COPING WITH LONGEVITY –

THE NEW GERMAN ANNUITY VALUATION TABLE DAV 2004 R

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ABSTRACT

This article depicts the derivation of the German annuity table DAV 2004 R, which consists of a base mortality table for the period 1999 and a trend function for projected future mortality improvements. Both best estimation values and the choice of safety margins are discussed. The table is compared with other current international annuity tables.

KEYWORDS

Mortality table, annuity, Germany, trend function, mortality improvement.

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1 INTRODUCTION

1.1 Background

The German life insurance market has undergone several dramatic changes in recent years. One of those changes has affected the product structure of new business. Traditional endowments were the backbone of the German life industry for decades and they experienced an unprecedented boom as late as shortly after German reunification. From around that time onwards unit-linked policies started gradually replacing endowments in new business. Although the emergence of the unit-linked policy represented a significant change for the industry, the biometric risk inherent in endowments and unit-linked covers is very similar. In both lines of business life insurers run the risk of more insured lives dying than expected.

Since the early 1990s another line of business has been capturing an increasing share of new business: annuities. In the most recent past annuities have even become the leading line in terms of new business. The following figure illustrates this development.

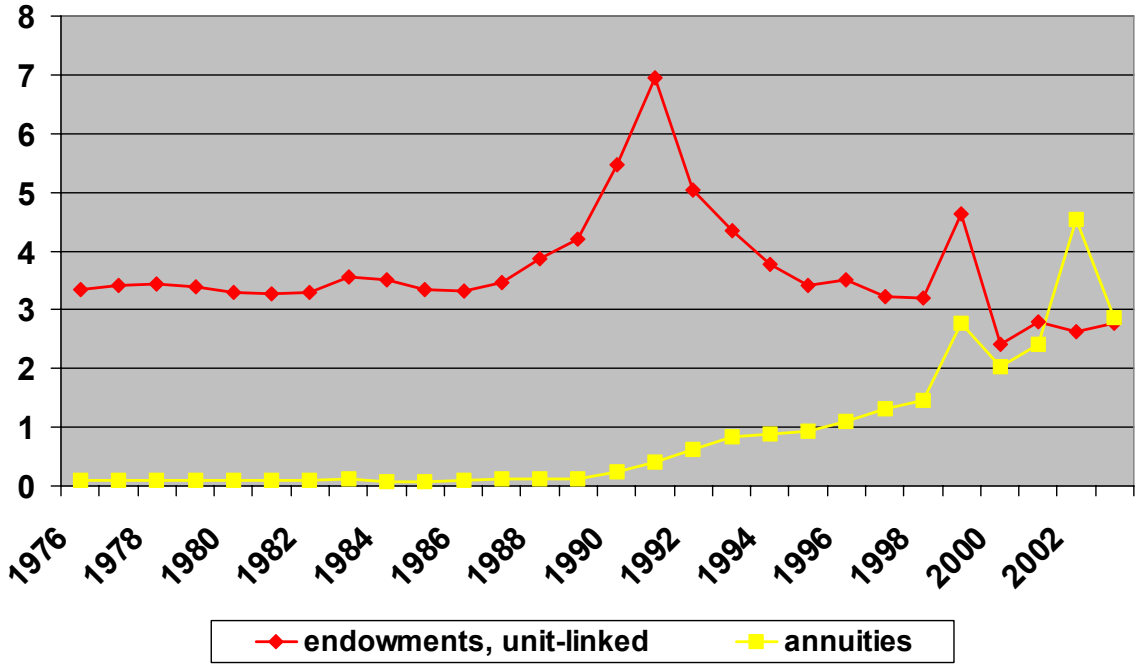


Figure 1: Number of new policies per year (in millions). Source: see [G]

The biometric risk underlying annuities is very different from that of endowments and unit-linked policies. In issuing an annuity policy the life insurer exposes itself to the longevity risk, i.e. the risk that fewer people will die than expected. This paper presents the latest German approach to tackling this risk: the new German annuity valuation table DAV 2004 R.

The German annuity market is dominated by two basic products, immediate annuities and deferred annuities. Deferred annuities account for a large share of the total annuity business. They are normally regular level premium products which are taken out long before the benefit payment starts. However, the annuity rates applicable at the beginning of the benefit payment period are often already guaranteed at the outset. The policyholder usually has the op-

tion of annuitizing the policy at the end of the deferment period or receiving the corresponding lump sum.

Both products normally entail no or very little death benefit. With the limited death benefit for a deferred annuity there is normally a positive reserve for the contract in the event of death that is not paid to the beneficiaries. It is therefore standard industry practice to take explicitly into account that a number of insured lives will die before the beginning of the benefit payment period. In a way, the surviving policyholders "inherit" the reserves of the deceased persons. This means that insurance companies are already running a longevity risk during the deferment period because fewer insured lives may actually die than expected.

German insurance companies are required by law to use prudent rates for the valuation of their business. It is explicitly forbidden to use only best estimate pricing assumptions, which are called 2nd order mortality rates in Germany. The mortality rates, including appropriate provision for adverse deviations from the best estimate assumptions, are referred to as 1st order mortality rates. It should be mentioned that valuation mortality tables are normally also used for pricing purposes in Germany.

1.2 Mortality tables for annuities

Mortality tables for a fixed calendar year do not represent a best estimate for calculating annuities, since they do not reflect the mortality improvement trend.

This is why it has become standard international practice to use generation mortality tables for annuity business. Generation mortality tables are a combination of mortality rates per birth year and a trend assumption relating to the future mortality improvements or, alternatively, a combination of mortality rates for a fixed calendar year, called the base table, and such a trend assumption. Sometimes the latter systems are approximated by a table for a fixed generation and age shifts for other generations in order to facilitate IT implementation.

The use of mortality tables for annuities has a long history in the German market.

1.2.1 Rueff 1955

In 1955 Fritz Rueff, inspired by findings that mortality tables for different generations can be satisfactorily aligned with the help of age shifts, published the first mortality table for annuities, called Rueff 1955 [R]. He used the German population table from 1950 as a starting point for the base table. The population table was adjusted to reflect insured lives' mortality based on the experience of one company. After this adjustment the base table rates were about 20% below those of the population mortality table. The trend assumption relating to the future mortality improvements was derived from past German population tables. Rueff expected his table to remain in use until about 1980. In fact it proved to be the industry standard even beyond that point. Actually, the application of the table had been producing longevity losses before it was replaced, but those losses were not very significant given the past stable profits from the investment side and the small business volume.

1.2.2 DAV 1987 R

The longevity losses eventually became unacceptable, leading to the development of the table DAV 1987 R in the mid-1980s [L2]. Basically, an approach similar to Rueff's method in

1955 was used: the trend assumption relating to the future mortality improvement was derived from German population tables covering the period between the beginning of the 20th century and the early 1980s and the mortality rates for the generation born in 1950 were chosen as the base table. Those mortality rates were taken from the rates determined at the population level, reduced by an insured lives deduction of about 15% to 20%, which was not based on actual annuitants' experience.

1.2.3 DAV 1994 R

In the early 1990s it was felt that the DAV 1987 R table had to be replaced fairly quickly because the actual mortality improvements in the 1980s strongly exceeded the future mortality improvements reflected in DAV 1987 R. Newly available insured lives annuity data also indicated that the difference between insured lives and population mortality had previously been underestimated. In the new table DAV 1994 R [SS] the strong mortality improvements from the 1980s were extrapolated up to the year 2000. From that point forward, however, a much lighter trend assumption based on population tables from the late 19th century to the mid-1980s was applied. The base table was the population table for the generation born in 1955, adjusted by the insured lives' mortality differential found in the experience data.

1.2.4 Another new table

With the increasing importance of annuity business, the statutory valuation requirements, the low interest-rate environment and the long-term guarantees given in annuity business, insurance companies have a growing vested interest in using adequate mortality rates for pricing and evaluating annuity business. In 2003/2004, a committee of the German Actuarial Society (DAV) thus re-examined the question of whether a new mortality table was necessary for annuity business.

