"THE IMMUNISATION OF A WORKERS' COMPENSATION FUND"

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Portugal

Summary

The aim of this study is to present a comparison of the results of the immunisation of a Workers' Compensation Pension Fund in Portugal obtained by using a deterministic model with those obtained by using a stochastic model developed by Longstaff and Schwartz (1993,1992).

We observe that the use of a mandatory interest rate of 6% and the French Mortality Table TV 73-77 to calculate the liabilities, as obliged by law, undervalues the real amount of the pension flows. This is because the upward and downward movements of the interest rate are parallel and affect simultaneously all the terms of the structure of the interest rate.

Therefore, we recalculated these obligations using the Longstaff and Schwartz model, in which the variations of the interest rate are stochastic, and we conclude that the amount of the liabilities is higher than that resulting from the deterministic model, due to the volatility of the short-term interest rate.

Key Words: Workers' Compensation Pensions; Immunisation; Stochastic duration; Affine term-structure models.

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1. Introduction

In Portugal, Workers' Compensation insurance is a legal obligation on the part of employers (included in the Social Security benefits). Employers are obliged to transfer liability as defined by law to bodies authorised and specialised in the management of this form of insurance (i.e. general insurance companies – third-party liability insurance), should the former lack sufficient funds to assume, on their own account, the cost of temporary or permanent physical disability arising from accidents in the work-place. Likewise, as of 1 November 1999, the self-employed are obliged to abide by the same legal requirements.

The insurance guarantees that those insured are covered for accidents that occur as a consequence of their carrying out their occupational activity.

The purpose of the coverage is to assure the conditions of survival and recovery of the victims of an accident, as well as the welfare of their relatives.

As a consequence of the accident, the worker can die, be temporarily disabled or acquire a permanent disability. In the first case, his/her relatives will be entitled to receive either a lump sum or a life annuity. In the second case, the injured person will receive an allowance for temporary incapacity to work and also medical, surgical, pharmaceutical or other care, if needed. In the latter case, the injured will also receive a pension.

Workers' Compensation contracts are drawn up company by company, regardless of the number of employees to be covered, and premiums are negotiated case by case.

A Workers' Compensation fund aims to provide compensation for disabilities suffered, by means of benefit payments as defined by law, which, in the case of permanent incapacity, can only be attributed in the form of a pension for life[*1], decided by the Labour Tribunal and which, on reaching the age of retirement, continues to be paid in addition to the State Social Security pension.

[*1] In fact, it is an annuity calculated as in life assurance, taking into account the age of the beneficiary and a mortality table.

Should the accident result either in a permanent reduction of the injured party's work-or earning-capacities or, indeed, the death of the victim, then Decree-Law nr. 100/97, 13.9 stipulates an entitlement to specified benefits (i.e. pensions) to the victim or his/her surviving marital partner. This benefit is annulled in the event of remarriage.

These pensions can, under certain, stipulated circumstances, also be attributed to surviving, dependent parents and/or children of the deceased.

The effects of the variations of the interest rate on the duration of the liabilities of a general insurance company, were analysed by Babbel and Klock [1994][1], identifying and quantifying the cash-flows of the liabilities based on the application of the development factors of the reserves. They propose the calculation of modified duration of the liabilities, based on the definition of transformed cash-flows. This methodology was also applied to the Workers' Compensation business, taking into consideration the long-term nature of the insurance company's commitments, namely, in the pensions domain, in which the duration of the liability is, obviously, unspecifiable.

Therefore, it is crucial to analyse the conditions for the immunisation of a mutual fund for Workers' Compensation in Portugal, based on the deterministic principles proposed by Redington [1952][2] and revised by Boyle [1976][3] for the hypothesis of stochastic movements of the interest rates in accordance with the modelisation by Cox, Ingersoll and Ross [1979][4].

These results will subsequently be compared with the model by Longstaff and Schwarz [1992], which considers two factors in the structure of the interest rate, namely, the inherent dynamics of the short-term interest rate and its volatility.

The present work is organised as follows:

In section 2, we analyse the effects of the changes either in the mortality rate or in the interest rate, which, in turn, determine changes to the cash-flows and to the measures of duration and convexity in a deterministic model. In section 3, the deterministic model of immunisation of a Workers' Compensation fund is proposed, with the inclusion of an empirical application. This

[1] Babbel, D. and D. Klock (1994), Measuring Interest Rate Risk of Property/Casualty Insurers Liability in "*Insurance, Risk Management, and Public Policy*" Gustavson and Harrington editors, Kluwer Academic Publishers.

[2] Redington, F. M. (1952), "Review of the Principles of Life-Office Valuations", *Journal of the Institute of Actuaries*, 18, pp. 286-315.

[3] Boyle, P. (1976), Immunisation under Stochastic Models of the Term Structure, *Journal of the Institute of Actuaries*, Vol. 105, pp. 177-187.

[4] Cox, J., Ingersoll, Jr. and Ross, S. (1979), Duration and the Measurement of Basic Risk", *Journal of Business*, 52, Nº1, pp. 51-61.

model is compared with the stochastic models of Cox, Ingersoll and Ross and of Longstaff and Schwarz in section 4.

The data and the methodology are explained in section 5 and an empirical application of the stochastic model to a real fund is presented in section 6.

The main conclusions are given in section 7.

2. Fund Management

In this section, we briefly describe the historical results of the Workers' Compensation line of business.

The funds corresponding to the permanent disability pensions are managed by insurance companies authorised to operate general lines of business in Portugal.

In 1997, the mathematical provisions corresponding to the underwritten liabilities (pensions, presumed pensions and pension reinforcement) totalled 141.336 million contos[*1] (705 million euros).

Approximately 40 million contos (200 million euros) of provisions were added to this figure to cover the evolution of Workers' Compensation claims and unearned premiums, raising the total of funds invested to approximately 181.333 million contos (904 million euros) in a portfolio of financial assets composed primarily of Government and corporate bonds.

2.1. The Cash-Flows of a Workers' Compensation Fund

In general terms, we can say that the periodical cash-flow of a Workers' Compensation fund managed by an insurance company, calculated by the subtractive method, is based on the following:

+	*Premiums*
-	*Cash payments* (reimbursement of expenses)
-	*Payments of indemnities (temporary incapacity)* Replacing the salary.
-	*Pension payments (permanent incapacity, death)*
-	*Variation of the technical reserve for outstanding claims*
-	*Operating costs of the company*
+	*Financial income* Interest, dividends, gains from transactions on the portfolio of assets.
-	*Reinsurance*
=	*Final result*

The earned premiums are equal to the written premiums, less reimbursements and cancellations deducted from the unearned premiums reserve (premiums to cover risks in the following year).

[*1] *(n.b. 1 conto=1000 PTE).*

The technical reserve for outstanding claims is defined as the predicted total of future costs of all claims having been incurred up to the date of the balance sheet. Moreover, those claims which have been incurred, but not yet reported *(i.e.* IBNR) by the date of closure of the balance sheet must also be taken into account.

