Mortality forecasting using logistics equation

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Budapest, 27.04.17

Problem

Well known methods for forecasting usually take into account the "internal" information obtained form historical as well as from experimental data. Assumptions being based on analysis of tendency often became with time incorrect due to:

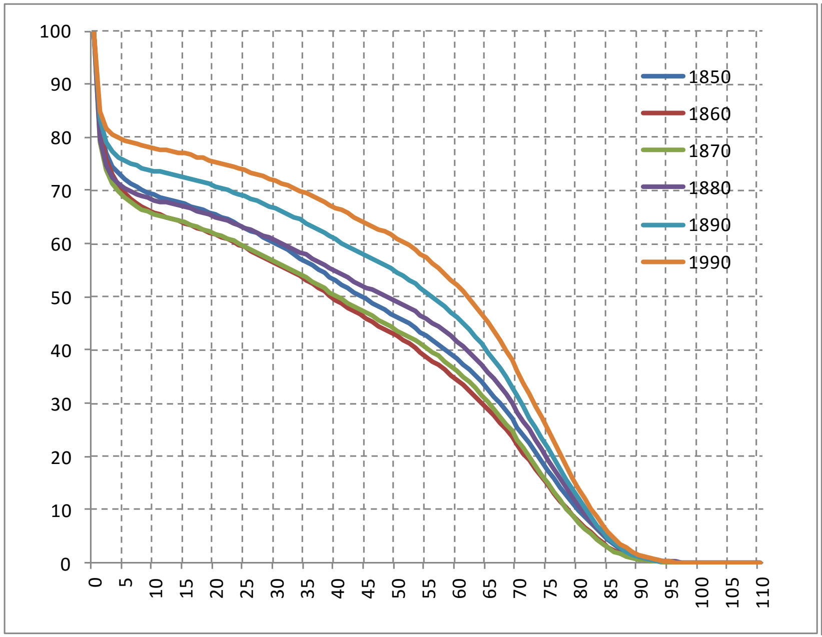
- **Appearing of the new factors which can break the tendency;**
- **Changing influence of some parameters with time due to non-linear solution;**
- **etc.**

