above mentioned shift being unknown.
In the case of health covers, such as LTC, risk emerges further from uncertainty concerning the time spent in the disability state. Actually, when living benefits are paid in case of disability it is not merely important how long one lives, but also how long he/she lives in a condition of disability.
Although it is reasonable to assume a relationship between mortality and morbidity, the relevant definition is difficult due to the complexity of such a link and to the impossibility of defining and measuring disability objectively. Three main theories have been formulated about the evolution of senescent morbidity (as pointed out, for example, in Swiss Re (1999)).
(i) “Compression theory” (see Fries (1980)): chronic degenerative diseases will be postponed until the latest years of life because of medical advances. Assuming there is a maximum age, these improvements will result in a compression of the period of morbidity.

(ii) "Pandemic theory" (see Gruenberg (1977) and Kramer (1980)): the reduction in mortality rates is not accompanied by a decrease of morbidity rates; hence, the number of disabled people will increase steadily.

(iii) "Equilibrium theory" (see Manton (1982)): most of the changes in mortality are related to specific pathologies. The onset of chronic degenerative diseases and disability will be postponed and the time of death as well.

The scenarios depicted by the above mentioned theories produce rather different consequences for the insurer; in particular, Compression theory suggests optimistic views, whilst Pandemic theory pessimistic ones. Although the most extreme scenarios can reasonably be looked at as unfeasible hypotheses, the deep differences among such theories imply a high level of uncertainty about the evolution of senescent morbidity. The adoption of projected tables for the evaluation of insured benefits seems then appropriate. However, since the three theories imply rather different scenarios, the mentioned uncertainty should be included in the actuarial model used for evaluating benefits.

It should be pointed out that uncertainty in future mortality and senescent disability trends implies the risk of systematic deviations from the scenario used to calculate expected values (and in particular premiums and reserves), hence a model risk. The demographical risk inherent in LTC covers consists of a random fluctuation component (similarly to any insurance cover) and a systematic deviation component as well.

In order to appraise the risk inherent in LTC covers, uncertainty of the future evolution of mortality and disability must be explicitly taken into account. To this aim, several scenarios must be considered, each one including a specific projection of mortality and disability trends which represents a possible realization of the actual future scenario.

The traditional actuarial approach to demographical projections consists in extrapolations of (recent) trends as far as these can be perceived from observed data. However, the approach we adopt in this paper (commonly followed by demographers) leads to models expressing the basic characteristics of the evolving scenario in which demographical changes take place. In this latter approach, the use of analytical laws for mortality and disability rates is required, whose parameters are functions of the calendar year. The adequacy of the projection model can be checked comparing the behaviour of some quantities with the scenario characteristics suggested by the three theories mentioned above. So, possible scenarios have been defined (and their adequacy tested) in terms of the evolution of the expected time spent in the healthy state and in the disability state.

Let us consider a person in state i at time t, where t denotes the time elapsed since policy issue. In a non-projected multistate context, denote by $e_{ij}(t)$ the time expected to be spent in state j from time t, i.e. from age x+t (x representing the age at entry). Referring to the multistate model in Fig. 2.1, the following quantities can be defined:

- $e_{11}(t)$: expected time spent in the healthy state for a healthy person, i.e. healthy life expectancy for a healthy person;
- $e_{12}(t)$: expected time spent in the disability state for a healthy person, i.e. disability life expectancy for a healthy person;
- $e_{22}(t)$: life expectancy for a disabled person.

The total life expectancy for a healthy person, $e_{1.}(t)$, is simply given by the sum of the time spent in the healthy state and in the disability state, i.e.

$$e_{1.}(t) = e_{11}(t) + e_{12}(t) \tag{3.1}$$

In a projected context, functions depend on the calendar year. Let y denote the calendar year in which the person enters insurance. So, expected values $e_{ij}(t;y)$ (instead of $e_{ij}(t)$) shall be considered. In this framework, the theories mentioned above can be expressed in terms of the evolution of life expectancy, for any fixed t, as follows:

(i) $e_{1.}(t;y)$ increases as y increases, with a major contribution (in relative terms) from $e_{11}(t;y)$;

(ii) $e_{1.}(t;y)$ increases as y increases, with a major contribution (in relative terms) from $e_{12}(t;y)$;

(iii) $e_{11}(t;y)$ and $e_{12}(t;y)$ increase as y increases, at similar rates.

In terms of life expectancies, the evolutionary theories on mortality and senescent morbidity mentioned above suggest the behaviours sketched in Fig. 3.1.

