# PENSIONS IN A SHRINKING ECONOMY

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# PENSIONS IN A SHRINKING ECONOMY

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**Abstract**

An aging society with an absolutely or relatively declining labour force can – with constant capital intensity – experience a reduction in the national capital stock thereby in the short term disengaging resources which can be transferred to the elderly for consumption purposes. Hence, net savings are negative: dissavings of the pensioners exceed savings of the working age population. An aging society can indeed survive by 'eating up part of its capital stock' for some time.

**Keywords:** shrinking population, pensions, economy, savings and investments

## 1. Introduction

Both developed and underdeveloped countries are undergoing in the 21st century a major secular demographic transition. The transitional process consists of a continuing fall in fertility and expansion of life expectation and it determines a progressive aging of most of the world population. In developed countries the two persistent determining factors have been at work for many decades. In these countries the fertility rate fell from 2.8 in 1950-55 to 1.5 in 2000-05, far below the replacement level of 2.1 children per woman[1]. Furthermore, in the first three or four decades of the 21st century the developed countries will face the retirement of their baby-boom generations. A number of developed countries, particularly those that recently started economic take-off and practised further development are also undergoing the demographic transition from higher fertility rates towards below-replacement rates leading to a continuing shrinking of the total population. In the developing countries thus aging will become apparent in the second half of the 21st century.
It is generally agreed upon that in an aging and shrinking population the forthcoming labour force scarcity – particularly in Southern and Eastern European countries, where between 2010 and 2050 the working-age population is likely to decline by about one-third – will increasingly place economies under strain, thereby jeopardizing the sustainability of old-age pension systems. After years of concern about the high growth rates of the world population nowadays in many developed countries the aging and the shrinking of the population is increasingly felt as a major problem[2].

Population aging has potentially substantial implications for the performance of the economy. There is a widespread fear that an increasing proportion of the elderly combined with a decline in the work force will deteriorate economic development, leading to a lower level of GNP and per capita GNP, hence lower net income for the workers and the pensioners. In particular an unstable age structure, e.g. a low proportion of working-age persons in the total population means that production likely will decrease relative to consumption and impede economic growth. On the other hand a relatively smaller number of workers may lead to a higher retirement age and longer working days resulting in increased labour force participation. The crucial question western countries are confronted with therefore remains whether, and if so, how population aging with corresponding declining working age population will affect the economic well-being of their citizens? Or, in other words, can sustainable prosperity be ensured in an aging and shrinking society?

The present paper deals with this important question and concludes that an aging and shrinking society can maintain welfare for some time by not replacing part of its capital stock and transfer the resources becoming free for provision of additional income to the retirees. Our analysis begins with describing the adjustment of the production factor capital in a shrinking economy (paragraph 2). Subparagraph 3.1 (with appendix) describes in a highly stylized framework the working of a mature pension scheme based on funding as a process of transfer of income between generations regulated by the market transaction of capital assets that workers buy when active and sell when they retire. Subparagraphs 3.2 and 3.3 consider the effects of a one-off and of a secular demographic decline respectively in an economy with an embedded pension system (financed by funding) on the main economic variables, size of capital stock, capital intensity, national income, national savings, wage rate, interest rate, per capita consumption of workers and of pensioners. Paragraph 4 draws conclusions.

## 2. Capital in a shrinking economy

It is generally argued that Western welfare states can neutralise the detrimental effects of population aging and subsequent population decline on productive capacity up to a certain extent by an increase of the retirement age, longer working days and/or a further increase of participating of the working age population – in particular those groups which now suffer from low employment – in the labour force[3]. But above all labour force productivity growth is of crucial importance. Immigration can be beneficial to the economy only if the integration of the migrants into the workforce and into the society as a whole is succesful[4]. The productive capacity of countries is declining with the size of the labour force but increases with the productivity rate of the smaller labour force. If smaller cohorts of workers have higher levels of human capital – because the preceding generation has invested more in each child – standards of living may rise further despite aging and a higher amount of transfers to the elderly[5].
It can be shown that even a modest rate of technological progress is (necessary and) sufficient to offset the adverse consequences of population aging and a declining labour force on welfare and the consumption level[6]. Note that in those analyses the capital effect in a shrinking economy is ignored.
An aging society (opposed to a younger society) can hold and probably will hold most of the time a larger capital stock, hence a higher capital intensity (or capital-labour ratio), a declining capital productivity and a rising labour productivity and higher real wages. Therefore, an enhanced capacity is created to finance old-age pensions. In an aging society with an absolutely or relatively declining labour force also new relative scarcities of factors arise. Labour is now scarcer opposed to capital, leading to higher levels of wages and lower interest rates.

