Market Value of Liabilities Mortality Risk: A Practical Model

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Abstract

Market values of the invested assets are frequently published. For most insurance liabilities, there are no published *market* values and, therefore, these have to be constructed. This construction can be based on a best estimate and a price for the risks in the liabilities. This paper presents a model explaining how the best estimate and the price of mortality risk can be constructed. Several methods to describe the risks are already known. The purpose of this paper is to describe a method to determine the mortality risk in a practical way.

1. Introduction

This paper describes a methodology that could be used to calculate a fair value for mortality risk. It offers a practical solution to analyzing the mortality risk and deriving a market-value margin. The market value of assets is, in most cases, publicly available through markets for financial instruments. This is not the case for the liability side of the insurer's balance sheet.

The market value of the insurance liabilities is not publicly available and, in most cases, not readily available or even unknown. Therefore, a market value of liabilities (MVL) has to be constructed. The model presented in this paper also can be useful for pricing, profit testing, and reserve testing purposes. Also, for other kinds of risk, like morbidity, similar models can be developed.

In most cases the best estimate liability (BEL) is available or can be determined. A reward for risk should be added to the BEL—the market value margin (MVM)— to come to a MVL, which is

$$MVL = BEL + MVM \quad (1)$$

This paper presents a model to determine the best estimate mortality (BEM). As with all the other risks, the basis for the "mortality risk" is the BEM. The BEM will give an expected loss level, but around this best estimate will be a distribution of possible loss-levels. This distribution consists of several parts: the volatility, calamity, and parameter uncertainty components.

The volatility component represents the normal fluctuations around the best estimate. The calamity component represents the probability of extremely adverse fluctuations attributable to a certain event, such as an epidemic (like the Spanish Flu) or a catastrophe (like an earthquake). There is also a parameter uncertainty that occurs during the estimation of each parameter, that is, the uncertainty that the parameter has been estimated properly.

2. Best Estimate Mortality

2.1 Introduction

For most purposes, such as pricing, embedded value, profit testing, risk-adjusted return on capital, and fair value, "as good as possible" estimation of the expected mortality is needed. The BEM assumption should fit the expected mortality as well as possible for the applicable group of insured persons now and in the future. Consequently, the BEM contains two main parts: (1) the current mortality for a specified group of (insured) persons and (2) the expected changes in the level of this mortality in the future. The first part we call the *level* of the mortality, the second part the *trend.*

This paper describes several issues that can be

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important in estimating mortality, with attention to the data needed for reliable estimation of both the level and trend of mortality. Finally, it will provide some practical models that can be used to calculate a BEM assumption.

2.2 Important Issues in Finding the BEM

2.2.1 The Level

Insured population mortality (for life insurance) is not the same as population mortality. In general, the mortality within the insured population is lower because of underwriting. In some countries (e.g., the United States and the United Kingdom), special mortality tables have been developed for the insured population that vary depending on the product type and based on statistics from insured populations. In other countries, population tables are used with adjustments made to reflect insured experience. These adjustments can be related to all ages (e.g., insured mortality age x = population mortality $x - 5$) or to adjust the mortality rates ("qx's") for a particular age.

Because aggregate age adjustments are less exact, age dependent factors are used to adjust the qx's. Furthermore, if industry (i.e., insured) tables are used, the average industry mortality must be compared with the company's mortality experience to find how they differ.

The ideal estimation of the level requires a statistical analysis covering several aspects of mortality experience:

- The mortality within the specific company's *own* portfolio.
- The difference between the mortality of an insured population and the entire population depending on *age, gender* and, in some countries, *smoking status.*
- The *product type* and goal of the product and market involved for example mortgage or pensions, term insurance, whole life or annuity.
- The *issue year* (select period).
- *Underwriting* procedures, for example, guaranteed issue, no medical exam, paramedical or medical.
- Measurement in *sums at risk* (instead of numbers of policies).

In most cases, all these statistical relationships cannot be precisely measured or only can be partly measured. If that is the case, the most important factors might be the use of the mortality of the specific portfolio and the use of sums at risk as the amounts exposed. If an estimation of mortality using age-dependent factors per product type cannot be made (because the estimation cells are too small), it may be possible to use age-independent factors for product groups. For example, products can be broadly grouped into those with positive risk (term insurance, endowment, and permanent life) and negative risk (pure endowment and annuities).

Section 3.2 contains an example of how to translate population mortality table into an insured population mortality table.

2.2.2 The Trend

Because mortality rates change over time, it is important to account for this in the BEM assumption. For centuries, and especially in the 20th century, the human life expectancy has increased and should continue to increase in the future. Of course the questions are, for how long and at what level? An increasing life expectancy means decreasing mortality rates.

The historical decrease of the mortality rates is a result of positive and negative impacts on the health and mortality of humans. The change in mortality in the past is mainly caused by several factors, sometimes positive (+) and sometimes negative (−):

- Medical developments (+).
- Environment (+ or −).
- Behavior (+ or −).
- New diseases, like AIDS (−).

The change in historical mortality was not constant. Several changes in trends have occurred, including periods of increasing mortality rates for some age groups, for example, mid- to late-20s, as accidental deaths became the primary cause of death.