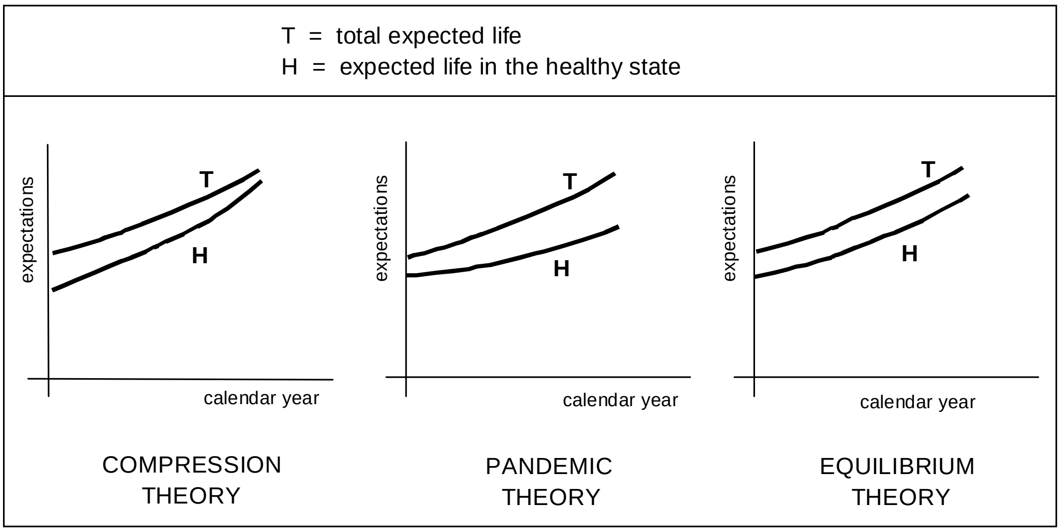

Figure 3.1 – Scenarios suggested by main theories

In what follows, we focus on one generation of insureds only, entering the LTC covers (at age x) in the same year. Hence, the calendar year can be omitted in the notation.

With reference to males aged 65 in current year, we have adopted five projected scenarios, whose relevant life expectancies are quoted in Table 3.1. Scenario H_C has been taken as a starting point for the construction of projected scenarios H_1, H_2, …, H_5, since it comes from cross-sectional observations of mortality and disability of elderly people. In particular, data on Italian mortality (taken from ISTAT surveys) and on British disability rates (taken from OPCS surveys) have been referred to. Analytical models have been used for representing mortality and disability; more precisely, mortality has been represented in terms of the Weibull law and disability in terms of the Gompertz law (details of the assumptions and models adopted are described in Ferri, Olivieri (2000)). For disabled lives, mortality rates have been obtained from those of healthy lives, assuming that the former keep higher than the latter (although specific data at this regard are lacking, such hypothesis seems quite reasonable).

H	$e_{11}^{(H)}(0)$	$e_{12}^{(H)}(0)$	$e_{1.}^{(H)}(0)$	$e_{22}^{(H)}(0)$	$\frac{e_{11}^{(H)}(0)}{e_{11}^{(H_C)}(0)}-1$	$\frac{e_{12}^{(H)}(0)}{e_{12}^{(H_C)}(0)}-1$	$\frac{e_{1.}^{(H)}(0)}{e_{1.}^{(H_C)}(0)}-1$
H_C	14.428	1.566	15.995	15.307	0.00%	0.00%	0.00
H_1	15.156	1.435	16.591	15.931	5.05%	-8.41%	3.73%
H_2	16.042	1.563	17.605	16.983	11.19%	-0.25%	10.07%
H_3	15.844	1.749	17.593	16.983	9.82%	11.65%	10.00%
H_4	15.501	2.073	17.574	16.983	7.43%	32.36%	9.88%
H_5	16.577	2.366	18.943	18.397	14.89%	51.05%	18.44%

Table 3.1 – Life expectancies

As it emerges from Table 3.1, the five projected scenarios have been built with reference to their impact on the expected time spent in the healthy and in the disability state. Firstly, note that in any projected scenario an increase of total life expectancy with respect to cross-sectional observations has been assumed. This is suggested by mortality trends in populations that include both healthy and disabled people. In absolute terms, and with comparison to scenario H_C, the changes in $e_{1.}(0)$ mainly depend on the changes in $e_{11}(0)$. Therefore, consequences suggested by Compression, Equilibrium and Pandemic theory must be checked by looking at the relative contributions of $e_{11}(0)$ and $e_{12}(0)$ (as shown, for example, by the quantities $e_{1j}^{(H)}(0)/e_{1j}^{(H_C)}(0)-1$, j=1,2, where $e_{1j}^{(H)}(0)$ is the life expectancy calculated according to the generic scenario H).
For the insurer, H_1 represents a scenario that should involve lower costs than the others for two reasons. Firstly, there is a slight increase in total expected life. Secondly, the change in the time expected to be spent in the healthy state is larger in percentage than the one related to disability, which actually falls down to a negative value. This evolutionary hypothesis has therefore been chosen because it seems one of the best suited to represent consequences depicted by Compression theory. On the other side, H_5 represents the scenario with presumably the highest costs involved since it assumes a fall in mortality rates accompanied by a substantial rise in disability rates, which imply a considerable increase in disability life expectancy as well as total life expectancy. Therefore, this scenario can reasonably express the Pandemic theory evolutionary hypothesis, although the latter does not lead to any particular increase in disability rates. However, this more pessimistic scenario can also reasonably include the risk of an underestimation of disability rates, given that cross-sectional observations have been obtained from population medical data and not from insurance experiences. Scenario H_3 assumes a "medium" decrease in mortality rates in respect of H_C. Moreover, no change in respect of H_C has been considered for disability rates. The result is a projection which is somehow intermediate between the above depicted scenarios, with a change in total life expectancies which receives almost equal (relative) contributions from changes in healthy and disability life expectancies. For this reason, scenario H_3 can be considered as reflecting the evolutionary projection of Equilibrium theory. Finally, scenarios H_2 and H_4 depict projections that are intermediate between scenario H_3 and the other two "extreme" scenarios H_1 and H_5. As far as the hypotheses are concerned, it must be mentioned that an unchanged link has been assumed between mortality rates for healthy and disabled people. This choice has been suggested by the lack of specific observations on mortality of active and disabled lives.

