Stochastic Projections of the Financial Experience of Social Security Programs: Issues, Limitations and Alternatives

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For well over a century now, actuaries have projected the financial experience of social security systems around the world. Policymakers – and actuaries themselves – have debated various aspects of those financial projections, including: (a) the duration of the projections, which range around the world from just a few years to 75 years in the case of Canada and all the way to *infinity* in the case of the United States (since 2003); (b) the fundamental economic and demographic assumptions, including mortality improvement and fertility trends but obviously not limited to those; and (c) methodology, with respect to which an important controversy focuses on deterministic versus stochastic methods. This paper focuses on that last point but touches on the previous ones, also, in the course of the discussion.

Deterministic models

In the simplest terms, deterministic projections are the output of models that apply a set of assumptions regarding the values of important, relevant variables to represent future experience. These assumptions are expressed in terms of "best-estimate" or expected future values. These deterministic assumptions are applied to the initial state, which to a large extent is known for the system under consideration. Then, the deterministic, mathematical models can produce as many years of projected future experience as are desired for any intended purpose. Future cash flows are usually discounted back to the present to determine a system's financial condition and associated financial values. All of this methodology is reasonably well understood by actuaries and policymakers, alike.

Stochastic models

Stochastic models are much more complicated than deterministic ones and became feasible only in recent years, due to tremendous increases in computer power. Stochastic models, as the name implies, recognize that all of the important and interesting economic and demographic variables are themselves *stochastic* (or probabilistic) in nature and should be treated as such. Thus, their future paths are not determined with certainty but, rather, depend on the random (or seemingly random) behavior of those variables. (Note that some important variables are important but *uninteresting* in a stochastic sense because they are very stable, exhibiting little probabilistic behavior. An example might be retirement age: In the U.S., most retired workers claim Social Security benefits at the earliest possible age, 62, and if they claim at any other age before 70, actuarial reduction and increase factors make the lifetime value of benefits very similar.)

All models are uncertain

Both deterministic and stochastic models share certain weaknesses, most of which are simply inherent to modeling and, thus, unavoidable. No interesting or important aspect of future experience can be known with certainty. After all, it's in the future! Yet actuaries are best placed and qualified to make projections into that unknown future.

Both deterministic and stochastic models usually rely on past experience to some degree, at least as a starting point, to represent future experience. But how much past experience is relevant and reliable (and available) for that purpose? If, for example, price inflation has averaged 3 percent annually over the most recent prior 30 years, then a competent, professional actuary might well assume 3-percent annual inflation over the *next* 30 years, based on that 30 years of past experience. But what if inflation averaged 4 percent annually over the past *60* years (and, therefore, 5 percent annually over the first 30 years of that 60-year period)? Should the actuary assume 4-percent annual inflation over the next 30 years? What should he or she assume about the next 60 years? Couldn't one argue that 5 percent (the average inflation rate for one 30-year period) is just as reasonable an assumption as 3 percent or 4 percent? (Note that one way to reduce the sensitivity of the projections to differing assumed inflation rates is to assume "real" rates of earnings increases, interest rates and other related things, making the nominal inflation rate not so important.)

Does the length of the projection period dictate in any way the amount of past experience that must be considered when setting assumptions? Does using a 60-year projection period imply that more past data are required to be taken into consideration than does using a 30-year projection period? If so, why? And how much additional weight, if any, should be given to the *most recent* experience? What if inflation averaged only *1* percent annually for the past 10 years? Or zero for the past year or two? And how many years of past experience must be considered when projecting farther out into the future than past experience data exist?

Distinguishing a blip from a trend is difficult for everyone, including actuaries. And then actuaries must insert elements of their own judgment regarding why the future might *not* look quite so much like the past. Recognition of past and anticipated changes in law, regulation and administrative practices is very important in this context. Returning to the example of price inflation, past experience may have led to new laws or policies designed to dampen inflation. Actuaries must consider whether those new policies will be effective, and to what extent, as a result, future inflation may be lower than past inflation.