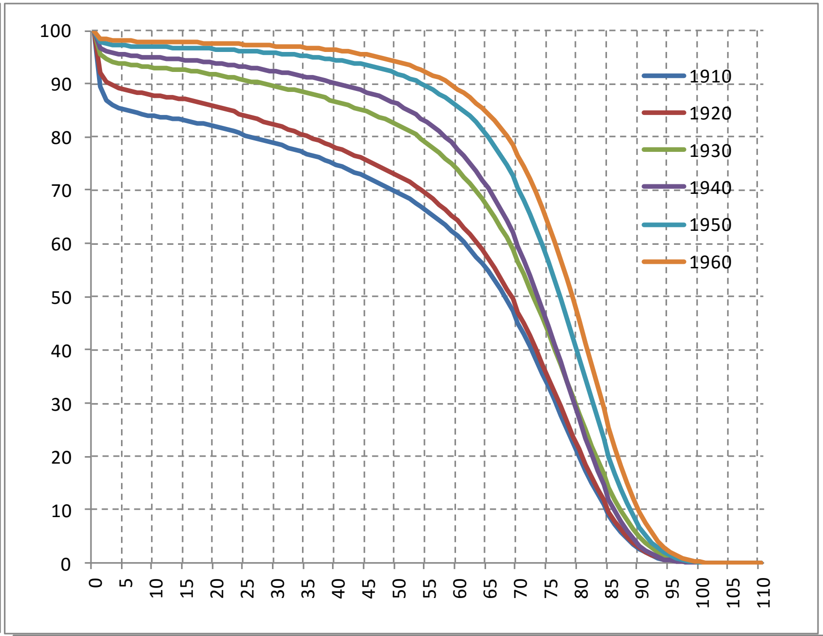
Idea

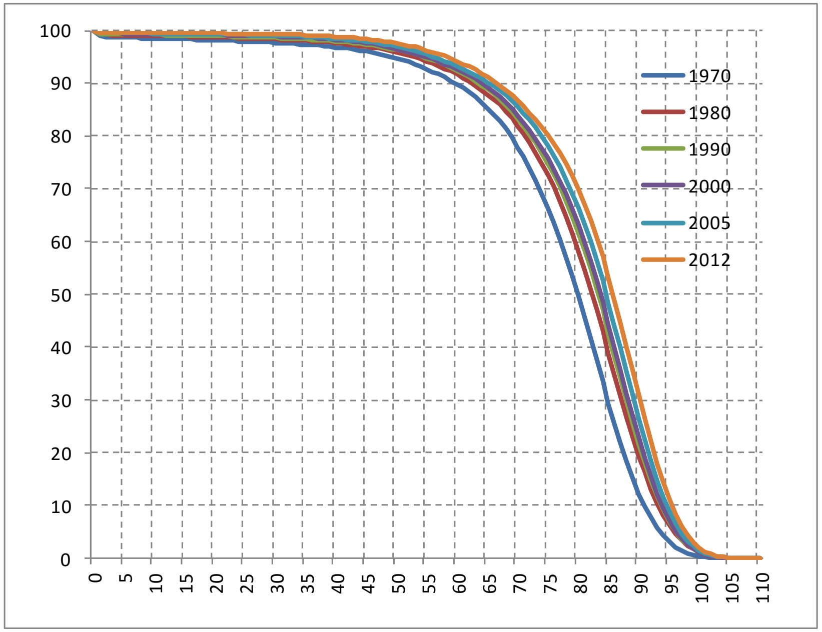
Why not to search the analogue in another areas of our knowledge? Let us try to find the similar shape of the survival function among well-known solutions and models. For example, one can see that solution of logistics equation has similar shape as survival function. Is it possible playing with coefficients of logistics equation to close its solution to the shape of survival?

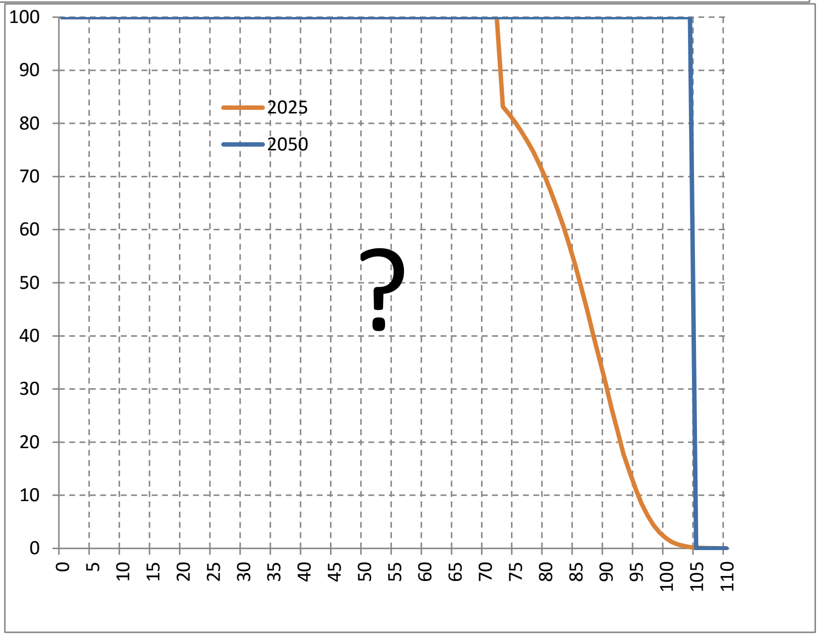
Using as example historical data dynamics for females of Holland population (www.mortality.org) coefficients of logistics equation were found for criterion function and forecast of life expectancy was built.

Historical data lx (Holland, Females)

