Option Pricing with Actuarial Techniques

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Sumit Narayanan

Abstract

In this paper we consider pricing of maturity guarantees for a unit-linked contract using the Esscher transform, a traditional actuarial technique. We consider three stochastic processes for the interest rate underlying the unit fund. The paper illustrates that the choice of central assumptions is far more important than the choice of model.

Keywords

Esscher transform; Maturity guarantees; Option pricing; Financial economics.

1 Introduction

1.1 Developments in financial economics have recently evoked a great deal of interest among the actuarial community, especially in the wake of recent developments in the life insurance industry. For many years, actuarial science and financial economics have been going their separate ways, despite the similarities in the nature of problems addressed.

1.2 The press release announcing that Merton and Scholes were awarded the Nobel Prize in Economics in 1997 stated:

“The method adopted by this year’s laureates can therefore be used to value guarantees and insurance contracts. One can view insurance companies and the option market as competitors.” [7].

1.3 Options and guarantees offered by life insurers are nothing new, yet a lot more attention is now being drawn to the way they are priced and reserved for. There may be several reasons for this including:
certain high profile debacles in the insurance industry (e.g. Equitable Life);
- a move towards fair value accounting for insurers;
- a falling interest rate regime witnessed in many countries;
- an increased attention being drawn to the balance sheet of all corporates, including insurers, in this era of corporate failures;
- focus on executive stock options and how they are reflected in the financial statements;
- advances in option valuation theory.

1.4 With an increased focus on transparency, actuaries have now embraced financial economic techniques whole-heartedly. Many applications of financial economics have been found in actuarial science. However, actuarial science also has much to contribute to the growth of financial economics.

1.5 This paper reviews one such application of a traditional actuarial technique, the Esscher transform, to value option contracts. A maturity guarantee on a unit-linked contract offered by a life insurer can be likened to a European put option sold by the insurer, which can then be valued using the put option price. A reader new to financial economics may want to read Appendix A prior to proceeding with the other sections of the paper.

1.6 In Section 2, we introduce the Esscher transform and show how it can be used to value put options under a general form for the underlying stock price. In Section 3 we show how a maturity guarantee can be valued under the following three different assumptions for the underlying unit fund growth:

- Wiener Process;
- Shifted Poisson Process; and,
- Shifted Gamma Process.

1.7 Section 4 contains a few graphs to study the nature of the maturity guarantee and its sensitivity to various models and parameters. Apart from the Esscher transform, there are other actuarial techniques that have also been employed to value option contracts. These are briefly discussed in Section 5. Another interesting application of an option contract is to value surrender guarantees, which are much harder to value as they involve a discretionary element to the time of surrender. The paper is concluded in Section 6 with some final remarks.
2. **Esscher Transform**

2.1 In this section we show how an actuarial technique can be fruitfully used to value option contracts. The Esscher transform is a powerful tool invented by actuarial science. Its initial applications included numerical computation of the distribution of aggregate claims and simulation of rare events.

**Definition**

2.2 For \( t \geq 0 \), let \( S(t) \) denote the price of a non-dividend paying stock at time \( t \). We assume that there exists a stochastic process, \( \{X(t)\}_{t \geq 0} \), with stationary and independent increments, initial value \( X(0) = 0 \), such that

\[
S(t) = S(0)e^{X(t)}, \quad t \geq 0.
\]

2.3 The stock price, \( S(t) \), is thus growing with interest, which has a stochastic process as denoted by \( X(t) \).

2.4 We also assume that the random variable \( X(t) \) has a probability density function \( f(x,t) \). The moment generating function \( M(z,t) \) of \( X(t) \) is defined by

\[
M(z,t) = E[e^{zX(t)}] = \int_{-\infty}^{\infty} e^{zx} f(x,t)dx
\]

and the Esscher transform with parameter \( h \) of \( f \) is defined by

\[
f(x,t;h) = \frac{e^{hx} f(x,t)}{M(h,t)}
\]