These reserves for this particular line of business can be divided in two main categories:

1. Reserve for indemnities not related to pension payments (e.g. medical expenses, salaries indemnities for partial disability, etc). This reserve was calculated traditionally in the Portuguese market as a percentage of at least 25% of the commercial premiums and handling charges in the financial year, less reimbursements and cancellations;
2. The mathematical reserves corresponding to the pensions due to permanent disabilities. These reserves are equal to the present value of the pensions to be paid out owing to Workers' Compensation claims, calculated in accordance with the regulations in force.

In calculating the outstanding claim reserves provision, the expenses incurred in handling the claims are also to be taken into account, irrespective of their origin.

The problem, therefore, is to transform the technical reserves, included in the balance sheet as a single amount, into periodical flows.

With regard to the reserves for indemnities other than pensions (i.e. operating costs, payments of allowances, medical and hospital expenses and others) and the pension payments, this transformation is simple.

In contrast, as far as the mathematical reserves for pensions are concerned, this transformation will obey a specific actuarial methodology.

2.2. The Market Operating Account for Workers' Compensation Insurance

The operation of this line of business presented the following evolution during the period analysed:

TABLE 1

Workers' Compensation Line

Market			U: millions PTE	
	1990	**1995**	**1996**	**1997**
Written Premiums	**50.718**	**75.327**	**78.432**	**79.310**
Earned Premiums		74.626	77.692	79.068
Loading	40.724	58.808	63.774	66.244
Underwriting Margin	9.994	15.818	13.918	12.824
Operating Costs	20.378	21.991	23.852	22.626
Net Margin	-10.384	-6.173	-9.934	-9.802
Reinsurance balance	**107**	**-136**	**-42**	**-88**
Operating Result	**-10.277**	**-6.310**	**-9.976**	**-9.890**
Financial Income	**11.952**	**12.801**	**14.947**	**21.649**
Final Result	1.675	6.491	4.971	11.759
Number of Claims	n.a.	94.490	113.569	231.092

Sources:
1990 - *ISP,"Actividade Seguradora em Portugal 1990"*
1995 – ISP, *" Estatísticas de Seguros 1995"*
1996 – ISP, *"Estatísticas de Seguros 1996"*
1997 - APS, *"Relatório de Mercado 1997"*

As we can see in Table 1, the net margin of the line is always negative, but the final result is always positive. This occurs because of the financial gains that cover completely the operating result.

Taking the premiums as a base 100, we can confirm that the underwriting margin never covers the operating costs in all the period analysed, owing fundamentally to the technical costs and within these, to the costs of the provision for future pensions. However, the net margin remains stable over the last two years because of the decrease in the rate of the operating ratio. We must stress that due to the consequences of the management of the portfolio of financial assets, the final result more than quadruples the result of 1990.

TABLE 2

Workers' Compensation Line

Market	1990	1995	1996	1997
Written Premiums	100,0%	100,0%	100,0%	100,0%
Earned Premiums		99,1%	99,1%	99,7%
Loading	80,3%	78,1%	81,3%	83,5%
Underwriting Margin	19,7%	21,0%	17,7%	16,2%
Operating Costs	40,2%	29,2%	30,4%	28,5%
Net Margin	-20,5%	-8,2%	-12,7%	-12,4%
Reinsurance Balance	0,2%	-0,2%	-0,1%	-0,1%
Operating Result	**-20,3%**	**-8,4%**	**-12,7%**	**-12,5%**
Financial Income	23,6%	17,0%	19,1%	27,3%
Final Result	3,3%	8,6%	6,3%	14,8%

This result is all the more striking, given that the line of business is a social, mandatory insurance.

We stress that the beneficiaries have no participation in the financial results, other than the mandatory interest rate (6% until the end of the Financial Year 1999).

3. The Model of Deterministic Immunisation of Workers' Compensation Pensions

One of the main purposes of this section is to assess the effect of the variations of mortality and/or the interest rate on a Workers' Compensation pensions portfolio.

This line of business is characterised by the existence of a time-lapse between the payment of the premium by the policyholder and the dates of payments for claims, in particular those claims which will give rise to the life annuities which will continue to be paid even when the pensioner becomes eligible for a State Social Security pension.

The existence of this time-lapse permits the insurance company to obtain returns from the management of the portfolio of covering assets from the mathematical reserves. Since the insurance contracts involve cash-flows at different points of time, the models traditionally used in finance are a logical point of departure for an analysis of the immunisation problem.

To cover the liabilities derived from the life annuities, the insurance company is obliged to include in its balance sheet an amount equivalent to a single premium corresponding to a life annuity, calculated at a technical interest rate of 6% and in accordance with the official mortality rate table[5].

In this way, Workers' Compensation pensions are the equivalent of immediate annuities sold by Life Insurance companies.

This equivalence makes it possible, therefore, to respect the two conditions of Redington's immunisation. The first condition is that the duration of the assets must be equal to that of the liabilities; and the second states that the dispersion of the cash-flows of the assets should be greater than the dispersion of the cash-flows of the liabilities.

However, in order to obtain the second condition, the majority of the works published on the subject consider that this condition is verified immediately if, as proposed by Bierwag, Kaufman and Toevs [1983][6], certain rules can be empirically proven. According to the latter authors, in order to have immunisation, both of the following are required:

-the durations of the sub-sets of assets and those of the sub-sets of the liabilities should be equal;

[5] French female population for the period 1960-64

[6] Bierwag,G., Kaufman,G. and Toevs,A. (1983), Immunisation Strategies for Funding Multiple Liabilities, Journal of Financial and Quantitative Analysis, Vol.18, Nº1.

- the durations of the sub-sets of assets are respectively shorter than the shortest term of the sub-set of the liabilities and longer than the longest term of the sub-set of the expected liabilities.

Furthermore, taking into account the influence of the reinsurance, the correction of the cash-flows can have a decisive bearing on the figures of duration and convexity, since the variations of the cash-flow derived from reinsurance could be negatively correlated with the variation generated by fluctuations in the interest rate, thereby neutralising the correction.

3.1. The Assets/Liabilities Immunisation

The evaluation of the liabilities arising from the payment of life annuities depends mainly on two factors:

- the interest rate, r, used in the calculation of the present value of the cash-flows;
- the probability of survival, ${}_t p_x$, implicit in the calculations made with a given mortality table.

For a particular individual, if his/her life expectancy were *n*, he/she would receive the series of certain annuities:

$$\overline{\boldsymbol{a}_1}\,|_r, \overline{\boldsymbol{a}_2}\,|_r, \overline{\boldsymbol{a}_3}\,|_r, \ldots \overline{\boldsymbol{a}_n}\,|_r$$

with probability:

$$\boldsymbol{P}(\hat{\boldsymbol{a}}_x = \overline{\boldsymbol{an}}\,|_r) = (\frac{\boldsymbol{d}_{x+n}}{\boldsymbol{l}_x}), (\boldsymbol{n} \geq 0)$$

Since it is a question of a random value, Pollard et al. [1969][7] define the variable moments as the moment of order *r*, in relation to the origin and given by the expression:

$$E((\tilde{a}_x)^r) = \sum_{n=0}^{\infty} (\overline{a_n}\,|)^r (\frac{d_{x+n}}{l_x})$$

and the moment of order one is the value of a_x.