This all makes it difficult, if not impossible, to predict the future mortality over a long period. Although there is uncertainty, a prediction of a future trend is necessary. Several methods to predict the future mortality exist and all are based on analyzing historical data, with the addition of expert opinions (i.e., medical world expertise). Some of the methods available are (1) cause of death, (2) structure, (3) child mortality, (4) accidents, (5) constant part, (6) exponential part (aging), (7) general model (independent of cause of death), and (8) expert opinion.

The advantages and disadvantages of using each method as an indicator of future mortality are given in the following subsections.

2.2.2.1 Modeling by Cause of Death

- *Advantages:* High level of science and better analyses of observed effects.
- *Disadvantages:* Model hard to handle; difficult to get the data and limited observations for some cases; only a few years of predictions are possible; when a cause of death disappeared in the past, a new one always appeared (this is, of course, difficult to model).

2.2.2.2 Modeling by Structure

- *Advantages:* A good track on the mortality development is possible; a change in the structure of the mortality table is better to model; high level of science.
- *Disadvantages:* Lots of parameters have to be estimated; only a few years of predictions are possible; difficult to get the necessary data.

2.2.2.3 General Model

- *Advantages:* Simple to handle and, in most cases, needed data is available.
- *Disadvantages:* Low level of science and the past shows normal changes in trends.

2.2.2.4 Expert Opinion

- *Advantages:* Real medical changes in the future are not reflected in the figures of the past and expected shocks can be modeled.
- *Disadvantages:* No two experts expect the same outcomes with respect to future mortality; the opinion of the expert will be rather general, for example an increase of the life expectancy at age zero of x years; the result is not yet a mortality table.

In practice, combinations of the above methods can be used to estimate future trends. For example, the Dutch model is a general model with corrections for AIDS.

For all models, it is important to check the results. *Don't be just a statistical analyzer.* Always compare the results with already published results. Always check with experts and look over the border. Compare the results with results in other countries. Be aware when comparing the future mortality of several countries that the mortality is more likely to converge rather than diverge.

It is better to use raw population tables to predict future mortality trends. The use of industry tables or special tables for a certain product to establish trends is not advisable because changes in underwriting procedures and the insured population can disturb the model. Also, using smoothed tables based on Makeham models is not recommended to estimate trends. Mortality development using these models spreads special circumstances only applicable for a certain age group over the whole table.

Appendix A presents an alternative smoothing method. With this model, the structure of the original table stays intact, but the stochastic noise is smoothed away. Section 3 presents a simple model, based on a general approach, that can be used to estimate future mortality trends.

3. BEM, an Example

This model demonstrates a practical way to develop a BEM assumption. In this example, population mortality is used and the trend model is based on population mortality. The first step is to make a model for the future mortality because the result has to be used to estimate the level.

3.1 Model to Estimate Trend

We want a generation mortality table, where the mortality rates depend on age (x), gender and calendar year (t): $q(x; t)$. To estimate the trend for each age/gender separately, define a function, $f(x)$, such that the development of the mortality occurs in the following way:

$$q(x; t+1) = f(x) \times q(x; t). \qquad (2)$$

Therefore,

$$q(x; t+a) = f(x)^a \times q(x; t). \qquad (3)$$

The trend factor $f(x)$ can be estimated using historical data. As mentioned in Section 2, it is better to use raw population mortality data. This data can be smoothed using the algorithm in Appendix A.

It is useful, especially for smaller countries, to average several years (three to five years, depending on available data and size of the data set) in order to reduce the volatility between years because sometimes mortality in a country has some extra volatility attributable to cold winters, the flu, and so forth. By observing experience over several years, this extra volatility will be smoothed away, at least partially.

The factor $f(x)$ can be estimated in the following way:

$$f(x) = \sqrt[p]{\frac{q(x;\, t)}{q(x;\, t-p)}}, \tag{4}$$

where t is the year of the last available table, p is the number of years in the observation period and $t - p$ is the year of the last significant change in trend observed in the data.

3.1.1 Trend Analyses

To define a significant change in trend, several methods are available, depending on the data. Two methods are:

- *Spline functions:* Through the use of a statistical technique and a certain confidence level, past data can be analyzed. A significant change in trend is evident, if the qx's in a certain year are not within a certain confidence interval, based on an estimation using the observations from earlier years. The method requires the availability of sufficient data for each calendar year. This method can also be applied using life expectancies instead of mortality rates. The advantage of using life expectancies is that only one change of trend year is found instead of one for each age. Using qx's can provide several change of trend years.
- *Graphical analysis:* In this method, a change in trend is found by pictorial analyses rather than statistical analyses. Of course, this is less accurate than the method described above, and the result will include some subjective conclusions.

When adding new data annually the earlier estimated trend will change. If the change continues in the same direction there is an indication of a significant change in trend. Still it is not clear when the trend has changed.

Sometimes, good analyses of the data are not possible, for example, when only few data are available, or if data are only available over long intervals. This would occur if mortality rates were published only once in 10 years. In that case, it is possible to look at surrounding countries.

3.1.2 Generation Table

Using one of the methods described in 3.1.1, the t and $t - p$ from formula (3) are defined and $f(x)$ can be estimated. Using the analysis described in 3.1.1, a generation table can be derived.

There is the danger that if each country develops its own future mortality table, they will not be comparable. As mentioned before, the difference in mortality between several countries is expected to converge in the future, not diverge. Therefore the method developed for Europe starts with the local trend but grades into an average European mortality by 2040.

