

## **The Fair Value of Insurance Liabilities**

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### **ABSTRACT**

The fair value of insurance liabilities is currently a highly disputed topic around the globe, particularly among US property and casualty insurers. Interest in the topic was sparked by the release of FAS115 [1993], a ruling which resulted in inconsistent accounting measurement of assets and liabilities in the USA. While assets are measured at market value, liabilities are recorded at historical cost under FAS115.

This paper seeks to review the concept of fair valuation from a life insurance perspective. Two methods of fair valuation are considered: the embedded value methodology and the options pricing technique. Although algebraically reconcilable, the practical equivalence of the two methods is questioned. In considering the two methodologies, consideration is given to the treatment of risk, the choice of discount rate and the impact of an insurer's credit rating in a fair valuation.

### **KEYWORDS**

“Life Insurance Liability Valuation” “Fair Value Methods”

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## ***Part 1- Preliminaries***

### *1(i) Background of essay*

Subsequent to the issue of statement Financial Accounting Standard 115 [1993] in the USA, there was a discrepancy in the measurement of insurance assets and liabilities, with most of the former category marked to market but the latter still wholly measured by traditional accounting means. While there is unanimity among accounting and actuarial bodies that this standard has introduced artificial volatility of equity into the balance sheet of US life insurers, there has been no consensus resolution. International Accounting Standard 39 (IAS39) [1998], although designed to move accounting for financial instruments toward a fair value system, has failed to solve the problem since it explicitly excludes insurance liabilities from fair valuation requirements. Measuring the fair value of liabilities is a non-trivial procedure, currently disputed globally by accountants, actuaries and regulators.

The term “fair value” was initially coined for use in situations where no active or deep market exists for the security or liability in question. IAS32 [1999] defines fair value as “*The amount for which an asset could be exchanged, or a liability settled, between knowledgeable, willing parties in an arm’s length transaction.*” Most importantly, the amount is that applicable in a current transaction between ready buyers and sellers, not a forced sale. Perhaps the best market for observing the fair value of life insurance liabilities is the reinsurance market.

In 1999, subsequent to the developments in the USA, The International Accounting Standards Committee (IASC) published an issues paper, entitled “Insurance”, addressing the issue of fair value in greater detail. Questions raised include (IASC reference notation is given in brackets):

- (a) Whether insurance contracts should be included in a fair value standard (11B);
- (b) The appropriate discount rate (11G);
- (c) The need for a risk provision (11H);
- (d) The impact of the insurer’s creditworthiness on the valuation (11I); and
- (e) The appropriateness of the embedded value methodology for fair valuation (11K).

*1(ii) Aims and approach*

In accordance with the views espoused by the IASC's Steering Committee, it is agreed that the method used for insurance liability valuation should parallel the market valuation of assets. With only one side of the balance sheet "marked to market", the volatility of shareholders' equity risks being unrepresentative of a company's exposure to interest rate risk. Moreover, given the increasingly volatile economic environment, the need for a reporting system that ensures stable and consistent valuations of equity and earnings is critical. Thus, in light of the perceived need for such a liability valuation structure, the objective of this paper is this:

*To examine the two main methods of fair valuation, these being the embedded value and the direct valuation/option pricing approaches. In the process, appropriate consideration will be given to issues (b) - (e), as detailed above. The equivalence of the two methodologies will be demonstrated, both algebraically and practically.*

The fair valuation of several different insurance products was conducted in the development of this paper. For illustration purposes, it was chosen to focus wholly on a participating whole of life product with annual distributable policyholder dividends. Such a product is admittedly more familiar to the American life insurance market than other insurance markets but has been selected because it allows for a more complete exposition of the key fair value concepts in the extant literature, with both market-linked (policyholder dividends) and market-independent (benefits and expenses) charges. An attempt has been made to use realistic assumptions in the modelling of this product, with the exception of certain simplifications necessary for ease of presentation. A detailed description of the product and underlying assumptions is given in Appendix 1.

### *1(iii) Parameters of essay*

In exploring the aims listed in *Part 1(ii)*, a benchmark fair valuation approach was required. With fair valuation of insurance liabilities being a relatively recent and popular concept, there are myriad views emerging on the topic but little consistency of approach. It was chosen to base the model fair valuation specifications and calculations on Luc Girard's paper "*Market value of insurance liabilities*" published in January, 2000. This article also provided the conceptual framework for the algebraic reconciliation of the two fair value methodologies in *Part 2(v)*. Finally, it should be noted that the aim of this essay is not to dwell on the algebra, but rather to provide a balanced practical and theoretical overview of the key valuation concepts. As such, much detail has been omitted from the final paper, both in the algebraic proofs and with regards to the model calculations.

## ***Part 2 - Exposing the fair value framework***

Despite the IASC's efforts to achieve market valuation of financial liabilities via the prescription of IAS39 [1998], little progress has been made in the development of a consensus fair valuation framework for life insurance liabilities. The main complications have been the long-term nature of the life insurance contract, the high levels of uncertainty and the need for large initial expenses to be recouped from subsequent revenue. Regulators have been unwilling to abandon traditional conservative reporting methods and as such reported earnings and equity remain artificially volatile in many jurisdictions.

In recognition of the need for a fair value framework to replace US GAAP legislation, the American Academy of Actuaries' Fair Valuation Taskforce held a dedicated conference in 1998. Arnold Dicke presented a paper cataloguing ten different methods for the fair valuation of life insurance liabilities. His two "type A" methods remain the most commonly embraced in the actuarial field, these being the actuarial appraisal/embedded value approach and the options pricing methodology.