The adjustment of the economy to a decling labour force involves the following mechanism, an adjustment of the capital intensity or, in other words, an adjustment of the per capita capital stock: larger, constant or contracting. In respect of capital formation or per capita net investments we may therefore distinguish three situations,

a. no reduction in net per capita investment, leading to a further increase in the capital intensity of production. Consequently, today less consumption goods can be produced. On the other hand, the higher capital intensity gives rise to a larger labour productivity and a larger future resource base. High flexibility of the economy, viz. a high elasticity of substitution between capital and labour and a high elasticity of savings to the interest rate, makes that a larger part of the capital stock – becoming redundant as a result of the declining labour force – can be absorbed by an increase in the capital intensity, resulting in a higher per capita level of output, hence of workers' income and even of pension benefits;
b. a reduction in net investments occurs but they remain positive – the capital stock and the capital intensity are still increasing – thereby augmenting in the medium term the volume of consumption for the whole population. In this case savings of the younger population still exceeds dissavings of the elderly, but to a diminishing extent. Eventually net investments and net savings become zero: the *copper rule zero growth* path arises[7]. Note that the case of zero net savings is only consistent with zero population growth and constant capital-labour ratio or consistent with a declining population with higher capital-labour ratio.

   Note also that the scale of the benefits of a lower investment level and a lower required savings rate likely will be less than figures might suggest. Only new capital investments are in most cases incorporating new technologies of production, which are of crucial importance for further productivity growth. Relying on the old capital stock alone means a productivity loss. Capital accumulation combined with good institutions, policies and technological progress will without diminishing marginal capital returns result in continued economic growth;

c. a reduction in the stock of capital – with constant capital intensity – thereby (in the short term) disengaging resources which can be transferred to the aged. In this case net investments are negative or, in other words, dissavings of the elderly exceed savings of the working population. An aging society can indeed 'live on its capital' for some time. Part of the capital stock is not reproduced and consumption goods are produced instead of replacing worn-out capital goods[8].

   As far as the allowable rate of capital stock reduction does not exceed capital depreciation, redundant capital can be disengaged rather easily by diminishing or halting gross investments and, hence, can be easily 'turned back into consumption'. The fastest feasible decline is given by the rate of deplacement. On the other hand, when allowable disinvestment exceeds capital depreciation practical problems arise in respect of the (physical) feasibility of consuming part of the existing capital stock. When allowable disinvestment equals capital depreciation – that means a steady state contraction of the capital stock –, the economy with a decreasing labour force moves along the equilibrium negative growth or decline path. We may speak of the *iron rule decline* path[7]. As a consequence there is no need for new pension savings apart from the case of compensating dissavings elsewhere[9].

   Ultimately capital equipment invested abroad (running surpluses on the current account) can be sold, thereby acquiring foreign currency by which consumer goods can be bought (implying sustained current account deficits)[10].

In the sixties (when Aaron wrote his famous article) there were no limits to growth (or we were not aware of it) of the world economies. The first report of the Club of Rome (1971) warned humanity for depletion of resources of energy and of other raw materials. They were largely ignored. At the beginning of the 21st century scarcity of energy, many other exhaustible resources and food products are resulting in substantial price increases that may impediment further economic development.

In conclusion, in an aging society it is far from sure that the golden rule growth path will continue for ever. By reducing the capital stock in relative or even in absolute terms per capita welfare nonetheless can be maintained for some time. In a later stage growth rates of national product can become zero or even negative. Unluckily we might notice a quite new trend, viz. that many economies after some point of time in the 21st century will be moving from a golden rule growth path via copper rule zero growth to an iron rule decline path.

## 3. Pensions in a shrinking economy

In the present paragraph we consider some illustrative numerical examples that show how an economy with an incorporated pension scheme based on funding evolves and adjusts to a shrinking of the population. We utilize the basic neoclassical model where the capital is owned by the elderly via pension funds.

### *3.1 The model*

We consider a stationary neo-classical economy with two identical overlapping generations. The three conditions of equilibrium are full capacity, full employment and equality of savings and investments. Thers is no technological progress or productivity growth. In the stationary state net savings are zero. There are two equal-length life periods: each person works in period 1 and is retired in period 2. The elderly lends the total amount of savings accumulated during their working years at the beginning of each period to the pension institutions, which in turn invest them in productive activities. At the beginning of each period the firms which borrowed the financial capital from the pension funds hire the labour force. At the end of each period net per capita national product is composed of the wage rate and the return to capital. The workers use their wages for consumption and buying assets from the old generation. The old generation consumes all its income (their original capital plus interest earned on it) and dies.

The demography thus is characterized by a stationary or zero growth population with two overlapping generations – groups of workers and retired non-workers – of size N equal to 100 individuals each. In the next sections 3.2 and 3.3 due to a fall in fertility the working population in period 1 – note that the length of a period is roughly 30 years – shrinks from 100 to 90. In period 2 demographic equilibrium is restored resulting into 90 individuals in both groups of workers and retirees. In section 3.4 population decline continues untill in period 15 the size of the groups of workers and retirees stabilizes at 50 individuals each. Furthermore in a stationary population the economy with zero net savings is stationary also. Workers and retirees consume according to their preferences equal amounts of goods and services. Dissavings of the elderly are precisely matched by savings of the working population. The pension funds system functions as an intermediary between the overlapping generations.