3.1.3 An Alternative Extrapolation Model for Mortality

In modeling the mortality trend in several countries, for example, the European countries, we must ensure that the final result for the separate countries should not differ too much. If we use a separate extrapolation model up to 2040 for each country, we will get tables with too much spread in the life expectancy. So, for countries with an unclear trend or those with not enough information, we determine that the mortality in 2040 will be the same as the average mortality in Europe. The present mortality is graded to the 2040 average mortality by interpolating, using a factor $f(x)$, such that

$$f(x) = \sqrt[2040-j]{\frac{q(x;\, 2040)}{q(x;\, j)}}, \tag{5}$$

where j is the year of the latest local table.

The generation table is defined by

$$q(x;\, j+t) = q(x;\, j) \times f(x)^t. \tag{6}$$

The problem with this method is that the trend, which is observed at the local level, is completely ignored. That is why another model is created in which the mortality development factor is time dependent, giving us $f(x,\, t)$. In this case, the generation table is defined by

$$q(x;\, j+t) = q(x;\, j) \times \prod_{i=1}^{t} f(x;\, j+i), \tag{7}$$

where $f(x; j)$ is the trend derived from the local observations

$$f(x; j) = \sqrt[a]{\frac{q(x; j)}{q(x; j-a)}}. \quad (8)$$

a is derived from an analysis of the latest change in trend in the observations ($f(x; j)$ can, in this case, be a "short-term trend").

The development of $f(x; j+i)$ is in such a way that

$$q(x; j+t) = q(x; j) \times \prod_{i=1}^{t} f(x; j+i) \quad (9)$$

and defining

$$f(x; j+i) = f(x; j) \times e^{i \times \alpha(x)} \quad (10)$$

so

$$q(x; j+t) = q(x; t) \times \prod_{i=1}^{t} f(x; j) e^{i\alpha(x)} \quad (11)$$

and

$$q(x; j+t) = q(x; j) \times f(x; j)^t \times e^{\alpha(x)t(t+1)/2}. \quad (12)$$

In Equation (12), it is clear that as t increases, the third factor will dominate the middle factor. In other words, the local trend, $f(x; j)$, will become less important as time goes on.

We can solve $\alpha(x)$ from Equation (12) by taking the logarithm

$$\log q(x; j+t) - \log q(x; j) = t \times \log f(x; j) + \alpha(x) \times \frac{1}{2} \times t \times (t+1)$$

$$\alpha(x) = \frac{\log q(x; j+t) - \log q(x; j) - t \times \log f(x; j)}{\frac{1}{2} \times t \times (1+t)}. \quad (13)$$

After doing this, some extra conditions (to control the model) can be added:

- If $q(x; j) < q(x; j+t)$, use for $j < p \leq j+t$: $q(x; p) = q(x; j)$
- If $q(x; j) > q(x; j+t)$ and $q(x; j+p) < q(x; j+t)$, use for $j+p \leq r < j+t$: $q(x; r) = q(x; j+t)$.

The results are that, in several cases, we analyzed the impact of this alternative approach to mortality trends. These cases depend on the difference between the local and average qx at this moment and the difference in local ($= f(x; j)$) and average trend. With the average $q(x)$ at this moment at 0.01, we analyzed the cases shown in Table 1. Figures 1–4 show the impact of this model.

3.2 Model to Estimate the Level

The level of mortality (this is the mortality for the specified insured group) will be expressed in terms of the entire population mortality using a correction factor on the qx's. Therefore,

$$q_{ins}(x; t) = fac(x) \times q_{pop}(x; t), \quad (14)$$

where $q_{pop}(x; t)$ is calculated using the methods mentioned under 3.1.

The factor $fac(x)$ can be found using observations in the most recent years. Use as much information as possible, but be sure that the observed populations in former years are similar to the recent population. The factor is the ratio between the loss because of mortality and the risk premium based on $q_{pop}(x; t)$. When $X_{i,j}$ is the possible loss for risk i in year j (so claim-reserve) and L_j is the actual loss in year j, we get (in this case, the factor in age-independent):

$$fac = \frac{\sum_j L_j}{\sum_j \sum_i q_{pop}(x; j) X_{i,j}}. \quad (15)$$

3.3 BEM Assumption

The BEM assumption can now be defined as

$$q_{BE}(x; t) = fac \times q_{pop}(x; t), \quad (16)$$

or, when it was possible to find an age-dependent level,

$$q_{BE}(x; t) = fac(x) \times q_{pop}(x; t). \quad (17)$$

Table 1
Examples

Case	Local Trend	International Trend	Local $q(x; j)$
1	0.99	0.98	0.012
2	1.01	0.99	0.012
3	1.01	0.99	0.008
4	0.98	0.99	0.012