## 2(i) Embedded value methodology

Appraisal and embedded value methodologies are traditionally used by UK and Australian actuaries. Consequently there exist readily established parameters and procedures. While appraisal values of future distributable earnings typically reflect both earnings from existing liabilities *and* franchise value (a capitalisation of future expected business), fair valuations of liabilities should, by definition, refer to existing policy liabilities only. Thus for this paper, the title “Embedded Value” Methodology (EVM) is used to focus attention on the fact that fair valuations refer to a closed block of business.

The method is a deductive approach, first evaluating enterprise value and then deducting this from the known value of assets to determine the fair value of liabilities. Ignoring taxes, the net fair value of liabilities (*FVL*) may be expressed as:

$$FVL = CapL + MVA - EV \quad (1)$$

where

*CapL* = the market value of a portfolio of assets with book value to the required risk based capital supporting the product liabilities.

*MVA* = the market value of a portfolio of assets with book value equal to the policy liabilities.

(Note that *CapL* and *MVA* could be combined to equal the total assets held in respect of the liabilities. They are separated here to facilitate later calculations.)

*EV* = embedded value of the business, equivalent to the discounted sum of future free cashflows.

There is an obvious circularity in the above methodology. Derivation of *FVL* requires *EV*, which is a function of the cash flows to the shareholder and in turn of *FVL* itself. This problem may be eliminated using backwards recursion and the assumption that  $FVL = EV = 0$  at  $t=T$ , the end of the product period.

Defining the liabilities to include an outflow of profit to shareholders gives the implied recursive EVM specification of  $FVL$ :

$$FVL_{t-1} = \frac{FVL_t + L_t + RP_t}{(1+i_t)}; \text{ where } FVL_t = 0 \quad (2)$$

where

$L_t$  = the net policyholder cash flows, including benefits, claims, premiums.

$RP_t$  = Required Profit, or the outflow to shareholders. This is the payment to shareholders, that, when added to interest earned on invested capital, equals the shareholders' cost of capital;

$$= (k - j) * CapL_{t-1} + (k - i_t) * (MVA_{t-1} - FVL_{t-1})$$

where

$k$  = cost of capital

$j$  = interest rate earned on risk based capital

$i_t$  = vector of risk adjusted discount rates

In applying the EVM it should be stressed that  $CapL$  reflects the risk-based capital requirements to support the liabilities. This will not necessarily concur with statutory capital requirements, as used by most actuaries in calculating embedded values. In the event that valuations are based on excessive statutory capital requirements, a compensatory adjustment should be applied to the discount rate to ensure fair value.



## 2(ii) Options pricing methodology

Options pricing methodologies (OPM) for fair valuation might be more appropriately termed direct value of risk methods, given that they are readily applied in both a static and uncertain economic world. By calculating the present value of the future liability cash flows, such methods provide a straightforward and direct means of liability fair valuation; hence their categorisation as a “constructive” approach in fair valuation literature. Specifications of the OPM are varied but typically reconcilable. Two examples follow.

Girard’s [2000] specification of the OPM in a static world and ignoring expenses is:

$$FVL = \sum \frac{L_t}{(1 + r_{ft} + \theta_t^L)^t} \quad (3)$$

where

$r_{ft}$  = risk free interest rate

$\theta_t^L$  = liability spread

While Girard favours a one-off risk adjustment to the discount rate, IAAust [2000] incorporates an additional risk margin (*FVR*) into its OPM specification. *FVR* is defined as a stream of additional cash flows equal to the fair value of liability risks:

$$FVL = PVCF + FVR \quad (4)$$

where

*PVCF* = anticipated value of liability cash flows

*FVR* = fair value of liability risks

Applying the notation used earlier and defining  $RM_t$  to be the required risk margin at time  $t$ , one may write:

$$FVL = \sum \frac{L_t}{(1 + r_t^*)^t} + \sum \frac{L_t * RM_t}{(1 + r_t^*)^t} = \sum \frac{L_t(1 + RM_t)}{(1 + r_t^*)^t} \quad (5)$$

In view of the inclusion of  $FVR$ , one might expect the discount rate to be risk-free. However, IAAust specifies a “risk-adjusted” discount rate, denoted here by  $r_t^*$ . The obvious implication is that the risk provision,  $FVR$ , values the actuarial uncertainty in forecasting future liability cash flows, while  $(r_t^* - r_{ft})$  embodies residual non-actuarial risk, such as the default option held by the insurer. The reasoning for this interpretation will be examined more thoroughly in *Part 2(iii)*. In any case, the two OPM specifications may easily be reconciled by defining  $FVR$  as a “catch-all” variable equal to the difference between liability cash flows discounted at  $r_t^*$  and  $r_{ft} + \theta_t^L$ .

Note that were  $r_t^*$  defined to be the asset earning rate, the expression above would directly reconcile with Girard’s specification of the EVM for fair liability valuation. The risk margin would then be equivalent to the Required Profit charge.

### *2(iii) Accounting for risk*

Debate is rampant among actuaries over the appropriate method to incorporate risk into the liability valuation. Under the EVM, risk is incorporated by the required profit components or risk capital charges that must be added to the liability cash flows. These are determined via specification of a risk-adjusted cost of capital discount rate applicable to the distributable earnings. When applying the OPM, as defined by Girard, risk is accounted for by the specification of the liability spread,  $\theta^L$ . Since, as will be shown in *Part 2(v)*, the EVM and OPM are essentially equivalent, debate over the appropriate method of incorporating risk will result only in computational ease, not greater accuracy. The same factors will drive the Required Profit margins in the EVM as the liability spread in the OPM.