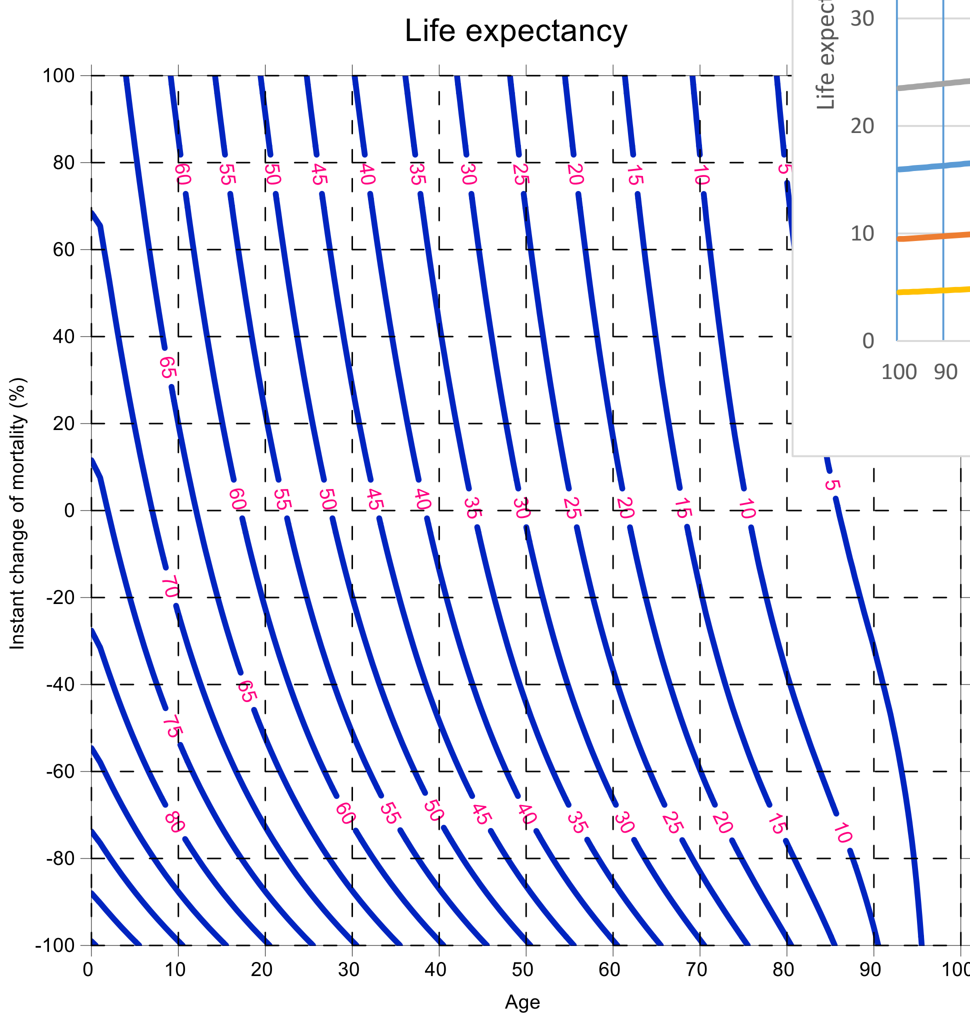


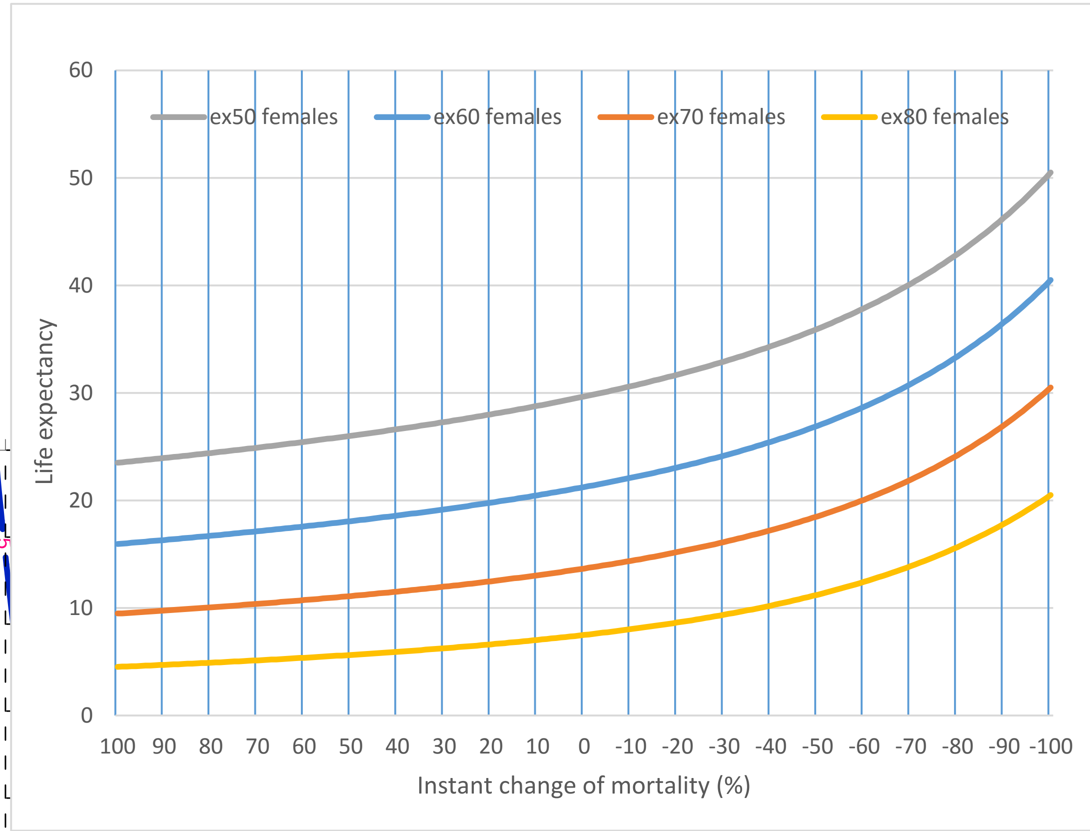