The committee came to the conclusion that the post-2000 DAV 1994 R mortality improvement trend assumption did not appropriately reflect the mortality improvements in the last three decades of the twentieth century. The development of a new table, called DAV 2004 R, was therefore decided. All mortality tables prior to the 1971/73 mortality table were disregarded for the purpose of projecting the future mortality improvements contained in DAV 2004 R. The development prior to the 1970s was no longer considered representative for the future. For the first time it was also assumed that annuitants' mortality improvements exceed those of the general population.

Munich Re and Gen Re have been collecting and analyzing insured lives data from individual annuity business for several years. Between 1995 and 2002, more than 20 life insurance companies contributed to this observation material, which was one of the main data sources for the committee's activities. This observation material, which encompasses 13.7 million years' exposure, is not only much more voluminous than the data which was available for the DAV 1994 R table, it is also much more comprehensive with regard to contractual details. The base table of DAV 2004 R was derived from this data source. According to the data, the difference between the mortality rates of insured lives and the general population is even slightly more pronounced than was assumed for DAV 1994 R, although although the old mortality differential curve has largely been confirmed.

The German Actuarial Society (DAV) has recommended using this new table for evaluating and pricing new business with effect from 1 January 2005. The German version of this paper [DAV] is the official reference for the new table DAV 2004 R.

The new DAV 2004 R table consists of the following components:

- 2nd order base table: a best estimate of the insured lives mortality rates in 1999,
- 1st order base table: the 2nd order base table reduced to take into account provisions for adverse deviation,
- 2nd order mortality trend: best estimate of the future mortality improvements, and
- 1st order mortality trend: the 2nd order trend increased to take into account provisions for adverse deviation.

These components will be explained in more detail in the following chapters. In the remainder of this chapter we will focus on the long-term development of premiums for annuities in the German market.

1.3 Development of premiums for annuities

We will first look at deferred annuities where the replacement of DAV 1994 R by DAV 2004 R in 2005 caused a premium increase of about 10% to 20% depending on age and gender.

For an examination of the long-term development of premiums for annuities we picked a sample case, a 35-year-old buying a deferred annuity policy with a deferment period of 30 years, i.e. benefit payments commence and regular premium payments stop at age 65.

We then calculated premiums for policy commencement years from 1960 to 2015, based on the table valid at each time, as outlined in section 1.2. However, we applied the current market standard interest rate of 2.75% rather than the various actual past interest rates so that the pure biometric assessment would not be distorted by interest rate effects.

When looking at the following table, one must bear in mind that premiums for annuity policies with identical contractual features keep rising from commencement year to commencement year because of the trend assumption relating to the future mortality improvements.

Period	Mortality table at...		Total premium increase on 2.75% interest rate basis		Annualised premium increase on 2.75% interest rate basis	
	beginning of period	end of period	male	female	male	female
1960 - 1990	Rueff 1955	DAV 1987 R	69.8%	102.0%	1.8%	2.4%
1990 - 1995	DAV 1987 R	DAV 1994 R	41.3%	22.6%	7.2%	4.2%
1995 - 2000	DAV 1994 R	DAV 1994 R	0.0%	2.4%	0.0%	0.5%
2000 - 2005	DAV 1994 R	DAV 2004 R	22.5%	10.3%	4.1%	2.0%
2005 - 2010	DAV 2004 R	DAV 2004 R	2.4%	1.8%	0.5%	0.4%
2010 - 2015	DAV 2004 R	DAV 2004 R	2.2%	1.7%	0.4%	0.3%
1960-2005	Rueff 1955	DAV 2004 R	193.9%	179.9%	2.4%	2.3%
1990-2005	DAV 1987 R	DAV 2004 R	73.1%	38.5%	3.7%	2.2%

Table 1: Development of premiums for deferred annuities from 1960 to 2015

Based on the uniform interest rate, the premiums for males, for example, rose by about 73% from 1990 to 2005. Had the DAV 1987 R table that was valid in 1990 been retained until 2005, the premium increase would only have been 5%. If only the DAV 1994 R table had been introduced during the 1990 to 2005 period but not the latest DAV 2004 R table, the premium increase from 1990 to 2005 would have amounted to 46%.

In immediate annuities, the replacement of DAV 1994 R by DAV 2004 R in 2005 caused a premium increase of about 5% to 10%. The following table shows the long-term premium comparison for immediate annuities. Here the sample case is a 65 year-old buying an annuity with immediate beginning of benefit payments.

Period	Mortality table at...		Total premium increase on 2.75% interest rate basis		Annualised premium increase on 2.75% interest rate basis	
	beginning of period	end of period	male	female	male	female
1960 - 1990	Rueff 1955	DAV 1987 R	19.8%	44.6%	0.6%	1.2%
1990 - 1995	DAV 1987 R	DAV 1994 R	25.4%	11.0%	4.6%	2.1%
1995 - 2000	DAV 1994 R	DAV 1994 R	3.0%	2.8%	0.6%	0.6%
2000 - 2005	DAV 1994 R	DAV 2004 R	11.6%	10.5%	2.2%	2.0%
2005 - 2010	DAV 2004 R	DAV 2004 R	2.9%	2.3%	0.6%	0.4%
2010 - 2015	DAV 2004 R	DAV 2004 R	2.7%	2.1%	0.5%	0.4%
1960-2005	Rueff 1955	DAV 2004 R	72.6%	82.4%	1.2%	1.3%
1990-2005	DAV 1987 R	DAV 2004 R	44.1%	26.1%	2.5%	1.6%

Table 2: Development of premiums for immediate annuities from 1960 to 2015

As can be seen, premiums for deferred annuities increased by more than 2% a year on a long-term basis and have even risen by close to 4% a year for males within the last 15 years, whereas premium increases for immediate annuities tended to be more moderate. It can be assumed that any inflation that occurred in the past was reflected in the annuity amount that policyholders insured. The premium increases shown in tables 1 and 2 therefore come on top of any inflation. They provide clear evidence of both actual changes in mortality rates and dramatic changes in the projection of future mortality improvements. On the one hand, mak-

ing provision for the pension age through the vehicle of annuities has clearly become much more expensive over the years. On the other hand, these premium increases are just a reflection of the increased longevity risk facing every individual. People who try to make provision with financial products other than annuities are very much exposed to the risk of outliving their assets.

For insurance companies these premium increases also mean considerable challenges in the form of past and future reserve strengthenings. While we are confident that the new table represents a reasonable approach, constant future monitoring of the mortality improvements of both the general population and insured lives is essential now more than ever in order to detect any adverse experience.

2 BASE TABLE

The insured lives data from exposure years 1995 to 2002 collected by Munich Re and Gen Re is used to derive the base table. The appropriate base year for a table derived from this observation material is 1999.

An analysis of the observation material demonstrates the existence of a strong relationship between mortality and annuity levels. Several annuity level phases were defined and the mortality level in the different phases was measured against the overall mortality level. The results are shown in the following table:

Annual annuity amount (euros)	Males	Females
0 – 600	117%	111%
601 – 1200	110%	105%
1201 – 2000	101%	99%
2001 – 3500	90%	88%
3501 – 6000	89%	91%
> 6000	86%	91%
Aggregate	100%	100%

Table 3: Mortality level in different annuity level phases

In the light of these results, mortality rates weighted by lives are not appropriate for annuity business. Accordingly, where possible, the mortality rates in the base table have been derived as mortality rates weighted by annuity level.

2.1 The selection table for the benefit payment period

The annuity payment period is divided into six selection phases by the number of years lapsed since the start of the benefit payment: 1st year,..., 5th year, 6th+ years ("ultimate").

For the purpose of deriving selection factors, in a first step we compare the mortality rates in the different selection phases with the population mortality. To this end, we calculate per selection phase the ratio of the actual number of deceased persons in the data material over the

number of expected deceased persons using the population mortality. The following figure shows the various ratios.