Figure 1
Example Case 1

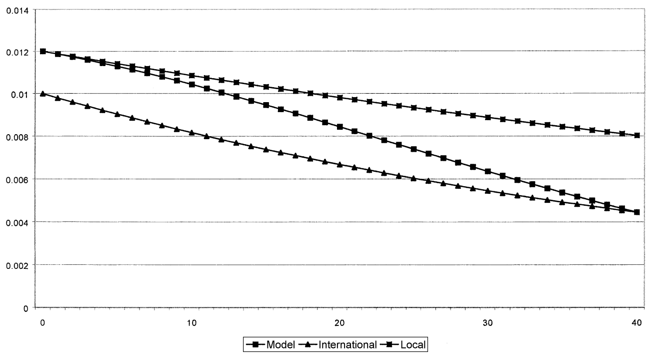
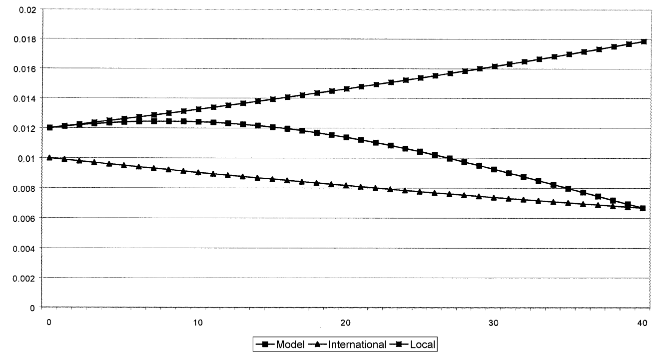


Figure 2
Example Case 2



4. THE MARKET VALUE MARGIN OF THE MORTALITY RISK

The literature indicates that MVMs are only added to the best estimate liabilities when the risk is not diversifiable. I would like to add the word "fully" to this definition. There is no MVM when the risk diversified fully away adds more of the same risk. Suppose that the standard deviation $\sigma(n;..;..)$ reflects n different risks. Then, in formulas

$$\lim_{n\to\infty} \frac{\sigma(n; \ldots ; \ldots)}{n} = 0, \qquad (18)$$

there is no MVM. The solvency margin and the economic capital cover these risks.

There are several ways to calculate the MVM or MVL: using the cost of capital, the β-method (known from the financial world), the PH-transform, and the Esscher transform.

I used an MVM based on a factor times the standard deviation of the underlying distribution. This factor led to a 90% one-sided confidence interval. Of course, this 90% depends on the risk aversion of the buyer and also the whole market situation at that moment (see Hardy and Panjer 1998).

Other than in the PH-transform and Esscher methods, the distribution itself doesn't change because of adding an MVM. The MVM in the method presented in this paper can be seen as just consisting of a buffer.

5. DEFINITION OF THE MORTALITY RISK

Mortality risk can be divided into four sub risks: volatility, calamity, uncertainty level, and uncertainty trend.

5.1 Volatility

Volatility is the risk that actual size of claims differs from expected, assuming the estimated parameters (in the BEM) is true. Volatility is the

Figure 3
Example Case 3

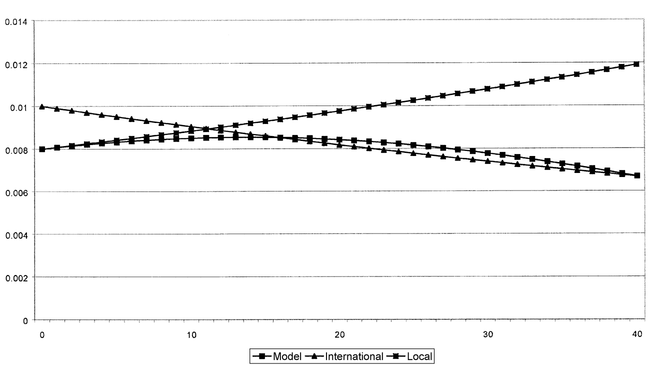
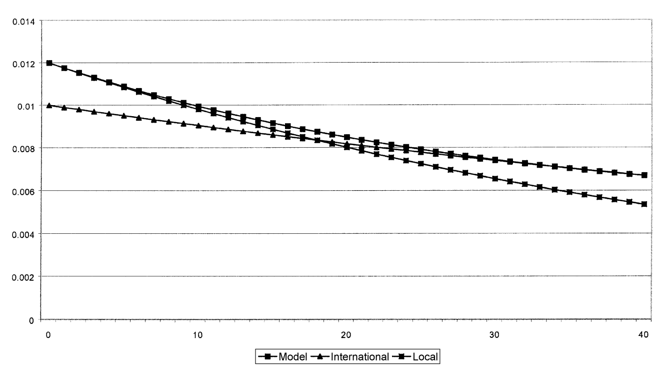


Figure 4
Example Case 4



result of the randomness of the claim process. Normally it is assumed that individual claims are mutually independent. This allows the assumption that the number of deaths can de drawn from a binomial distribution, which simplifies the modeling process. In reality, the number of claims is not completely independent when people are exposed to the same risk. Examples include more than one person in a car accident or plane crash, cold winters, and small epidemics.

These nonindependent risks are difficult to model. Therefore, I recommended using a somewhat more dangerous distribution that has a fatter tail. An example of such a distribution around the number of claims would be a Poisson distribution.

Another important factor to consider is the spread of the sums at risk (net amounts at risk). This can have a huge impact on the volatility.

5.2 Calamity

Calamity is the risk of a one-time claim level of extreme proportions attributable to a certain event. This event will not change the parameters directly, but can be seen as a one-year shock. Examples of calamity risk are:

- Epidemics like the Spanish Flu in 1918. Millions of people died all over the world because of this disease. In the United States alone, an estimated 500,000 extra deaths occurred.
- Natural catastrophes, such as an earthquake, flood, or meteorite strike.
- Terrorist attacks with biochemical or chemical weapons.