2.5 For the corresponding moment generating function of \( f(x,t;h) \)

\[
M(z,t;h) = \int_{-\infty}^{\infty} e^{zr} f(x,t;h)dx
\]

\[
= \left( \frac{M(z+h,t)}{M(h,t)} \right)
\]

2.6 Since \( M(z,t) \) is continuous in \( t \), it can be shown that

\[
M(z,t;h) = [M(z,1;h)]'
\]

**Risk-neutral Esscher transform**

2.7 As we want to ensure that stock prices are internally consistent, we seek \( h=h^* \) so that
the discounted stock price process, $e^{-\delta S(t)}$, is a martingale with respect to the probability measure corresponding to $h^*$. In particular,

$$S(0) = E^*[e^{-\delta S(t)}] = e^{-\delta} E^*[S(t)]$$

where $\delta$ denotes the constant risk-free force of interest.

2.8 Using the equation in 2.2, the parameter $h^*$ is the solution of the equation

$$l = e^{-\delta} E^*[e^{X(t)}],$$

or

$$e^{\delta} = M(1, t; h^*).$$

2.9 Setting $t=1$ we get,

$$\delta = \ln[M(1,1; h^*)].$$

2.10 Gerber and Shiu [2] call the Esscher transform of parameter $h^*$ the risk-neutral Esscher transform, and the corresponding equivalent martingale measure the risk-neutral Esscher measure. The merit of the risk-neutral Esscher measure is that it provides a general, transparent and unambiguous solution.

Valuing derivatives using Esscher transform

2.11 The value of a derivative is the expected discounted value of the implied payoffs. Let us consider a European call option on the stock with exercise price $K$ and exercise date $\tau, \tau > 0$.

2.12 The value of this option (at time 0) is

$$E^*[e^{-\delta}(S(0) - K)_+].$$

2.13 Defining $\kappa = \ln[K / S(0)]$, the above equation becomes

$$e^{-\delta \kappa} \int_{-\infty}^{\infty} [S(0)e^x - K] f(x, \tau; h^*)dx$$

2.14 Gerber and Shiu [2] show that the value of the European call option with exercise price $K$ and exercise date $\tau$ is
2.15 Applying the put-call parity theorem (see Appendix A), we can deduce the value of the European put option with exercise price $K$ and exercise date $\tau$ as,

$$K e^{-\delta \tau} [F(\kappa, \tau; h^*)] = S(0)[1 - F(\kappa, \tau; h^*)]$$

2.16 The Esscher transform:
- allows us to find a deflator which achieves the martingale property;
- does not assume a particular form of distribution underlying the stock price; and,
- can also be used where the underlying stock price exhibits jumps (e.g. Poisson Process).

3. Valuing Maturity Guarantees

3.1 In the previous section we derived formulae for call and put options without assuming any specific distribution for the interest rate process $X(t)$. In this section we illustrate the European put option results obtained previously to value maturity guarantees.

3.2 A maturity guarantee on a unit-linked product can be likened to a European put option, where the insurer sells the option to the insured to ‘put’ the insurance contract at a predetermined value at maturity. The insurer can estimate the value of the guarantee by assuming a certain underlying stochastic process for the unit fund and the probability of survival to maturity.

Parameters

3.3 The following table sets out the parameters used in the models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Description/ comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>$x$</td>
<td>Age of the policyholder at policy inception</td>
</tr>
<tr>
<td>Single premium</td>
<td>$P$</td>
<td>Single premium paid by the policyholder</td>
</tr>
<tr>
<td>Allocation rate</td>
<td>$AllocationRate$</td>
<td>Percentage of premium allocated to the unit fund.</td>
</tr>
</tbody>
</table>
### 5th Global Conference of Actuaries

