# Risk Willingness
# in Non-Life Insurance

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# View of the Supervision Authority

150 years ago: insurance business **very risky**
many insurance companies went bankrupt

legislative provisions and
mathematical methods changed this

e.g. in Austria:
1860: first legislative measures
1909: sixth ICA in Vienna: presentation of the mathematical model by Filip Lundberg

in Europe:
1973: European Council Directive
Solvency of a company measured by the **equity capital (Eigenkapital)**

two Problems:

1. only one key figure
2. accuracy of the technical provisions

The relation between technical provisions and equity capital is in many cases something like 10:1

a change of the provisions of +/- 5 %
means a change of the solvency margin by -/+ 50 %

"Slightly manipulate the technical provisions and you can make an insolvent company solvent and vice versa!"

**equity capital (Eigenkapital)**

Proposal: additional key figures

**unbalancedness of the risk**
**(Unausgeglichenheit der Risken)**
generated by the fluctuations of the claims costs

**premium loading (Sicherheitszuschlag)**

**retention (Eigenbehalt)**

**risk willingness (Risikobereitschaft)**
measuring the risk behavior of the entrepreneur

**relationship** between the items:

risk willingness will be
**high** if "retention" and "unbalancedness" is high
**low** if "capital" and "premium loading" is high

heuristic conclusion:

$$risk\ willingness = \frac{retention * unbalancedness}{capital * premium\ loading}$$

for the **quantification** of the five items:

a few denotations:

u  equity capital

Z(t)   the **gross** total **claims costs** in the year t

S(t)   the **net** total **claims costs** in the year t

P(t)   the **net premiums** (retention premiums) in the year t

Y(t) = P(t) - S(t)   the **net result** of the company in the year t quantification of the items:

**unbalancedness of the risk (Unausgeglichenheit der Risken):**

$$v[Z] = \frac{Var[Z]}{E^2[Z]} = \text{square of the coefficient of variation of } Z$$

a measure for the fluctuation of the claims costs

**premium loading (Sicherheitszuschlag):**

$$\frac{E[Y]}{E[Z]} = \text{a measure of the premium loading}$$

**retention (Eigenbehalt):**

$$\frac{Var[S]}{Var[Z]} = \text{a measure for the efficiency of the applied reinsurance program}$$

**equity capital (Eigenkapital):**

$$\frac{u}{E[Z]} = \text{a measure for the equity equipment}$$

our marvelous formula:

$$risk\ willingness = \frac{retention * unbalancedness}{capital * premium\ loading}$$

becomes:

$$risk\ willingness = \frac{\dfrac{Var[S]}{Var[Z]} * \dfrac{Var[Z]}{E^2[Z]}}{\dfrac{u}{E[Z]} * \dfrac{E[Y]}{E[Z]}} = \frac{Var[S]}{u * E[Y]}$$

the risk willingness will become
**high** if the variance of the net claims costs is high, and
**low** if the equity equipment is high and
**low** if the expected value of the net result is high

the main entrepreneurial decisions:

the extend of the involved equity capital u

the retention program, measured by $\frac{Var[S]}{Var[Z]}$

the premium loading, measured by $\frac{E[Y]}{E[Z]}$

they describe the risk willingness of the entrepreneur.

$$risk\ willingness = \frac{\frac{Var[S]}{Var[Z]}}{u * \frac{E[Y]}{E[Z]}} = \frac{Var[S]}{u * E[Y]}$$

**sophisticate approach** to a quantification of the **risk behavior**:

the risk process could be described by:
Y(t) = u + P(t) - S(t)