Babbel and Merrill [1998] argue that liabilities are affected by three sources of uncertainty: actuarial risks, market risks and non-market systematic risks. The obvious difficulty lies in determining which of these factors are best modelled as impacting the cash flows and which as affecting the discount rate or liability spread. Consistent with the literature, a hybrid OPM/EVM approach is suggested whereby the actuarial risks (mortality, morbidity, lapse, surrender) are incorporated into the cash flow using either a certainty equivalent approach or an explicit profit charge, with residual risks accounted for by the liability spread. This approach is similar to IAAust's OPM specification (Equation 5) and avoids the confusing situation wherein a deduction from the discount rate is required for actuarial risks and an addition is applied for market risks. For simplification purposes, taxes and expenses have not been considered but these could be readily incorporated in either the cash flows or the liability spread.

Proponents of asset-liability matching argue that the OPM discount rate for liabilities should be fixed to the investment earnings rate, that is  $\theta_t^L = \theta_t^A$ , where  $\theta_t^A$  is the spread earned on product assets. For liabilities directly dependent on asset performance, this argument appears validated. To the extent, however, that a life insurer's asset and liability portfolios are traditionally not perfectly matched, it is argued that general correlation between the two portfolios is sufficiently accounted for in the risk free rate. For asset-independent liabilities, additional risks should be treated as liability specific. In *Part 1(v)*, it will be shown that the liability spread may in fact better be defined as an adjustment to the spread earned on product assets.

Indeed the common argument for asset-independent liabilities is to use a market discount rate for a corporate fixed interest security with like duration. In assessing the validity of this choice of discount rate, it is useful to consider a more detailed breakdown of market risks affecting the valuation. The remainder of this subsection is dedicated to this task.

The Casualty Actuarial Society's (CAS) Risk Premium Project [2000] suggests that there are two major paradigms used to compute risk loads or liability spreads: these being the finance perspective and the actuarial perspective. The fundamental difference in the two approaches lies in the treatment of diversifiable and non-diversifiable risk. Under traditional economic asset pricing theory, the appropriate discount rate for a given project is greater than the risk-free rate of interest only when expected cash flows from the project bear systematic risk. Thus, given that the cash flows associated with insurance liabilities are caused by events largely uncorrelated with market factors, the systematic risk for an asset independent insurance liability should be negligible; and the economist would advocate a risk-free discount rate. Furthermore, economists assume that insurance company shares are held by diversified investors operating in perfect markets, hence any diversifiable risk may be eliminated by portfolio choice.

While it remains a supported tenet of corporate finance that the market prices only systematic risk, this concept may not be practically applied to the actuarial valuation of insurance liabilities. Instead, it is advocated that insurance liability valuations incorporate various non-systematic risks and market imperfections, namely credit risk and illiquidity. This step is justified by recognising that spread components such as illiquidity and credit risk, unpriced in a pure CAPM world, are typically unavoidable or at least not diversifiable in the life insurance liability marketplace. The fundamental difficulty with life insurance liability valuations is that the liabilities are not freely traded; perfect capital markets do not exist and thus each individual risk should be priced on a standalone basis. Equivalently stated, uncertainties associated with insurance cash flows are inherently costly for the firm to bear, thus the appropriate discount rate should be set above the risk free rate. Gutterman's [1999] general theorisation that the "less efficient the market, the greater the weight that should be placed on entity-specific assessment" is analogous to this reasoning. The concept of reinsurance provides further support for this argument. If shareholders were diversified, as the corporate finance view purports, why would

insurance companies pay reinsurers to remove the non-systematic risk from their portfolio? Appendix 2 provides a theoretical justification for inclusion of a positive default risk premium in the discount rate.

Support for a liquidity premium is notably absent in current fair value literature. One might assume this to be due to the fact that traditional insurance liabilities are not “at call.” Nevertheless, given that no deep market exists for insurance liabilities, the bid-ask spread for insurance portfolio sales is likely to be wide. Consequently there appears some grounds for inclusion of such a premium when estimating fair value.

Given this discussion on the need for credit and liquidity spreads, it seems almost paradoxical that the bulk of the extant literature on OPM discusses the concept and applicability of “replicating portfolios”. It is suggested that if a market price does not exist for an insurance contract in its entirety but markets exist for securities that duplicate component parts of the insurance contract, fair value may be constructed as the sum of the aggregate components. That is, certainty equivalent interest rate sensitive cash flows may be discounted at the rates applicable to Arrow-Debreu benchmark securities to avoid naïve estimation of an aggregate liability spread for the contract. However, such techniques would, by earlier arguments, provide erroneous results, given that the hypothetical benchmark securities are required by definition to be freely traded in liquid asset markets. More precisely, by failing to incorporate additional discount spread components for liquidity and default risk, the estimated value of the replicating portfolio will exceed the fair value of the liability contract. Furthermore, given recent suggestions by IAAust [1999] that replicating portfolios typically overvalue corporate bonds, it is likely that discount rates derived from replicating portfolios will be lower than true OPM fair value rates for insurance liabilities.

Amongst those who support an adjustment for credit risk, there remains contention as to whether the default risk premium should be calculated as an industry wide amount or on an insurer-specific basis. In the mergers and acquisitions market, sale of a liability to a third party represents an accounting transfer from the balance sheet of one insurer to that of another. To this end, it seems incongruous that the liability valuation should depend on either party’s credit

rating, especially in jurisdictions such as the USA, where life insurance liabilities are effectively guaranteed by State guaranty funds. Moreover, even while the liabilities remain on a life insurer's books, any discounting of liability estimates due to the insurer's individual credit rating may be misleading in third party or regulatory solvency assessments. Girard [2000], does not support an industry wide premium, however, claiming that such methodology fails the "no-gain-no-loss" test. His reasoning is that a highly rated company will recognise a gain upon reinsuring business to a lower rated company and vice-versa in the event that individual credit ratings are not considered. In passing judgement Girard fails to recognise the true goal of liability fair valuation, that is to define a measure that may readily be implemented both in liability sales and on the balance sheet. In the event that fair valuation of liabilities is compulsorily required on insurer's balance sheets, a standardised industry credit rating is then the only means of obtaining true comparability of equity. Indeed the amount paid in a merger or acquisition may well differ from fair value, in that the purchase price will include synergistic benefits to the acquirer.