<table>
<thead>
<tr>
<th>Bid-offer spread</th>
<th>$BOS$</th>
<th>The amount by which the offer price exceeds the bid price.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(t)$</td>
<td>$U(t)$</td>
<td>Value of the unit fund at time $t$</td>
</tr>
<tr>
<td>$K$</td>
<td>$K$</td>
<td>Guaranteed maturity value. Defined as return of single premium with guaranteed interest rate.</td>
</tr>
<tr>
<td>Delta</td>
<td>$\delta$</td>
<td>Risk-free force of interest</td>
</tr>
<tr>
<td>$i$</td>
<td>$i$</td>
<td>Guaranteed interest rate</td>
</tr>
<tr>
<td>Tau</td>
<td>$\tau$</td>
<td>Term</td>
</tr>
<tr>
<td>Kappa</td>
<td>$\kappa$</td>
<td>$\ln(K / U(0))$</td>
</tr>
<tr>
<td>Mu</td>
<td>$\mu$</td>
<td>Mean of the stochastic process underlying the interest rate at which the unit fund is growing.</td>
</tr>
<tr>
<td>Sigma</td>
<td>$\sigma$</td>
<td>Standard deviation of the stochastic process underlying the interest rate at which the unit fund is growing.</td>
</tr>
<tr>
<td>$k$</td>
<td>$k$</td>
<td>Shift parameter for the Shifted Poisson process</td>
</tr>
<tr>
<td>Lambda star</td>
<td>$\lambda^*$</td>
<td>Mean of the Shifted Poisson process</td>
</tr>
<tr>
<td>$c$</td>
<td>$c$</td>
<td>Shift parameter for the Shifted Poisson and Shifted Gamma processes.</td>
</tr>
<tr>
<td>Alpha</td>
<td>$\alpha$</td>
<td>Shape parameter of the original Gamma process</td>
</tr>
<tr>
<td>Beta</td>
<td>$\beta$</td>
<td>Scale parameter of the original Gamma process</td>
</tr>
</tbody>
</table>

3.4 The unit fund is modelled in the following manner. Let $U(t)$ denote the value of the unit fund at time $t$. The initial value of the unit fund is obtained as:

$$U(0) = P \times (1 - BOS) \times AllocationRate$$

3.5 The unit fund at time $t$ is then obtained as:

$$U(t) = U(0) \times e^{X(t)} \quad for \quad 0 \leq t \leq \tau$$
where \( \{X(t)\} \) is the stochastic process underlying the interest rate at which the unit fund is growing.

3.6 We assume three different processes for \( \{X(t)\} \):

- Wiener process;
- Shifted Poisson process; and,
- Shifted Gamma process.

**Wiener process**

3.7 The Wiener process implies that the change in the logarithm of the unit fund is distributed normally. It is a continuous time model and is also the basis of the Black-Scholes model. The results derived for this model correspond to the Black-Scholes option pricing formula.

3.8 Let the stochastic process \( \{X(t)\} \) be a Wiener process with mean per unit time \( \mu \) and variance per unit time \( \sigma^2 \).

3.9 The Esscher transform (parameter \( h \)) of the Wiener process is

\[
M(z,t;h) = \exp\{[\mu + h\sigma^2]z + \frac{1}{2}\sigma^2 z^2]\ t
\]

which is a Wiener process with modified mean per unit time \( \mu + h\sigma^2 \) and unchanged variance \( \sigma^2 \).

3.10 It can be shown that the value of the maturity guarantee is given by the following equation

\[
A = Ke^{-\delta\tau} \Phi \left( \frac{-\kappa + \left(\delta - \frac{\sigma^2}{2}\right) \tau}{\sigma\sqrt{\tau}} \right) - U(0) \Phi \left( \frac{-\kappa + \left(\delta + \frac{\sigma^2}{2}\right) \tau}{\sigma\sqrt{\tau}} \right)
\]

3.11 The above equation ignores the probability of survival to maturity. Taking probability of survival into account, the value of the maturity guarantee is

\[
A \ast \tau p_s
\]

**Shifted Poisson process**

3.12 The Poisson process is a discrete time model and is commonly encountered in actuarial work. For example, the surplus distribution process of an insurance company is classically
modelled assuming that claims have a Poisson distribution. One criticism of the Wiener process is its continuous nature, whereas in reality stock prices do exhibit jumps.