At the outset of the process, the company's expected net present value, according to Pollard et al, is equal to:

$$\mathrm{E}(a_{\overline{n}|} - \sum_{k=0}^{w} a_{\overline{n}|} * \frac{d_{x+n}}{l_x})$$

[7] Pollard, A., Pollard, J (1969), A Stochastic Approach to Actuarial Functions, *Journal of The Institute of Actuaries*, Vol. 95, pp. 79-113 (1969).

$$= a_{\overline{n|}} - a_x$$

with n as the life expectancy (e_x) of the pensioner of initial age x.

It is demonstrated that the value of the uncertain annuity a_x is always lower than the value of the certain annuity calculated with maturity, n, equal to the pensioner's life expectancy at age[8] x.

On the other hand, Jordan [1975][9] demonstrates that when there is constant change in the force of mortality, its effect on an annuity is equivalent to that of the same change on the continuous interest rate. This being so, we can treat the effect of the variation of the interest rate by acknowledging that an identical variation is observed in the mortality.

In an insurance portfolio, the above expression represents the net value of the Workers' Compensation fund, and we observe that, under the conditions described below, this value is always positive.

Let us suppose that the flows from the pensions portfolio of the insurance company are covered by the flows from a portfolio of bonds with the same maturity.

Traditionally, the conditions required to ensure the perfect match between assets and liabilities are associated with Redington's results [1952]. A brief description of these is now presented.

(1) The present value (PV) of both assets and liabilities needs to be equal, which means

$$VA = VL$$

where $VA = \int_0^\infty A_T e^{-rT} dT$, $VL = \int_0^\infty L_T e^{-rT} dT$ and r represents the flat discount rate under continuous compounding and a deterministic interest rate setting.

So, the profit is equal to the value of the assets from which the value of the liabilities has been deducted. The profit is designated by $\boldsymbol{S(r) = A(r) - L(r)}$.

[8] cf. Neil, A. (1989), *Life Contingencies*, The Institute of Actuaries.

[9] Jordan, C. (1975). *Life Contingencies*. Society of Actuaries.

By using a Taylor series expansion to determine the impact of small changes of r on the PV of assets and liabilities, Redington proves that as long as:

(2) $$\frac{\partial VA}{\partial r} = \frac{\partial VL}{\partial r} \quad \text{(Duration)}$$

(3) and $$\frac{\partial^2 VA}{\partial r^2} > \frac{\partial^2 VL}{\partial r^2} \quad \text{(Convexity),}$$

the PV of the assets will remain higher than that of the liabilities. It is important to note that by using standard duration and convexity, the previous conditions only ensure a perfect match between assets and liabilities under parallel shifts of the yield curve. In this situation, only the level risk is offset (this case is often mentioned as the portfolio dedication in the literature on the subject).

Referring to Bierwag [1987][10], it is proven that when n>=0, the derivative

$$\boldsymbol{\frac{dS}{dr}} = -\boldsymbol{v}\Big(\sum_{\boldsymbol{t}=1}^{\boldsymbol{n}} \boldsymbol{t} * \boldsymbol{CF} * (1+\boldsymbol{r})^{-\boldsymbol{t}} - \sum_{\boldsymbol{t}=1}^{\infty} \boldsymbol{t} * \boldsymbol{CF} * (1+\boldsymbol{r})^{-\boldsymbol{t}} \, {}_{\boldsymbol{t}}\boldsymbol{p}_{\boldsymbol{x}}\Big)$$

is nil when the probability is one, by which Redington's first condition of immunisation is respected, that is:

$$DA * a_{\overline{n|}} = DL * a_x$$

with

$$DA = Duration \quad of \quad assets$$

$$DL = Duration \quad of \quad liabilities$$

The second condition of immunisation is also verified, since the second derivative is positive when $DA = DL(\frac{L}{A})$ and $(-IL + IA + DL^2 - DA^2) \geq 0$:

$$\frac{d^2 S}{di^2} = v^2 * (DA * A - DL * L - IL + IA + DL^2 - DA^2) \geq 0$$

[10] Bierwag, G. (1987), *Duration Analysis*, Ballinger Publishing Company.

If the two conditions are satisfied, it is proven that there are arbitrage profits derived from the effective structure of the interest rate (cf. Shiu [1987])[11] since, using the Taylor series expansion, the following is obtained:

$$\boldsymbol{S}(\boldsymbol{r}+\Delta \boldsymbol{r}) \cong \boldsymbol{S}(\boldsymbol{r})+\boldsymbol{S}'(\boldsymbol{r})\Delta d+\frac{1}{2}\boldsymbol{S}''(\boldsymbol{r})(\Delta \boldsymbol{r})^2$$

As $\boldsymbol{S}''(\boldsymbol{r})>0$ thus $\boldsymbol{S}(\boldsymbol{r}+\Delta \boldsymbol{r}) > \boldsymbol{S}(\boldsymbol{r})$.

3.2. Empirical Application of the Deterministic Model

The model was applied to the Workers' Compensation pensions portfolio data of an insurance company that has a 3% share of this line of business, operating in the Portuguese market in the Financial Year 1998, embracing all the pensions which were initiated in that year, or in the preceding years.

The projection was made of the future cash-outflows (pensions) for a ten-year period for all the liabilities in force.

The corresponding portfolio of financial assets is created with the sum of the mathematical provisions, this being the stock of assets re-evaluated at market prices at the end of each year.

3.3. The Portfolio of Pensions Liabilities

The structure of the pensioners' portfolio in Workers' Compensation according to age classification was as follows, over the last three years in the operating account:

TABLE 3

Age	Total of Monthly Pensions (U:PTE)			Number of Pensioners by age		
Group	31.12.96	31.12.97	31.12.98	31.12.96	31.12.97	31.12.98
0 – 9	5.963.507	5.789.778	5.001.927	32	30	24
10 – 19	11.096.762	13.125.704	15.807.849	96	95	111
20 – 29	22.090.004	23.465.486	29.433.143	201	201	215
30 – 39	34.742.899	45.978.308	51.515.490	269	320	344
40 – 49	42.894.331	43.748.392	53.804.631	370	361	435
50 – 59	31.275.784	39.700.702	47.208.064	348	405	412
60 – 69	32.359.698	35.196.545	39.264.599	407	412	431
70 – 79	7.356.627	8.070.257	10.469.115	227	226	246
80 – 89	1.123.112	1.745.811	1.872.598	84	86	90
90 – 96	7.597	17.316	17.565	4	9	10
Total	**188.910.321**	**216.838.299**	**254.394.981**	**2.038**	**2.145**	**2.318**

[11] Shiu, E. (1987), "Asset/Liability Management: From Immunisation to Option Pricing Theory" in *Financial Management of Life Insurance Companies*. Ed. by Cummins and Lamm-Tennant.