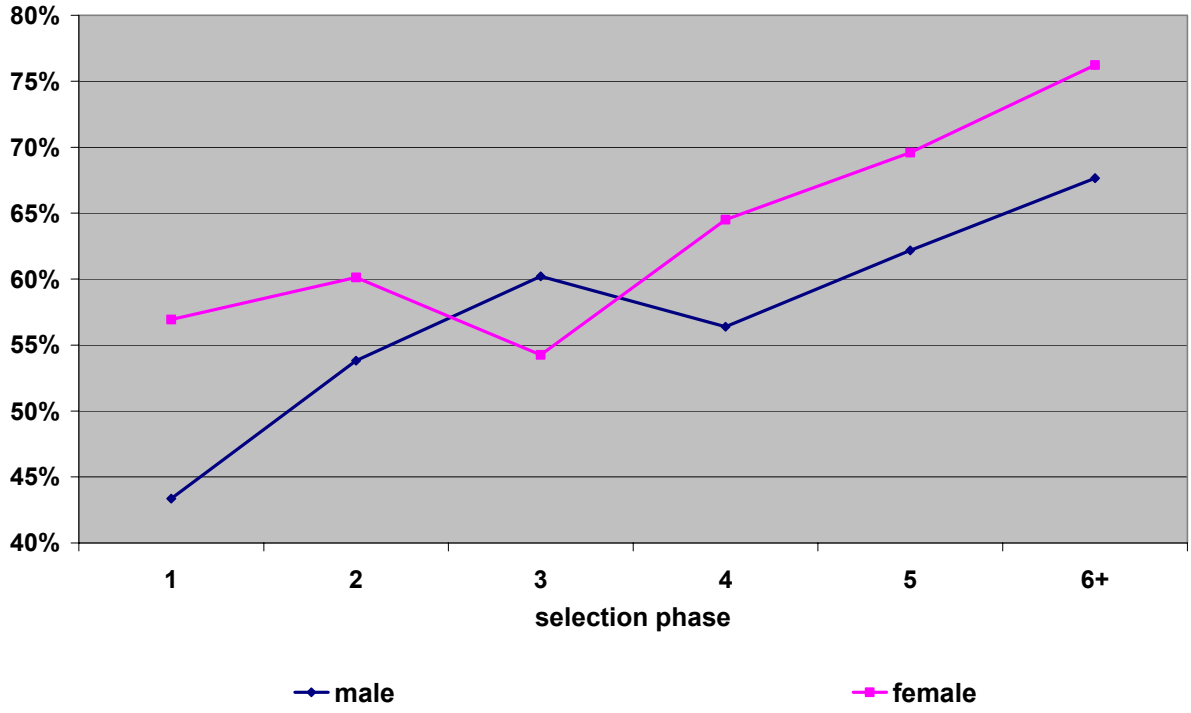


Figure 2: Mortality rates by selection phase relative to the population mortality

The following observations can be made:

- Male mortality in selection phase 1 is lower than in any other selection phase. Female mortality in selection phase 1 is lower than in selection phases 2, 4, 5 and 6+.
- Mortality in selection phase 6+ is highest, both for males and females.
- Mortality in selection phases 2 to 5 fluctuates.

The following model is thus adopted for the selection table:

- There is an ultimate mortality table for selection phase 6+.
- Mortality in selection phases 1 to 5 is a factor of the ultimate mortality table. This factor depends not on age, but on gender. There is a factor for selection phase 1 and a common factor for selection phases 2 to 5.

In order to determine appropriate selection factors we first calculate crude ultimate mortality rates. The crude mortality rates are graduated for the age band 60 to 99 using the weighted Whittaker-Henderson method (see [KBLOZ] or [L1]) with weight 0.5 on the smoothness measured by second differences. The selection factors are then defined as the ratio of the actual number of deceased persons in the respective selection phase to the number of deceased persons which would be expected if the graduated ultimate mortality rates were applied to the exposure of insured lives in the respective selection phase.

We obtain the following values:

	Male	Female
Selection phase 1	0.670538	0.712823
Selection phases 2 to 5	0.876209	0.798230

In the absence of other available data it is reasonable to use the graduated ultimate mortality rates for the purpose of deriving selection factors. However, the data in the ultimate selection phase is insufficient for deriving the final ultimate mortality rates for the new table, particularly for the age band 60 to 65 at which typically benefit payment commences. After adjusting the relationship between the number of deceased persons and the number of insured lives by means of the selection factors, we therefore use the data from all the selection phases to derive the ultimate mortality rates. For full details, see [DAV].

The crude rates are again graduated using the weighted Whittaker-Henderson method with weight 0.5 on the smoothness measured by second differences. These graduated rates are the final best estimate ultimate mortality rates.

We compared the ultimate mortality rates based upon the data from all the selection phases with the ultimate mortality rates which were derived using data from the ultimate selection phase only. It was found that for ages 70+ the differences between the two sets of mortality rates are negligible. More significant differences at younger ages are attributable to the small volume of ultimate data which exists and which is also influenced by early retirees who are not representative of the portfolio of all pensioners.

Outside the age band 60 to 99 there was insufficient insured lives data, with the result that we are forced to extrapolate the ultimate mortality rates.

For the ages up to 59 we assume that the ratio of the ultimate mortality rate at age 60 q_{60}^{6} to the 1999 population mortality rate at age 60 $q_{60,1999}^{pop}$ can be transferred to the younger ages. We thus define:

$$q_x^6 = q_{x,1999}^{pop} \cdot \frac{q_x^6}{q_{60,1999}^{pop}} = q_{x,1999}^{pop} \cdot \begin{cases} 66.6\% \ for \ men \\ 85.2\% \ for \ women \end{cases}$$

As the maximal age of the German population mortality tables is 89, we cannot use the population mortality to extrapolate at the oldest ages. Therefore, we follow the method set out in [TKV] to extrapolate the ultimate mortality rates at ages above 99. In [TKV] six extrapolation approaches are examined:

- The Gompertz model: $q_x = 1 - \exp(-\exp(a + bx))$
- The Quadratic model: $q_x = 1 - \exp(-\exp(a + bx + cx^2))$

- The Heligman and Pollard model: $q_x = \frac{a\exp(bx)}{1 + a\exp(bx)}$

- The Weibull model: $q_x = 1 - \exp(-a(x + \frac{1}{2})^b)$

- The Kannisto model: $q_x = 1 - \exp\left(-\left(\frac{a\exp(bx)}{1 + a\exp(bx)} + c\right)\right)$

- The Logistic model: $q_x = 1 - \exp\left(-\left(\frac{\beta\exp(bx)}{1 + \alpha\exp(bx)} + c\right)\right)$

We first fit all six models to the ultimate mortality rates at ages 85 to 95 using the Maximum-Loglikelihood method, leaving the age band 96 to 99 to assess the accuracy of the calibration with various stochastic criteria:

- The number of the expected deceased persons applying the extrapolated mortality rates to the actual number of deceased persons

- The loglikelihood

- The chi-square statistic

All three criteria consistently show that the Logistic, the Kannisto ($c \neq 0$) and the Quadratic model produce the best results. However, since mortality rates start decreasing at age *x=-b/2c* with the Quadratic model, it is disregarded. Given the actual ultimate mortality rates q_x^6 at ages 85 to 95, this point is reached much earlier than age 120, particularly for females.

For the purpose of deciding between the Logistic and the Kannisto model we compare the extrapolated mortality rates with the Japanese 1999 mortality rates at ages 105 and 109:

	Males		Females	
	x=105	X=109	x=105	X=109
Japanese population	46.1%	52.2%	41.9%	49.6%
Logistic model	42.2%	50.0%	36.1%	43.8%
Kannisto model	40.9%	46.1%	33.4%	38.3%

As the level of the Kannisto mortality rates seems to be too low we use the Logistic model for extrapolating the ultimate mortality rates at the ages 100 to 120. For full details, see [DAV].

2.2 The aggregate table for the deferment period

A table for the deferment period is also needed. Munich Re and Gen Re have collected details of more than 12 million years' exposure from more than 20 insurance companies in the observation period 1995 to 2002 which can be used to derive such a table.