3.13 Let \( N(t) \) be a Poisson process with parameter \( \lambda \) and \( k \) and \( c \) are positive constants. We assume that the stochastic process \( \{ X(t) \} \) follows

\[
X(t) = kN(t) - ct
\]

3.14 Let

\[
\Lambda(x; \theta) = \sum_{j=0}^{\infty} \frac{e^{-\theta} \theta^j}{j!}
\]

be the cumulative Poisson distribution function with parameter \( \theta \). The cumulative distribution function of \( X(t) \) is

\[
F(x, t) = \Lambda\left(\frac{x + ct}{k}; \lambda t\right).
\]

3.15 The Esscher transform (parameter \( h \)) of the Poisson process is

\[
M(z, t; h) = \exp[\lambda e^{hk} (e^{z^k} - 1) - cz] t
\]

which is also a shifted Poisson process with modified Poisson parameter \( \lambda e^{hk} \).

3.16 The maturity guarantee can be valued as

\[
\{ Ke^{-\delta \tau} \Lambda((\kappa + c\tau)/k; \lambda^* \tau) - U(0) \Lambda((\kappa + c\tau)/k; \lambda^* e^{k\tau}) \} \ast \tau \ p_\tau
\]

**Shifted Gamma process**

3.17 The Gamma process is the continuous time case of the Poisson process. This is exhibited in the graphs in Section 4.

3.18 Let \( \{ Y(t) \} \) be a Gamma process with parameters \( \alpha \) and \( \beta \) and the positive constant \( c \) is the third parameter. We assume that the stochastic process \( \{ X(t) \} \) follows

\[
X(t) = Y(t) - ct
\]
3.19 The cumulative distribution function of $X(t)$ is

$$F(x,t) = G(x + ct; \alpha, \beta)$$

where $G(\cdot)$ is the cumulative Gamma distribution function.

3.20 The Esscher transform (parameter $h$) of the shifted Gamma process is

$$M(z,t;h) = \left( \frac{\beta - h}{\beta - h - z} \right)^{\alpha} e^{-\beta z}, \quad z < \beta - h$$

which shows that the transformed process is of the same type, with $\beta$ replaced by $\beta - h$.

3.21 The value of the maturity guarantee can be calculated as

$$\{ Ke^{-\delta \tau} [1 - G(\kappa + c\tau; \alpha \tau, \beta^*)] - U(0) [1 - G(\kappa + c\tau; \alpha \tau, \beta^*)] \} * p_x.$$

4 Model Output

4.1 In this section we illustrate the value of the maturity guarantee under the three different models. The values of the parameters used in the models are set out in Appendix C.

Term

4.2 The following graph shows the value of the maturity guarantee as a function of term.
4.3 The maturity guarantee is not a monotonically increasing function of term. This is due to the maturity guarantee applying at a point in time and not during the entire duration of the bond. It is interesting to note that a similar guarantee on surrender values would be a monotonically increasing function of time as the policyholder has the choice of surrendering the policy at any time before maturity. The interested reader is referred to [5, Chapter 7].

4.4 The point of inflexion for the models considered is around 10 years. Interestingly, single premium bonds sold in the market today commonly have a term of 10 years.

4.5 The Shifted Gamma process is a continuous case of the Shifted Poisson process which explains the shape of the two curves.

4.6 The choice of model becomes more critical for longer-term guarantees as the value of the guarantee implied by the different model diverges.

**Volatility**

4.7 The following graph shows the value of the maturity guarantee as a function of volatility.

4.8 The value of the maturity guarantee increases approximately linearly with volatility. The rationale being that the insurer selling the maturity guarantee loses from decreases in the unit fund, but has limited upside risk in the event of an increase in the unit fund.
4.9 Hence, the cost of a similar guarantee would be higher in markets that experience greater volatility, e.g. India, where the annual volatility observed on the BSE Sensex for the period 1990-2001 is around 30%. This compares with an annual volatility of around 15% for the Dow Jones Industrial Average for the same period.