The mathematical provisions constituted to cover the liabilities of the pensions to be paid, according to the age groups, were as follows over the last three years:

TABLE 4

(U:PTE)

Age Group	Mathematical Provision		
	31.12.96	31.12.97	31.12.98
0 –9	67.263.026	65.686.882	57.335.136
10 – 19	83.214.636	99.824.475	129.365.870
20 – 29	318.645.525	339.478.562	395.522.103
30 – 39	544.254.855	720.671.206	808.149.830
40 – 49	636.612.475	687.791.903	799.476.160
50 – 59	428.040.715	521.632.326	646.834.548
60 – 69	372.285.794	366.051.927	448.002.513
70 – 79	59.358.399	54.690.202	84.419.860
80 – 89	5.748.030	7.978.001	8.803.490
90 – 96	21.563	20.299	50.367
Total	**2.515.445.018**	**2.897.387.831**	**3.377.959.877**

Appendix I presents a table of the evolution of the mathematical provisions and the pensions to be paid to Workers' Compensation pensioners as of 31.12.98 until 2009[12], calculated on a basis of all the pensioners registered on 31.12.98 and taking into account the mortality of the pensioners in each year according to the Mortality Table TV 73/77, with a technical interest rate of 6%.

3.4. The Cash-Flows of Financial Assets and Liabilities

The portfolio cash-flow of financial assets of the insurance company to cover the liabilities of the Workers' Compensation line was as follows:

TABLE 5

Years	Cash-flow of assets	Net value at interest rate of 6%	Cash-flow of liabilities	Net value at interest rate of 6%
1	253.401.230	239.057.764	289.740.687	273.340.271
2	934.552.496	831.748.395	287.640.064	255.998.633
3	177.780.870	149.268.247	281.977.871	236.754.058
4	378.584.511	299.874.392	279.660.274	221.517.131
5	169.185.170	126.425.001	276.240.783	206.423.183
6	169.185.170	119.268.869	272.995.333	192.450.938
7	569.081.738	378.471.858	269.080.233	178.953.723
8	323.501.038	202.968.553	266.077.870	166.940.547
9	553.931.038	327.870.930	261.290.751	154.657.594
10	2.491.591.420	1.391.291.635	3.068.552.293	1.713.463.573
	Total	**4.066.245.643**		**3.600.499.651**

[12] This is due to the portfolio of assets we used to cover the liabilities

As can be observed, the initial net value of the fund is **465.745.992 PTE.**

3.5. The Assets/ Liabilities Immunisation

The durations[13] of the liability and asset cash-flows and the respective dispersions are presented in the following table:

TABLE 6

	Duration	Duration*Present Value	Dispersion
Assets	**6,04**	**26.034.401.230**	**12**
Liabilities	**6,73**	**24.229.475.746**	**10,88**

So, we can confirm that, in the case of the deterministic model, the fund is immunised because the two conditions mentioned in section 3.2, i.e. VA*DA>=VL*DL and IA>=IL are verified. This means that the net value due to the variations of the interest rate is always positive.

[13] The duration and the convexity of the liabilities were calculated for the total number of pensioners on 31 December 1998, taking into account the mortality of the pensioners in each year, according to the Mortality Table TV 73/77, with a technical (mandatory) interest rate of 6%.

4. The Stochastic Approach

4.1. Cox, Ingersoll and Ross (1985) and Longstaff and Schwartz (1992) models.

We will now evaluate the impact on the net present value of the insurance liabilities of the assumption that interest rates follow a stochastic process. In this sense, the first point that needs to be established relates to the choice of the stochastic models to be implemented. In the early 1980s, one of the best known models to explain the term structure of interest rates was Cox, Ingersoll and Ross [CIR (1985)]. The main assumption behind this model is that the volatility of the short rate increases with the square root of the level of the rate. Taking this formulation into account, the existence of negative interest rates is excluded from the set of the model results and more variability is allowed at times of high interest rates and less variability when rates are low. The stochastic process followed by the short-term interest rate ("r") under the CIR model is:

$$dr = a(\bar{r} - r)dt + s\sqrt{r}\,dz\,,$$

which implies that the general solution for pure discount bond prices, spot yields and their volatility structure are, respectively:

$$P(r,t) = A(t)e^{-B(t)r}\,,$$

$$Y(t) = -\frac{\ln A(t)}{t} + \frac{B(t)}{t}\,,$$

$$s_R(t) = \frac{s\sqrt{r}}{t}B(t)$$

where

$$t = s - t, \qquad A(t) = \left(\frac{f_1 e^{f_2(t)}}{f_2(e^{f_1(t)} - 1) + f_1} \right)^{f_3}, \qquad B(t) = \left(\frac{e^{f_1(t)} - 1}{f_2(e^{f_1(t)} - 1) + f_1} \right),$$

$f_1 \equiv \sqrt{a^2 + 2s^2}$, $f_2 \equiv \dfrac{(a + f_1)}{2}, f_3 \equiv \dfrac{2a\bar{r}}{s^2}$.

Despite the fact that the CIR model continues to be widely used at the present time, there are strong constraints on it. The most important of these relates to the incapacity of the model to fit different types of shapes for the yield curve. Given its functional form, especially its limited number of degrees of freedom, the CIR model presents only a good fit for upward term structures of interest rates. In fact, this is the main reason why this model has been so widely used over the last decade by European core markets, but not on the peripheral markets (for further details see Rebonato [1997] Chapter 9). Since the current yield curve for Euroland rates allows its projection, we decided to use this. Nevertheless, our main aim in using the CIR model was to compare its outcomes with those which we obtained from the Longstaff and Schwartz model (L&S) [1992]. This second model can be seen as an extension of the CIR model given that it uses two state variables. All the latest theoretical developments to explain the dynamics of the yield curve suggest the existence of three important explanatory variables: level, slope and curvature risks. According to the principal component analysis (PCA), these variables will correspond directly to the first, second and third components, which, in most cases, explains roughly 99% of the yield curve volatility. On the strength of these arguments, we found good support for the implementation of a two-factor model, such as that of Longstaff and Schwartz [1992]. Furthermore, given the high number of degrees of freedom presented by L&S' closed-form solution for the price of pure discount bonds, this model fits perfectly well across different yield curve shapes.