To better show the working of pension finance systems in a Cobb-Douglas framework we make use of the so-called transition equation, as employed e.g. by Auerbach and Kotlikoff [1995] [11]. See in more detail the appendix.

### *3.2 Shrinking population and pension scheme financed by funding*

#### OLD STEADY STATE (3.2.1)

In the equilibrium situation the capital stock remains unchanged from one period to the next. Hence, net savings are zero. In an stationary or steady state economy the dissavings of the retired elderly are exactly matched by the savings of the working population (Table **1** first line).

The retirees own the capital stock, 268.9 units of account that via the pension fund system has been lent to the firms. The firms pay the workers wages equal to the marginal product of labour. A paternalistic imperative says that consumption opportunities for workers and elderly are equal. The contribution rate of the pension scheme appears to be 0.2855. National product is 1345.5. The per capita consumption is 6.73 units. For a retired individual the 6.73 units are equal to the value of the per capita capital stock (2.69 units) plus the interest on it of 1.50[12].

#### ONE-OFF DECLINE OF THE LABOUR FORCE; TRANSITION TO A NEW STEADY STATE (3.2.2)

We will next consider how a neo-classical stationary economy adjusts to a decline in fertility. Assume a baby bust, leading to a lower number of workers. In the example shown in Table **1** (: transition to a new steady state) we assume that in period 1 a decline in fertility level reduces the working population from 100 to 90. Labour is becoming more scarce. As a result the wage rate tends to increase and – given the labour supply of 90 workers and the unaffected capital supply of 268.9 units – becomes equal to 9.72 units. The new capital-labour ratio is higher than before, 2.99 opposed to 2.69, leading to a lower interest rate, 1.39 opposed to 1.50 and a higher wage rate, 9.72 against the pre-baby bust value of 9.42 and a higher level of consumption of the workers of 6.95 (in period 0: 6.73). This phenomenon is referred to as the demographic dividend. Due to the fall in the interest rate per capita consumption of the elderly decreases from 6.73 to 6.44 units[13].

Total wages in period 1 amount to 874.8 (that is 9.72 * 90) and workers' savings are accumulated to 249.8 (that is 874.8 * 0.2855), which is lower than before 268.9 (: 942 * 0.2855) because the number of workers is lower. The capital stock at the beginning of period 2 is 19.1 units less than the capital stock at the beginning of period 1. Or, in other words, the dissavings of the retired elderly are not completely matched by the savings of the working population. Aggregate net savings are -19.1. Part of the capital stock has not been replaced but instead consumption goods are produced, which are consumed by the pensioners (0.19 unit per head)[14]. Remember that the retirees have the ownership of the capital stock and, thus, are exclusively entitled to income supplements from dissavings. Total consumption in period 1 is 1268.9, which is higher than national product (that is 1249.8). The difference is again 19.1, that is the decline in the capital stock which is eaten up.

In the second period the lower savings of the smaller working population lead to a rise in the interest rate and consequently to a lower capital intensity of national production. In this period and in following periods also the savings supply of the workers is insufficient to uphold the capital stock, which continues to decline from 268.9 to 242.

The new steady state is restored at a position of lower aggregate activity (Y = 1210.9), but with the same capital intensity of production, at the same wage rate (w = 9.42) and identical interest rate (r = 1.50). Per capita levels of consumption (both $c^w$ and $c^{ret}$ 6.73 units) are equal to those in the old steady state. The number of both workers and retirees is 90, opposed to 100 earlier. In the new steady state the fertility rate is back on its replacement level.

*Age structure*

We may observe that age structure is ignored in the model used. Age structure directly determines the relative size of the working-age population given that labour force participation rates vary by age. A decline in fertility rates reduces the number of children and raises the share of the working-age population and labour supply in the short run. Persistently low fertility levels (below replacement value) imply a relatively large size of the elderly and a smaller size of the young working generation compared with the stationary (zero-growth) population. In period 1 birth rates are assumed to return to their replacement level of 2.05

Finally note the crucial assumption made that the adjustment of the economy, viz the capital stock to the fertility shock always will take place smoothly. In period 1 the capital supply is relatively abundant, interest rate falls and the capita-labour ratio increases. Actually dissavings take place. Later the production factor capital becomes relatively scarcer again and the interest and the capital-labour ratio restore to the old steady state level. Such an adaptibility highly likely does not exist in the real world. The model leaves practically no scope for distortions.

Summarizing, the potentially adverse effects of a one-off decline of the labour force for welfare and consumption can easily be absorbed by not replacing part of the capital stock (or dissavings), keeping per capita consumption of both workers and pensioners, per worker capital and the contribution level constant from one steady state to the other.