The observation material neither shows a strong selection effect in the deferment period nor is the impact of any selection effect on premiums and reserves for deferred annuities very substantial. That is why an aggregate table is derived for the deferment period which is not graded in terms of the years lapsed since the commencement date.

From ages 65 onwards the observed exposure for annuities in payment strongly outweighs the exposure for deferred annuities, which rapidly becomes sparse at these ages. It is thus reasonable to revert to the observation material relating to annuities in payment for the ages 65+.

In a first step we calculate crude mortality rates which are not graded separately in terms of the years lapsed since the commencement date or the start of the benefit payment, both from the observation material relating to the deferment period $\left(q_x^{def}(crude)\right)$ and from the observation material relating to the benefit payment period $\left(q_x^{ann}(crude)\right)$.

Both sets of crude mortality rates are then graduated using the weighted Whittaker-Henderson method with weight 0.5 on the smoothness measured by second differences. The graduated rates q_x^{def} and q_x^{ann} are then put together at age 65 in order to obtain aggregate mortality rates. q_{65}^{def} and q_{65}^{ann} differ by less than 1%, meaning that no additional graduating is needed.

As the ultimate mortality rate from the selection table and the aggregate mortality rate at age 99, q_{99}^{6} and q_{99}^{agg}, are almost identical, the extrapolation for ages 100+ from the selection table can also be used for the aggregate table.

2.3 Safety margins

2.3.1 Margin for volatility risk

For the calculation of the margin for volatility risk we use the following denotations:

q_x^{agg} The best estimate aggregate mortality rates derived in section 2.2,

s^{α} The relative margin for volatility risk for confidence level $1-\alpha$,

L_x^M The insured lives aged x in the model portfolio,

T_x The random variable number of the deceased at age x in the model portfolio,

V_x The mathematical reserve for an x-year-old's contract,

The margin s^{α} is designed to reduce the volatility risk when the table is applied. The idea is to provide protection against a maximum loss at a defined confidence level. This basic concept has been widely used in German mortality and morbidity tables (cf. [L1], [SS] and more generally [P], [PS]). In the context of German annuity products, the most appropriate measurement for loss is the amount of the mathematical reserve that can be released in the event of death. If fewer insured lives die than was originally expected, then less mathematical reserve can be released and the insurance company may suffer a loss. Thus, what is required is that

with confidence $1-\alpha$ it is not possible for less mathematical reserve to be actually released for the deceased than was originally expected:

$$P\left(\sum_x T_x \cdot V_x \geq \sum_x \left(1-s^{\alpha}\right)\cdot q_x^{agg} \cdot L_x^M \cdot V_x\right) \geq 1-\alpha$$

In order to solve the equation an underlying model portfolio is defined as follows:

The size of the annuity portfolio, 200,000 insured lives (50% male, 50% female), corresponds to the projected average size which German annuity portfolios will feature in a few years' time. The structure of the portfolio is geared to the observation material with constant annuity amounts (observation material average). It is furthermore assumed that 10% of the policies are annuities in payment, whereas 90% are deferred annuity policies, and that for all policies, benefit payment commences at age 65. Finally it is assumed that there are no death benefits such as guaranteed periods of benefit payment or survivorship annuities.

We additionally assume that T_x is independently distributed. Given these assumptions the Central Limit Theorem allows us to approximate the mathematical reserve which can be released at death by a normal random variable with known expectation and standard deviation. The margin s^{α} can then be calculated for any confidence level 1-α using this random variable's expectation and standard deviation as well as the $(1-\alpha)$ quantile of the standard normal distribution. For full details, see [DAV].

If 1-α =95%, we obtain a margin for volatility risk of 6.26% for males and 7.22% for females. Even if it were the case that we needed to assume a specific model portfolio for this step, sensitivity calculations have demonstrated that the actual confidence level does not change significantly if the age structure, the proportion of male insured lives or the proportion of policies already in payment varies.

2.3.2 Margin for level parameter risk

In order to derive the table, a certain model was postulated and parameters needed deriving. There are several sources of level parameter risk:

- Structural differences between the observation material and any actual portfolio to which the table is applied.
- A difference between the mortality levels at individual companies and that in the observation material.
- Structural differences in future new business, particularly due to changes to the political and taxation frameworks.
- The actual observation material having also been subject to statistical fluctuations.

A 10% flat-rate margin for level parameter risk is thus defined, meaning that there is a total deduction of 15.6% for males and 16.5% for females.

2.4 Depiction of the results

The following figure shows the selection mortality rates from section 2.1 and the aggregate mortality rates from section 2.2 as a percentage of the population mortality rates in the year 1999. The strong degree of self-selection exercised by the insureds at ages of around 60 is clearly evident from the aggregate mortality rates.

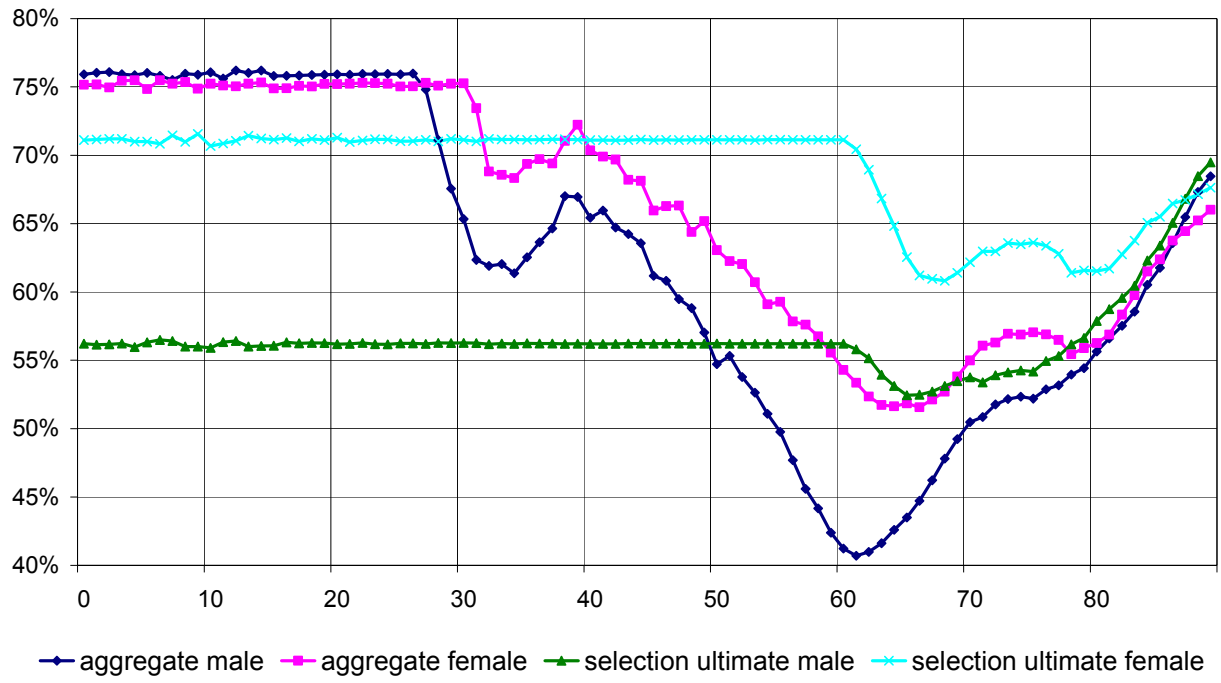


Figure 3: Comparison between the derived mortality rates and population mortality rates

3 MORTALITY IMPROVEMENT

3.1 Model choice

An age-dependent mortality improvement model was used for the previous German annuity table DAV 1994 R (see [SS]):

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp\left(-F(x)\right)$$

with a trend function $F(x)$ depending on age x. In the following, this model is referred to as the traditional model.