4. Risk analysis for Stand Alone covers

A Stand Alone LTCI provides a fixed amount annuity in the case of LTC need. The amount of the annuity can be defined as a function of the disability level. Since we have considered one level of disability only, we assume that only one level of benefit is provided. For brevity, we adopt a constant and unitary amount of yearly benefit. Of course, variable benefits could be considered, in particular due to mechanisms of benefit indexing. However, in order to interpret numerical results more easily, such aspects have been disregarded. Financial risks have also been disregarded; in particular a constant investment yield has been considered.

The risk for the insurer is measured through the variance of its loss function, which has been evaluated at time 0 (i.e. at policy issue). A homogenous portfolio has been considered, i.e. a group of policies entering at the same time and age, with the same amount insured, similar in terms of risk class, etc. The same amount of premium is required to such policies, to be paid at entry.

The variance of the loss function can be evaluated either disregarding or considering uncertainty concerning the future evolution of mortality and disability. In the former approach (which can be defined "deterministic"), a (projected) scenario is assigned; different scenarios can be compared in terms of the different value they imply for the chosen measure of risk. The range of variation of such a measure gives an idea of how different a result can be from what we expect it is. In the latter approach ("stochastic" approach) a set of scenarios is considered, to which a probability distribution is assigned. In this case, it could be interesting to analyse not only the variance of the loss function, but also the probability of being the portfolio loss function greater than a given value, i.e. of events like $L^P(t) > \lambda$, where λ is a given value.

Let us assume that N homogeneous insurance covers are issued at time 0; the entry state is state 1 (healthy). Adopting at first a deterministic approach, let us consider a given scenario H, i.e. a given set of hypotheses concerning mortality and morbidity. The variance of the individual loss function at time 0, $\text{var}(L(0) \mid H)$, can be easily calculated (details for this result and the following ones are described in Ferri, Olivieri (2000)). Assuming that the N insureds are independent risks, it turns out that the variance of the portfolio loss function, $\text{var}(L^P(0) \mid H)$, is proportional to that of the individual loss function

$$\text{var}(L^P(0) \mid H) = N \, \text{var}(L(0) \mid H) \tag{4.1}$$

In particular, if we consider the following index (briefly referred to as risk index)

$$r^{(H)} = \frac{\sqrt{\text{var}(L^P(0) \mid H)}}{N\pi} \tag{4.2}$$

where π is the individual single premium, we find out

$$r^{(H)} = \frac{1}{\sqrt{N}} \frac{\sqrt{\text{var}(L(0) \mid H)}}{\pi} \tag{4.3}$$

Expression (4.3) in particular shows that the risk borne by the portfolio, per unit of single premium, reduces as the initial size of the portfolio increases. This is due to the fact that when a given demographical scenario is considered, the only risk accounted for is that of random fluctuations, which is a pooling risk.

In Table 4.1 the expected value, variance and risk index for a given policy are quoted, calculated under the different projected scenarios described in Section 3. For comparison, also the values calculated under the cross-sectional scenario are shown. A person aged 65 at time 0 is considered; an annual investment yield equal to 3% has been assumed.

	Expected value $E(L(0) \mid H)$	Variance $var(L(0) \mid H)$	Risk index $r^{(H)}$
H_C	$0.9182 - \pi$	6.7486	2.8291
H_1	$0.8530 - \pi$	6.3709	2.9591
H_2	$0.9292 - \pi$	6.9278	2.8328
H_3	$1.0370 - \pi$	7.5455	2.6488
H_4	$1.2261 - \pi$	8.5930	2.3909
H_5	$1.3871 - \pi$	9.6543	2.2400

Table 4.1 – Stand Alone; individual loss function

Under the heading "Expected value", the numerical values represent the expected value of liabilities for the various scenarios. Adopting the equivalence principle, the single premium π is, in each case, equal to such values whence the expected value of the loss function is zero. A safety loading can however be added, especially if the projected scenarios are meant as realistic representations of the possible future actual scenario. The expected value and variance of the portfolio loss function can be found simply by multiplying both quantities by the initial size of the cohort. The risk index has been calculated assuming that the single premium is equal to the expected value of liabilities under each scenario. At the portfolio level, the risk index can be found dividing by $\sqrt{N}$ what quoted in Table 4.1.

As far as the consequences of the various projections are concerned, moving from scenario H_1 to H_5 implies an increasing expected cost of benefits and an increasing risk for the insurer as well, as shown by the variance of the loss function. However, in relative terms risk declines with the severity of the projection, as shown by the index $r^{(H)}$, because a higher premium results. The Stand Alone is anyway a risky product, since the variance of the loss function has a large magnitude when compared to premiums.

Figures 4.1 and 4.2 show the variance profile of a policy in the healthy and in the LTC state, respectively, under the different scenarios. In the disability state the variance of the loss function is affected by mortality only; hence in Figure 4.2 only scenarios involving different mortality levels are considered (actually, in scenarios H_2 and H_4 the same mortality levels than scenario H_3 have been assumed, such scenarios differing in terms of disability levels only – see Table 3.1). Risk, in terms of variance of the loss function, decreases with time. Moreover, the variance profile varies with the projection, the lowest being implied by H_5. This witnesses that in our projections the mortality assumptions have been chosen so that they imply a stronger concentration of the curve of deaths and forward shift of its mode (see Section 3). Risk decreases with time also when the healthy state is considered. However, the different projections induce profiles that cross each other. This is due to the interactions between mortality and disability levels. Figure 4.1 shows that in the first years of the coverage, the variance of the loss function is mostly affected by disability levels (actually, the variance of the loss function for a healthy person is higher under scenarios H_4 and H_5, which imply the highest inception rates); as time passes, the effect of mortality concentration becomes predominant (in the latest years scenario H_5, which is based on the highest mortality levels as well, produces the lowest variance).