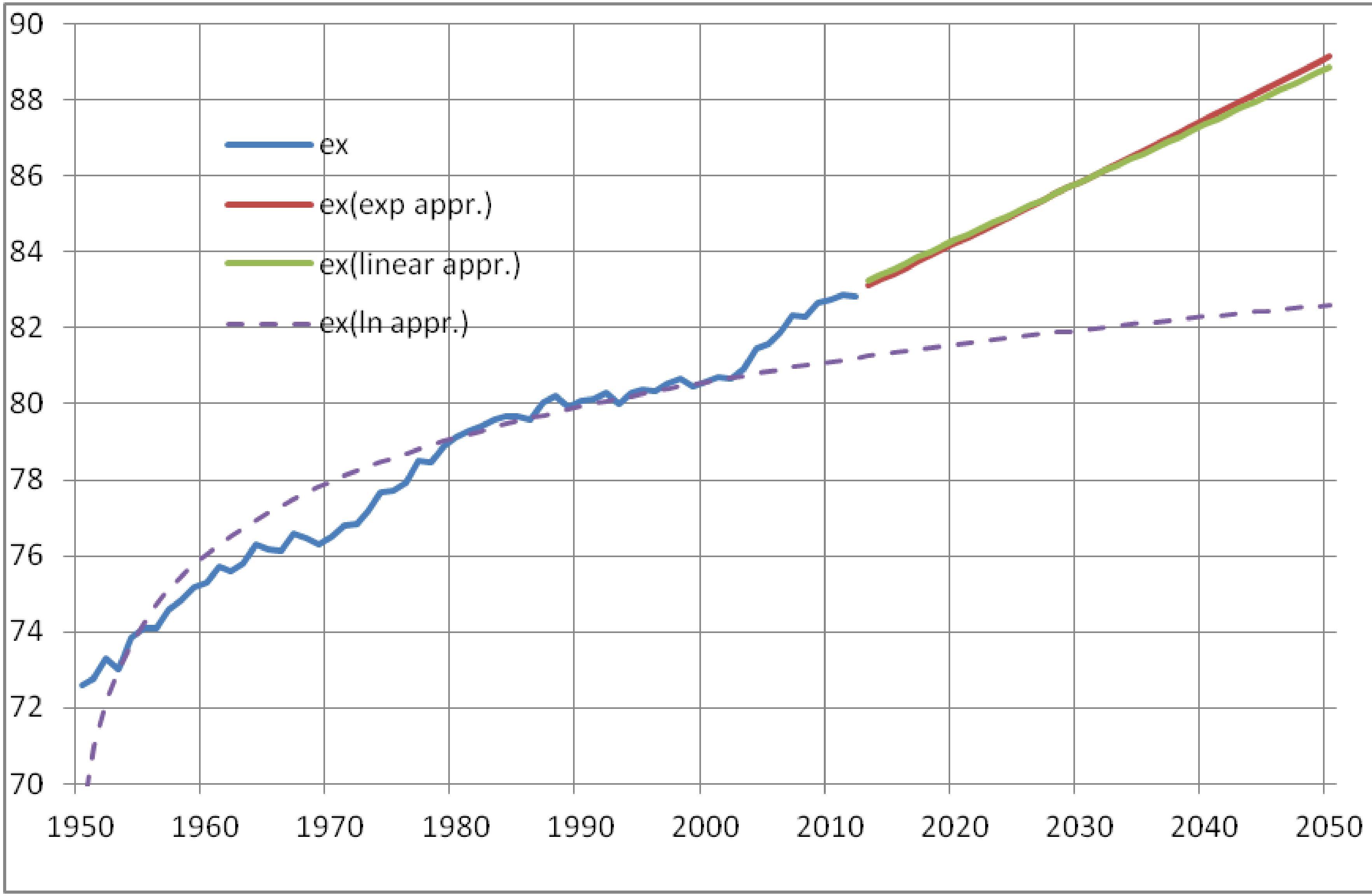
Does the limit of life expectancy exist?