Projecting future experience is not a mechanical process, obviously – and shouldn't be. Analysis of why the past may not be representative of the future must be performed with respect to each and every important assumption in the model, whether deterministic or stochastic.

Stochastic modeling: a useful tool to measure uncertainty

In 2010, the International Actuarial Association, working with several of its member organizations, produced an excellent, 400-page book entitled, *Stochastic Modeling: Theory and Reality from an Actuarial Perspective*. That book explains in enormous detail the advantages and disadvantages of stochastic (versus deterministic) modeling. In the first chapter, on page I-2, the following excerpt lays out some situations where stochastic modeling is helpful and appropriate:

> - **When analyzing extreme outcomes or "tail risks" that are not well understood**
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> Scenarios intended to represent one-in-100-year events or one-in-1,000-year events are typically used to stress-test asset or liability portfolios. In some cases, such as a "Black Tuesday" stock market event, the modeler may even believe that he or she has a handle on the likelihood of an extreme event. But recent experience has proved differently in many cases. How many would have included the "perfect storm" scenario of low interest rates and low equity returns of the early 2000s or an event that could cause the catastrophic losses of the World Trade Center terrorist attack? How many would have contemplated the 2008/2009 credit crisis that has affected the United States and European markets? Stochastic modeling would likely have revealed these scenarios, even if indicating they were extremely unlikely.
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> Yet financial risks are among the best understood risks with which actuaries deal. Mortality and morbidity risks are often much more complex, especially when considering the scope of assessing pandemic risk such as the 1918 Spanish flu event. Such an event could represent a 1-in-1,000 event, or it might not. Insufficient frequency [of] data, rapidly evolving healthcare technology, and increasing globalization could alter the likelihood of a contemporary pandemic and call into question the credibility of using a few select stress scenarios to understand the true exposure. Another approach would be to develop a stochastic model that relies on a few well-understood parameters and is appropriately calibrated for the risk manager's intended modeling purpose.

Social insurance programs, on the other hand, are generally *not* subject to significantly large "tail risk." Financial projections for such programs appropriately focus much more on the *means* of the assumed probability distributions for important variables, not the *extremes* (or "tails") of those distributions. For example, everyone recognizes that small numbers of retired workers will either die young or live a very long time (say, to age 100 or older). These extreme cases are, first of all, very uncommon and, secondly, have financial effects insufficiently different from the average case to cause more than a negligible impact on the program as a whole. Unless substantial numbers of retired workers start exhibiting these characteristics, the average age at death will not move very much from year to year. Thus, in the context of social insurance projections, the probability distributions for many assumptions are tighter than in many other actuarial applications. However, social insurance programs' tail risk arises due to the length of the projection period and to a great extent depends upon the amount of growth of the economy and certain aspects of extreme economic experience, e.g., hyperinflation, stagflation, etc. Typically, when a nation encounters significant adverse economic changes, its social insurance programs are modified to take those "tail events" into account. The ability to change the underlying program is not always an option outside of government-sponsored programs like social insurance. Finally, certain non-economic risks, like flood or earthquake risk (as was seen recently in Japan), may be quite severe in certain countries. So tail risk exists, but it cannot necessarily be brought out well by any model.

Continuing the example of flood risk, flood insurance is subject to huge tail risk, because a rare, extremely severe hurricane or cyclone striking a heavily developed area can easily cause claims that exceed all the costs in multiple previous years. In the social insurance case, tail risk is relatively low and difficult to model; in the flood insurance case, tail risk, i.e., the high cost of rare events, is significant and must be modeled. When tail risk is low, rare events may be safely disregarded in the financial model (even if recognized at some other level of analysis); when tail risk is high, rare events must be explicitly reflected in financial projections. Stochastic models do that very well.