The L&S model originates from a general equilibrium framework where the short-term rate r is defined by $r = ax + by$ with $a \neq b$. Alongside this, the short rate volatility v is given by $v = a^2 x + b^2 y$. Finally, the state variables x and y follow the stochastic differential equations presented below:

$$dx = (g - dx)dt + \sqrt{x}dz_1$$

$$dy = (h - qy)dt + \sqrt{y}dz_2$$

Combining these two equations with the identity for r and v allows the authors to determine the processes for r and v and to reach the following closed-form solution for the price of pure discount bonds:

$$P(r, v, t) = A^{2g}(t) B^{2h}(t) e^{(kt + C(t)r + D(t)v)},$$

where

$t = s - t,$

$$A(t) = \frac{2f}{(d+f)(e^{(ft)}-1)+2f},$$

$$B(t) = \frac{2y}{(q+y)(e^{(yt)}-1)+2y},$$

$$C(t) = \frac{af(e^{(yt)}-1)B(t) - by(e^{(ft)}-1)A(t)}{fy(b-a)},$$

$$D(t) = \frac{y(e^{(ft)}-1)A(t) - f(e^{(yt)}-1)B(t)}{fy(b-a)},$$

and

$$q = x + l\,,\quad f = \sqrt{2a + d^2}\,,\quad y = \sqrt{2b + q^2}\,,\quad k = g(d+f) + h(q+y)\,.$$

The yield of a bond maturing in T, $\mathrm{Y}(T)$, is simply obtained as:

$$Y(T) = -\frac{\log(P(T))}{T} = -\left(\frac{kT + 2g\log A(T) + 2h\log B(T) + C(T)r + D(T)v}{T}\right).$$

Finally, the instantaneous volatility of a bond return is given by:

$$s_R(t) = \frac{1}{t}\sqrt{\left(\frac{aby^2(e^{ft}-1)^2A^2(t) - ab(e^{yt}-1)^2B^2(t)}{f^2y^2(b-a)}\right)r + \left(\frac{bf^2(e^{yt}-1)^2B^2(t) - ay^2(e^{ft}-1)^2A^2(t)}{f^2y^2(b-a)}\right)v}\,.$$

4.2. Matching of Assets and Liabilities under CIR and L&S Models

In a stochastic interest rate environment, only the first order conditions (defined in terms of "stochastic durations") are relevant for formulating immunisation rules. In order to generalize the analysis, while encompassing the previously two-term structure models as special cases, the short-term interest rate, r, is defined as an affine function of a Markovian vector $\underline{X} \in \Re^n$ of n state variables:

$$\mathrm{r} = \mathrm{f} + \underline{G}'\cdot\underline{X},$$

where $\mathrm{f} \in \Re$ and $\underline{G} \in \Re^n$ are model' parameters to be estimated, and "·" denotes the inner product in $\Re^n$. Let us further assume that, under the probability space (Ω, F, Q), the state vector $\underline{X}$ satisfies the following stochastic differential equation (SDE):

$$d\underline{X} = \underline{\mu}(\underline{X})dt + \sigma(\underline{X}) \cdot d\underline{W^Q},$$

where $\underline{\mu}(\underline{X}) \in \Re^n$ and $\sigma(\underline{X}) \in \Re^{n \times n}$ satisfy the usual Lipschitz and growth conditions required for a strong solution to exist, while $\underline{W^Q} \in \Re^n$ is a standard, Q-measured Brownian motion. The martingale measure Q is obtained when the "money-market account" is taken as the numeraire of the inter-temporal stochastic economy under analysis, and it is assumed to exist.[14] From Duffie and Kan (1996), it is well understood that pure discount bond prices are only exponential-affine if, and only if, both the drift and the instantaneous variance of the above diffusion process are written as affine functions of the state vector, i.e:

$$\underline{\mu}(\underline{X}) = a \cdot \underline{X} + \underline{b},$$

and

$$\sigma(\underline{X}) = \Sigma \cdot \mathrm{diag}\left\{\sqrt{v_1}, K, \sqrt{v_n}\right\},$$

where $a, \Sigma \in \Re^{n \times n}$, $\underline{b} \in \Re^n$, and

$$v_i = \alpha_i + \underline{\beta_i}' \cdot \underline{X}, \quad i = 1, K, n,$$

with $\alpha_i \in \Re$ and $\underline{\beta_i} \in \Re^n$. Note, for instance, that the Longstaff and Schwartz (1992) model can be fitted into the Duffie and Kan (1996) framework through the following restrictions: $n = 2$, $f = 0$, matrix "a" is diagonal, matrix Σ is a (2x2) identity matrix, $\alpha_i = 0$, $\forall i$, and $\underline{\beta_i}$ is a vector of zeros except for its i^{th} entry which is equal to one. The CIR model can also be fitted into the Duffie and Kan (1996) specifications, since this is the most general formulation for the affine class of term structure models.

Taking into account the general SDE satisfied by the state-vector, and applying Itô's lemma to the present value of both assets (VA) and liabilities (VL), it follows that[15]

$$dVA = r \times VA \times dt + \frac{\partial VA}{\partial \underline{X}'} \cdot \sigma(\underline{X}) \cdot d\underline{W^Q},$$

and

$$dVL = r \times VL \times dt + \frac{\partial VL}{\partial \underline{X}'} \cdot \sigma(\underline{X}) \cdot d\underline{W^Q}.$$

[14] That is, the market is assumed to be arbitrage-free.

[15] The drift specification is a direct consequence of Q being a martingale measure.

Joining the previous two equations, we obtain the dynamics of the financial institution rate of return:

$$\frac{dVA}{VA} - \frac{dVL}{VL} = \left(\frac{\partial VA}{\partial \underline{X}'} \frac{1}{VA} - \frac{\partial VL}{\partial \underline{X}'} \frac{1}{VL} \right) \cdot \sigma(\underline{X}) \cdot d\underline{W^Q}.$$

It is then clear that, starting from perfect matching between assets and liabilities, i.e., with VA = VL, the immunisation against interest rate risk is obtained if the instantaneous variance is equal to zero, that is, if:

$$\left\| \frac{1}{VA} \frac{\partial VA}{\partial \underline{X}'} \cdot \sigma(\underline{X}) \right\|^2 = \left\| \frac{1}{VL} \frac{\partial VL}{\partial \underline{X}'} . \cdot \sigma(\underline{X}) \right\|^2,$$

where $\|\cdot\|$ represents the Euclidean norm in $\Re^n$. Hence, hedging against interest rate risk involves matching the assets and liabilities sensitivities with respect to each model factor. The vectors $\frac{\partial VA}{\partial \underline{X}'} \frac{1}{VA}$ and $\frac{\partial VL}{\partial \underline{X}'} \frac{1}{VL}$ are usually known as the “stochastic durations” of assets and liabilities, respectively. Each element of such stochastic duration vectors is easily shown to correspond to a weighted average of the stochastic durations of its assets or liabilities components, with respect to a model state variable. Therefore, all that it is required is to know how to compute each stochastic duration for a discount factor. In the CIR model, $\frac{\partial P(r,t)}{\partial r} = -P(r,t)B(t)$. In the L&S model,

$$\frac{\partial P(r,v,t)}{\partial r} = P(r,v,t)C(t) \quad \text{and} \quad \frac{\partial P(r,v,\tau)}{\partial v} = P(r,v,\tau)D(\tau).$$

In order to assign a temporal dimension to the above-specified stochastic durations, as well as to maintain consistency with the deterministic analysis made in section 3, the empirical analysis that will follow uses the stochastic duration definition of Munk (1999). That is, the stochastic duration of any asset or liability is rewritten as the time-to-maturity of a pure discount bond, with the same instantaneous variance of relative price changes. This measure is then directly compared with the Macaulay durations computed in section 3.