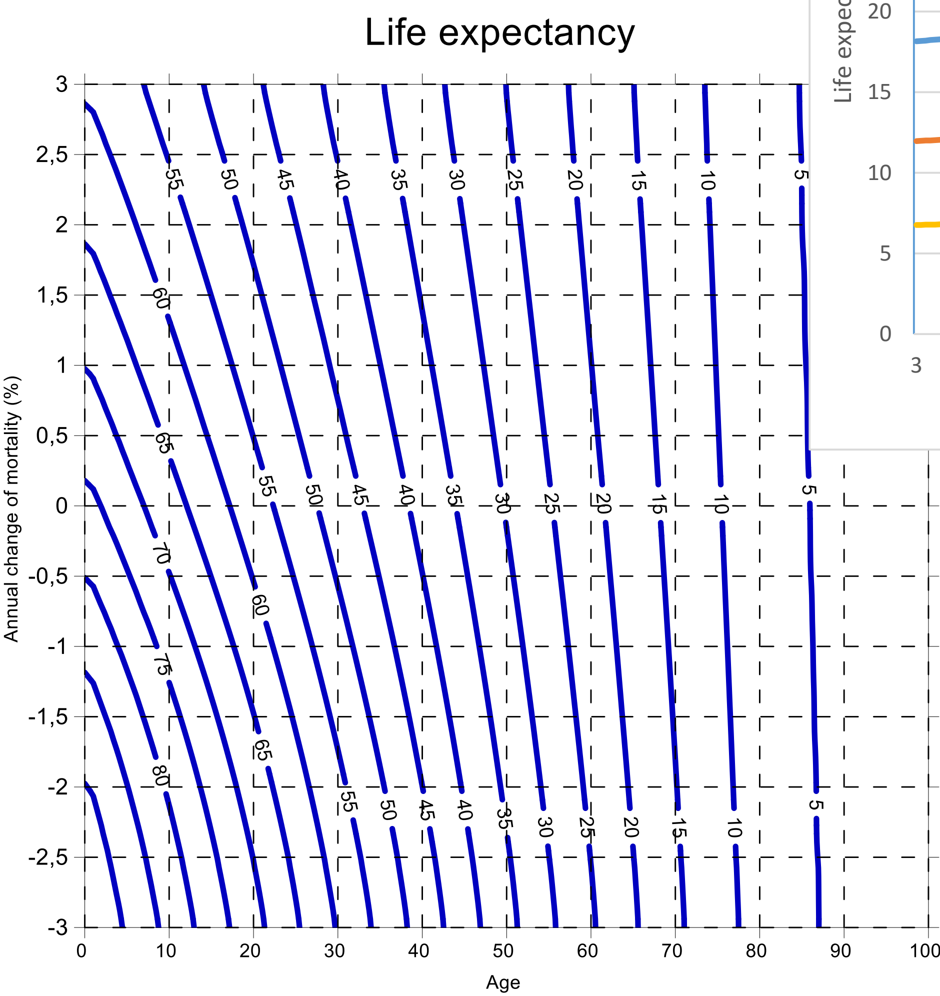


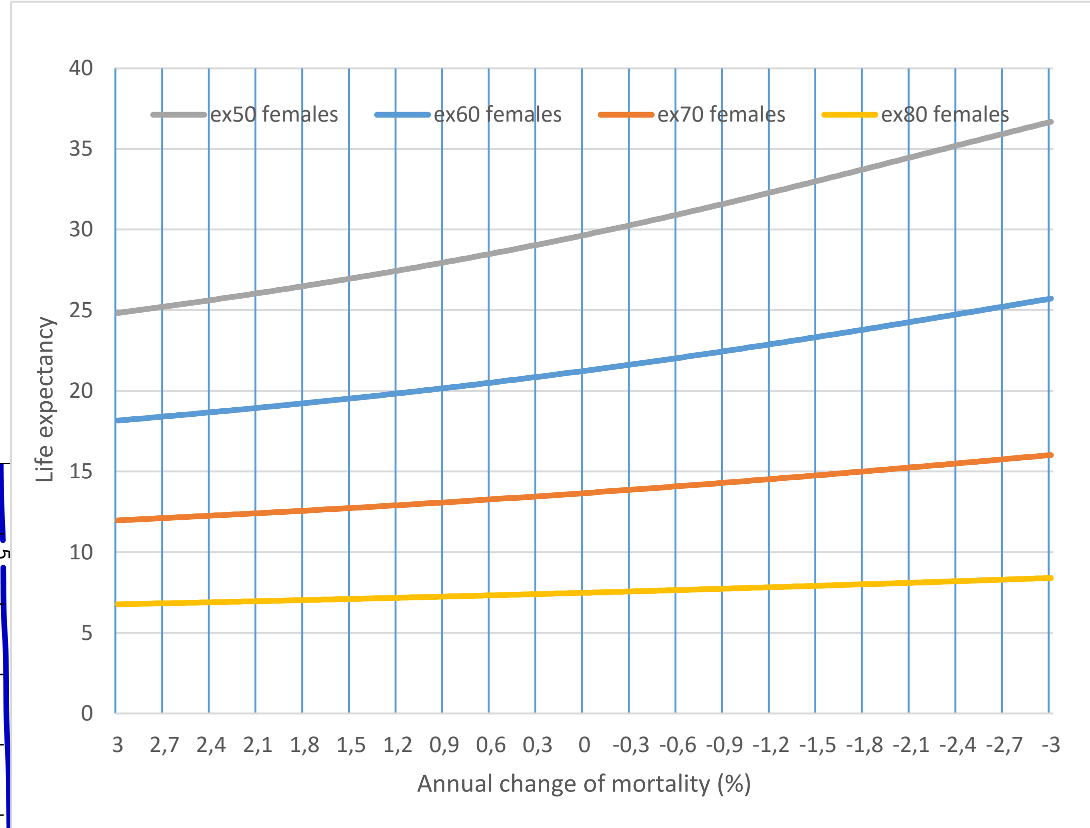
How long could be continued effect of mortality improvement?

Forecast of e_0, using trend models (females, Holland)



Consequences of annual changes of mortality rate in old ages are more real.





But for young ages it looks like a challenge.