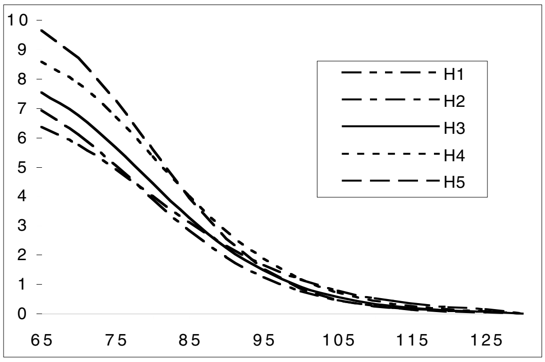

Figure 4.1 – Stand Alone; variance of the loss function; healthy state

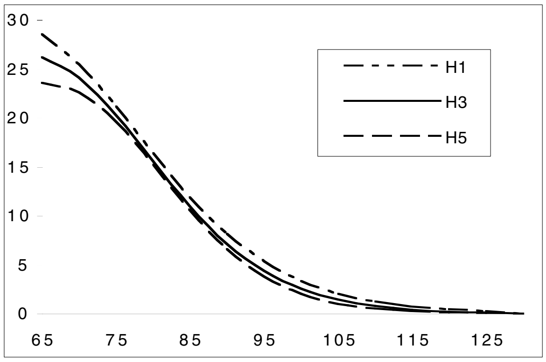

Figure 4.2 – Stand Alone; variance of the loss function; LTC state

As far as the risk profile at portfolio level is concerned, the variance of the portfolio loss function at time t is affected both by variability of payments at individual level and by variability of the portfolio composition. It is therefore difficult to interpret the behaviour of $\mathrm{var}(L^P(t) \mid H)$ in terms of the consequences produced by the different scenarios. We have then preferred to calculate the variance of the portfolio loss function only in relation to three assigned patterns of the number of healthy and disabled people, based on the number expected to be in each state and on a normal approximation of deviations from the expected value (we denote by min, med and max such patterns, according to the hypotheses assumed for their calculation). In Figure 4.3 the profiles of the variance of the portfolio loss function are depicted, for a portfolio of initial $N = 1{,}000$ insureds.

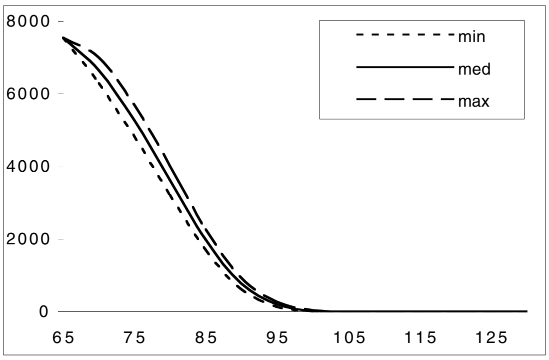

Figure 4.3 – Stand Alone; variance of the portfolio loss function for given patterns of the size of the portfolio

In the sequel, scenario H_3 is adopted as premium and reserving basis. Hence $\pi = 1.0370$. This choice is suggested by the fact that H_3 is the intermediate scenario, in terms both of the expected value and variance of liabilities. Obviously a safety loading could be explicitly included in this premium considering either the risk of the cover (in terms of the variance of the loss function) or an appropriate function of the different premium levels resulting from the other projected scenarios. In Fig. 4.4 and 4.5 the reserves for a healthy and disabled insured are respectively depicted.

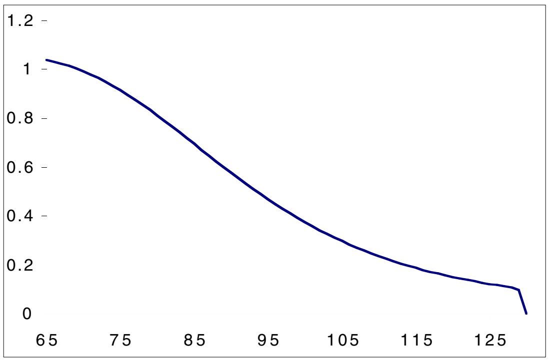

Figure 4.4 – Stand Alone; reserve for a healthy person

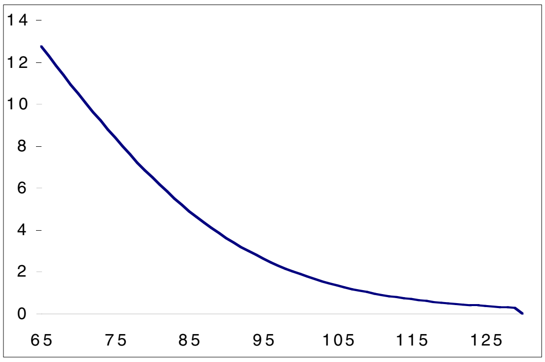

Figure 4.5 – Stand Alone; reserve for a disabled person

Let us now adopt a stochastic approach. The risk analysis is therefore performed considering explicitly uncertainty in demographical projections, but still adopting scenario H_3 as pricing and reserving basis. To this aim we consider the five projected scenarios H_k ($k = 1, 2, \dots, 5$) of Table 3.1, interpreting each of them as a possible outcome of the actual future mortality and disability rate levels. Then, a weight ρ_k is linked to scenario H_k ($k = 1, 2, \dots, 5$), representing the "degree of belief" of such hypothesis. Clearly, $\sum_{k=1}^{5} \rho_k = 1$. Risk analyses are still performed at time 0 only.