*2(iv) Modelling the default risk premium; and the sensitivity of a fair valuation to changes in the discount rate*

To better understand the effect of accounting for individual credit risk in determining the fair value of an insurer’s liabilities, sensitivity analysis was performed on the block of participating whole of life policies described in Appendix 1. An EVM style valuation was employed, with the liabilities and Required Profit margins estimated using Equation 2. Risk spreads, intended to approximate those observed in the corporate bond market, were applied to the discount rate to derive the fair value of liabilities for an AAA and BBB company. All other assumptions, including risk profit components and market value of assets were held constant. The following sensitivity table displays the results.

**Table 2.4.1**

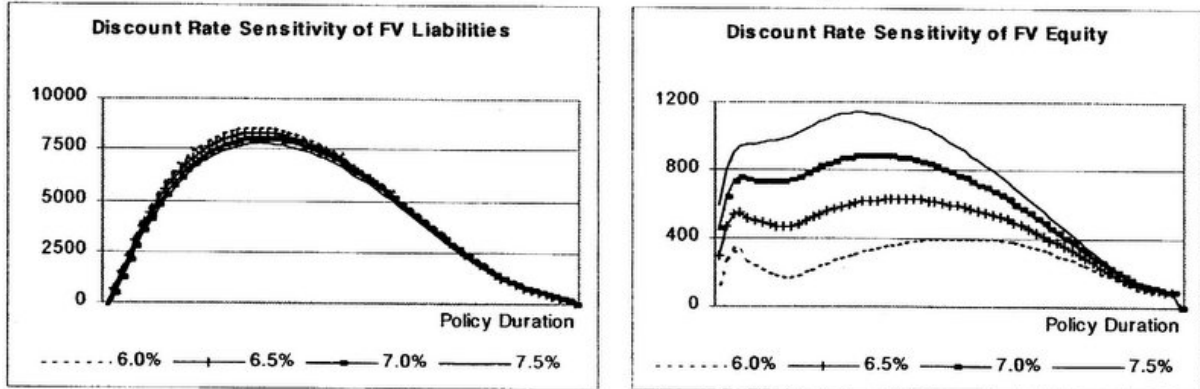
	Earned	BBB	AAA	Risk-Free
Year 10 Discount Rate	7.5%	7.0%	6.3%	6.0%
FV Asset	6,453	6,453	6,453	6,453
FV Liabilities	5,466	5,716	6,125	6,271
FV Equity	987	737	469	182
% of EVM Liabilities	100%	105%	112%	115%
% of EVM Equity	100%	75%	33%	18%

The incongruity discussed earlier bears out. The fair value of liabilities for a AAA company is approximately 7% higher than that for a BBB company, holding the same block of business and supporting assets. More disturbingly, the Fair Value of Equity for the company with the AAA rating is over 50% lower than the equity of the BBB company. To the layman investor, simple balance sheet investigation would imply the BBB company to be in the stronger financial position.

The plots below in Figure 2.4.1 show the effect of applying different discount rates to the liability cash flows over the entire duration of the product. While the fair value of liabilities displays significant sensitivity to the discount rate, the change in FV Equity when the discount rate is reduced from the asset earned rate to the risk free rate is extreme, with an 82% divergence

in year 10. This high degree of sensitivity is attributable to the leveraging effect with FV Equity calculated as a residual value, with an order of magnitude approximately one fifth of the FV Liabilities.

Figure 2.4.1 Sensitivity plots of FV Equity and FV Liabilities to changes in discount rate





2(v) *The algebraic equivalence of the EVM and OPM*

In “Two paradigms for the Market Value of Liabilities,” [1997], Dr. Reitano suggests that the OPM and EVM will only provide equivalent valuations in the “simplest, most contrived, hypothetical” instances. The aim of this subsection is to show that, assuming consistency of assumptions; the OPM and EVM are in fact theoretically equivalent methods for fair valuation.

To see the equivalence of the EVM and OPM, it should first be shown that the discounting of future free cash flows is analogous to discounting the individual asset and liability cash flows. For this step, the reader is referred to Girard’s [2000] decomposition by induction.

Also, by definition:

$$EV_{t-1} = \frac{EV_t + DE_t}{1+k}, \text{ where } EV_T = 0$$
$$MVA_{t-1} = \frac{MVA_t + A_t}{1+i_t}, \text{ where } MVA_T = 0$$

where

$A_t$  = the cash flows from product assets

$DE_t$  = the distributable earnings / free cash flows

Applying the result that  $FVL = CapL + MVA - EV$ , the EVM specification of  $FVL$ , as given by Equation 2, may be derived. Rewriting this in terms of the risk free rate and an option adjusted asset spread,  $\theta_t^A$ , gives:

$$FVL_{t-1} = \frac{FVL_t + L_t + RP_t}{(1+r_t + \theta_t^A)}, \text{ where } FVL_T = 0 \quad (6)$$

The OPM specification, as given by Equation 3, may also easily be expressed using backwards recursion:

$$FVL_{t-1} = \frac{FVL_t + L_t}{(1+r_t + \theta_t^L)}, \text{ where } FVL_T = 0 \quad (7)$$