### *3.3 Secular decline of the population and pension scheme under funding*

In par. 3.2 we considered a one-off decline of the labour force after which the steady state is restored. In the present paragraph a sequence of one-off declines of labour supply is presented till the total population has been halved to return to its new secular level in period 15 (Table **2**). The economy adapts to a process of population decline partly through a change in the capital intensity of production and partly by replacing only part of the capital stock. Net savings are negative and the unreplaced capital goods are reconverted into consumption goods. The dependency ratio deteriorates from 1.0 to 1.052. Thanks to a higher capital intensity per capita consumption during the transition despite the higher dependency ratio is somewhat higher than in equilibrium[15]. In period 1 total consumption of young and old respectively sums up to $6.83^3$ * 95 + 658.3 = 1307.44, which is higher than national income (that is 1298.0). The difference is the level of dissavings, 9.44 units. The contribution rate remains at 0.2855 the same as in section 3.2.2 before.

In the new steady state both the working population and retired population are 50, half of the population size in the old steady state[16]. Also national income is reduced by 50 percent to 672.7. All per capita economic variables are at their original level. After a very long period of relatively low fertility rates it finally is restored at its replacement level around 2.05.

Conversely, if we imaginarily face continued low fertility rates assuming no net immigration and constant longevity population size declines persistently. The economy is shrinking proportionally and ultimately comes to a complete standstill approaching zero level. That can be considered as a sustainable, but highly likely not as a desired equilibrium.

#### *Higher longevity*

To complete the discussion we consider how a economy with a shrinking population might face higher longevity[17]. Suppose an expected increase in longevity of about 20 percent in period 1 (Table **3**). The support ratio deteriorates from 1.0 to 1.26 to return to its new secular value of 1.20. As a result the contribution rate rises from 28.55 percent to 35.05 percent.

> In the new steady state the capital-labour ratio is 3.60 vis-à-vis 2.69 before. As a result labour productivity rises from 13.45 to 14.69. Hence, despite considerable aging the per capita consumption is only slightly less than in the old steady state, 6.68 against 6.73.

Summarizing, it appears that the potentially adverse result of a relative or even absolute decline in the labour force on the economic well-being (consumption level) of the population can at least partially, but virtually always fully be offset by dissavings (or the effect of non replacing part of the capital stock). In addition, due to a higher capital intensity of the economy during the transitional period a higher level of consumption of the workers can be realized. The pensioners' consumption at that time is slightly less.

## 4. Concluding remarks

In the present paper we examined the effects of a one-off and a secular decline in population on the main economic variables using a basic neo-classical framework and a Cobb-Douglas production function with unitary elasticity of factor substitution. Stylized numerical examples are utilized as an illustration.

We may conclude that the economy adapts to a demographic decline in the first place through a rise in the capital intensity leading to a lower interest rate and a higher wage rate, enabling higher consumption for the workers, but not so for the retired. By not replacing part of the capital stock additional resources become available for the elderly for maintaining at least in part their level of consumption. A highly flexible economy makes it more likely that some part of the capital made redundant by a decline in the labour force will be absorbed by an increase in the capital-labourratio and another part will become free for supplementing the consumption of the pensioners. Up to the present time this so-called Solow-effect has not been a major factor, but it can be an important one in the coming decades in aging countries.

The economic effects of population decline will be more in line with the aforementioned findings when the country is larger, while for a smaller country like the Netherlands the economic effects of a shrinking population may be either amplified or reduced due to specific economic characteristics of the smaller country, e.g. the interest rate is a exogenous variable, hence independent of a changing capital-labour ratio. In addition, there are differences among member states in their preferences for leisure and consumption over time and pension arrangements vary considerably in scale and scope.

Finally, it is (too) easy to forget how rapidly real incomes have grown over the 20th century, viz. about a sevenfold increase. At current trend growth rates GDP will double in real terms roughly every 30 to 35 years. It can be concluded that without doubt we can afford to grow older. We in the European countries (and elsewhere in the more developed world as well) are rich enough to sustain an old-age pension for a more numerous older population. A fortiori, it can be said that we are approaching saturation in the consumption of material goods, not only of necessities but of luxury goods as well, which were only science fiction or dreams during the first half of the 20th century[18].

As individuals and nations grow more affluent they tend to increase the share of total spending devoted to some commodities and reduce the share devoted to others. Food is a clear example of the first category. Outlays for health, education and provision for incomes in retirement, sickness/disability and unemployment will all increase their share as per capita national income rise. As individuals and nations continue to grow more affluent it seems likely that they will wish to spend – and can afford to do so – more on pension income and health care. Hence, the share of expenditures for pension provisions and health care facilities in national income will grow further and as a result is not anything to be concerned about.

## References and notes

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1 UN Population Division, *World Population Prospects: the 2008 Revision*, New York, 2009.

2 It is impressing that the hypothetical continuation of the current fertility rates would lead to a population from 6.1 billion in 2000 to 12.8 bilion in 2050, 43.6 billion in 2100, 244 billion in 2150 and 134,000 billion in 2300. Attention to a declining population was already given bij J.M. Keynes [Some economic consequences of a declining population, in: *The Collected Writings of John Maynard Keynes*, D. Moggridge (ed.), Macmillan, London, 1937].