The cohort model of birth-year-dependent mortality improvement is defined in [W], chapter 6.6:

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp\left(-G(t+1-x)\right)$$

with a trend function $G(t+1-x)$ depending on birth year $t+1-x$. [W] contains studies of mortality data from England and Wales showing a cohort effect. For the purpose of choosing an appropriate model for mortality projections of the German population, the traditional model and the cohort model were examined as well as the synthesis model stemming from a combination of the two:

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp\left(-F(x) - G(t+1-x)\right)$$

Although Likelihood ratio tests suggest that the synthesis model is better suited than the other two models for the purpose of modeling German population mortality data from the past (see 3.2.1), the resulting projections of the traditional model are much stabler than those of the other two models. For full details, see [DAV].

Therefore, the traditional model is chosen for projecting mortality.

3.2 2nd order mortality improvement trend

Mortality improvement trends for the population are examined in section 3.2.1 for different periods. The reasons for assuming a decreasing trend over time are stated in section 3.2.2, which also includes a description of the linear trend reduction method. The 2nd order DAV 2004 R mortality improvement trend is defined in section 3.2.3.

3.2.1 Mortality improvement trends for the population

In order to study changes in population mortality improvement over the last decades the following crude mortality improvement trends are considered:

- Short-term trend of 10 abbreviated population mortality tables for West Germany from St 1989/91 to St 1998/2000,
- Medium-term trend of 28 abbreviated population mortality tables for West Germany from St 1971/73 to St 1998/2000 (for 1986/88 the general population mortality table 1986/88 is used),
- Long-term trend of 11 general population mortality tables from ADSt 1871/1880 to ADSt 1986/88 and the abbreviated population mortality table for West Germany St 1998/2000.

In December 1969 and January 1970 German population mortality increased due to a influenza epidemic. This could result in an incorrect assessment of the mortality improvement trend. Therefore, the medium-term trend is based on mortality tables from St 1971/73 onwards.

Age-dependent mortality improvements are calculated according to the traditional model. The following figure shows the annual mortality improvements for males (for females the results are similar):

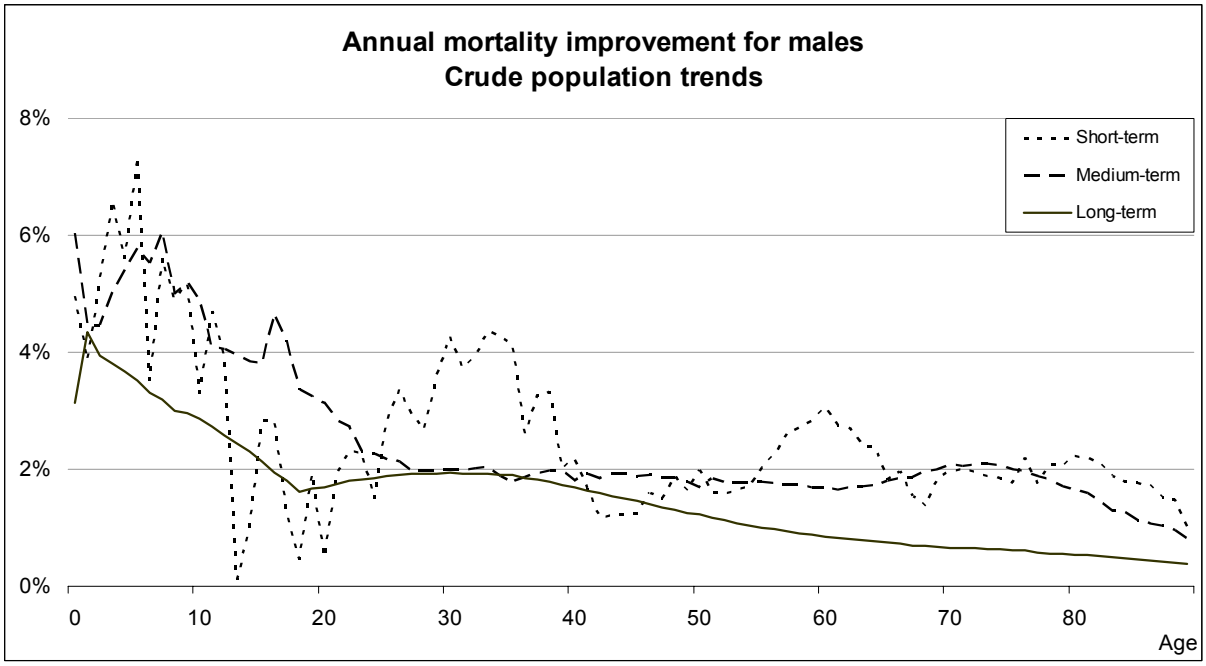


Figure 4: Annual mortality improvements of males

The annual mortality improvements are smoother for longer periods than for shorter periods.

In order to compare these trends the arithmetic means of annual mortality improvement for ages from 60 to 89 are considered:

	Males	Females
Short-term trend	1.97%	2.00%
Medium-term trend	1.67%	2.05%
Long-term trend	0.62%	1.04%

Table 4: Arithmetic means of annual mortality improvement for ages from 60 to 89.

The long-term trend is significantly lower than the medium- and short-term trends. For males, the short-term trend is higher than the medium-term trend.

3.2.2 Linear trend reduction

In the previous section it was noted that the long-term trend is significantly lower than the medium- and short-term trends. In Japan, where the mortality level is lower than in Germany, a reduction in the mortality improvement trend has been observed since 1970. Given these findings it seems inappropriate to use the high short-term trend for projecting mortality to the long-term future. Instead, a reduction in the mortality trend over time is used for projecting mortality. This is modeled by the linear trend reduction method, which was also used for the Austrian annuity table AVÖ 1996R (see [JLPS]):

The trend function depends on age and calendar year and is denoted by $F(x,t)$. The connection between trend function and mortality is given by

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp\left(-F(x,t)\right).$$

The so-called initial trend $F_1(x)$ is used for the first years of the mortality projection. This initial trend is reduced linearly to the target trend $F_2(x)$ in a transition period. The target trend $F_2(x)$ is used after the transition period. The time $t = 1999$ corresponds to the start of the mortality projection.

The following parameters need to be determined:

- Initial trend $F_1(x)$,
- Target trend $F_2(x)$,
- Period T_1 up to the commencement of the transition period and
- Period T_2 up to the end of the transition period.

3.2.3 Mortality improvement trend for insured persons

The initial trend is based on the level of the short-term trend. Due to the fact that it is smoother, the medium-trend is used for defining the short-term trend: The crude medium-term trend function $F(x)$ is graduated using the Whittaker-Henderson method. The annual mortality improvement of the graduated medium-term trend for males is increased by 0.3%, which is the medium difference between short- and medium-term trends for ages from 60 to 89 years. For females, the graduated medium-term trend is used because this trend has approximately the same level as the short-term trend.

A loading for insured persons is needed as adjustment for differences between the mortality improvement of the population and the mortality improvement of insured persons.

Various international studies (see for example [V]) and the results based on the Munich and Gen Re data and the German social insurance data have shown that the mortality improvement of upper socio-economic groups is greater than mortality improvement of lower socio-economic groups.

Given that private annuities are mainly (especially if weighted by annuity amount) purchased by people belonging to upper socio-economic groups, the second finding confirms that the mortality improvement of annuitants is greater than the mortality improvement of the population.

This difference is taken into consideration in the annuity tables of some countries (for example, the UK and Switzerland) by determining the mortality improvement trend based on insured persons' data rather than population data. In some cases, the difference between annuitant and population mortality improvement is fairly large: In Switzerland, for example, the annual mortality improvement of males aged 70 years is 1.33% for the population and 2.41% for annuitants (see [SVV]).

In Germany, there is not enough data on annuitant mortality to determine a mortality improvement trend based on the data of insured persons. Therefore, a loading for insured persons is derived from German social insurance data.

The mortality improvement of white-collar workers is 0.14% to 0.23% higher than for the total collective. Therefore, the loading for insured persons is defined as an increase in annual mortality improvement $1-\exp(-F(x,t))$ of 0.2%.

