On the Pricing of Top & Drop Excess of Loss Covers

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Agenda

- Definition
- Numerical examples
- Modelization
- Exact solution
- False solution
- Approximate solution
- Conclusion
Top & Drop covers: definition

- Top layer covered
- Cover may drop to a lower layer
Numerical example 1

- Layer 1: 300 XS 100
- Layer 2: 400 XS 400
- Assume the cedent fears
  1) Big losses > 800
  2) Accumulation of losses > 20 and < 100
Numerical example 1

- 200 XS 800
- OR
- 200 XS 200 aggregate for claims > 20
  fgu with max 100 per claim
- No reinstatement
Numerical example 2

- Layer 1: 200 XS 200 with one reinstatement
- Layer 2: 400 XS 400
- Assume the cedent fears
  1) Big losses > 800
  2) Accumulation of claims in layer 1
Numerical example 2

- 200 XS 800
- OR
- 200 XS 200
- with an annual aggregate deductible = 400
- Unlimited free reinstatements
Modelization

- X: large claims
- N: number of large claims
- Y: small claims
- M: number of small claims
- Mutual independence
Example 1

\[ X_{i \, Re-top} = \min(200, \max(0, X_i - 800)) \]
\[ X_{i \, Re-drop} = \min(100, X_i I_{X_i \geq 20}) \]
\[ Y_{i \, Re-drop} = \min(100, Y_i I_{Y_i \geq 20}) \]
Example 1

\[
S = X_1^{Re-top} + \ldots + X_N^{Re-top}
\]

\[
T = X_1^{Re-drop} + \ldots + X_N^{Re-drop}
\]

\[
U = Y_1^{Re-drop} + \ldots + Y_M^{Re-drop}
\]

\[
Cover = \min(200, S + \max(0, T + U - 200))
\]
Example 2

\[ X_{i}^{\text{Re-top}} = \min(200, \max(0, X_{i} - 800)) \]
\[ X_{i}^{\text{Re-drop}} = \min(200, \max(0, X_{i} - 200)) \]
\[ Y_{i}^{\text{Re-drop}} = \min(200, \max(0, Y_{i} - 200)) \]
Example 2

\[
S = \sum_{i=1}^{N} X_i^{\text{Re-top}} + \ldots + X_N^{\text{Re-top}}
\]

\[
T = \sum_{i=1}^{N} X_i^{\text{Re-drop}} + \ldots + X_N^{\text{Re-drop}}
\]

\[
U = \sum_{i=1}^{M} Y_i^{\text{Re-drop}} + \ldots + Y_M^{\text{Re-drop}}
\]

\[
\text{Cover} = \max(0, S + T + U - 400)
\]
Multivariate Panjer’s algorithm

Let $N$ belong to Panjer’s family of distributions

Let $X$ and $Y$ be possibly dependent

Let $S = X_1 + \ldots + X_N$

Let $T = Y_1 + \ldots + Y_N$

Then a multivariate version of Panjer’s algorithm exists
Numerical example

- **Severity**: limited Pareto
- **Frequency**: Poisson
- **Large claims**: $\lambda = 0.3$, $A=400$, $B=1000$, $\alpha=0.9$
- **Small claims**: $\lambda=2.5$, $A=20$, $B=400$, $\alpha = 1.4$
Exact pure premiums

- Example 1 : 20.519
- Example 2 : 2.252
Assumption of independence

Example 1: \( 21.131 > 20.519 \) (conservative)

Example 2: \( 1.153 < 2.252 \) (not conservative)
Fréchet space and theorem

- $R(F_1, F_2)$: space of all random vectors (distribution $F_{12}$) with fixed marginals $F_1$ and $F_2$
- Let $F_{\text{min}} = \max[F_1 + F_2 - 1, 0]$
- Let $F_{\text{max}} = \min[F_1, F_2]$
- Then $F_{\text{min}} \leq F_{12} \leq F_{\text{max}}$
Correlation order

\[(X_1, X_2) <_c (Y_1, Y_2) \text{ iif } F_{X_1, X_2} \leq F_{Y_1, Y_2}\]
Fréchet bounds

- Using some lemmas and Fréchet theorem we arrive at

- Example 1: $19.469 < E[\text{cover}] < 21.279$
- Example 2: $0.952 < E[\text{cover}] < 5.471$
Assumption of independence

- Using some lemmas we arrive at

  Example 1: 21.131 (better upper bound)
  Example 2: 1.153 (better lower bound)
Summary

- Example 1: 19.469 < 20.519 (exact) < 21.131 (indep) < 21.279
- Example 2: 0.952 < 1.153 (indep) < 2.252 (exact) < 5.471
Correlation order extends to supermodular order.

Example 2 may be treated within that framework.

However, example 1 may not: a specific framework was necessary in dimension 2.
Conclusion

- Danger of falsely assumed independence.
- Existing bounds may be crude.
- Exact model is time-consuming but provides an exact solution.