3 Policy measures may include a higher retirement age, reducing older workers' wage gains – so that older workers are no longer such more expensive than younger workers – and eliminating basic held views about the employment of older people. Recommended are policies directed at increasing employment opportunities for older workers by recontracting them in less prominent jobs with lower wage rates. Jensen, S.E. and O.H. Jorgensen [*Low fertility, labour supply and retirement in Europe*, WDA-HSG discussion paper 2008/3, University of St. Gallen, Sankt Gallen, 2008] argue that in European countries a higher retirement age tends to decrease labour supply. This is exactly the opposite of what policy measures intend to make happen.

4 Note that e.g. immigration from Eastern European countries apart from brain drain is making the aging of the population in those countries even more serious.

5 Despite higher participation rates the working population in Western countries ultimately will decline. As long as productivity growth is exceeding population decline GNP continues to grow. Only a stagnating productivity growth in a more distant future will be unable to offset the decline in the employed working age population.

6 See e.g. Kuné, J.B., *On Global Aging; old-age income systems in the EU and other major parts of the world*, Springer Verlag, Heidelberg, April 2003 and Kuné, J.B., Population aging and welfare: the case of the Netherlands, *Pensions, an international journal*, *14*, no. 4, 2009, pp. 231-241.

7 Opposed to the *golden rule growth* path, well known from economic growth theory.

8 This effect is also called the Solow-effect: society can permit to consume more while it is able to invest less and still maintain the per worker production level. Again, innovative technology can not or only to a minor extent be introduced as they are often incorporated in reproduced capital goods. The situation can get worse if the baby boom generation in the coming decades want to sell their assets to the smaller younger cohorts resulting in an 'asset market meltdown'.

9 As a consequence the elderly also lack the ownership of the pension capital stock that can be considered as a more secure basis for retirement income than the willingness and ability of the current labour force for paying pension contributions under a pay-as-you-go system.

10 Increased investment abroad may alleviate the aging problem in the 'western' developed countries. Note however that virtually all the major (and smaller) developed countries will have the same problem in nearly the same time period ahead. The aging 'western' world as a whole has only a rather small foreign sector. Or, in other words, not all societies can play the same game at the same time. Japan, for instance, has during the last few decades invested its current account surpluses abroad, to a large extent however in countries which also face an aging problem. Realising overseas assets Japan – on a macro-level – may probably solve at least in part its aging problem but at the expense of making the problem of those other countries worse.

11 The Auerbach-Kotlikoff framework (see note 19) – applied by themselves for presenting the transition path following an epidemic – can be employed in answering the question whether pension schemes can be helpful in the face of labor force decline and whether part of the capital stock can be transformed smoothly into consumption for the retired. The A-K framework is used by S. Cesaratto [*Pension*

*reform and economic theory; a non-orthodox analysis*, Edward Elgar, Cheltenham, UK] to criticize the neo-classical approach in pension reform measures. Evidently integrated analyses are preferential to investigate the effects of a demographic transition on the economy. Noteworthy are the INGENUE-studies of CEPII; e.g. M. Aglietta, *Macro-economic consequences of pension reform in Europe: an investigation with the INGENUE world model*, CEPII, working paper 17, Paris, 2001.

12 The interest rate r is not an annual interest rate. The length of a period in the model of paragraph 3.1 is roughly 30 years. The corresponding annual interest rate is 3.1 percent.

13 Note that the effects of a demographic change on human capital is ignored. If parents invest more in child quality when there are less children a declining labour force is accompanied by more human capital per worker leading to a higher level of production and welfare.

14 Only part of the capital stock that wears out is replaced. In Table **1** the replacement rate in period 1 is 7.1% (19.12 divided by 268.89). Since we consider periods of about 30 years this percentage is very low. Depreciation generally will be much higher.

15 Note that in the converse situation of a persistently growing population the capital intensity is lower. Also per capita consumption during the transition to a new steady state is lower than in equilibrium due to positive net savings.

16 The recovery of countries with rather low fertility levels (less than 1.5) to a new steady state of population size on replacement level appears to be increasingly difficult [Lutz et al., The low fertility trap hypothesis: forces that may lead to further postponement and fewer births in Europe, *Vienna Yearbook of Population Research 2006*, Austrian Academy of Sciences, Vienna, 2006].

17 For simplicity it is assumed that 100 retired people living (1+x) periods can be approximated by 100*(1+x) retired people living 1 period.

18 In most western countries nowadays the poverty line is at a level of real income that was attained by only those in the highest 10 percent of the income distribution about 1900.

19 As in Auerbach & Kotlikoff [*Macroeconomics; an integrated approach*, South-Western College Publishing, Cincinnati, Ohio, 1995]. A slightly different approach is followed by Seidman [*Funding social security; a strategic alternative*,Cambridge University Press, Cambridge, UK, 1999].