Referring to a homogeneous portfolio, with the same characteristics described above, and assuming that the insureds are independent risks under each scenario, the expected value and variance of the portfolio loss function at time 0 are now given by the following expressions

$$E(L^P(0)) = \sum_{k=1}^{5} E(L^P(0) \mid H_k)\, \rho_k \tag{4.4}$$

$$\text{var}(L^P(0)) = \sum_{k=1}^{5} \text{var}(L^P(0) \mid H_k)\, \rho_k + \sum_{k=1}^{5} \left(E(L^P(0) \mid H_k) - E(L^P(0))\right)^2 \rho_k \tag{4.5}$$

It turns out

$$E(L^P(0)) = N \sum_{k=1}^{5} E(L(0) \mid H_k)\, \rho_k = N\, E(L(0)) \tag{4.4'}$$

$$\begin{aligned} \text{var}(L^P(0)) &= N \sum_{k=1}^{5} \text{var}(L(0) \mid H_k)\, \rho_k + N^2 \sum_{k=1}^{5} \left(E(L(0) \mid H_k) - E(L(0))\right)^2 \rho_k \\ &= N\, E(\text{var}(L(0) \mid H)) + N^2\, \text{var}(E(L(0) \mid H)) \end{aligned} \tag{4.5'}$$

It must be stressed that now the variance of the portfolio loss function depends on N^2. More precisely, the first term of $\text{var}(L^P(0))$, which is proportional to N, gives account of random fluctuations around expected values, whilst the second term, which is proportional to N^2, expresses systematic deviations of observed values from expected ones. The relevant systematic risk is due to the uncertainty about the future demographical scenario and is a generalization of the longevity risk, which affects long term life insurance products such as annuities (see, for example, Olivieri (2001)). In terms of the risk index (now denoted simply by r, since it does not depend on a particular scenario) we have

$$r = \frac{\sqrt{\mathrm{var}(L^P(0))}}{N\,\pi} = \left(\frac{1}{N} \frac{E(\mathrm{var}(L(0)\,|\,H))}{\pi^2} + \frac{\mathrm{var}(E(L(0)\,|\,H))}{\pi^2} \right)^{1/2} \tag{4.6}$$

The first term in the expression of the risk index r is similar to (4.3); actually it relates to the (pooling) risk of random fluctuations, which reduces with the size of the portfolio. The second term is constant with respect to the size of the portfolio; it shows the risk of systematic deviations, which is a non-pooling risk. Note also that as N increases, the risk index r tends to a positive value, represented by its systematic component.
In Table 4.2 the expected value and variance of the loss function are quoted. As an example, we have chosen the following weights: $\rho_1 = \rho_5 = 0.05$, $\rho_2 = \rho_4 = 0.15$, $\rho_3 = 0.6$; note that the most realistic scenario has been adopted for pricing and reserving, whilst the extreme scenarios (i.e. scenario H_1 and H_5) occur with a low probability. What emerges from Table 4.2 is the heavy riskiness linked to systematic deviations. As mentioned above, the so-called longevity risk, related to changes in mortality and disability levels, can be measured with the systematic component of the variance of the loss function. However, to a large extent the risk of random fluctuations keeps higher than that of systematic ones. It can be found out that, in our example, when $N > 528$ the non-pooling component of the variance is predominant, whilst when $N < 528$ the pooling component is larger than the other.

N	Expected value $E(L^P(0))$	Variance $\mathrm{var}(L^P(0))$	Pooling risk $\frac{N\,E(\mathrm{var}(L(0)\,\|\,H))}{\mathrm{var}(L^P(0))}$	Systematic risk $\frac{N^2\,\mathrm{var}(E(L(0)\,\|\,H))}{\mathrm{var}(L^P(0))}$
1	0.020	7.671	99.81%	0.19%
10	0.205	78.017	98.14%	1.86%
100	2.048	910.743	84.07%	15.93%
528	10.814	8,087.207	49.99%	50.01%
1,000	20.480	22,164.267	34.55%	65.45%
10,000	204.802	1,527,326.669	5.01%	94.99%
100,000	2,048.021	145,841,666.960	0.52%	99.48%

Table 4.2 – Stand Alone; portfolio loss function

It must be stressed that the stochastic approach leads to a rather different risk assessment than the deterministic one. However, the risk of systematic deviations although neglected in the deterministic approach is obviously present any time a future behaviour is concerned. The advantage of the stochastic approach is that it allows to explicit such risk, whilst in a deterministic setting only random fluctuations can be formally represented.
With reference to Table 4.2, note that having adopted H_3 for premium calculation, the expected value of the loss function is not zero. At this regard, as mentioned above, a loading could be explicitly included into the premium so that $E(L^P(0)) = 0$. Obviously, a (further) loading could also be determined in relation to the variance of the loss function.
In Table 4.3 the risk index is quoted for different initial sizes of the portfolio, both in the deterministic and in the stochastic approach. Note that whilst for $N = 1$ the two approaches lead to a similar risk evaluation (the magnitude of the relative risk is the same), for $N > 1$ the latter approach leads to a relative riskiness which is higher and has a positive limiting value. As already explained, this is due to the systematic component of the variance of the loss function.
We finally point out that arguments similar to the above ones hold when a different set of weights ρ_k is chosen (unless they lead back to the deterministic case).