Equations 6 and 7 thus represent two expressions for  $FVL$  that differ only in terms of the

additional cash flow item  $RP$  in the numerator and the option-adjusted spread used for discounting. But, equating the two expressions gives:

$$\begin{aligned} \frac{FVL_t + L_t + RP_t}{(1 + r_t + \theta_t^A)} &= \frac{FVL_t + L_t}{(1 + r_t + \theta_t^L)} \\ \Rightarrow \theta_t^L &= \theta_t^A - RP_t * \frac{(1 + r_t + \theta_t^L)}{(FVL_t + L_t)} \\ \Rightarrow \theta_t^L &= \theta_t^A - \frac{RP_t}{MVL_{t-1}} \end{aligned} \quad (8)$$

Hence, if the liability spread is defined as the option-adjusted spread, less a Required Profit margin, the two expressions for  $FVL$  are equivalent. Intuitively, the profit margin may be interpreted as the required risk adjustment to the assumed rate of investment earnings to discount the liability cash flow. Recognising  $RP_t$  as the shareholder component of the liability cash flows, this margin could then be regarded as a transfer from the policyholders to the shareholders for the additional risk borne by the shareholders, which would otherwise have been borne by the policyholders themselves.

Girard notes that as the risk level of the product asset portfolio increases, the option-adjusted asset spread should increase. Likewise the cost of capital will rise, increasing the required profit margin,  $RP/MVL$ . He thus concludes that the net effect on  $\theta^L$  is indeterminate. Expanding on this issue further, the net effect should depend on the investor's aversion to risk function, since this will determine the speed at which the investor's cost of capital adjusts to changes in total product risk. Finally, it bears noting that since the risk free rate, market liquidity, and the insurer's default risk are generally regarded as determinants of the cost of capital, the decomposition of  $\theta^L$  given by Equation 8, is consistent with that discussed in *Part 2(iii)*.

To summarise, providing there is consistency of assumptions it would appear that any difference between the EVM and OPM is purely aesthetic and lies in the definition of the spread. Under the OPM, the spread is explicitly defined. Under the EVM, the spread is implicitly calculated so as to incorporate the Required Profit margin.

2(vi) A practical reconciliation of the EVM and OPM

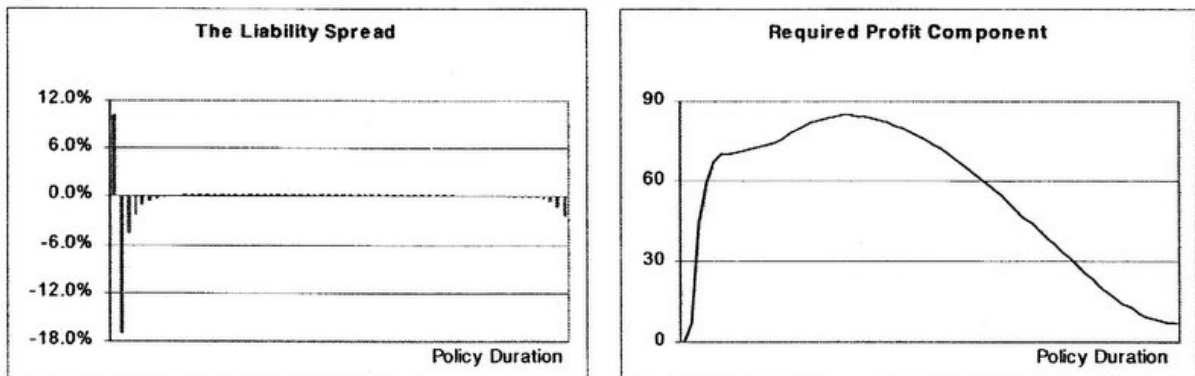
The same block of participating whole of life business was used to manually demonstrate the equivalence of the two fair value methodologies. For brevity, only the first ten product years are given in Table 2.6.1:

**Table 2.6.1**

Year	EVM			OPM				Implied $\theta_t^L$
	$MVE_{(t)}$	$MVA_{(t)}$	$FVL_{(t)}$	$L_{(t)}$	$RP_{(t)}$	$L_{(t)}+RP_{(t)}$	$FVL_{(t)}$	
1	599	90	-509	501	8	509	-509	10.3%
2	799	1,121	322	-913	45	-868	322	-17.0%
3	903	2,012	1,109	-822	59	-763	1,109	-4.6%
4	942	2,807	1,865	-740	67	-673	1,865	-2.3%
5	944	3,540	2,595	-661	70	-591	2,595	-1.2%
6	952	4,239	3,287	-567	70	-496	3,287	-0.7%
7	960	4,875	3,916	-453	71	-383	3,916	-0.3%
8	968	5,455	4,487	-349	71	-278	4,487	-0.1%
9	977	5,980	5,002	-251	72	-179	5,002	0.0%
10	987	6,453	5,466	-161	73	-88	5,466	0.2%

As Table 2.6.1 shows, by discounting the liability cash flows at the asset earned rate and including an appropriate Required Profit component, the EVM and OPM produce equivalent valuations. The implied liability spread, calculated by recursive substitution into Equation 7, is given in the final column. For completeness, plots of the Required Profit components and implied liability spread over the 65-year product duration are given in Figure 2.6.1.

**Figure 2.6.1 Plots of the implied Liability Spread and Required Profit Components**



Analysis of the liability spread is anything but enlightening with the spread following no obvious pattern or time trend. The large positive spread in the first year is intuitively consistent with the high risk of policy launch, but this is followed by five years of significant negative spreads. Interestingly a period of stability is reached for the bulk of the product duration with an average spread of circa 1%.