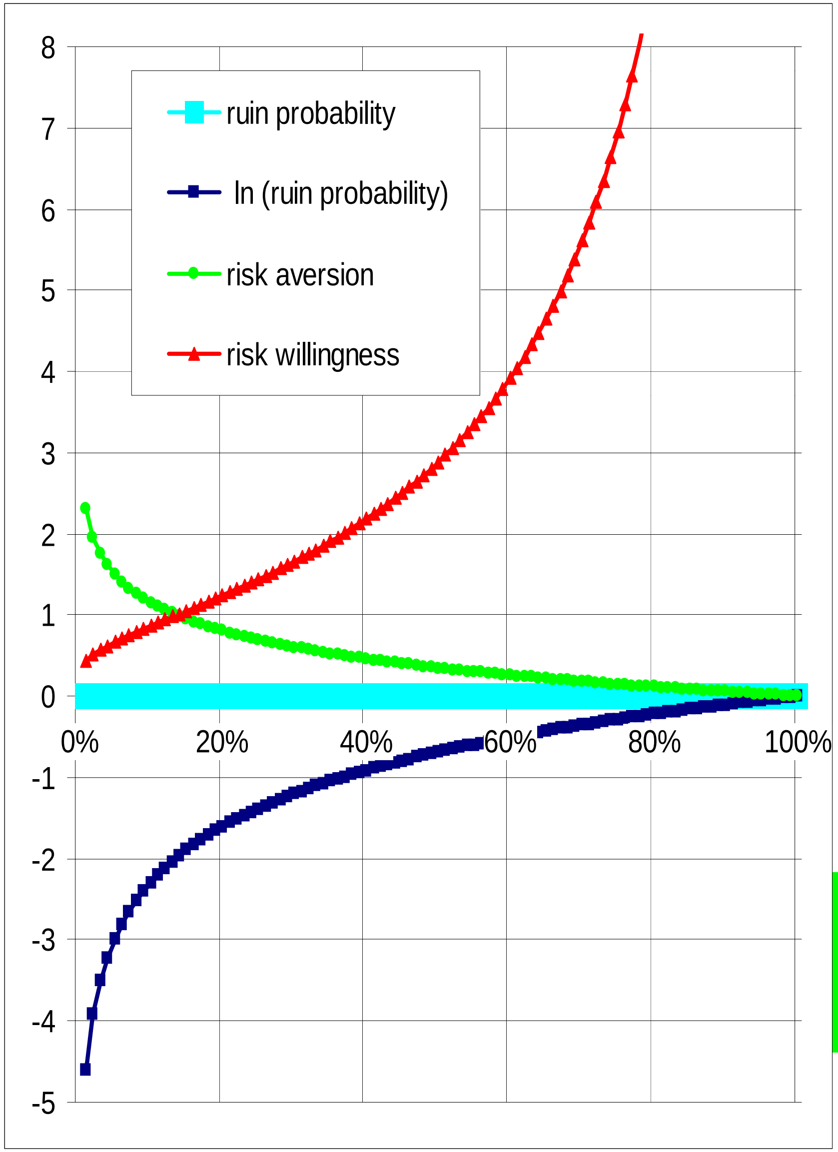
$$\psi(u) = \mathrm{Prob}[Y(t) < 0 \;\; \textit{for any } t \;\Big|\; \textit{equity capital } u \;]$$

this probability (usually called ruin probability) is bounded:

$$\psi(u) \le e^{-\kappa u} \quad \textit{where} \;\; \kappa \;\; \textit{satisfies} \quad \ln E[e^{-\kappa Y}] = 0$$

taking the first terms of power expansion we obtain: $\kappa \approx \frac{2 * E[Y]}{Var[S]}$

finally by putting equality in the inequality
(as it is correct in the Erlang case) we obtain: $\psi(u) \approx e^{-\frac{2*u*E[Y]}{Var[S]}}$




ruin probability:

$$\psi(u) \approx e^{-\frac{2*u*E[Y]}{Var[S]}}$$

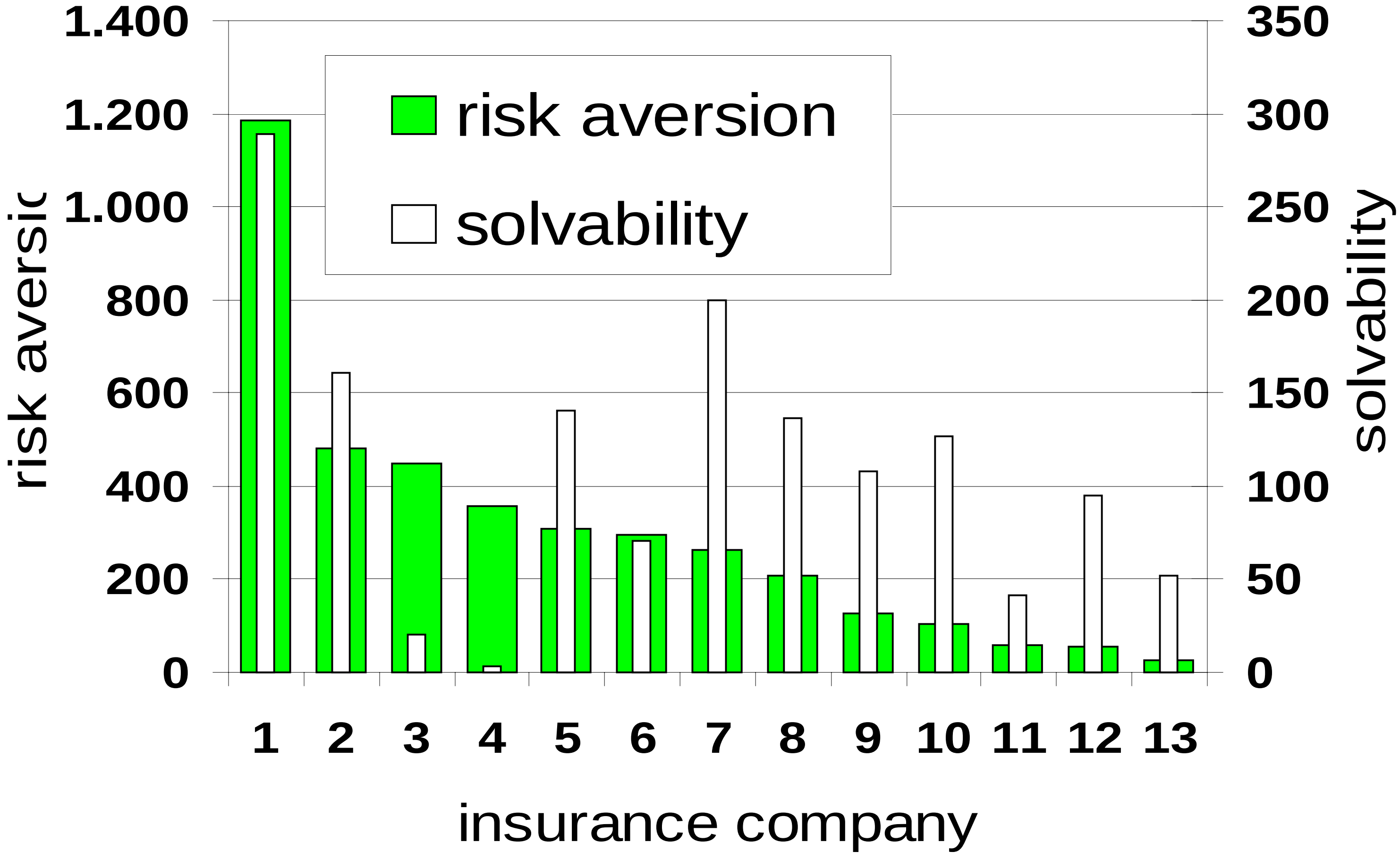
the linear scale dose not describe the tremendous increase of afford to come closer to the value 0 %.
So use another scale:
approaching 0 % asymptotically:

$$\ln\psi(u) \approx -\frac{2*u*E[Y]}{Var[S]}$$

$$\text{risk aversion} = -\frac{1}{2}\ln\psi(u) = \frac{u*E[Y]}{Var[S]}$$

risk willingness = 1/risk aversion

risk aversion and solvability
for 13 companies, period 1987 - 1996



proposal:

have a look on more than one key figure

but don’t forget the relationship between the figures!

The challenge is to improve the quantification of the observed key figures!

for example by assuming a distribution for Z
(e.g. gamma distribution or a fat tailed distribution)

or by taking an additional term in the power expansion, that means taking in account the skewness of Z.