The mortality improvement trend can only be determined directly from population data up to age 89. For ages 90 to 120 the trend needs extrapolating. For the purpose of obtaining an idea of a reasonable trend level for high ages, the trends for some other data were examined:

Graduating the mortality improvement trend based on German social insurance data for ages 66 to 98 results in an annual mortality improvement of approximately 1% at age 95 for both males and females.

Based on data from Japan (see [RSJ]) for ages 100 to 104, annual mortality improvements of 0.82% (males) and 1.25% (females) respectively were calculated.

Annual mortality improvement is extrapolated for ages 90 to 120 with a polynomial of degree 2 in age band $90 \le x \le x_0$ and with 1% in age band $x_0 \le x \le 120$.

As a result the annual mortality improvement is 1% for males from age 97 upwards and 1% for females from age 99 upwards, which seems plausible.

For ages 0 to 22 the annual mortality improvement of the initial trend exceeds 3%.

However, the short-term trend for these ages is lower: The short-term average annual mortality improvement for ages 0 to 22 is 1.74% for males and 1.85% for females if the average is calculated by weighting the ages according to the age distribution of the model portfolio of German annuity data described in section 2.3.1.

Therefore, the annual mortality improvement of the initial trend is limited to 3% These limits affect ages from 0 to 22 for both females and males. It should be noted that the trend limit for low ages has virtually no effect on the calculation of premiums and reserves.

The loading for insured persons of 0.2% annual mortality improvements is added to the initial trend. Finally, the trend is extrapolated for high ages to a level of 1% annual mortality improvement and limited for low ages to a level of 3% annual mortality improvement. This defines the initial trend $F_1(x)$ for insured persons.

The target trend $F_2(x)$ for insured persons is defined as follows: the annual mortality improvement of the target trend $F_2(x)$ is 75% of the annual mortality improvement of the graduated (and for high ages extrapolated and for low ages limited) medium-term trend, which was increased by the loading for insured persons, but not by the medium difference between short- and medium-term trends for males.

The time parameters T_1 and T_2 need to be chosen appropriately depending on the purpose for which the 2nd order mortality is used. The Parameter combination $T_1 = 5$, $T_2 = 10$ seems to be appropriate.

3.3 1st order mortality improvement trend

The fundamental risk in determining a mortality improvement trend is the principal uncertainty of estimating future mortality improvement based on data from the past. The main risks which estimating future mortality improvement entail are the model and trend parameter risks.

3.3.1 Model risk margin

A linear trend reduction is assumed for the 2nd order mortality improvement– see section 3.2.2. The model risk particularly consists of the risk that mortality improvement will not decline in the future. For the purpose of making allowances for the model risk a safety margin is incorporated into the trend, which is defined by omitting a trend reduction assumption. This means that the initial trend defined in 3.2.3 is used for the whole future. It corresponds to a target trend increase $F_2(x)$ of at least 34% (namely, 34% for females and between 34% and 72% for males).

3.3.2 Trend parameter risk margin

An allowance was made for the risk of an increase in the mortality improvement trend by means of a safety margin of an additional 0.25% annual mortality improvement for all ages.

This margin was determined using stress scenario considerations similar to those used in the Swiss annuitant table ER 2000 (see [K] and [SVV]). The consequences of certain stress scenarios for the reserves in 2005 were examined. The stress scenarios are defined by a trend function increase (including the model risk margin) of 50% for a period of 10 years[1]. For the model portfolio these stress scenarios result in an average increase in reserves of approximately 2%. The risk of change margin of an additional 0.25% mortality improvement also results in an approximate 2% increase in reserves.

3.3.3 Other risks

There is no explicit additional safety margin for other risks (for example, from the estimation of the trend parameters). It is assumed that the safety margins for the model and trend parameter risks implicitly make allowance for other risks.

4 INTERNATIONAL COMPARISONS

4.1 Examples of international tables with trends

4.1.1 Swiss table ER 2000

Both the base Swiss annuity table ER 2000 (see [SVV]) and the trend function $F(x)$ for ages $x \geq 50$ were derived from mortality data on Swiss individual annuities from the period 1961

to 1995. Its trend is based on the traditional model $\frac{q(x,t+1)}{q(x,t)} = e^{-F(x)}$. Both the 2nd and 1st order trends are kept constant in the table ER 2000.

4.1.2 UK table IA 92 mc

The "92" series of UK tables was published in 1999 (see [CMI1]). Again, both the base table and the age-dependent trend function $F(x,t)$ for table IA 92 by amount for immediate annuities were derived from mortality data on insured persons from the period 1955 to 1994. The age-dependent trend decreases over time:

$$\frac{q_{x,t}}{q_{x,0}} = \alpha(x) + [1-\alpha(x)].[1-f(x)]^{\frac{t}{20}}$$

with

$$\alpha(x) = \begin{cases} c & x < 60 \\ 1+(1-c).\frac{(x-110)}{50} & 60 \le x \le 110 \\ 1 & x > 110 \end{cases}$$

and

$$f(x) = \begin{cases} h & x < 60 \\ \frac{(110-x).h+(x-60).k}{50} & 60 \le x \le 110 \\ k & x > 110 \end{cases}$$

where $c = 0.13$, $h = 0.55$ and $k = 0.29$.

During the course of the last few decades the mortality improvement trends of UK mortality tables have been increased several times. This is evident from the following chart from [I], p. 90 on projections of life expectancy for males aged 60 according to UK tables 1955, 1968, 1980 and 1992:

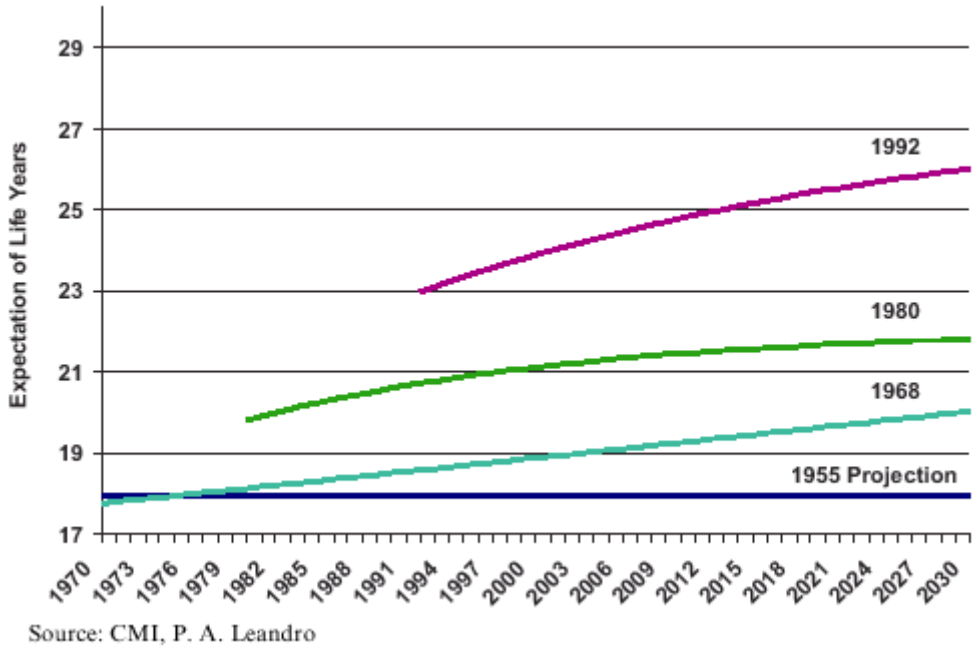


Figure 5: UK projections of life expectancy for males aged 60

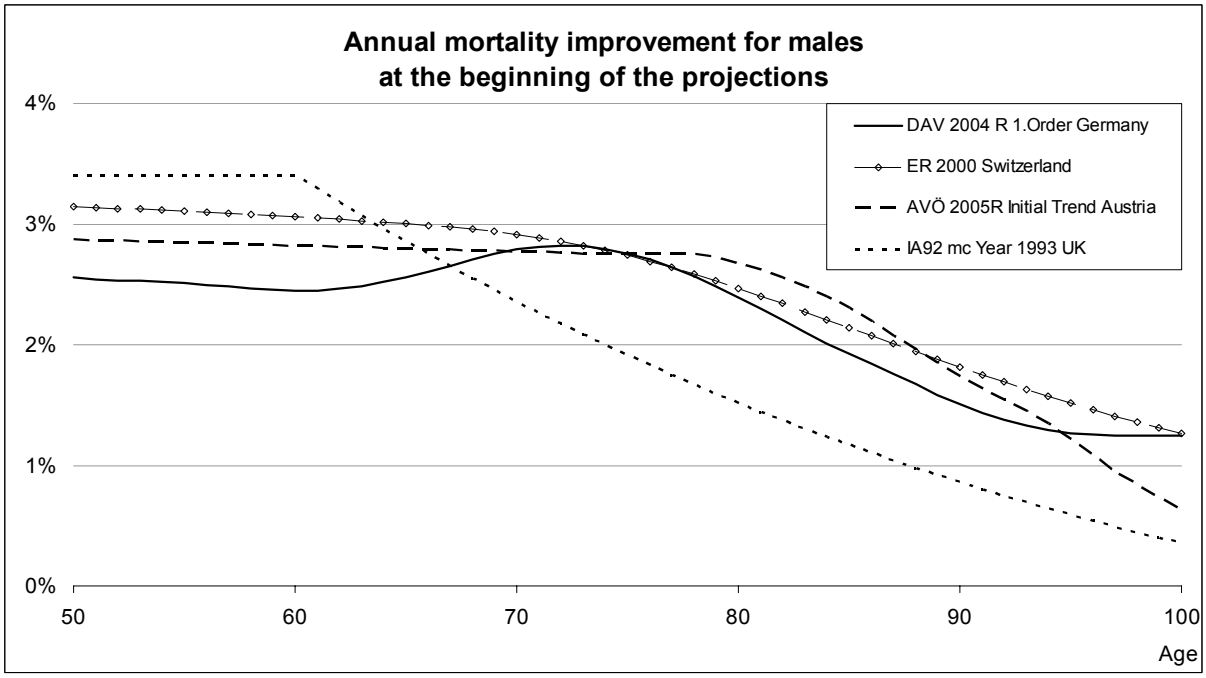
In 2002 cohort-dependent mortality improvements were superimposed onto the IA 92 age-dependent mortality improvements (see [CMI2]). These cohort-dependent mortality improvements are used for years up to 2010, 2020 or 2040 (short, medium and long variants). The variant medium cohort (IA 92 mc) is used for comparisons in the following sections.

4.1.3 Austrian table AVÖ 2005 R

The Austrian annuity table AVÖ 2005 R is based on the traditional linear trend reduction model as described in section 3.2.2. The base table and the trend function of AVÖ 2005 R were derived from data on population mortality in Austria. The initial trend is based on data from the period 1972 to 2002. In the transitional period from 2002 to 2102 the trend is reduced on a linear basis to the target trend which is 50% of the initial trend.

4.2 Comparison of trends

The following two figures show the DAV 2004 R trends for males compared to the trends of the Swiss, UK and Austrian tables. The results for females are essentially the same. The upper figure shows annual mortality improvements $1-\exp(-F(x,t))$ at the beginning of the projections. In particular, $1-\exp(-F(x,t))$ refers to the year $t=1993$ for IA 92 mc and to the year $t=2001$ for AVÖ 2005 R. In the lower figure $1-\exp(-F(x,t))$ refers to year $t=2030$ for IA 92 mc and AVÖ 2005 R.



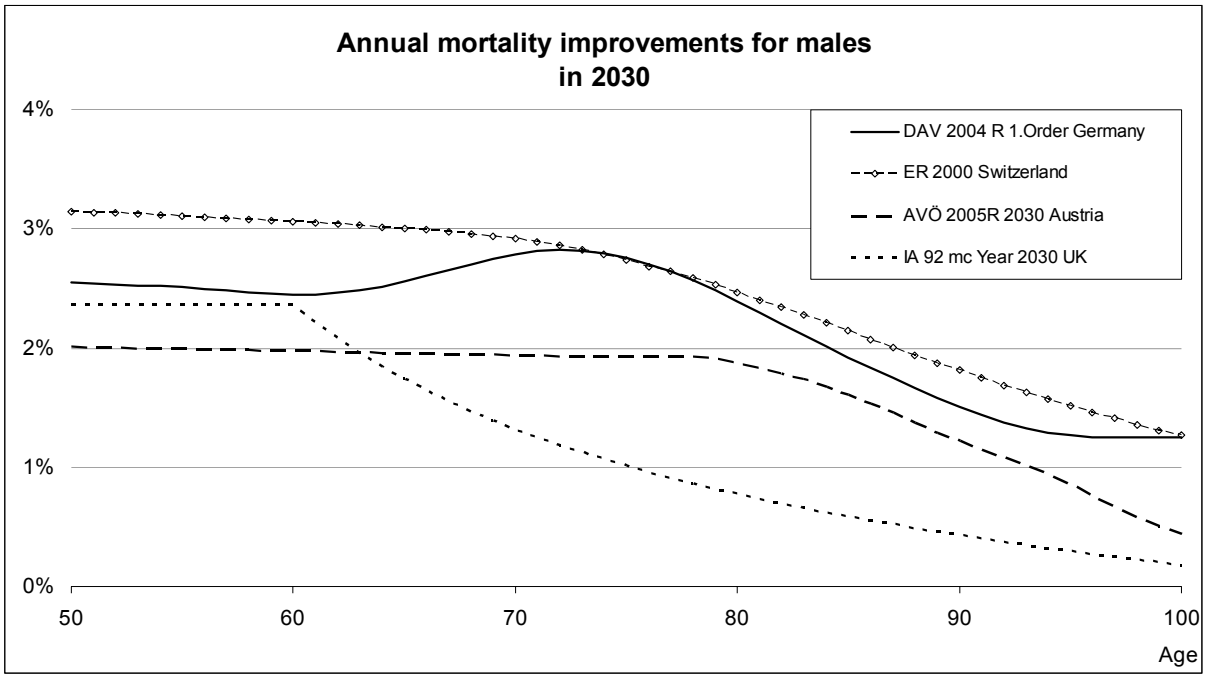


Figure 6: Annual mortality improvement, males

4.3 Comparison of mortality rates

The following figure compares the mortality rates for a person aged 65 in 2005 projected by DAV 2004 R with the corresponding mortality rates projected by the Swiss table ER 2000, the Austrian table AVÖ 2005 R and the UK table IA 92 mc.

As can be seen, there are huge differences in the projected rates. The Swiss mortality rates are by far the most conservative. The kinks at ages 61 and 66 are due to the selection factors in DAV 2004 R.