This kind of risk is difficult to model because there are a very limited number of observations. Therefore, we can not estimate the capital by looking at the tail of any mathematical distribution. We must on expert opinions for this.

5.3 Uncertainty Level

Estimating the right level of mortality (see Section 3.1) will bring an uncertainty with it. Observations needed to produce the estimations will be volatile. This volatility (in the past) makes it difficult to find the right level. One rule is: The more observations we have, the smaller this uncertainty will be. Because uncertainty level is, in fact, a result of volatility in the past, the distribution used to estimate this uncertainty is taken from existing mathematical distributions and are similar to the volatility models. In calculating margins from the uncertainty level, it is also important to consider the extra risk because of the spread of the sums at risk.

5.4 Uncertainty Trend

In the estimation of the trend, there will be uncertainty. As shown in Section 2, the best estimate trend is based on one of the past trends or can be a partial average of the trends in the past. Medical developments and other factors can make an estimated trend obsolete. One example is finding a cure for a (at this moment) fatal and frequent disease, which will cause a downward shock in the mortality rates.

The only possibility we have to model the uncertainty trend is to use several possible trends experienced in the past. A problem in modeling the trend uncertainty is the fact that the trends used in the different ages will be correlated in a rather complex way. To avoid this complexity, the liabilities are modeled, not the qx's. By modeling the liabilities, the correlation between the ages will automatically be included in the model.

6. THE MARKET VALUE MORTALITY RISK

6.1 The MVM Volatility

If we take out all the extra fluctuations, like the impact of cold winters and calamity, the limit of formula (18) will go to zero. The volatility, calculated in this way, will not have an MVM; the solvency margin and the economic capital will cover the volatility.

6.2 The MVM Calamity

It is difficult to find the existing mathematical distributions for the calamity risk because there are only a limited number of observations. The only real calamity the civilized world experienced in the last century was the Spanish Flu in 1918. That epidemic doubled the expected number of deaths for age groups important to insurance companies. Medical and reinsurance experts have indicated that such a calamity could occur only once in every 100-year period.

Of course, the real risk depends on the geographic location and the geographic spread of the risks. The MVM can be based on the rates provided by reinsurers or calculated as an average market price to hold the money needed (cost of capital) for the calamity risk (for example, 10% over the impact of 100% extra mortality).

6.3 The MVM Level

Because of the volatility observed in the past, there is never a 100% certainty that the level of the BEM is correct. As with volatility, a mathematical distribution, compound Poisson can be defined and used to calculate the MVM for uncertainty level.

Let's look at a method to determine the MVM for uncertainty level using the compound Poisson distribution based on a 90% confidence interval. The level uncertainty is nothing more than a translation of a possible mistake in estimation caused by the volatility in observations from the past.

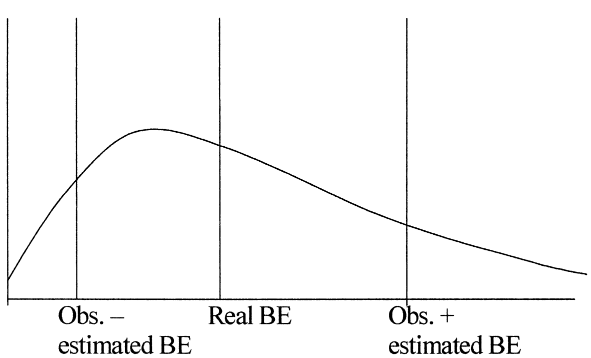
We are concerned that the results we have observed in the past are not representative of true mortality. To capture a nearly worst-case scenario, assume that the results are those obtained in Figure 5 at the 10^{th} or 90^{th} percentile (whichever is the adverse result for the given situation). Further assume that the true mortality rates are a constant multiple f of the observed rates.

For each life let X_i be the sum at risk and let I_i be zero if the life lives and one if the life dies. Total losses are then $\sum_i I_i X_i$. The expected value of the total losses is $\sum_i q_i(x) X_i$, where $q_i(x)$ is the "true" probability for life i.

Assuming the number of deaths is Poisson distributed (Poisson is a bit conservative and leads to simple answers), the total loss will be compound Poisson distributed.

Figure 5

Best Estimate Could Be Wrong



In actuarial theory, there are several ways to calculate a 90^{th} percentile in a compound Poisson distribution. Some examples are: Panjers recursion; the use of a shifted gamma, Esscher approximation, and the normal power approach.

Here we will use the normal power approach. Special in the case of the compound Poisson distribution, it leads to very simple formulas and, in most cases, to accurate results. The normal power approach is based on the Cornish-Fisher expansion. (See Kendall and Stuart 1977). In most cases, we use three moments; in very skewed distributions, it is advisable to use more (e.g., four) moments. The model in this paper is based on three moments.