N	Deterministic approach $r^{(H_3)}$	Stochastic approach r
1	2.64885	2.67082
10	0.83764	0.85175
100	0.26488	0.29101
1,000	0.08376	0.14356
10,000	0.02649	0.11917
100,000	0.00838	0.11645
…	…	...
∞	0	0.11615

Table 4.3 – Stand Alone; risk index

Finally, in Figure 4.6 three possible profiles of var($L^P(t)$) are depicted, related to three patterns of the size of the portfolio, which have been chosen considering the expected size of the portfolio and a normal approximation of deviations from the expected portfolio size itself.

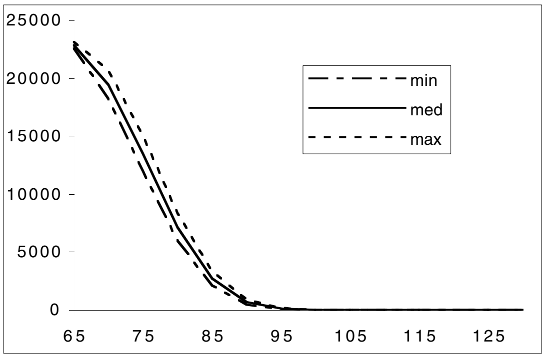

Figure 4.6 – Stand Alone; variance of the portfolio loss function for given patterns of the size of the portfolio

5. Risk analysis for Enhanced Pension covers

The Enhanced Pension product consists in a straight life annuity uplifted in case the annuitant becomes disabled (according to a given definition of LTC disability). For a given amount of single premium, the cost of the uplift is met by a reduction in the initial pension income.

With reference to the multistate model of Fig. 2.1 assume that a healthy person at retirement is eligible for a traditional straight life annuity (basic pension), with annual amount b. He/she can switch this benefit to a combination of an annuity while he/she is in state 1 with annual amount b_1, $b_1 < b$, and an LTC annuity (enhanced pension) with annual amount b_2, $b_2 > b$. The single premium is given by the actuarial value of the basic pension; once $b - b_1$ is chosen, it is then possible to calculate the LTC benefit b_2, which obviously depends on the technical basis adopted for performing such calculation. The individual loss function is the (random) present value of benefits payable in state 1 and state 2. The portfolio loss function is the sum of the individual loss functions.

Let us consider a homogeneous portfolio of Enhanced Pension covers, which are independent under any given demographical scenario. We have adopted scenario H_3 as pricing and reserving basis. Taking b = 1, we have π = 13.1496. Choosing b_1 = 0.9, we then find b_2 = 2.2110. Note that a great increase in benefits can be obtained in case of LTC disability, which is funded by a modest decrease in the current benefit.

Table 5.1 shows the expected value and variance of the individual loss function under the various scenarios (all hypotheses are the same than in Section 4). In particular, the risk index has been obtained relating $\sqrt{\text{var}(L(0)\,|\,H)}$ to the single premium, calculated according to scenario H_3. Considerations similar to those concerning the Stand Alone cover can be expressed. Note in particular that, due to the different levels of benefits, H_4 is the most risky scenario. However, the relative riskiness of the Enhanced Pension is significantly lower than that of the Stand Alone. This is due to the fact that in the Enhanced Pension cover benefits are paid starting from policy issue, whence a high initial funding, i.e. a high single premium, is required. In the Stand Alone case, on the contrary, the occurrence of benefit payment is random, whence a lower premium is necessary.

	Expected value $E(L(0)\,|\,H)$	Variance $\text{var}(L(0)\,|\,H)$	Risk index $r^{(H)}$
H_C	$12.0136 - \pi$	47.8556	0.5261
H_1	$12.3126 - \pi$	43.2333	0.5000
H_2	$13.0130 - \pi$	41.6292	0.4907
H_3	$13.1496 - \pi$	43.7139	0.5028
H_4	$13.3891 - \pi$	47.2853	0.5229
H_5	$14.3708 - \pi$	46.3433	0.5177

Table 5.1 – Enhanced Pension; individual loss function

Examples in Fig. 5.1 to 5.5 are similar to those considered in Section 4. Similar arguments hold.