Table 1. Population decline in period 1 in a neoclassical economy; pensions financed by funding. Equal consumption of workers and pensioners in equilibrium (effective contribution rate 0.2855)

| **period** | **working population** | **retired population** | **capital stock** | **per worker capital** | **national income** | **national savings** | **wage rate** | **interest** | **per capita consumption** | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | **workers** | **pensioners** |
| | **( $N^w$ )** | **( $N^r$ )** | **( K )** | **( $K/N^w$ )** | **( Y )** | **( S )** | **( w )** | **( r )** | **( $c^w$ )** | **($c^r$)** |
| **old steady state** | 100 | 100 | 268.89 | 2.69 | 1345.5 | 0.00 | 9.42 | 1.50 | 6.73 | 6.73 |
| **1** | 90 | 100 | 268.89 | 2.99 | 1249.8 | -19.12 | 9.72 | 1.39 | 6.95 | 6.44 |
| **2** | 90 | 90 | 249.77 | 2.78 | 1222.5 | -5.47 | 9.51 | 1.47 | 6.79 | 6.85 |
| **3** | 90 | 90 | 244.31 | 2.71 | 1214.4 | -1.62 | 9.45 | 1.49 | 6.74 | 6.76 |
| **4** | 90 | 90 | 242.69 | 2.70 | 1212.0 | -0.48 | 9.43 | 1.50 | 6.73 | 6.73 |
| **5** | 90 | 90 | 242.21 | 2.69 | 1211.2 | -0.14 | 9.42 | 1.50 | 6.73 | 6.73 |
| **6** | 90 | 90 | 242.06 | 2.69 | 1211.0 | -0.04 | 9.42 | 1.50 | 6.73 | 6.73 |
| .. | .. | | .. | .. | .. | .. | .. | .. | .. | .. |
| .. | .. | | .. | .. | .. | .. | .. | .. | .. | .. |
| **new steady state** | 90 | 90 | 242.00 | 2.69 | 1210.9 | 0.00 | 9.42 | 1.50 | 6.73 | 6.73 |
| | | | | | | | | | **total consumption** | **1210.9** |

Table 2. Proportional shrinking population in a neoclassical economy; pensions financed by funding. Equal consumption of workers and pensioners in equilibrium (effective contribution rate 0.2855)

| period | working popul. ( $N^w$ ) | retired popul. ( $N^r$ ) | capital stock ( K ) | per worker capital ( $K/N^w$ ) | national income ( Y ) | national savings ( S ) | wage rate ( w ) | interest ( r ) | per capita consumption workers ( $c^w$ ) | pensioners ( $c^r$ ) |
|---|---|---|---|---|---|---|---|---|---|---|
| **old steady state** | 100 | 100 | 268.89 | 2.69 | 1345.5 | 0.00 | 9.42 | 1.50 | 6.73 | 6.73 |
| **1** | 95 | 100 | 268.89 | 2.83 | 1298.0 | -9.44 | 9.56 | 1.45 | 6.83 | 6.58 |
| **2** | 90.3 | 95 | 259.41 | 2.87 | 1238.8 | -11.83 | 9.61 | 1.43 | 6.87 | 6.64 |
| **3** | 85.7 | 90.3 | 247.58 | 2.89 | 1178.5 | -12.05 | 9.62 | 1.43 | 6.88 | 6.66 |
| **4** | 81.5 | 85.7 | 235.53 | 2.89 | 1120.1 | -11.68 | 9.63 | 1.43 | 6.88 | 6.67 |
| **5** | 77.4 | 81.5 | 223.84 | 2.89 | 1064.2 | -11.16 | 9.63 | 1.43 | 6.88 | 6.67 |
| **6** | 73.5 | 77.4 | 212.68 | 2.89 | 1011.0 | -10.63 | 9.63 | 1.43 | 6.88 | 6.67 |
| | .. | .. | .. | .. | .. | .. | .. | .. | .. | |
| | .. | .. | .. | .. | .. | .. | .. | .. | .. | |
| **14** | 50 | 51.3 | 140.70 | 2.81 | 682.1 | -5.24 | 9.55 | 1.45 | 6.82 | 6.74 |
| | .. | .. | .. | .. | .. | .. | .. | .. | .. | |
| | .. | .. | .. | .. | .. | .. | .. | .. | .. | |
| **new steady state** | 50 | 50 | 134.45 | 2.69 | 672.7 | 0.00 | 9.42 | 1.50 | 6.73 | 6.73 |
| | | | | | | | | | **total consumption** | **672.7** |

Table 3. Shrinking population and living longer in a neoclassical economy; pensions financed by funding. Equal consumption of workers and pensioners in equilibrium (effective contribution rate 0.3505)