The obvious implication to be drawn from the liability spread plot is that, despite their algebraic equivalence, the EVM and OPM are unlikely to be reconciled in practice under Girard's specifications. Reitano's conclusion bears weight, at least in practice if not in theory. A priori the pattern of liability spreads is unpredictable, which leads one to the conclusion that working with explicit risk capital charges is preferable to incorporating all risk charges in the liability spread.

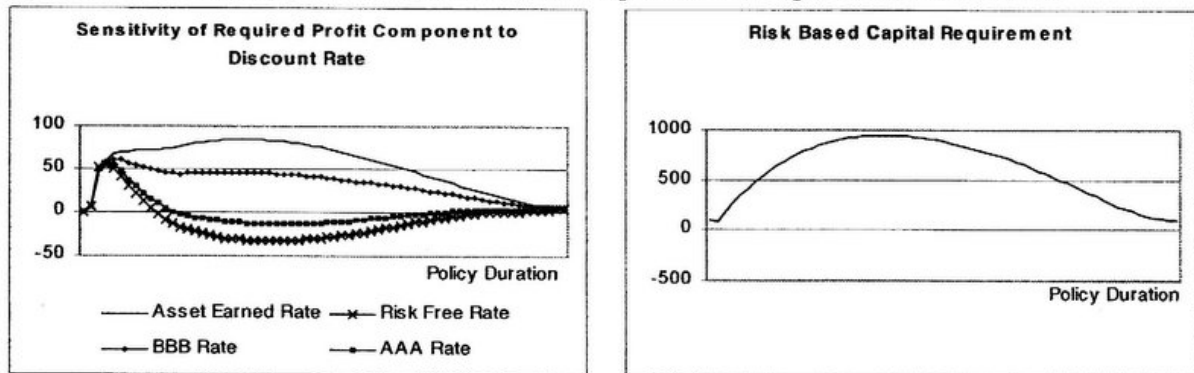
Consideration of the assumptions used in the model reveals two obvious deficiencies, which may be creating the bizarre spread pattern. Firstly, the assumed constant asset spread of 1.5% might better be set as time dependent. Secondly, the cost of capital could be modelled as leverage adjusted and/or time and interest-rate dependent.

Further more detailed studies of fair value would do well to examine the impact of such adjustments.

2(vii) The relationship between the Required Profit Component and the discount rate

Although Girard's specification of the *FVL* under the EVM uses the asset earned rate together with the appropriate Required Profit charges, it is conceivable that Required Profit components could be calculated for discount rates other than the asset earned rate. Figure 2.7.1 below shows the Required Profit components needed to ensure equivalence of the EVM and OPM methodologies for discount rates other than the asset earned rate.

Figure 2.7.1 Sensitivity plot of Required Profit Component to changes in the discount rate



While the pattern of Required Profits roughly approximates the risk based capital component (*CapL*) when the discount rate equals the asset earned rate, the Required Profit components for both the AAA and risk-free discount rates are counterintuitive. Both are negative for a substantial fraction of the product duration. The obvious implication is that working with a risk-free rate may not always be practical.

To date, no theoretical basis has been established to determine the EVM Required Profit charges associated with a discount rate other than the asset earned rate. Obviously, were a discount rate other than the asset earned rate to be prescribed, more research would need to be conducted into determining and explaining patterns of Required Profits.

2(viii) Sensitivity of reported profits to the cost of capital assumption

Finally to highlight the dependence of the fair valuation framework on the cost of capital, the sensitivity of reported profits to changes in this factor was examined.

In the base valuation case, the hurdle rate on the participating policy was set to equal the pricing internal rate of return, 15%. The plots in Figure 2.8.1 show the effect of  $\pm 5\%$  adjustment to the cost of capital. The effect of increasing (decreasing) the cost of capital from the pricing IRR is to “upfront” all subsequent losses (profits). When applying a 20% cost of capital to a product priced to return 15%, the large negative earnings in the first year, together with the small value of equity, result in a -221% return on equity in Year 1. The ROE then reverts to stabilise at 20%. Similarly, a 10% cost of capital gives a return on equity of 502% in Year 1 but this instantaneously reverts to the required 15%.

Figure 2.8.1(a) Sensitivity plots of book profits to changes in the cost of capital