The normal power approach uses the first three central moments of the distribution to compare with the standard normal distribution.

$$P\left[\frac{S-\mu}{\sigma} \leq s + \frac{\gamma(s^2-1)}{6}\right] \approx \Phi(s). \quad (19)$$

In case of a one-sided 90% confidence interval: $\Phi(s) = 90\%$, so $s = 1.28$ and $\frac{s^2-1}{6} = 0.11$. Therefore, where c represents the total loss because of mortality,

$$c_{obs} = c_{real} + \sigma_{real}(1.28 + 0.11\gamma_{real}). \quad (20)$$

The "real" parts in this formula are, of course, not known yet so we have to translate them into known figures. Therefore, we define a translation factor f, which is *age independent,* to translate known mortality rates into the "real" ones. In the following, X is the loss attributable to the death of an insured person:

$$\begin{aligned} c_{real} &= \sum q_{real}(x)X \\ &= f \sum q_{obs}(x)X. \\ &= fc_{obs} \end{aligned} \quad (21)$$

Also, the best estimate mortality (see Section 2) derived from the observations can be used.

As known from the compound Poisson distribution:

$$\sigma_{real} = \sqrt{\Sigma\, q_{real}(x)X^2} = \sqrt{f\, \Sigma\, q_{obs}(x)X^2} = \sqrt{f}\, \sigma_{obs}. \quad (22)$$

The σ_{obs} can be calculated using the population mortality.
Also:

$$\gamma_{real} = \frac{\Sigma\, q_{real}(x)X^3}{\sigma^3_{real}} = \frac{f\, \Sigma\, q_{obs}(x)X^3}{(\sqrt{f}\sigma_{obs})^3} = \frac{\gamma_{obs}}{\sqrt{f}}. \quad (23)$$

Then using formula (20),

$$c_{obs} = fc_{obs} + \sqrt{f}\sigma_{obs}\left(1.28 + 0.11\,\frac{\gamma_{obs}}{\sqrt{f}}\right)$$
$$0 = fc_{obs} + 1.28\sigma_{obs}\sqrt{f} + (0.11\sigma_{obs}\gamma_{obs} - c_{obs}). \quad (24)$$

The $\sqrt{f}$ can be solved using the quadratic formula. Define

$$D = \sigma^2_{obs}1.28^2 - 4c_{obs}(0.11\sigma_{obs}\gamma_{obs} - c_{obs}).$$

$$\text{Then,} \quad f = \left\{\frac{-1.28\sigma_{obs} + \sqrt{D}}{2c_{obs}}\right\}^2. \quad (25)$$

If the unwanted observation is in the left tail (as in case of positive risks) we should use −1.28 instead of 1.28. All the numbers needed in the formula can easily be found using the portfolio and a known mortality table.

Using f we can calculate the MVL by transforming the qx's as follows:

Table 2
Example Observation of Level Mortality (Whole Population: q_x = 0.007)

	Observed Mortality Rate		
Portfolio	1	2	3
Observation	0.006797	0.005789	0.004543

Table 3
Example of Level Uncertainty

Caculation Level Uncertainty				
	Portfolio	1	2	3
	σ/μ	0.038	0.044	0.138
	γ	0.038	0.049	0.155
	Observed	0.971	0.827	0.649
	MVL Neg risk	0.925	0.777	0.519
f	MVL Pos risk	1.020	0.879	0.805

$$q_{mvl}(x;\, t) = f \times q_{obs}(x;\, t). \quad (26)$$

As an example, three different portfolios are created with a simulation approach. Each of these portfolios will have its own risk profile. Portfolio 1 has 100,000 risks, all the same insured sum; portfolio 2 has 100,000 risks, insured sum uniform spread between 1 and 1,000; and portfolio 3 has 100,000 risks, 90% insured sum between 1 and 1,000, 10% between 1 and 10,000,000.

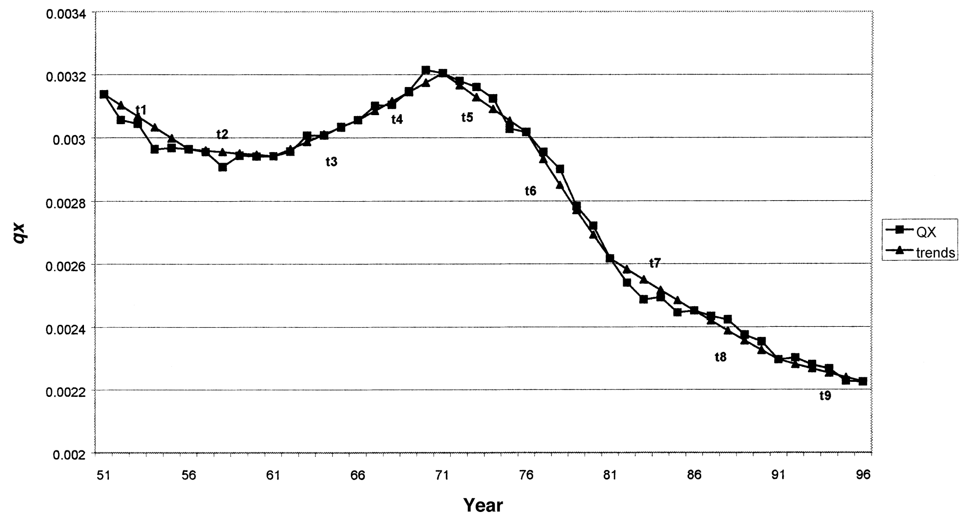
The average population mortality rate is 0.007. Several deviations from the average mortality depending on sum insured are included. The observation simulation over one year gives us the results in Table 2. The results of the calculations are shown in Table 3. It is clear that the MVM (difference between observed and MVL) is highest when using the fat tailed portfolio 3.

