Number of IBNR Claims and Multivariate Compound Poisson Distribution

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Plan of Paper

• 1- Introduction and notation
• 2- The pgf of the claims number
• 3- A Poisson model
• 4- A negative binomial model
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1- Introduction

Claim incurred but not reported: IBNR
Jewell’s model: homogeneous Poisson Process for number of claims incurred.
Generalizations:
  1- delay probabilities varying over years
  2- non-homogeneous Poisson process
  3- N~Compound Poisson distribution
Assumptions of model

Discrete time period model with
Exposure period \( \{1,2,\ldots,T\} \)
Observation period \( \{1,2,\ldots,k\}, k>T \)
Reporting independent of incurral
Maximum possible value \( m \) for lag
Notation

\( N_i \): # of claims incurred in accident period \( i \)
\( R_{ij} \): # of claims incurred in accident period \( i \),
reported \( j \) periods later (\( j=0,1,\ldots, m \))
\( U_i \): # of IBNR claims at end of obs. period
\( R_i \): # of claims reported by end of obs. period
2- The pgf of the claims number

Pgf of \( N_i \): \( \exp \{ \lambda [P_i(z)-1] \} \)

Prop. 1: Joint pgf of r.v. \( R_{ik}, k=0,1,\ldots,m-1 \)

Marginal pgf of \( R_{ij} \)

Expressions for mean, variance, covariance

Specific distributions for \( N_i \): Poisson, NB
Multi-period model

Assume $N_1, \ldots, N_T$ independent with compound Poisson distribution

Derive pgf of total # of claims in exposure period (Compound Poisson distribution)
3- A Poisson model

Expression for MLE’s of parameters

Identifiability problem with:
- non-parametric distribution for reporting lag W
- certain continuous distributions for W

Use modified discrete distribution for reporting lag
4- A negative binomial model

- \( N_i : \) Negative binomial \((s, 1-p)\)
- Joint distribution of \((R_{i0}, \ldots, R_{i,k-i})\):
  Negative multinomial

Results: Marginal distribution of \( R_{ij} \sim \)
Negative binomial

Components \( R_{ij} \) not independent of each other, as with Poisson assumption
4- A Negative binomial model

- Marginal distribution of $U_i$: NB
- Covariance between $U_i$ and $R_{ij} > 0$
- Total number of IBNR claims $U_1 + \ldots + U_T = \text{Sum of ind. NB}(s, -- )$
  Representable as a C.P. distribution
- Conditional distribution of $U_i$ given $(R_{i0}, \ldots, R_{i, k-i})$ also Negative binomial
5- References