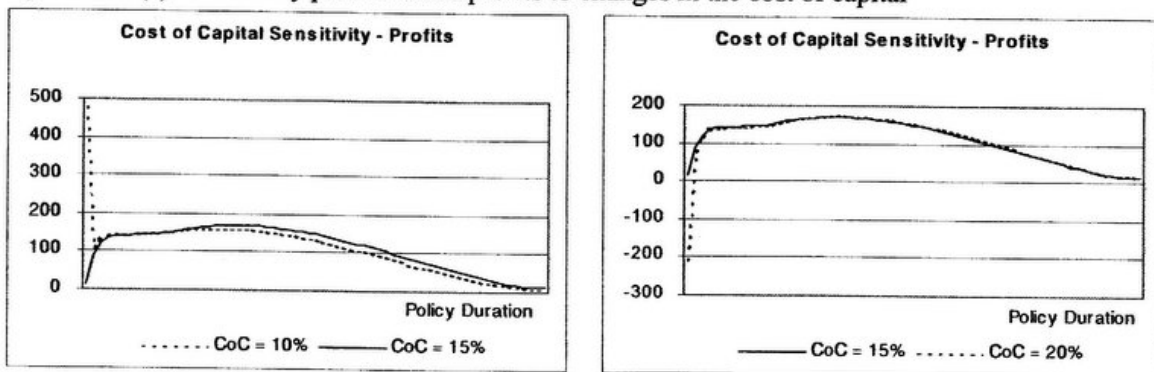
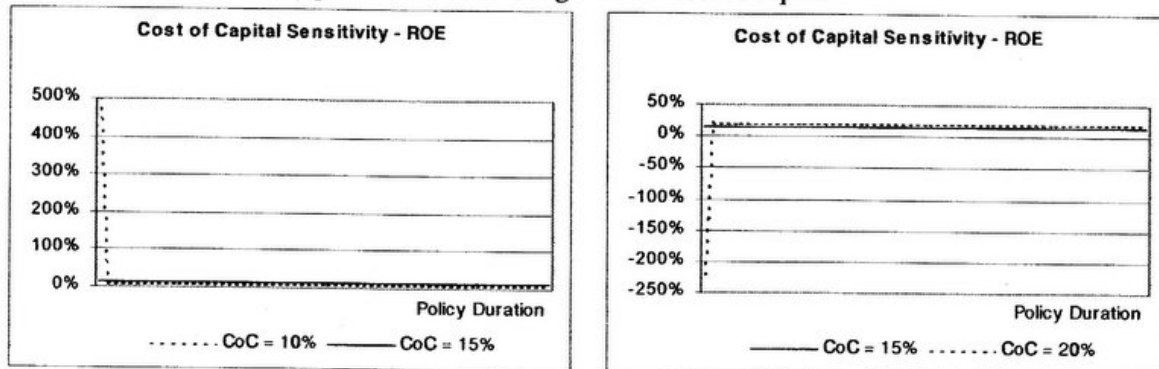
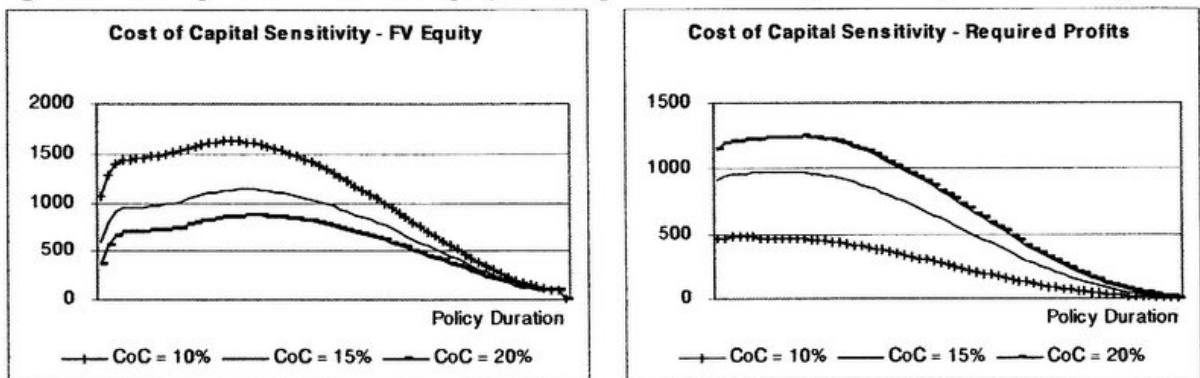


Figure 2.8.1(b) Sensitivity plots of ROE to changes in the cost of capital



In addition to the effect on profits, a change in the cost of capital affects both the FV Equity component and the Required Profit charges, as shown in Figure 2.8.2. The change in the Required Profit component is intuitive. In setting a higher cost of capital, the return on equity will be higher over the duration of the product, as demonstrated in Figure 2.8.1(b). Thus the Required Profits or risk-capital charge that must be added to the liability cash flows will increase accordingly. In turn FV Liabilities increases and FV Equity falls at each duration.

**Figure 2.8.2 Responsiveness of FV Equity and Required Profit Charges to changes in cost of capital**



### ***Part 3 - Conclusions***

The IASC has received countless responses to its Issues Paper “Insurance”, the majority of which recognise the need for a liability valuation framework that is consistent with market value asset measurement. However, the IASC’s avocation of a “fair value” measurement basis remains highly contentious.

This paper has examined two fair valuation techniques, the embedded value approach and the options pricing methodology. Where the latter is a constructive approach to valuation, the former uses a deductive methodology. The key difference between the two methodologies lies in the treatment of risk. EVM valuations employ explicit required profit margins, while under a pure OPM valuation, all risk is implicitly incorporated via the liability discount spread. In practice, a hybrid OPM/EVM style valuation is usually preferred. Although reconcilable in theory, the practical equivalence of the two fair valuation techniques remains in question given the unforeseeable pattern of observed OPM liability spreads at policy issue.

In examining the fair value framework, the paper highlights the subjectivity of such valuation methodologies. Analysis undertaken demonstrates the volatility of fair value equity and earnings valuations in response to changes in the discount rate, cost of capital and default risk. While the pricing of non-systematic risk is argued as desirable in estimating a life insurance liability’s fair value, it is found that accounting for entity-specific credit risk produces incongruities in the financial statements. An industry wide default risk factor is deemed necessary for financial statement comparability.

Ultimately, in selecting a liability valuation basis it is important to consider the purpose and users of the resultant financial statements. Although a market valuation basis for liabilities is conceptually desirable, the illiquidity of the insurance market suggests such an objective is not practicable, at least not in the foreseeable future. Until reinsurance or insurance merger and acquisition markets develop sufficiently to determine appropriate valuation parameters, reporting bases which limit the scope for judgement, will most likely be of greater use to regulators and individual investors in assessing a life company’s worth.