5. Data and Methodology

After the presentation of the theoretical background required for our study, the next stage was to estimate the parameters of the CIR and L&S models. As the benchmarks for Euroland interest rates, we used the O/N, 1-m, 3-m, 6-m, 1-yr Euribor rates and prices of German tradable debt on the following dates: 3 April and 7 November 1999. These dates were chosen because the European Central Bank changed its repo rate by 50bp, respectively, by a cut and a raise. In order to ensure the quality of the fitness, we considered only a 10-year investment horizon, which does not change the validity of the conclusions. All estimation routines were carried out through an adjustment to the dirty prices of the bonds and not in yield-to-maturity terms. This procedure obliged us to create synthetic assets for the Euribor rates. For both models, the parameters were estimated using non-linear least squares and no additional constraints were imposed, besides those strictly needed for the achievement of consistent theoretical solutions (and their implementation). The CIR model only requires that all parameters need to be positive and that $a + b < 1$. However, the parameterisation of the L&S model is substantially more difficult. The model has 9 parameters, the two state variables r and v plus 7 parameters, a, b, g, h, d, l and x. Because of convergence difficulties, we followed the same procedures that Dahlquist and Svensson [1996] adopted to resolve this problem. Since the state variable associated with the short-term rate, r, can be associated to the overnight rate, the restriction that r equals the overnight rate was imposed. This same constraint was also imposed on the CIR model. For the state variable associated with the volatility of the short-term rate, v, we simply determined the 1-year average volatility of the overnight rate (equal to 0.0015 per year on the assumption that for rates before January 1999, the D-Mark overnight rate acts as a proxy). Finally, the following restrictions were still imposed to preserve the assumptions taken by the L&S economy. All parameters need to be positive, with the exception of l (which is obliged to be negative in order to ensure that term premia is always non-negative), b needs to be greater than a, and at last,

$$a < \frac{v}{r} < b .$$

Based on the estimates obtained for the CIR and L&S parameters, the final step was to apply the closed-form solutions of both models to the insurance liabilities to determine its present value and stochastic duration, and, thereafter, to determine the immunisation requirements.

6. Results Analysis

The following table presents the estimates obtained for the parameters of CIR and L&S models on both dates (except the CIR parameters on 7 April 1999, since the model did not fit the shape of the curve at that time):

TABLE 7

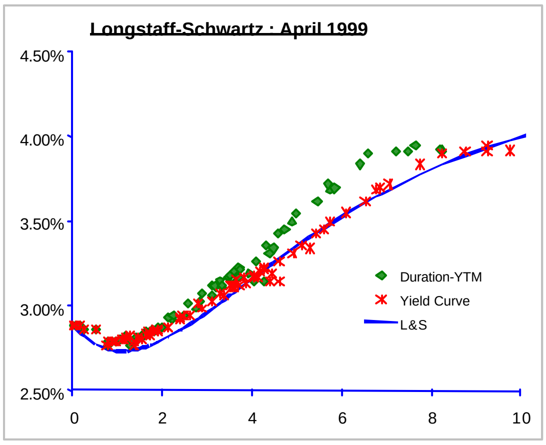
Longstaff & Schwartz

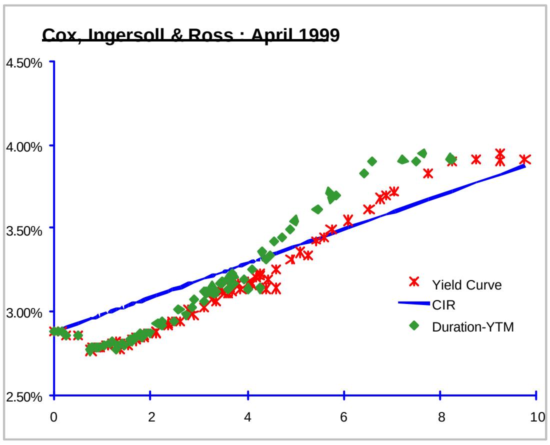
	03-Nov-99	07-Apr-99
r	0.0288	0.0309
V	0.0015	0.0015
α	0.0200	0.0350
β	0.0621	0.1310
γ	0.0000	0.0000
δ	0.7558	0.6440
η	0.5483	0.0950
λ	-2.0560	-0.1000
ξ	2.6470	0.1000

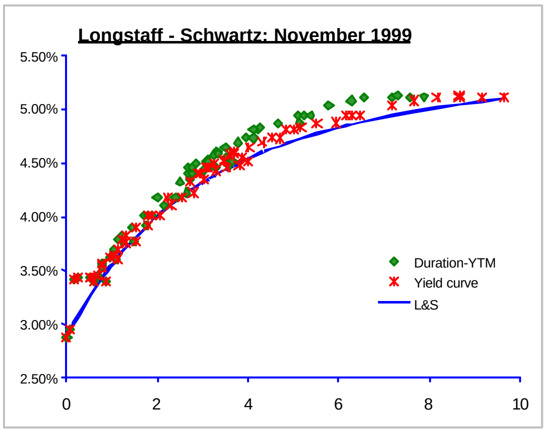
Cox, Ingersoll & Ross

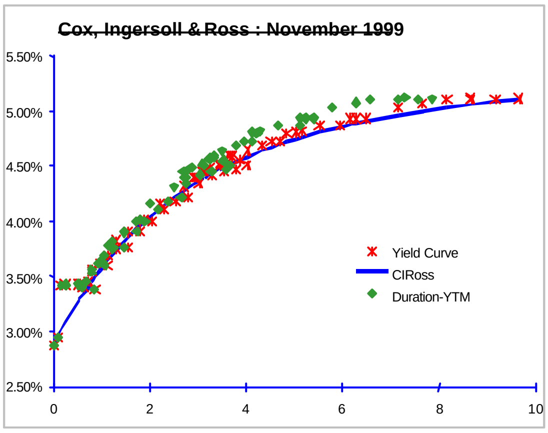
	03-Nov-99
r	0.0288
$\bar{r}$	0.0533
σ	0.0015
α	0.6484

From the charts presented below, it is evident that the L&S model performs better than the CIR model regarding the quality of fitness.

