Dynamic asset allocation techniques

Stuart Jarvis

BLACKROCK
Agenda

• Motivations
• Formulation of problems
• Techniques
• Results

Joint work with Adrian Lawrence & Sheng Miao
Motivations
Funding challenges

- UK plan funding worsened since mid-2007
- Widely accepted that asset allocation needs to be more dynamic & liability-led

Estimated s179 aggregate assets minus liabilities

Source: Pension Protection Fund
Eyes on the prize

- If can afford to take less risk, then do so!
Pension investment

- How should plans respond to intra-valuation performance?
  - Eg funding ratio breaches 100%
- Do option collar strategies make sense?
  - If so, what’s the best design?
- What’s the right context for exploring this?
Insurance investment

• Insurers encouraged to manage downside risk
  – VaR, CVaR

• What strategies optimise the trade-off between return and downside risk?
Themes

• Strategies are inherently multi-period
  – Allocation varies by time and state
  – Dynamic rather than static

• Dimensionality of problem appears rather hard
  – Analytic tractability rare
  – Numerical approaches often required
Problems considered

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Caveats

• We assumed lognormal distributions
  – this is clearly wrong, but then so is any model
  – advantage is tractability and transparency
  – so seek qualitative results: how should strategies evolve, not precise prescriptions

• Rules of thumb can then be tested
  – In more complex situations e.g. fat tails, stochastic volatility, backtests etc
Assets v liabilities
Lifetime portfolio allocation

• Merton et al
  – Expected-utility-maximising investors
  – Optimal allocation is unchanging if the market opportunity set is unchanging
  – Consumption (drawdown) determined by relative preference relative to bequest motive

• Nielsen (ICA 2006) a good survey
Two analytical directions

- We looked at two ways to evolve this classical theory:
  - Change the utility function to reflect investor’s situation (target; minimum)
  - Express target in terms of a stochastic liability rather than in money terms
Results

• Stochastic liability
  – Mathematics remains tractable
  – The structure of the portfolio is clarified: a ‘liability hedge’ portfolio arises naturally

• Non-CRRA utility curves
  – Family of adjusted curves discussed
  – Dynamic policies can be replicated with small number of appropriate options
Target level

- Straightforward idea: no additional benefit from surplus once full funding achieved
Naïve strategy

• Risk budget proportional to deficit \( \div \) time
Bellman approach

- Dynamic strategy consists of decisions to be made at each future time & state
- Suppose you know strategy at times > t
  - Then know expected utility in all states at t+1
  - Solve one-period problem at t: this is optimal
  - Nothing to do at time horizon, so done
- More complicated in continuous-time but same ‘Bellman principle’ can be applied
Cox-Huang approach

• Seek a payoff (and/or consumption policy) that maximises expected utility
  – Subject only to budget constraint
  – This is easier – calculus of variations

• Then derive strategy that gives this payoff
  – Price of a payoff familiar from option pricing

• This works when markets are ‘complete’ and no constraints on asset weights
Optimal payoff

- Compared to an investor without a liability target, just sell the unwanted upside
Unsurprisingly, take more risk if deficit larger; but risk budget is controlled.
Constrained naïve strategy

- Naïve strategy with capped risk budget (i.e. as would happen in practice) not too bad
Cumulative impact

- Distribution of outcomes no longer so wide; significant downside benefit
Life is more complicated

- Sponsor covenant
  - Investment policy
  - Contributions
Alternative: start with payoff

Strategy outcomes

Funding level

0%
100%
200%
300%
400%
500%
600%
700%
800%
900%
1000%
1100%
1200%
1300%
1400%

Cumulative growth asset return

Funding target

Target

Static weights

Funding target + floor
Optimal strategy

- Now, reduce risk (and perhaps increase contributions) if funding level falls
Asset allocation process

- **Level 1**: formulate investment objectives
  - Target funding level, tolerance for funding falls
  - Degree of reliance on sponsor covenant
  - Mixture of contributions and investment returns
  - Set long-term asset allocation to support this

- **Levels 2 and 3**: ongoing review
  - To what extent do assumptions in level 1 hold true?
  - Funding-level driven changes (journey management)
  - Medium term asset allocation

- **Level 4**: outperform strategic benchmark
  - Traditional role of alpha in the portfolio

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**Diagram:**

- **Level 1**: Strategic Asset Allocation/Asset Liability Modelling
- **Level 2**: Changes in asset allocation / contributions due to funding level moves
- **Level 3**: Changes in asset allocation due to market opportunities (if any)
- **Level 4**: Security Selection Against Indices (if any)
Utility v CVaR
Analytical technique

- Maximise utility subject to CVaR constraint
- Objective function framed in terms of statistics of payoff
  - Cox-Huang technique can still be applied
  - Ralf Gandy’s PhD thesis contains details
- Payoff profile & investment strategy different to previous cases
Optimal payoff profile

- CVaR-constrained optimal payoff
- Unconstrained optimal payoff

Stock total return vs. Optimal payoff

- Y-axis: Optimal payoff
- X-axis: Stock total return

Y-axis labels: 0.2, 0.4, 0.6, 0.8, 1
X-axis labels: 50%, 60%, 70%, 80%, 90%, 100%, 110%
Investment strategy

Variation of portfolio risk over time and portfolio value

Risk level (compared to unconstrained)
Comments

• Strategy makes sense given objective
  – Take more risk if things go well or (very) badly
• But doesn’t feel right along each path
  – If you’re in a ‘bad’ scenario, you can’t trade this off against good things that might have happened
• More time-coherent risk measure needed
• Extend to fatter-tailed distributions
Mean v CVaR
Numerical technique

• Maximise expected return subject to CVaR constraint
  – If we allow unlimited leverage, then problem is unbounded
  – Apply long-only, no-leverage constraints
• Numerical optimisation now needed
  – Construct a tree
  – Optimise the portfolio weights at each node
Possible numerical methods...

- Numerical
  - Finite difference (trees, PDEs)
  - Monte Carlo
  - Numerical integration

- “Use Monte Carlo for high dimensional problems, use finite difference where there are intermediate decision variables”

Wilmott
Monte Carlo -> trees

• So if you take standard MC, bifurcate at decision points then equate moments you get..

• … a tree
Recombining the tree

• Trick is to ensure that you can get to node C via node A or node B.
• This reduces considerably the total number of nodes
• Easy in a tree for one asset as in derivatives textbook.
• Multi-dimensional recombining trees are harder. The assets are correlated so probabilities of moving between nodes on one asset are linked to the other assets.
Weight matrix

• Defines the asset allocation at each node
  – Point on the efficient frontier
• Goal is to find best weight matrix
Tree summary

Tree geometry

Weight matrix

Optimization (to simultaneously optimize the whole weight matrix)
Implementing the PDE

- Weight matrix
- SDE
- PDE (Fokker Planck)
- Recurrence relationship (derivatives become finite differences)
- PDE solution
  - Initial conditions
Sample results

Static and Dynamic Efficient Frontiers for Tail VaR optimized portfolio
Results
Remarks

- Monte Carlo ill suited for dynamic optimization
- Change to an orthogonal basis may simplify calculations.
- PDEs can be made better behaved than trees.
- Worth looking to see whether there is a possibility of reducing the dimensionality of the problem.
Conclusions
Final remarks

• Dynamic strategies increase investors’ likelihood of meeting their goals
  – Indeed discussion helps clarify these goals

• Optimal strategies are often intuitive and practical versions need not be complex

• Further work
  – Fatter tails / EV distributions
  – Time-coherent downside risk objectives
Contact details

stuart.jarvis@blackrock.com
adrian.lawrence@blackrock.com