**Risk-free force of interest**

4.10 The graph below shows the value of the maturity guarantee as a function of delta.

![Graph showing the value of the maturity guarantee as a function of delta](image)

4.11 The value of the maturity guarantee is a monotonically decreasing function of the risk-free force of interest. As interest rates increase, the expected growth rate of the unit fund tends to increase. Also, the present value of any future cash flow decreases due to a higher discount rate. Thus, the value of the maturity guarantee will decrease.

4.12 The choice of model becomes more critical at lower risk-free force of interest.

**Further Research**

5.1 In this paper we have described how maturity guarantees can be likened to a European put option and valued using Esscher transforms. Several authors have found other actuarial techniques that allow valuation of more complex guarantees. As an example, consider a guarantee offered by an insurer on surrender values on a unit-linked contract. Such a guarantee allows the policyholder the choice to surrender the insurance policy
for a guaranteed amount any time before maturity, not just at maturity. The payoffs under such a guarantee are similar to that of an American put option.

5.2 An American option is trickier to value than European options and usually require a numerical approach, as analytical solutions cannot be found. For example, the Black-Scholes model assumes a European option and cannot be used to value American options. Further, if discontinuities are allowed by assuming a discrete stochastic process underlying the unit fund (for example the Poisson Process), then the optimal time to surrender the policy must also be found. This problem arises as the value of the unit fund will not just hit the guaranteed surrender value but will fall further below it due to a downward jump.

5.3 Gerber and Shiu [3] show that this problem can be tackled by a result from ruin theory. They use the concept of discounted probability by relating the stochastic process underlying the stock with the surplus process assumed in ruin theory.

5.4 Wilkie et. al. [8] discuss pricing, reserving and hedging of guaranteed annuity options by using two approaches, a stochastic investment model (based on the Wilkie model) and option pricing methodology. It is interesting to note that the Report of the Maturity Guarantees Working Party in 1980 considered the use of option pricing methodology to value such guarantees, but were not convinced of its applicability.

6 Conclusions

6.1 In this paper we have illustrated how a traditional actuarial technique can be used to value put options and then use the result to value maturity guarantees. The paper illustrates that the choice of central assumptions is far more important than the choice of model. However, in certain situations, for example, high volatility or low risk-free force of interest, the choice of model becomes more critical.

6.2 Many other challenging problems exist in this fascinating subject, and the authors hope that the paper inspires others to consider some of them.

6.3 As Keynes put it back in 1925:

“It is a task well adopted to the training and mentality of actuaries, and not less important, I fancy, to the future of the insurance industry than the further improvement of Life Tables.” [7]
A Preliminaries of Financial Economics

History

A.1 Mathematical modelling of financial markets arguably began with Louis Bachelier’s seminal theses, *Théorie de la Speculation* in 1900, wherein he modelled the French capital market as a fair game. He proved mathematically that the standard deviation of the distribution of future price changes is directly proportional to the square root of elapsed time, a result that was already empirically known to French actuaries and being exploited in the French stock market even before Bachelier found a mathematical proof for it.

A.2 Financial economics has since developed through advances in the theory of stochastic processes, especially diffusion processes, to model prices and risk. The celebrated Black-Scholes model of option pricing in 1973 was the most important breakthrough in financial economics after Bachelier’s work. The model, which is based on the assumption of no-arbitrage, has had a significant influence on the way traders’ price and hedge options. The Black-Scholes model has gained widespread acceptability in the derivatives market owing to its simple and elegant solution to what is essentially a complex problem. It has been pivotal to the growth of financial economics and the derivatives market in the 1980s and 1990s.

A.3 In the following sections, we describe some of the rudimentary theory behind financial economics.