Stochastic models are not the right tool in certain circumstances

Another excerpt from the IAA's book, on page I-3, explains when stochastic modeling may be inappropriate, and this passage seems particularly applicable to social insurance programs:

> ### When should use of stochastic models be questioned?
> In still other cases, stochastic modeling may not be of sufficient value to justify the effort. Examples include:
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> - **When it is difficult or impossible to determine the appropriate probability distribution**
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> The normal or lognormal distributions are commonly used for many stochastic projections because of their relative ease of use. However, there are times when these distributions do not properly describe the process under review. In these situations a distribution should not be forced onto the variable. Indeed, there are many other distributions available as described in other parts of this book; the analysis of which distribution best describes the available information is a critical part of the overall modeling exercise. However, there are still times when it is impossible to determine the appropriate distribution.
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> - **When it is difficult or impossible to calibrate the model**
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> Calibration requires either credible historical experience or observable conditions that provide insight on model inputs. If neither is available, the model cannot be fully calibrated and the actuary should carefully consider whether its use is appropriate. Sometimes, an actuary is required to develop a model when no credible, observable data exists. Such a situation requires professional actuarial judgment and the lack of data on which to calibrate the model should be carefully considered.
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> - **When it is difficult or impossible to validate the model**
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> Stochastic models are less reliable if the model output cannot be thoroughly reviewed, comprehended, and compared to expectations. If the actuary must use a model from which the output cannot be reviewed and validated, professional actuarial judgment is required and other methods of review and checking should perhaps be considered. The method of review and checking will necessarily involve a careful consideration of the specifics of the situation.

The first bullet above is especially applicable to the kinds of variables that must be projected in the context of social insurance. Taking a very simple example, a deterministic model might assume 3-percent annual price inflation, year after year. Inflation is an especially important variable with respect to social security systems that use price inflation to index benefit amounts after eligibility, as is done in many countries. And inflation projections would likely not be as simple as this illustrative example, even for a deterministic model. An equivalent, or perhaps comparable, stochastic model might assume the same 3-percent annual inflation level as the mean value but also assume some probability distribution around that mean, so that, for instance, inflation might be 2 percent in some years and 4 percent in others, but the average would be 3 percent – or tend toward 3 percent as the number of years in the projection period grows. That surely seems reasonable, but in reality, nobody knows – or even *can* know – the underlying probability distribution for a variable like price inflation (and, of course, that's only one example of many that could have been used to illustrate this point). An actuary must make certain assumptions as to future price inflation, recognizing that these are just assumptions, but the more one assumes (e.g., means versus probability distributions), the more uncertain the projections become. The IAA's book has a comment on that, too (on page I-5):

> ▪ **Uses of inappropriate distributions or parameters**
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> Users of stochastic modeling often assume – blindly or with very limited empirical evidence – that a variable has a particular distribution. While many processes can approximate a normal distribution, more often than not the possible outcomes are skewed. Users of stochastic modeling should be cautioned against the uninformed use of a particular distribution, which can generate scenarios producing completely useless results. Such a misguided assumption can be especially problematic when examining tail risk.

Stochastic models can be run again and again, hundreds or thousands of times, limited only by the time available. Because the important variables vary each time the model is run, according to their own assumed probability distributions, each running of the model can be expected to produce slightly different results. By ordering all of the runs in some meaningful way, probability distributions of the overall results can be obtained. Thus, after a thousand or so runs, one could state that the unfunded liability of a social security system is "no more than X with probability 0.95," and the like. Likewise, stochastic models can provide information such as the probability of the social security system's assets running out by, say, 2030. Whether policymakers actually want, need or can meaningfully apply such probabilities to policymaking decisions is an open question, the answer to which obviously must vary from country to country.

Because of their obviously greater technical sophistication and apparently closer resemblance to reality, which, of course, fluctuates, stochastic models tend to be favored by many analysts and users of these projections over deterministic ones, and the projections from stochastic models are sometimes given greater credibility. But is that really appropriate?