6.4 The MVM Trend

There are several ways to calculate an MVM for the uncertainty trend. As mentioned previously, it is difficult to calculate the MVM for an uncertainty around the separate mortality rates because of the complex correlation between these mortality rates. Therefore, it is advisable to calculate MVM for the uncertainty trend over the liabilities by calculating the impact of other possible trends on the liabilities.

To find the other possible trends, analyze the history. For example, suppose we have yearly mortality data from 1950 to 1995 (see Figure 6). We base the best estimate trend on the average trend between 1980 and 1995. Within the 45 years of observations, nine separate trends over five-year periods are observed, that is, the average

Figure 6
Example of Several Observed Trends (Male age 45)



trend between 1950 and 1955 ($i = 1$); 1955 and 1960 ($i = 2$), and so forth.[1]

Similar to the method used to calculate the best estimate mortality assumption, we find nine sets of factors: $f_i(x)$ ($i = 1$ to 9). With each set $f_i(x)$, a generation table can be calculated: $q_i(x; t + a) = f_i(x)^a \times q_{be}(x; t)$. With each generation table, i liabilities ($liab$) can be calculated. This results in nine different liabilities.

With these nine liabilities, a standard deviation can be calculated:

$$s_{trend} = \sqrt{\frac{9}{8}\left\{\left(\frac{1}{9}\sum_i liab_i^2\right) - \left(\frac{1}{9}\sum_i liab_i\right)^2\right\}}. \quad (27)$$

The trend uncertainty calculated is defined to be a Student t distribution with 8 degrees of freedom. This distribution is used because of the limited number of observations and the fact that a rather fat tail is needed to account for possible medical developments such as a cure against an important nowadays-deadly disease. In the Student (8) distribution the 90% confidence interval is based on 1.40 standard deviations. This gives:

$$MVM_{trend} = 1.40 \times s_{trend}. \quad (28)$$

[1] It is better to take just multiyear periods, rather than periods between significant changes in trends because the latter will overestimate the trend uncertainty.

Table 4 shows a simple example where this technique is used to calculate the uncertainty trend around a net-single premium for some products. The calculation is done for a term insurance, a pure endowment and an endowment (i.e., the sum of term and pure endowment). The effect of netting between the (positive) term insurance and the (negative) pure endowment is clear to see by summing the result of the two.

Table 4
Simple Example of Trend Uncertainty

Trend	1 Term	2 Pure End	1 + 2
51–56	0.091591	0.385952	0.477544
56–61	0.093998	0.383763	0.477761
61–66	0.108993	0.371087	0.480081
66–71	0.093744	0.384229	0.477973
71–76	0.082652	0.393559	0.476211
76–81	0.072914	0.401589	0.474502
81–86	0.076196	0.398958	0.475154
86–91	0.071073	0.402989	0.474062
91–96	0.070131	0.403957	0.474088
BE	0.070568	0.403498	0.474066
SD	0.013319	0.011226	0.002097
1,4*SD	0.018647	0.015717	0.002936←=MVM
BE + MVM	0.089215	0.419215	0.477002←=MVL
MVM/BE	26.4%	3.9%	0.6%
MALE AGE	45	45	45
DURATION	20	20	20
D.o.F	8		
Conf. Int.	90%		
Student	1.4		

REFERENCES

HARDY, M., AND H. PANJER. 1998. "A Credibility Approach to Mortality Risk," *ASTIN Bulletin* 28(2): 269–83.

KENDALL AND STUART. 1977. *The Advanced Theory of Statistitics,* Vol. 1. 4th ed. London: Charles Griffin.

APPENDIX A

THE VAN BROEKHOVEN ALGORITHM

In the actuarial world, several methods are known to smooth rough mortality tables. The method presented in this paper was made in the Netherlands in 1992 to smooth rough mortality rates for the purpose of extrapolating mortality rates into the future.

This algorithm is designed to smooth raw mortality date without losing the structure of the mortality table. A traditional smoothed table (for instance, with Makeham), will have lost part of the structure of the original raw table. This is because using Gompertz or Makeham functions increasing exponential functions are estimated over the whole or great part of the age range. In reality, mortality rates don't behave exponentially; they even can decrease over some small age groups.

One example is that the accident hump for males between the ages 20–25 leads, in some countries, to a decreasing mortality during some years above age 25. The Makeham function would lead to increasing mortality rates over all ages. Still, the development of mortality in time will be different in the range of ages where the accident hump appeared, compared to the other age groups.

The algorithm described in this note keeps deviations like the accident hump intact. Also, the resulting deviation of the mortality trend in this special age group will be more accurate. The algorithm is based on a moving average. However, the average will not be taken just linearly, but weighted with an exponential function of the second degree and after a transformation. In this way, a better fit to the original raw data results.