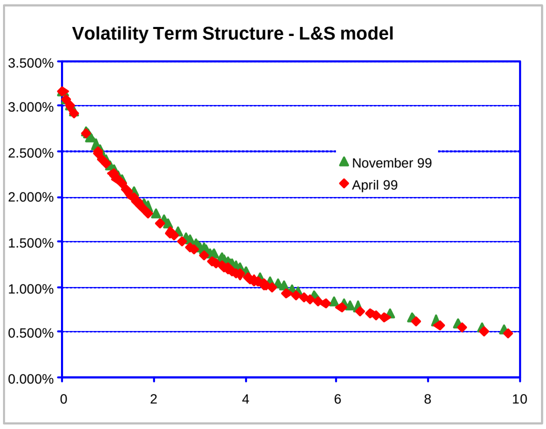


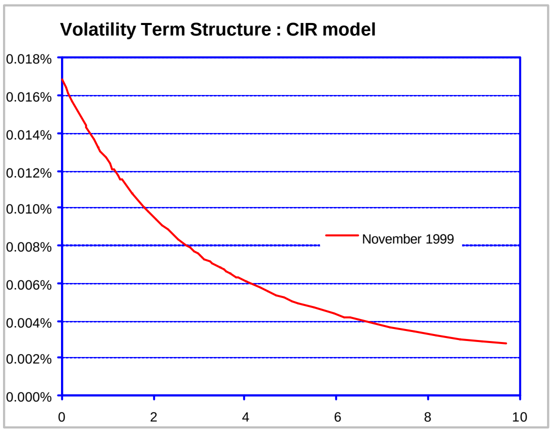






It is also important to analyse the volatility term structure of both models, since they will allow us to explain the allocation between short and long-term assets recommended by each model.





Based on the previous estimates for the L&S model on 7 April 1999, we determined the present value and stochastic duration of both assets and insurance liabilities as given in Table 5, which are presented below:

TABLE 8

Longstaff & Schwartz (07-Apr-99)

time (years)	discount factor	cash flows from assets	assets' present value	cash flows from liabilities	liabilities' present value
1	0.97054	253,401,230	245,934,763	289,740,687	281,203,478
2	0.94046	934,552,496	878,912,044	287,640,064	270,514,838
3	0.90777	177,780,870	161,383,785	281,977,871	255,970,488
4	0.87277	378,584,511	330,417,204	279,660,274	244,079,097
5	0.83651	169,185,170	141,524,410	276,240,783	231,077,072
6	0.79998	169,185,170	135,344,245	272,995,333	218,389,988
7	0.76391	569,081,738	434,728,938	269,080,233	205,553,888
8	0.72878	323,501,038	235,762,057	266,077,870	193,913,028
9	0.69485	553,931,038	384,901,197	261,290,751	181,558,923
10	0.66226	2,491,591,420	1,650,076,351	3,068,552,293	2,032,173,304
		Total	**4,598,984,993**		**4,114,434,105**
Stochastic duration (years)			4.297		4.856

The next two tables repeat the previous analysis, for both the CIR and the L&S term structure models, and is based on the parameters estimated on 3 November 1999:

TABLE 9

Cox, Ingersoll & Ross (03-Nov-99)

time (years)	discount factor	cash flows from assets	assets' present value	cash flows from liabilities	liabilities' present value
1	0.96533	253,401,230	244,616,823	289,740,687	279,696,536
2	0.92388	934,552,496	863,411,556	287,640,064	265,744,039
3	0.88023	177,780,870	156,487,700	281,977,871	248,204,817
4	0.83667	378,584,511	316,749,167	279,660,274	233,982,522
5	0.79428	169,185,170	134,381,074	276,240,783	219,413,634
6	0.75356	169,185,170	127,491,515	272,995,333	205,718,909
7	0.71469	569,081,738	406,715,320	269,080,233	192,308,144
8	0.67770	323,501,038	219,236,330	266,077,870	180,320,706
9	0.64257	553,931,038	355,937,251	261,290,751	167,896,553
10	0.60922	2,491,591,420	1,517,937,291	3,068,552,293	1,869,435,702
Total			**4,342,964,027**		**3,862,721,565**
Stochastic duration (years)			3.578		3.988

TABLE 10

Longstaff & Schwartz (03-Nov-99)

time (years)	discount factor	cash flows from assets	assets' present value	cash flows from liabilities	liabilities' present value
1	0.96532	253,401,230	244,612,769	289,740,687	279,691,900
2	0.92381	934,552,496	863,348,941	287,640,064	265,724,768
3	0.88011	177,780,870	156,467,433	281,977,871	248,172,672
4	0.83654	378,584,511	316,699,572	279,660,274	233,945,887
5	0.79417	169,185,170	134,361,110	276,240,783	219,381,038
6	0.75348	169,185,170	127,477,473	272,995,333	205,696,251
7	0.71465	569,081,738	406,695,402	269,080,233	192,298,727
8	0.67772	323,501,038	219,242,153	266,077,870	180,325,496
9	0.64264	553,931,038	355,977,688	261,290,751	167,915,627
10	0.60935	2,491,591,420	1,518,253,723	3,068,552,293	1,869,825,408
Total			**4,343,136,264**		**3,862,977,773**
Stochastic duration (years)			3.500		3.887

According to previous calculations, the PV of the insurance liabilities is significantly higher than that calculated at the official discount rate of 6%. This clearly points out that, for the time being, insurance liabilities are much higher than those presented on the balance sheets. This analysis also allows us to conclude that the gap between assets and liabilities tends to increase when business cycle conditions favour an economic slowdown. On 7 April, the mismatch was higher than 10%, which would certainly put the insurance company in a delicate situation, in the event of the assets being sold.

Consistently with the deterministic analysis, assets continue to possess a higher present value but a lower duration than liabilities.

7. Conclusions

The problem of immunising a Workers' Compensation fund can be examined by adopting the classic models currently employed in life insurance and pension funds. However, these models call for highly restrictive hypotheses to be taken into account, namely, that the temporal structure of the interest rates is horizontal, that the effects of a small change in the interest rate provoke parallel dislocations throughout the length of the interest curve and that there are positive arbitrage opportunities, as demonstrated by Shiu (1987). This latter restriction creates for the insurer the illusion that a perfect matching has opportunity gains which can be incorporated into the net value of the fund.

On the other hand, if we consider that the interest rate variations are stochastic, then the conclusions of the deterministic model could be unverifiable. The amount of the liabilities is significantly higher, causing a mismatch of approximately 10% in the case studied.

So, the main conclusion of this study is that the legal (mandatory) discount rate for insurance liabilities related to Workers' Compensation should be revised down, in order to accommodate the current environment of low interest rates induced by the EMU process. Otherwise, insurance companies will be forced to diversify their portfolios with low-grade investments in order to obtain yield pick-up. The risks associated with this solution would bring about a deterioration of the assets quality, which would mean an increase of the default probability. One other alternative would be a pro-active management of insurance companies' portfolios in order to reap the benefits of daily market volatility (mainly through trading positions).