Survival function & logistics equation

$$\frac{dS}{dt} = -k * S$$

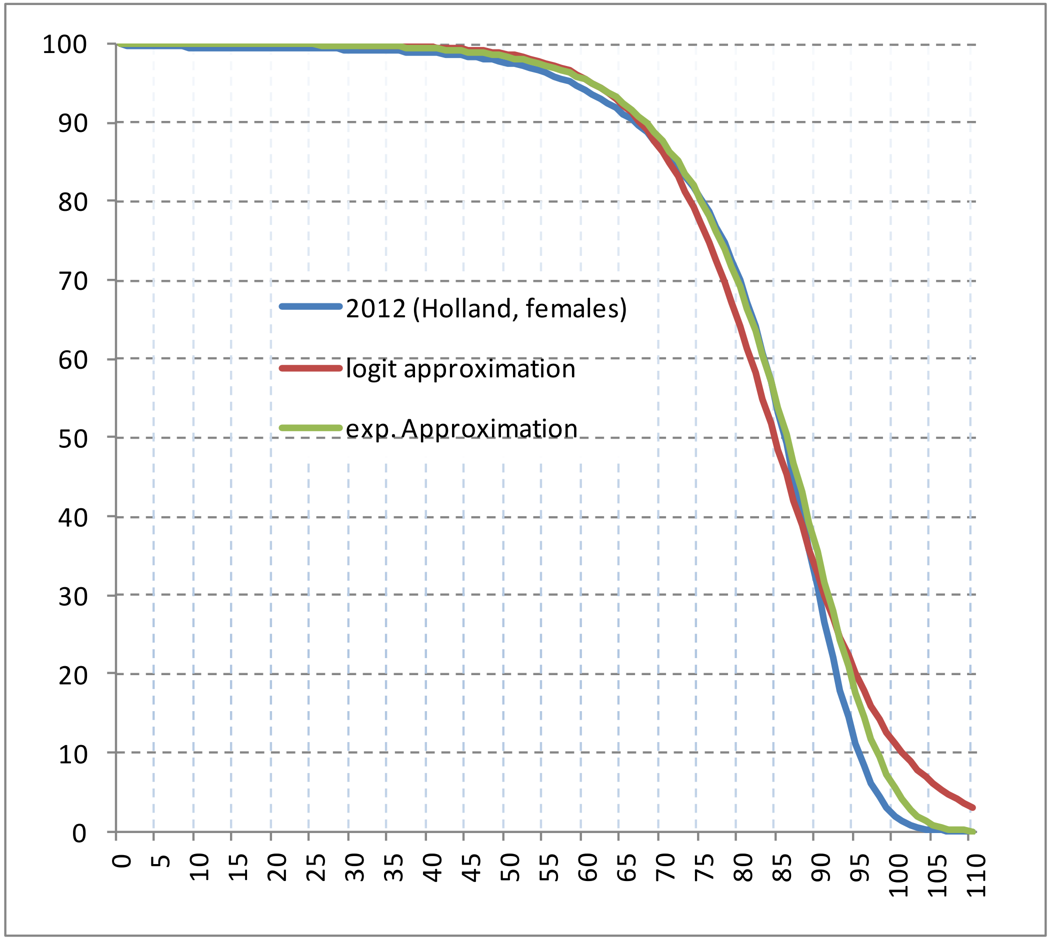
$$k = k_0 * \exp\ (a * x)$$

$$S = S_0 * \exp\left[\frac{k_0}{a} * (1 - \exp(a * x))\right]$$

$$\frac{dD}{dx} = a * D - b * D^2,$$

D- number of died