Starting with the raw mortality observations for age x and calendar year t, which are defined as $QR(x, t)$, we get the following transformation from the Makeham or Gompertz methods:

$$f(x, t) = \log[-\log\{1 - QR(x, t)\}]$$

We define the matrix $\boldsymbol{X}$ as

$$\begin{bmatrix} 1 & x-5 & (x-5)^2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & x+5 & (x+5)^2 \end{bmatrix}$$

and $\boldsymbol{Y}$ as

Figure 7

Smoothing Methods

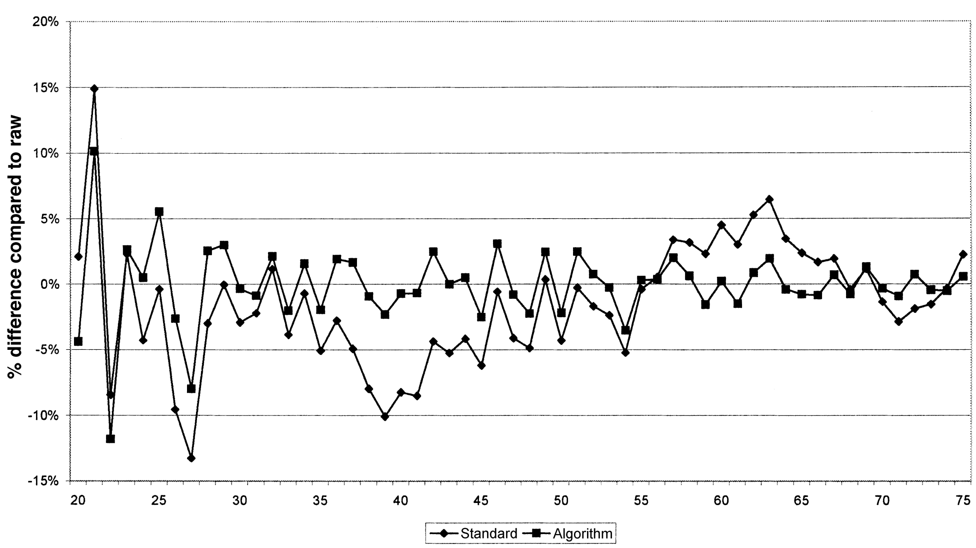
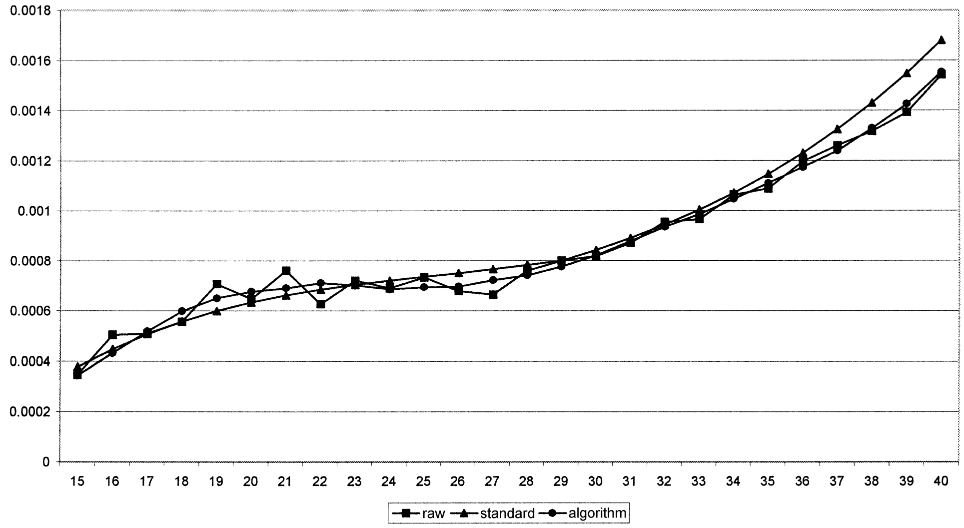


Figure 8
Smoothing Tables



$$\begin{bmatrix} f(x-5, t) \\ \cdot \\ \cdot \\ \cdot \\ f(x+5, t) \end{bmatrix}$$

Then $f'(x, t)$ follows:

$$\hat{f}(x, t) = [1, x, x^2](X'X)^{-1}X'Y$$

and

$$q(x, t) = 1 - e^{-e^{\hat{f}(x,t)}}.$$

In other words, the whole process can be seen as fitting a second-degree polynomial by least squares to the 11 observations surrounding the point of interest. Then the fitted value replaces the observed value. Using the raw mortality data for the Netherlands over the period 1991–95, this smoothing method is shown in Figures 7 and 8. Clearly, there is a high number of "changes of signs," and the smoothed data still follows the original structure of the raw data.

In the Figure 7 the difference (%) between the original raw data and the smoothed data is presented both for the algorithm and the standard-smoothing method, based on Makeham. In Figure 8, for the ages 15–40, the raw mortality rates and the smoothed mortality rates are given. Another test of how the smoothed data fit the original raw data can be seen in the calculation of life expectancy (see Table 5). Of course, the difference must be as small as possible.

Table 5
Test Fit of the Smooth Data Using Life Expectancy

Age	Life Expectancy Male (the Netherlands, 1991–1995)		
	Raw	Standard	Algorithm
0	74.240	74.808	74.245
45	31.274	31.303	31.275
65	14.599	14.590	14.596
90	3.373	2.932	3.373

Discussions on this paper can be submitted until October 1, 2002. The author reserves the right to reply to any discussion. Please see the Submission Guidelines for Authors on the inside back cover for instructions on the submission of discussions.