Stochastic models clearly *can* produce useful information but, like deterministic models, are subject to the "garbage in, garbage out" phenomenon. No model can be better than its input assumptions. And in the case of social security projections, the uncertainty increases exponentially as the projections extend decades into the future.

Central assumptions versus distributions

While difficulties in setting assumptions should be clear enough, the earlier discussion relates entirely to determining the *central assumption* for a projection period of some known length. The probability *distribution* needed to perform meaningful stochastic modeling adds another dimension of much greater complexity and therefore introduces much more uncertainty. Note that we are not concerned with the relatively easy task of defining a reasonable range of future values for key variables but rather assigning probabilities to those future values within the reasonable range, which is obviously much more complex, difficult and uncertain.

We have seen why the prior mean may not be the best estimate of the *future* mean for a relatively simple variable like price inflation. The same is true of the distribution about the mean. For one thing, as noted, government policies may have changed, becoming more or less tolerant of variations from inflation targets. The government may have set a target of 3-percent annual inflation and directed considerable resources toward dampening any fluctuations away from that target rate. If policies were different in the past – and they always are! – then prior experience may not be useful in projecting the future at all.

So, setting the mean, or central, assumption for a variable like inflation may be difficult already, but projecting the *distribution* of that variable going forward with any confidence becomes a daunting task (even if one believes that the past distribution *has* been determined somehow). Certainly, a distribution can be *assumed*, but can actuaries (and policymakers) really expect, or have a high degree of confidence, that it is the correct distribution? If, for instance, annual price inflation exceeded 4 percent in 12 of the previous 60 years, does that tell us anything useful at all about the probability of annual price inflation exceeding 4 percent in any of the *next* 60 years? Some would say yes, but should policymakers make decisions based on such assumptions? At a minimum, they should be well informed as to the limitations of the models that produced the financial projections.

These problems grow more severe as the length of the projection period increases. Taking the most extreme case possible, in 2003, the United States Government began publishing so-called "infinite-horizon" projections – using a deterministic model, not a stochastic one – for its Social Security system. At that time, the American Academy of Actuaries criticized the results as practically meaningless for policymakers – and potentially misleading, as well. As just one example of the shortcomings of such long-term projections, does a fixed normal retirement age of 67 (as provided under current law for workers born after 1959) make sense in the context of ever-increasing longevity? Should very long-term projections assume that the underlying law does not change? Does the length of the projection period require an equally long period of past experience? What if the program has existed for less than the length of the projection period, recognizing that the mere existence of a social insurance program affects many important variables? The questions are endless. Setting aside the technical issues associated with extremely long (even infinite) projection periods, the willingness of policymakers to base decisions on such projections should decline with the declining credibility of the projections themselves. How credible can infinite projections be, anyway? Those questions are related to the choice of the projection period and its inherent uncertainty, which applies to both deterministic and stochastic models.

Of course, that example of projecting to infinity is acknowledged to be extreme, but the principle demonstrated is still valid. The probability is small that any important variable, particularly very stable ones like mortality rates, will vary dramatically from one year to the next. Thus, for a projection looking forward just 5-10 years, most actuaries can have a reasonable (if not high) degree of confidence as to the range of future experience and even the probabilities of various outcomes within that range. But for very long-range projections, say 30, 50, 75 or even more years, such confidence must gradually diminish.

Covariances

The final challenge, embedded in stochastic models, is undoubtedly the most intractable, and it involves the relationships between and among variables, or covariances. As previously discussed, determining a central assumption for any important variable is difficult enough, and determining an appropriate future distribution for that variable is many times more difficult. But determining the interrelationship of any variable with all the other variables that must go into the model is simply impossible to do with any level of confidence – and many stochastic models avoid the problem by assuming that all covariances are zero! That's an assumption, too, but not an especially transparent or realistic one.

Some variables are related to each other in ways that are reasonably well understood – or at least well known and observed. Interest rates and inflation rates, for instance, are positively correlated for fairly obvious reasons. Wage growth and price inflation are similarly positively correlated, perhaps to a smaller degree. We understand logically why such relationships exist and can demonstrate them empirically, but can we express them mathematically in a way that will hold true for future years? And, again, are past relationships representative of future ones?