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Appendix I

Evolution of the mathematical provisions and of the pensions to be paid out to the beneficiaries of Workers' Compensation Pensions from 31.12.98

(U: PTE)

Year	Life Annuities		Temporary Annuities		IBNR		Total	
	Provisions	Pensions	Provisions	Pensions	Provisions	Pensions	Provisions	Pensions
1998	3.191.402.728	228.103.522	186.557.149	26.291.459	583.842.845	36.858.207	3.961.802.722	291.253.188
1999	3.143.562.799	226.648.427	169.784.402	26.280.120	580.047.924	36.812.140	3.893.395.124	289.740.687
2000	3.103.497.600	225.093.800	152.320.548	25.783.618	576.061.340	36.762.646	3.831.879.489	287.640.064
2001	3.062.177.040	223.434.379	138.038.771	21.834.908	571.921.426	36.708.584	3.772.137.237	281.977.871
2002	3.019.597.074	221.664.649	123.410.549	21.346.432	567.628.182	36.649.192	3.710.635.806	279.660.274
2003	2.975.789.928	219.778.817	109.472.760	19.877.116	563.104.943	36.584.850	3.648.367.631	276.240.783
2004	2.930.760.420	217.772.081	95.944.028	18.708.454	558.428.374	36.514.798	3.585.132.822	272.995.333
2005	2.884.528.412	215.638.549	83.428.516	17.004.172	553.560.142	36.437.512	3.521.517.070	269.080.233
2006	2.837.110.154	213.371.675	70.851.514	16.353.203	548.500.247	36.352.992	3.456.461.915	266.077.870
2007	2.788.540.288	210.964.231	59.976.656	14.064.901	543.248.690	36.261.620	3.391.765.635	261.290.751
2008	2.738.838.485	208.410.448	49.721.155	12.875.388	537.805.470	36.161.871	3.326.365.110	257.447.707
2009	2.688.036.064	205.704.142	40.736.584	11.118.054	532.132.255	36.053.366	3.260.904.903	252.875.562

Appendix II

The Market Operating Account for the Workers' Compensation Line

Workers' Compensation Line

Market U: millions PTE

	1990	1995	1996	1997
Written Premiums	**50.718**	**75.327**	**78.432**	**79.310**
Earned Premiums		**74.626**	**77.692**	**79.068**
Loading	**40.724**	**58.808**	**63.774**	**66.244**
Payments	30.202	43.143	48.739	
Outstanding Claims reserve (variation) (1)	10.522	15.820	14.553	
Other technical reserves(variation)	0	-155	483	
Underwriting Margin	**9.994**	**15.818**	**13.918**	**12.824**
Costs	**20.378**	**21.991**	**23.852**	**22.626**
Administrative Costs (2)	15.979	10.219	10.870	9.438
Commissions (3)	4.399	11.772	12.981	13.188
Net Margin	**-10.384**	**-6.173**	**-9.934**	**-9.802**
Reinsurance balance	107	-136	-42	-88
Operating Result	-10.277	-6.310	-9.976	-9.890
Financial Balance	**11.952**	**12.801**	**14.947**	**21.649**
Final Result	**1.675**	**6.491**	**4.971**	**11.759**
Number of Claims	**n.a.**	**94.490**	**113.569**	**231.092**

(1) In 1990 refers to Technical Provisions
(2) In refers to General Expenses
(3) In 1990 includes Commissions and Earning Expenses

Workers' Compensation Line

Market

	1990	1995	1996	1997
Written Premiums	**100,0%**	**100,0%**	**100,0%**	**100,0%**
Earned Premiums		**99,1%**	**99,1%**	**99,7%**
Loading	**80,3%**	**78,1%**	**81,3%**	**83,5%**
Payments	59,5%	57,3%	62,1%	
Outstanding claims reserve (variation)	20,7%	21,0%	18,6%	
Other Technical Reserves (variation)	0,0%	-0,2%	0,6%	
Underwriting Margin	**19,7%**	**21,0%**	**17,7%**	**16,2%**
Costs	**40,2%**	**29,2%**	**30,4%**	**28,5%**
Administrative Costs	31,5%	13,6%	13,9%	11,9%
Commissions	8,7%	15,6%	16,6%	16,6%
Net Margin	**-20,5%**	**-8,2%**	**-12,7%**	**-12,4%**
Reinsurance Balance	0,2%	-0,2%	-0,1%	-0,1%
Operating Result	-20,3%	-8,4%	-12,7%	-12,5%
Financial Balance	**23,6%**	**17,0%**	**19,1%**	**27,3%**
Final Result	3,3%	8,6%	6,3%	14,8%

Appendix III

The Operating Account of the company

The closing results of the company in question have always been negative, despite some improvement over the last two years.

Workers' Compensation Line
Insurance Company

U: Thousands PTE

	1995	1996	1997	1998
Written Premiums	**2.279.178**	**2.494.151**	**2.710.389**	**2.911.550**
Earned Premiums	**2.251.134**	**2.467.351**	**2.682.797**	**2.949.070**
Loading	**1.843.500**	**2.134.569**	**2.865.586**	**2.804.448**
Claims Costs	1.352.903	1.509.901	1.713.958	1.874.012
Outstanding Claims reserve (variation)**(*)**	474.126	637.015	1.155.752	794.064
Other Technical reserves (variation)	16.471	-12.347	-4.124	136.372
Underwriting Margin	**407.634**	**332.782**	**-182.789**	**144.622**
Costs (**)	**1.027.855**	**1.014.983**	**604.409**	**850.856**
Administrative Costs	591.451	552.288	258.039	416.213
Commissions	436.404	462.695	346.370	434.643
Net Margin	**-620.221**	**-682.201**	**-787.198**	**-706.234**
Reinsurance Balance	296.080	156.753	226.928	196.371
Operating Result	-324.141	-525.448	-560.270	-509.863
Financial Balance	**-310.204**	**208.424**	**485.906**	**465.359**
Final Result	**-634.345**	**-317.024**	**-74.364**	**-44.504**
Number of Claims	**7.542**	**8100**	**8829**	**9665**

Source: *Desdobramento de Ganhos e Perdas*
(*) Per Underwriting Year.
(**) Source: Company Reports and Accounts.

This trend is better observed in the table of the account when expressed as percentages of the written premiums.

Workers' Compensation Line

Insurance Company

	1995	1996	1997	1998
Written Premiums	**100%**	**100%**	**100%**	100%
Earned Premiums	**99%**	**99%**	**99%**	101%
Loading	**81%**	**86%**	**106%**	96%
Claims Costs	59%	61%	63%	64%
Outstanding claims reserve (variation)	21%	26%	43%	27%
Other Technical reserves (variation)	1%	0%	0%	5%
Underwriting Margin	**18%**	**13%**	**-7%**	5%
Costs (**)	**45%**	**41%**	**22%**	29%
Administrative Costs	26%	22%	10%	14%
Commissions	19%	19%	13%	15%
Net Margin	**-27%**	**-27%**	**-29%**	-24%
Reinsurance Balance	13%	6%	8%	7%
Operating Result	-14%	-21%	-21%	-18%
Financial Balance	**-14%**	**8%**	**18%**	16%
Final Result	**-28%**	**-13%**	**-3%**	-2%