Types of derivative securities

A.4 A *derivative* is a security that pays its owner an amount that is a function of the values of the underlying securities. That is, the value of the derivative is ‘derived’ from the underlying security. Forwards, futures, options and swaps are common examples of derivative securities.

A.5 A *forward* contract is an agreement to buy or sell an asset at a certain future time for a certain price. It is traded over-the-counter. These contracts are commonly used to hedge foreign currency risk.

A.6 A *futures* contract is, like a forward contract, an agreement to buy or sell an asset at a certain time in the future for a certain price. Unlike a forward contract, a futures contract
is normally traded on an exchange. As the two parties to the futures contract do not necessarily know each other, the exchange also provides a mechanism that gives the parties a guarantee that the contract will be honoured. To minimise risks of default, the exchange/broker will require both parties to deposit funds in what is termed a margin account. At the end of each trading day, the margin account is adjusted to reflect the parties’ gain or loss. This revaluing adjustment is called ‘marking-to-market’. A futures contract is commonly traded before maturity.

A.7 An option is an instrument that gives the holder a right (but not an obligation) to exercise his position at a certain price in the future. There are two basic types of options namely, call options and put options. A call option gives the holder a right to buy the underlying asset by a certain date for a certain pre-determined price. A put option gives the holder a right to sell the underlying asset by a certain date for a certain price. Options that can be exercised only on the expiration date are called European options. Options that can be exercised on any date before maturity are called American options.

A.8 A swap is an agreement between two parties to exchange cash flows in the future. The agreement defines the dates when the cash flows are to be paid and the way that they are to be calculated. Currency swaps and interest rate swaps are the most popular swap arrangements in the market.

**Martingale**

A.9 Martingale theory plays a central role in valuing derivative securities. A martingale is a stochastic process whose expected value at any time in the future is equal to its current value.

**Risk-neutral valuation**

A.10 Risk-neutral valuation is a technique employed in financial economics to value derivative securities. It introduces an equivalent martingale measure with respect to which the discounted stochastic process under consideration becomes a martingale. The importance of this valuation method lies in the fact that for every stochastic process there exists an equivalent martingale measure (not necessarily unique) with respect to which the discounted stochastic process becomes a martingale. The equivalent martingale measure is known as the risk-neutral probability measure.

A.11 This implies that under the risk-neutral measure the stochastic process grows at the risk-free rate of return. The risk-neutral measure is often referred to in literature as
the ‘Q-measure’ to distinguish it from the real world measure, often called the ‘P-measure’.

A.12 It is important to understand that risky assets (like equities, corporate bonds) will grow at interest rates different from the risk-free rate of return. The risk-neutral valuation framework does not assume that investors will be satisfied with risk-free rate of returns for risky assets.

A.13 Thus, under the risk-neutral valuation framework, the price of a derivative security is given by the expectation of the discounted payoff, where the expectation is taken with respect to the risk-neutral probability measure.

No-arbitrage valuation

A.14 Modern asset pricing theory is based on the law of one price, which states that portfolios with identical payoffs should have the same price, or there would be pure arbitrage opportunities in the market. This law is used to value derivative securities by finding ‘replicating portfolios’ consisting of stocks and bonds that yield the same payoffs in all scenarios. The value of the derivative security is then given by the value of the replicating portfolio.

Equilibrium pricing

A.15 There are situations when the no-arbitrage valuation approach cannot be applied. For example, when a new security is introduced and this security cannot be replicated by the existing traded securities. In these situations, we can use the equilibrium pricing approach.

A.16 This approach provides a more general framework for pricing securities. We assume that individuals (or agents) with fixed initial resources (or endowments) trade in a financial market to maximise their expected utility. When no individual has an incentive to trade at these prices, the market reaches a state of equilibrium. The prices that evolve in this state of equilibrium are called equilibrium prices.