The difficulties here are truly daunting. Most stochastic models simply allow all of the important variables to move around, or vary probabilistically, independently of each other. That is equivalent to assuming *no covariances* (or assuming covariances equal to zero). But such an assumption is incorrect in many cases. And make no mistake about it: Setting covariances equal to zero, even implicitly, is making an assumption, just as assuming zero inflation would be making an assumption, even if one argued that no assumption was being made.

Alternatives

Although stochastic models have shortcomings, they could still be very useful in calibrating the sensitivity tests, stress tests and tail-event tests for any social insurance program. An example of a good practice could be found in the most recent actuarial report on the Canada Pension Plan recently tabled before Parliament. The authors were clever enough to say, “The results should be interpreted with caution and a full understanding of the inherent limitations of stochastic time-series modeling.” The stochastic tool was even used to assess the financial market tail events. At the same time, policymakers do need to understand that both deterministic and stochastic projections have weaknesses and are not certainties. Even if policymakers and other users understand that obvious fact, they need to have some sense of the potential volatility of the projections. For example, if a deterministic model projects an unfunded liability of $6 trillion (approximately the current figure for the United States’ Social Security program over the next 75 years), are we 95-percent confident that the “true” figure is within 10 percent of that? Or is the range more like plus or minus 100 percent? Or even more? The solution might well be to limit the projection period for stochastic models to allow for their inherent limitations.

Such information has been provided for many years, even before today’s powerful computers made stochastic models a practical possibility. One way is to produce multiple deterministic projections: for example, low-cost, central (or “best estimate”) and high-cost sets of estimates, where the low-cost and high-cost projections are intended to represent the outer limits of likely future experience – though not to suggest that experience outside of that range is impossible, just unlikely. That tells observers nothing about the probabilities of various outcomes but may delineate a reasonable range fairly well.

Another way to show uncertainty is to perform so-called “sensitivity analysis,” where the central set of assumptions is used repeatedly, but one (or perhaps more than one) key assumption is allowed to vary, one at a time, within some reasonable range. That shows the sensitivity of the projections to changes in important variables. It has the shortcoming that many important variables tend to move in tandem (i.e., have positive covariances), and those covariances are implicitly ignored much of the time.

The combination of alternative scenarios and sensitivity analysis produces very useful information. Using stochastic models in a prudent way might even increase the usefulness of these alternative scenarios. The degree of uncertainty in the financial projections may not be expressed with precision, but it is still made clear to any interested observer.

Conclusion

While we now have the technical and computational ability to design and run ever-more complex stochastic models, the actuary has an increasing responsibility to translate into laymen’s language, for policymakers and others, the implications and limitations of using such powerful tools. The social insurance actuary must ascertain whether these models are appropriate or not depending on the specific circumstances surrounding their use. Is the output from such models any better for policymakers’ intended purposes than what they have already been relying on for so many years? Do the newer models provide better information, or just *more* information – along with the potential illusion of *better* information? Finally, we know from experience (supported by abundant academic literature) that just by creating a social insurance program, a country can expect its population life expectancy to rise and its fertility rate to fall. How does one model those effects?

The main objective in the context of social insurance is to obtain the best estimate projection of the financial experience of social insurance programs. And for that main objective, deterministic projections are the ones that give the output of models that apply a set of assumptions regarding the values of important, relevant variables to represent future experience. These assumptions are expressed in terms of “best-estimate” or expected future values.

Actuaries need to be very careful not to be drawn in by the capabilities of their computer models and must retain the ability to evaluate what they are seeing come out of those models. And, most importantly, actuaries need to inform users of their financial projections of the uncertainty inherent in them. With the seemingly endless capabilities of today's computer models, the actuary's judgment has never been so important and critical to advise policymakers. And that's good actuarial practice.