| **period** | **working popul.** ($N^w$) | **retired popul.** ($N^r$) | **capital stock** (K) | **per worker capital** ($K/N^w$) | **national income** (Y) | **national savings** (S) | **wage rate** (w) | **interest** (r) | **per capita consumption workers** ($c^w$) | **pensioners** ($c^r$) |
|---|---|---|---|---|---|---|---|---|---|---|
| **old steady state** | 100 | 100 | 268.89 | 2.69 | 1345.5 | 0.00 | 9.42 | 1.50 | 6.73 | 6.73 |
| **1** | 95 | 120 | 330.11 | 3.47 | 1380.4 | 8.57 | 10.17 | 1.25 | 6.61 | 6.20 |
| **2** | 90.3 | 114 | 338.69 | 3.75 | 1342.0 | -9.42 | 10.41 | 1.19 | 6.76 | 6.50 |
| **3** | 85.7 | 108.3 | 329.26 | 3.84 | 1283.8 | -14.29 | 10.48 | 1.17 | 6.81 | 6.60 |
| **4** | 81.5 | 102.9 | 314.97 | 3.87 | 1222.2 | -15.31 | 10.51 | 1.16 | 6.83 | 6.62 |
| **5** | 77.4 | 97.7 | 299.85 | 3.88 | 1161.7 | -14.81 | 10.51 | 1.16 | 6.83 | 6.63 |
| **6** | 73.5 | 92.9 | 285.03 | 3.88 | 1103.9 | -14.20 | 10.51 | 1.16 | 6.83 | 6.64 |
| | .. | | .. | .. | .. | .. | .. | .. | .. | .. |
| | .. | | .. | .. | .. | .. | .. | .. | .. | .. |
| **14** | 50 | 61.6 | 188.72 | 3.77 | 744.8 | -6.92 | 10.43 | 1.18 | 6.77 | 6.71 |
| | .. | | .. | .. | .. | .. | .. | .. | .. | .. |
| | .. | | .. | .. | .. | .. | .. | .. | .. | .. |
| **new steady state** | 50 | 60 | 180.22 | 3.60 | 734.6 | 0,00 | 10.28 | 1.22 | 6.68 | 6.68 |
| | | | | | | | | | **total consumption** | **734.6** |

## APPENDIX. neo-classical economics in a shrinking and growing older environment

*Lifetime budget constraint and the saving decision*

From the utility function

$$u_{t-1} = (c_{t-1}^{w})^{\alpha} \cdot (c_{t}^{ret})^{1-\alpha} \tag{1}$$

and the lifetime budget constraint,

$$c_{t-1}^{w} + \frac{c_{t}^{ret}}{1+r_{t}} = w_{t} \tag{2}$$

it follows that,

$$c_{t-1}^{w} = \alpha \cdot w_{t-1} \tag{3}$$
$$c_{t}^{ret} = (1-\alpha) \cdot w_{t-1} \cdot (1+r_{t}) \tag{4}$$

where $u_t$ stands for the utility derived from consumption of a typical member of generation $t$ when young and old given the time preference parameter $\alpha$; with $c_t^{w}$ and $c_t^{ret}$ worker's and elderly's consumption respectively, $w_t$ wage rate, $r_t$ interest rate and $a_t$ the per capita amount of assets.

At the end of each period net per capita national product ($y_t$) is composed of the wage rate and the return to capital, $y_t = w_t + r_t \cdot k_t$ where $k_t$ is the per capita capital stock.

The workers use their wages for consumption ($c_t^{w}$) and savings for pension purposes (saving rate appears to be 1-α). Savings occcur by buying assets ($a_t$) from the old generation, $w_t = c_t^{w} + a_t$. The old generation consumes all its income and dies, $c_t^{ret} = r_t \cdot k_t + a_t$.

*Production*

The neo-classical model with the embodied pension plan is given by the following set of equations. Further discussion follows below.

$$Y_t = A_0 e^{gt} \cdot K_t^{\beta} \cdot N_t^{(1-\beta)} \quad ; \quad 0 < \mu < 1 \tag{5}$$
$$DK_t = S_t \quad ; \quad DK_t = \dot{K}_t,\ K_o\ given, \tag{6}$$
$$N_t = N_0 \cdot e^{nt} \tag{7}$$

All variables are taken as continuous and differentiable functions of time. Y is total output per unit of time, K is capital stock and N is labour supply where n is the (working) population growth rate. There is Hicks-neutral technical progress at rate g. The Cobb-Douglas production function is characterized by constant returns to scale. Consider n = 0 and g = 0 in the next sections.

The equations (5), (6) and (7) are the three equilibrium conditions in respect of full capacity, the equality of (net) saving and investment and full employment.

In per workers terms equation (5) reduces to,

$$y_t = A_t \cdot k_t^{\beta} = f(k_t) \tag{8}$$

where $y_t = Y_t/N_t$, $k_t = K_t/N_t$ and $A_t = A_0$. The Cobb-Douglas function is characterized by a unitary elasticity of factor substitution. Following Auerbach and Kotlikoff [1995] A = 10 and β = 0.3. A constant A – also called the Solow-residual – means a zero rate of technological progress.

The marginal products of capital and labour are,

$$r_t = f'(k_t) = \beta \cdot A_t \cdot k_t^{\beta-1} \tag{9}$$
$$w_t = y_t - f'(k_t) \cdot k = (1-\beta) \cdot A_t \cdot k_t^{\beta} \tag{10}$$

When it can be assumed that production is based on profit maximization under perfect competition on both product and input markets, the profit rate and the wage rate at any time *t* are equal to the corresponding marginal products. The connection between a fall in the size of the current labour force (due to lower past fertility and/or due to a lower retirement age) and the wage rate is thereby established. The exponents β and 1-β are constants representing the relative share of capital and labour in national product *Y.* A production function with constant returns to scale guarantees that the payment of marginal products to the factors of production will exactly exhaust total production.