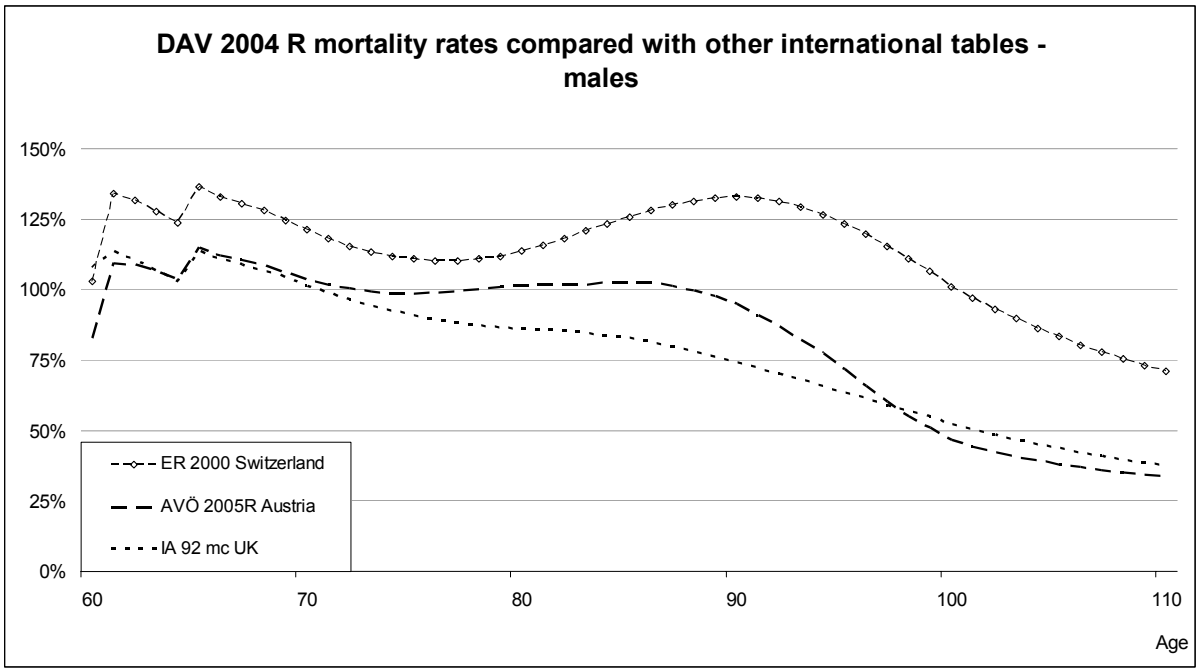


Figure 7: Comparison of DAV 2004 R mortality rates with international mortality rates for a male person aged 65 in 2005

4.4 Comparison of net premiums

The following table contains comparisons of net single premiums[2] and net annual premiums[3] for the the same mortality tables as in the previous sections.

The purpose is not a price comparison, but a compressed comparison of the different mortality tables.

Net premiums for males are lower for DAV 2004 R than for the Swiss mortality table. For females (not shown here), net premiums for DAV 2004 R and the Swiss mortality table are similar. Net premiums for DAV 2004 R are higher than for the UK and Austrian mortality tables.

	Germany DAV 2004 R	Switzerland ER 2000	UK IA 92 mc	Austria AVÖ 2005R
Immediate annuity				
Age at issue 60 / birth year 1945				
Net single premium	19,608	20,397	19,015	19,416
Deferred annuity - payout phase starting at age 60				
Deferment period 20 years / birth year 1965 / age at issue 40				
Net single premium	11,982	12,540	11,177	11,729
Net annual premium	776	810	720	761
Deferment period 30 years / birth year 1975 / Age at issue 30				
Net single premium	9,488	9,835	8,634	9,255
Net annual premium	461	479	418	450
Deferred annuity - payout phase starting at age 65				
Deferment period 20 years / birth year 1960 / age at issue 45				
Net single premium	10,522	11,074	9,655	10,213
Net annual premium	685	717	625	667
Deferment period 30 years / birth year 1970 / age at issue 35				
Net single premium	8,394	8,787	7,492	8,123
Net annual premium	409	428	364	397

Table 5: Net single premiums and net annual premiums for international mortality tables. Product parameters: Commencement date 2005, interest rate 2.75%. Net premiums in € for annual annuitiy payment of 1,000 € in advance. All values for males.

[1] This means $F(x,t) = F_1(x)$ is replaced by $F(x,t) = \eta \cdot F_1(x)$ for $\tau - 5 \le t < \tau + 5$ with $\eta = 150\%$. The centre of the 10-year period is varied from 2005 to 2054.

[2] Net single premiums for immediate annuities are given by $\ddot{a}_x$. Net single premiums for deferred annuities are given by ${}_{n|}\ddot{a}_x$.

[3] Net annual premiums for deferred annuities are given by ${}_{n|}\ddot{a}_x \big/ \ddot{a}_{x,n}$.

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Parts of this paper were previously published in a paper, "Coping with Longevity—The New German Annuity Valuation Table" by Ulrich Pasdika and Jürgen Wolff. Copyright 2005 by the Society of Actuaries, Schaumburg, Illinois. Reprinted with permission.

REFERENCES

[CMI1] Continuous Mortality Investigation Bureau: Standard tables of mortality based on the 1991-94 experience. *Continuous Mortality Investigation Reports,* 17, 1999. http://www.actuaries.org.uk/Display_Page.cgi?url=/library/cmir17/welcome.xml

[CMI2] Continuous Mortality Investigation Mortality sub-committee. An interim basis for adjusting the "92" Series mortality projections for cohort effects. *Continuous Mortality Investigation Working Paper*, 1, 2002. http://www.actuaries.org.uk/files/pdf/cmi/CMIWP1.pdf

[DAV] Herleitung der DAV-Sterbetafel 2004 R für Rentenversicherungen. Submitted for publication in *Blätter DGVFM*.

[G] Gesamtverband der Deutschen Versicherungswirtschaft e.V. Die Märkte für Altersvorsorge in Deutschland – Eine Analyse bis 2020. *Schriftenreihe des Ausschusses Volkswirtschaft 23*. Verlag Versicherungswirtschaft e.V., Karlsruhe.

[I] Institute of Actuaries discussion meeting, Living longer is bad for the nation's wealth', Abstract of the discussion. *British Actuarial Journal*, 7: 75-102, 2001.

[K] M. Koller. Lebensversicherungsmathematik, Skript zur Vorlesung LV1&2, Stand: 9. Oktober 2000. http://www.math.ethz.ch/finance/misc/Skript_LV1_2.pdf

[KBLOZ] P. Kakies,, H.-G. Behrens, H. Loebus, B. Oehlers-Vogel, B. Zschoyan. Methodik von Sterblichkeitsuntersuchungen. *Schriftenreihe Angewandte Versicherungsmathematik, Heft 15*. Verlag Versicherungswirtschaft e.V., Karlsruhe, 1985.

[L1] H. Loebus. Bestimmung einer angemessenen Sterbetafel für Lebensversicherungen mit Todesfallcharakter. *Blätter DGVM,* XXI: 497-524, 1994.

[L2] K.-H. Lühr. Neue Sterbetafeln für die Rentenversicherung. *Blätter DGVM,* XVII, 485–513 1986.

[P] M. Pannenberg. Statistische Schwankungszuschläge für biometrische Rechnungsgrundlagen in der Lebensversicherung. *Blätter DGVM*, XXIII: 35-64 1997.

[PS] M. Pannenberg, E. Schütz. Sicherheitszuschläge für biometrische Rechnungsgrundlagen in der Lebensversicherung. *Transactions of the 26th International Congress of Actuaries*, Volume 6: 481–511, Birmingham, 1998.

[R] F. Rueff. Ableitung von Sterbetafeln für die Rentenversicherung und sonstige Versicherungen mit Erlebensfallcharakter. Konrad Triltsch Verlag, Würzburg, 1955.

[RSJ] J.-M. Robine, Y. Saito, C. Jagger. Living and dying beyond age 100 in Japan. *Symposium Living to 100 and Beyond: Survival at Advanced Ages*. Lake Buena Vista, Florida, January 17–18, 2002.

[SS] B. Schmithals, E. Schütz. Herleitung der DAV-Sterbetafel 1994 R für Rentenversicherungen. *Blätter DGVM*, XXII: 29-69, 1995.

[SVV] Arbeitsgruppe des Schweizerischen Versicherungsverbands unter Leitung von M. Koller. Methodik zur Konstruktion von Generationentafeln.1998.

[TKV] A. R. Thatcher, V. Kannisto, J. W. Vaupel. *The force of mortality at Ages 80 to 120*. Odense University Press, Odense, 1998. http://www.demogr.mpg.de/Papers/Books/Monograph5/start.htm

[V] T. Valkonen. The widening differentials in adult mortality by socio-economic status and their causes. *Zeitschrift für Bevölkerungswissenschaft*, 23: 263-292, 1998.

[W] R. Willets. Mortality in the next millennium. 1999. http://www.sias.org.uk/papers/mortality.pdf