A.17 Equilibrium prices are related to the attributes of the agents in the economy, such as the endowments, beliefs and preferences, as well as to the type and structure of the traded securities. If any of these attributes change, the resulting equilibrium prices will, in general, change as well.
**Put-call parity**

A.18 The put-call parity theorem states that the payoff from a portfolio consisting of one share of the underlying stock and one European call option is equivalent to a portfolio consisting of a riskless zero-coupon bond and one European put option. Using the law of one price, the value of the two portfolios at any date prior to maturity must be equal as their payoffs at maturity are equal.

A.19 Hence, if we know the price of the European call option, the underlying stock and the riskless zero coupon bond, we can deduce the price of the European put option, and vice versa. For the put-call parity theorem to hold, the exercise date and exercise price of the European call and put options must be similar.


**B  BSE Sensitive Index Return and Volatility**

B.1 The following graph illustrates the movement of monthly averages of the BSE Sensitive Index (“Sensex”) for the period from April 1990 to November 2000. The annual volatility calculated for this period is around 30%.
B.2 The graph below shows the volatile average monthly returns on the BSE Sensex. The annualised returns calculated for this period are estimated to be around 16%.

![Monthly Returns on BSE Sensitive Index Base: 1978-79=100](image)

Source: www.indiastat.com

C Central Assumptions

C.1 The following table sets out the central assumptions used in the models. The values used are for illustration purposes only.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Wiener process</th>
<th>Shifted Poisson process</th>
<th>Shifted Gamma process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>35</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>Mortality</td>
<td>100% LIC 94-96</td>
<td>100% LIC 94-96</td>
<td>100% LIC 94-96</td>
</tr>
<tr>
<td>$P$</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Allocation Rate</td>
<td>99%</td>
<td>99%</td>
<td>99%</td>
</tr>
<tr>
<td>BOS</td>
<td>5%</td>
<td>5%</td>
<td>5%</td>
</tr>
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<td>$K$</td>
<td>1,350</td>
<td>1,350</td>
<td>1,350</td>
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<td>$\delta$</td>
<td>ln(1.06)</td>
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<td>10</td>
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<td>$i$</td>
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</tr>
</tbody>
</table>
References


About the authors :

Sanchit Maini

Sanchit graduated in Statistics from Delhi University in 1997 and later completed a postgraduate degree in Actuarial Science from Melbourne University in 1999. He qualified as an Associate of the Institute of Actuaries of Australia in 2001. Currently he is pursuing Fellowship exams of the Institute of Actuaries of Australia. He has one exam to sit.

Sanchit joined Watson Wyatt in January 2001. During an initial period in the UK, he was involved in a review of a life company’s embedded value for securitisation purposes. Since being in Delhi he has been involved in the preparation of market reports, the modelling of Indian products on VIP, pricing numerous products for clients and filing them with the IRDA and the preparation of business plans.

He has also been involved in several valuation projects in Singapore, Indonesia and the UK, and involved in modelling Hong Kong and Japanese products on VIP.

Prior to joining Watson Wyatt he was working for GE Capital, both in US and India. At GE Capital, he was working in product development for the universal life and fixed annuity lines of business.
Sumit Narayanan

Sumit joined Watson Wyatt in July 2001 after graduating with merit in M.Sc. Finance and Economics from London School of Economics. Currently he is pursuing exams of the Actuarial Society of India and the Institute of Actuaries, UK.

During his initial period in the UK Sumit underwent training on reserving, product pricing and valuations of life and non-life insurance products.

Since being in Delhi, Sumit has been involved in cash flow projections and profit testing of various Indian products on VIP, Watson Wyatt’s proprietary software. He has also supported other consultants on similar projects. He was involved in drafting the Indian section of the Asian life insurance market update for 2002. He has also been involved in market analysis of an Indian non-life product and a review on the state of healthcare financing in India. He was extensively involved in drafting a study on educational infrastructure in India for a leading multinational insurance company.

He has also been involved in the valuation of a UK Friendly Society, and involved in modelling Hong Kong and Japanese products on VIP.