*National saving and the national capital stock*

To keep things simple we assume that there are always N members of the working population and N elderly people (no population growth). Hence, the total population equals 2N. If $K_t$ stands for the national capital stock at time t, we have,

$$\begin{aligned} K_t &= N_t \cdot a_t = N \cdot (w_{t-1} - c_{t-1}^{w}) \\ &= N \cdot (1-\alpha) \cdot w_{t-1} \end{aligned} \tag{11}$$

with $\alpha$ the propensity to consume.

If $S_t$ indicates total national saving, then,

$$S_t = Y_t - N \cdot c_t^{w} - N \cdot c_t^{ret} \tag{12}$$

with
$$Y_t = N \cdot w_t + K_t \cdot r_t \tag{13}$$
$$S_t = K_{t+1} - K_t = I_t$$
$$\begin{aligned} S_t &= N \cdot (w_t - c_t^{w}) + (r_t \cdot K_t - N \cdot c_t^{ret}) \\ &= N \cdot a_{t+1} - N \cdot a_t \end{aligned} \tag{14}$$

This equation says that total national saving is the savings of the workers and of the retirees. Note that in the equilibrium (with n = 0 and g = 0) net savings are zero. Recalling that $a_{t+1} = w_t - c_t^{ret}$ (: the assets the workers at time t bring into their old age) and $c_t^{ret} = a_t \cdot (1 + r_t)$ (: the principal plus the interest at time t on the assets, owned by the retired), the savings of the retired – [$r \cdot K_t - N \cdot c_t^{ret}$] – equals to $N \cdot a_t \cdot r_t - N \cdot a_t \cdot (1 + r_t) = -N \cdot a_t$. Or, in other words, the retired dissave as they consume more than their capital income.

### *The transition equation*

To derive the relationship between the capital-labour ratio at times t and t+1 we start with the fact that $k_{t+1} = a_{t+1}$. Replacing $a_{t+1}$ with $(1–\alpha) * w_t$ equation (15), based on the utility maximization of the individual results,

$$k_{t+1} = (1 - \alpha) \cdot w_t \tag{15}$$

and replacing $w_t$ with $(1 - \beta) \cdot A_t \cdot k_t^{\beta}$ (: equation 10) we find,

$$k_{t+1} = (1 - \alpha) \cdot (1 - \beta) \cdot A_t \cdot k_t^{\beta} \tag{16}$$

Equation (16) is called the transition equation because it tells us how the economy's capital-labour ratio transits (changes) from one time period to the next[19].

### *The steady state*

The long-run position to which the economy converges is called its *steady state* of capital per worker. If the economy is in its steady state in period t, the capital-labour ratio a period later, at time t+1, will be the same as at time t. Let us call the steady-state value of the capital-labour ratio $\bar{k}$ . Hence,

$$\bar{k} = (1 - \alpha) \cdot (1 - \beta) \cdot A \cdot \bar{k}^{\beta} \tag{17}$$

and

$$\bar{k} = [(1 - \alpha) \cdot (1 - \beta) \cdot A]^{\frac{1}{1-\beta}} \tag{18}$$

### *Population growth or decline*

Rather than assume the number of people in each generation is constant over time, let us allow the number to increase or decrease each period at the rate n. The transition equation now becomes,

$$k_{t+1} = \frac{(1 - \alpha) \cdot (1 - \beta) \cdot A \cdot k_t^{\beta}}{1 + n} \tag{19}$$

Next, we find the steady state capital-labour ratio $\bar{k}$ ,

$$\bar{k} = \left[\frac{(1-\alpha)\cdot(1-\beta)\cdot A}{1+n}\right]^{\frac{1}{1-\beta}} \tag{20}$$

From (20) and (9) the steady state interest rate $\bar{r}$ ,

$$\bar{r} = \frac{\beta(1+n)}{(1-\beta)\cdot(1-\alpha)} \tag{21}$$

In the steady state aggregate capital and labour grow at the same rate n. The growth rate of capital also is *i / k*, where i = I / N (investment per worker).

The saving rate of the economy is,

$$s = \frac{S}{Y} = \frac{I}{Y} = \frac{i}{y} \tag{22}$$

By n = (i / k) and (8),

$$s = \frac{n}{A} \cdot k^{1-\beta} \tag{23}$$

Substituting (20) into (23) yields the steady-state saving rate $\bar{s}$ ,

$$\bar{s} = \frac{n}{1+n} \cdot (1-\alpha)\cdot(1-\beta) \tag{24}$$

*Pay-as-you-go financing*

The national capital stock is evenly owned by the younger working population and the older retired population and is administered by the Capital Supply Authority. At the end of each period when the retirees die, the capital stock is transferred to the next generation of workers who will instantly enter the labour market and of the next generation of elderly who will right away retire. Net savings are zero.

Hence, the transition equation,

$$k_{t+1} = k_t^{transfer} \tag{25}$$

Note that $k_0$ is equal in both pension finance systems.