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### ***Appendix 1 - Modelling assumptions***

Policies:	1,000
Face Amount:	\$100,000
Sex:	Male
Issue Age:	35
Plan:	Participating whole life product with premiums payable to age 100.
Valuation Interest Rate:	4.0%
Valuation Mortality:	1980 CSO
Net Premium Loading:	20.0%
Experience Interest Rate:	7.5%
Risk Free Rate:	6.0%
Experience Mortality:	45% of valuation mortality.
Expenses and Commission:	% of gross premiums: 125%, 25, 20, 15, 10, 9, 8, 7, 6, 5, 4% thereafter. The declining pattern is intended to reflect higher initial renewal commissions, which ultimately level off at 2%. Also, percentage of premium estimates have been used to approximate traditional fixed dollar expense charges.
Lapses:	10%, 9, 8, 7, 6, 5% thereafter. All surviving policies lapse at age 100.
Bonus interest charge:	1%
Bonus mortality charge:	15%
Risk-based capital (CapL):	10% of reserves + 100% of expected claims. (Girard model)
Base Cost of Capital:	Pricing IRR of 15%.

For simplification purposes, tax and inflation rates have been set to 0%. Also, a single interest rate has been used as opposed to spot rates derived from a yield curve and both the portfolio assets and risk-based capital have been assumed to earn interest at this single interest rate.

## *Appendix 2 - The need for a default risk premium*

The goal here is to frame an insurance policy as an option, and in doing so to justify the addition of a default risk premium to the risk free rate when discounting liability cash flows. The proof is in continuous time. To simplify, the focus here is on a single period non participating insurance policy and it is assumed that at the end of the contract period,  $T$ , all funds are either paid out in claims or distributed to shareholders as dividends. The theory could readily be extended to a multi-period setting.

Assume the policyholders pay aggregate premium  $P$  to acquire their policies and will in return receive an aggregate amount  $L$  at time,  $T$ . At the start of the period, shareholders also contribute an amount,  $E$ , such that the insurer must invest  $E + P$ . In line with Merton [1974], it is assumed that these two accounts — the premium and equity accounts — and the liability account accumulate according to geometric Brownian motion.

$$\begin{aligned}dP &= \alpha_p P dt + \sigma_p P dz_p \\dE &= \alpha_E E dt + \sigma_E E dz_E \\dL &= \theta L dt + \sigma_L L dz_L\end{aligned}$$

The policyholders' claims may be defined as:

$$PH_t(\tau) = P_t(\tau) - SH_t(\tau)$$

where

$$\begin{aligned}P_t(\tau) &= \text{the premium account, as defined above} \\SH_t(\tau) &= \text{the shareholders' claim} \\ \tau &= T - t\end{aligned}$$

There are essentially three possible outcomes:

1. The accumulation of  $P$  at time  $t=1$  is sufficient to meet claims. Policyholders' claims may be paid out entirely from the premium account.
2. The accumulation of  $P$  at time  $t=1$  is insufficient to meet claims but the accumulation of  $E+P$  exceeds total claims due. In this event the policyholders are paid out and shareholders receive any residual funds as dividends.
3. The accumulation of  $E+P$  is less than claims due. Here the shareholders are protected by limited liability status and may default on the excess of losses over  $E+P$ .

Each case is examined in turn.

1. This case is straightforward. The shareholders' distribution is a call option  $C_t$ , with payoff  $\max(P-L, 0)$ .
2. In this case the shareholders must liquidate that part of the equity account required to make up the difference between the accumulation of the premium account and total claims. This obligation is equivalent to shorting a put option,  $B_t$ , with payoff  $\max(L-P, 0)$ .

Should the liabilities exceed the total of the premium and equity account, the shareholders have a put option allowing them to default on any excess losses. This put is specified in the literature as the insolvency put,  $I_t$ , and depends on the total assets and liabilities of the firm. (See Phillips et. al. [1996] for a derivation of an insolvency put).

Using the notation above, the shareholders' claim is equal to the sum of the three cases.

$$SH_t(\tau) = C_t(\tau) - B_t(\tau) + I_t(\tau)$$

Substituting this into the expression for  $PH_t(\tau)$  gives:

$$PH_t(\tau) = P_t(\tau) - [C_t(\tau) - B_t(\tau) + I_t(\tau)]$$

This may be simplified by the application of put-call parity. Equivalently stated, the total shareholders' claim must equal the premium minus the discounted claims expected at time  $T$ . Discounting is at the risk free rate.

$$C_t(\tau) - B_t(\tau) = +P_t(\tau) - Le^{-r_f\tau}$$

Thus by substitution:

$$PH_t(\tau) = C_t(\tau) - B_t(\tau) + Le^{-r_f\tau} - [C_t(\tau) - B_t(\tau) + I_t(\tau)] = Le^{-r_f\tau} - I_t(\tau)$$

Completing the proof is now straightforward. In order to derive the risk-adjusted discount rate, one must determine the discount rate,  $r_d$  such that at time  $t$ , the total discounted claims equals the policyholders' claims. This gives:

$$\begin{aligned} PH(\tau) &= Le^{-r_d\tau} \\ \Rightarrow Le^{-r_f\tau} - I(\tau) &= Le^{-r_d\tau} \\ \Rightarrow Le^{-r_f\tau} &\geq Le^{-r_d\tau} \\ \Rightarrow r_d &\geq r_f \end{aligned}$$

Consistent with the discussion on the liability spread it is found that the effect of default risk is to increase the risk free discount rate. Thus the default risk premium,  $D_b$ , is a positive component of the liability spread determined as follows:

$$\begin{aligned} r_d &= -\frac{1}{\tau} \ln \left( e^{-r_f\tau} - \frac{I_t(\tau)}{L} \right) \\ D_t &= -\frac{1}{\tau} \ln \left( e^{-r_f\tau} - \frac{I_t(\tau)}{L} \right) - r_f \end{aligned}$$

The expression makes intuitive sense. The risk discount rate is the risk free adjusted by an amount equal to the insolvency put divided by total claims.