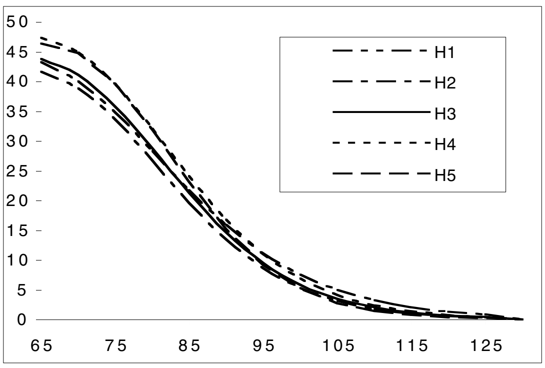

Figure 5.1 – Enhanced Pension variance of the loss function healthy state

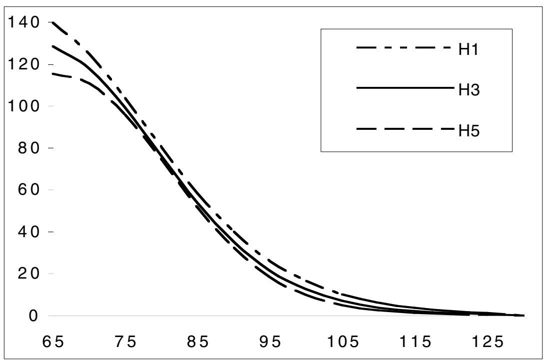

Figure 5.2 – Enhanced Pension; variance of the loss function LTC state

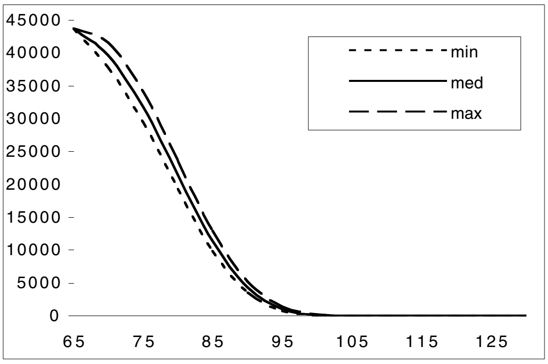

Figure 5.3 – Enhanced Pension; variance of the portfolio loss function for given patterns of the size of the portfolio

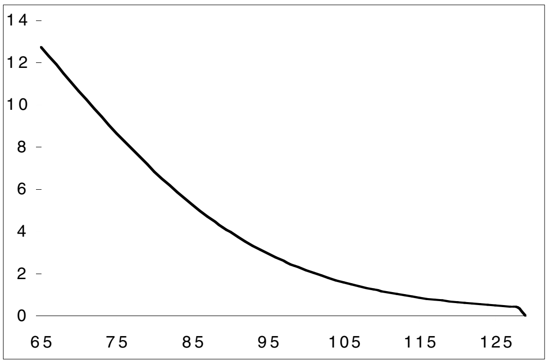

Figure 5.4 – Enhanced Pension; reserve for healthy people

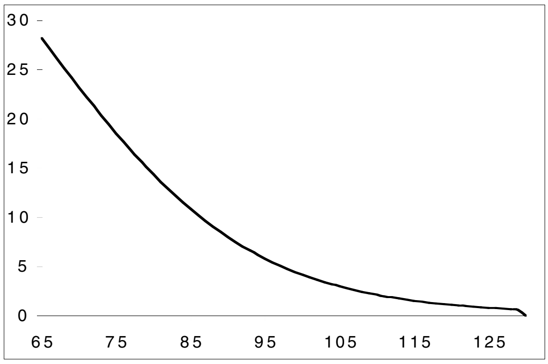

Figure 5.5 – Enhanced Pension; reserve for disabled people

Turning to the stochastic approach, the analysis is performed considering explicitly uncertainty of the future scenario. Numerical evaluations, performed under the same hypotheses of Section 4, are quoted in Tables 5.2 and 5.3 and in Figure 5.6. Similar considerations hold. In particular, the size of the portfolio that let random fluctuations and the systematic risk have the same contribution to the variance of the loss function is 368; if $N > 368$, the systematic component is more important, whilst the contrary holds when $N < 368$. Such result is not fully comparable to the Stand Alone case, due to the different level of benefits.

N	Expected value $E(L^P(0))$	Variance $var(L^P(0))$	Pooling risk $\frac{N\, E(var(L(0)\,\|\,H))}{var(L^P(0))}$	Systematic risk $\frac{N^2\, var(E(L(0)\,\|\,H))}{var(L^P(0))}$
1	0.035	44.164	99.73%	0.27%
10	0.346	452.422	97.35%	2.65%
100	3.464	5,602.351	78.62%	21.38%
368	12.748	32,431.008	49.98%	50.02%
1,000	34.641	163,836.229	26.88%	73.12%
10,000	346.408	12,419,634.573	3.55%	96.45%
100,000	3,464.082	1,202,323,573.901	0.37%	99.63%

Table 5.2 – Enhanced Pension; portfolio loss function

N	Deterministic approach $r^{(H_3)}$	Stochastic approach r
1	0.50280	0.50538
10	0.15900	0.16176
100	0.05028	0.05692
1,000	0.01590	0.03078
10,000	0.00503	0.02680
100,000	0.00159	0.02637
…	…	...
∞	0	0. 02632

Table 5.3 – Enhanced Pension; risk index

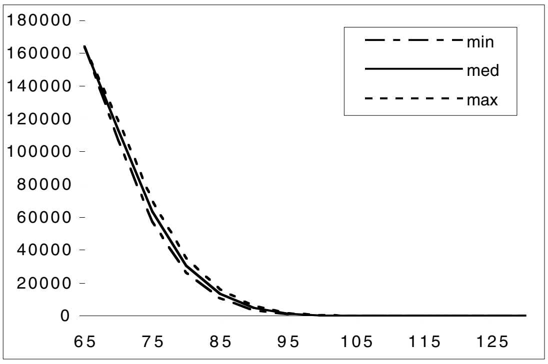

Figure 5.6 – Enhanced Pension; variance of the portfolio loss function for given patterns of the size of the portfolio

6. Tools to face the longevity risk

In the previous Sections the risk coming from the uncertainty of future mortality and morbidity levels (i.e. the longevity risk in a generalized sense) has been highlighted. Results obviously depend on the hypotheses adopted; however, the severity of the systematic risk is to a large extent independent of our choices. Hence, it is important to investigate how this risk can be faced.

Among the tools available to the insurer to face LTC risks, a simple increase in the safety loading does not seem easily feasible, due to competition on insurance markets. A primarily role should be played by the allocation of a suitable capital, i.e. tailored to the severity and features of the risks affecting the portfolio. This capital can therefore be referred to as solvency reserve; in this sense, its amount must be determined so that a given solvency requirement is fulfilled.
Usually, solvency requirements involve a comparison between assets and liabilities. Let us denote by A(t) the (random) value of assets pertaining to the portfolio at time t and by Y(t) the random value of liabilities at the same time. Referring to Olivieri, Pitacco (2001), we say that the required solvency reserve at time 0 is the initial value of assets A*(0) such that the probability that assets keeps higher than liabilities within a given time horizon is equal to an accepted level. In other words, the following condition must be satisfied

$$\Pr\left\{ \bigwedge_{t=0}^{T} A(t) - Y(t) \geq 0 \mid A^{*}(0), I(0) \right\} = 1 - \varepsilon \tag{6.1}$$

where ε is the accepted probability of ruin, T is the time horizon within which the insurer solvency is investigated and *I*(0) collects all information available at time 0 about the portfolio and the scenario in which the portfolio will evolve; in particular, *I*(0) embeds the available information on the demographical scenario. Denoting by n the maximum duration of the policies at time 0 (coinciding with the maximum age less the age at entry), where Y(n) = 0, it can be checked that

$$\begin{aligned} &\Pr\left\{ \bigwedge_{t=0}^{T} A(t) - Y(t) \geq 0 \mid A^{*}(0), I(0) \right\} \\ &= \Pr\left\{ A(T) - Y(T) \geq 0 \mid A^{*}(0), I(0) \right\} \\ &= \Pr\left\{ A(n) \geq 0 \mid A^{*}(0), I(0) \right\} \end{aligned} \tag{6.2}$$

Hence, the required initial reserve is the one which allows to get to a final positive value of assets, with an assigned probability. Finally, we call required solvency margin at time 0, M*(0), the difference between the required solvency reserve and the portfolio single premium, i.e.

$$M^{*}(0) = A^{*}(0) - N\,\pi \tag{6.3}$$

In Table 6.1 and 6.2 some results are shown, relating to a homogeneous portfolio of Enhanced Pension policies (see Olivieri, Pitacco (2001)); the results have been obtained through stochastic simulation. In Table 6.1 a deterministic approach is adopted (results have been obtained referring to scenario H_3), whilst in Table 6.2 a stochastic approach has been assumed, under the same hypotheses of the previous Sections. The huge difference between the results is therefore due to the types of risks faced by the solvency reserve; in Table 6.1 only random fluctuations are accounted for, whilst in Table 6.2 systematic deviations are also considered.

N	N π	ε = 0.01 A*(0)	$\frac{M^*(0)}{N\pi}$	ε = 0.025 A*(0)	$\frac{M^*(0)}{N\pi}$	ε = 0.05 A*(0)	$\frac{M^*(0)}{N\pi}$
100	136,035	151,443	11.326%	149,401	9.825%	147,179	8.192%
500	680,175	715,539	5.199%	709,534	4.316%	705,451	3.716%
1,000	1,360,350	1,402,735	3.116%	1,396,460	2.654%	1,390,785	2.237%
2,000	2,720,700	2,787,999	2.474%	2,780,099	2.183%	2,769,741	1.802%
3,000	4,081,050	4,174,577	2.292%	4,157,763	1.880%	4,141,550	1.482%
4,000	5,441,401	5,525,220	1.540%	5,512,469	1.306%	5,503,452	1.140%
5,000	6,801,751	6,906,187	1.535%	6,892,375	1.332%	6,876,162	1.094%

Table 6.1 – Enhanced Pension; required solvency reserve and margin (deterministic approach)

N	N π	ε = 0.01 A*(0)	$\frac{M^*(0)}{N\pi}$	ε = 0.025 A*(0)	$\frac{M^*(0)}{N\pi}$	ε = 0.05 A*(0)	$\frac{M^*(0)}{N\pi}$
100	136,035	154,963	13.914%	151,612	11.451%	148,973	9.511%
500	680,175	754,323	10.901%	741,187	8.970%	721,460	6.070%
1,000	1,360,350	1,501,214	10.355%	1,483,238	9.034%	1,437,263	5.654%
2,000	2,720,700	2,991,104	9.939%	2,965,069	8.982%	2,886,963	6.111%
3,000	4,081,050	4,480,350	9.784%	4,447,603	8.982%	4,296,875	5.288%
4,000	5,441,401	5,965,233	9.627%	5,929,172	8.964%	5,761,719	5.887%
5,000	6,801,751	7,452,583	9.569%	7,412,720	8.983%	7,226,563	6.246%

Table 6.2 – Enhanced Pension; required solvency reserve and margin (stochastic approach)

When dealing with ways to fund such required solvency reserve, proper reinsurance arrangements must be focussed. It goes beyond the scope of this paper to discuss such topic. We just mention that a stop-loss-like reinsurance treaty can represent a feasible solution in this respect, as suggested by Olivieri, Pitacco (2001).

7. Final remarks

Demographical risks affecting LTCI have been dealt with, with particular regard to those emerging from mortality and disability trends. The main contribution of the paper consists in discussing the necessity of constructing mortality and disability projected tables, which aspects must be considered to this scope and what analyses can be performed once projected tables are available. In the near future, further discussion is then required on tools to face LTC risks. As mentioned in the paper, tools to consider are capital allocation and reinsurance. Moreover, the examples presented show that when several benefits are granted, such as in the Enhanced Pension product, the magnitude of the premium required to the policyholder determines a reduction in the (relative) severity of the risk. So, a proper combination of benefits, which obviously involves commercial advantages, deserves deeper investigations as a way to face risks coming from mortality and disability trends.

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