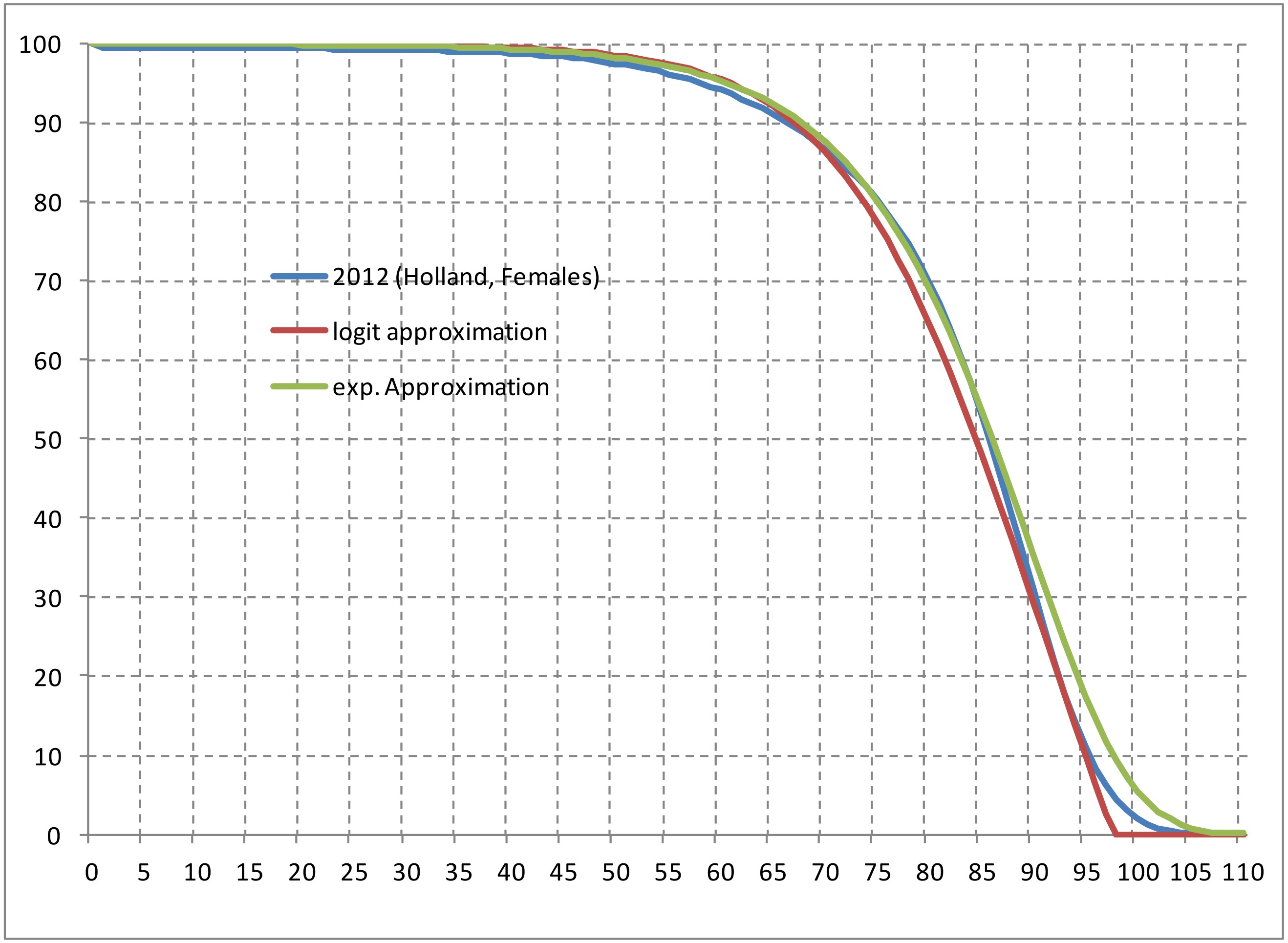
$$D = \frac{a * D_0 * \exp(a * x)}{a - b * D_0 + b * D_0 * \exp(a * x)}$$



Modified logistics equation

To eliminate the "tail" effect one more parameter was added.

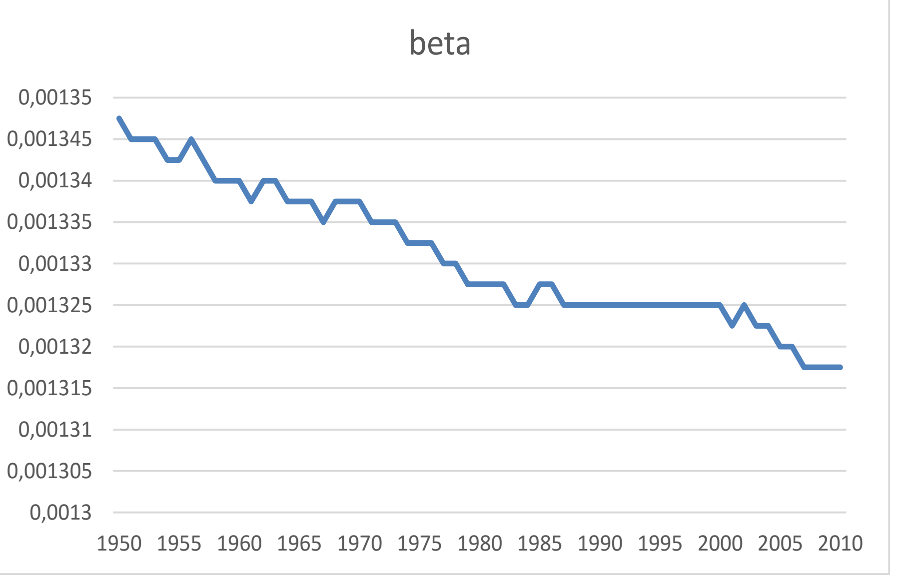
$$\frac{dD}{dx} = a * D - b * D^2 + c * D^3$$

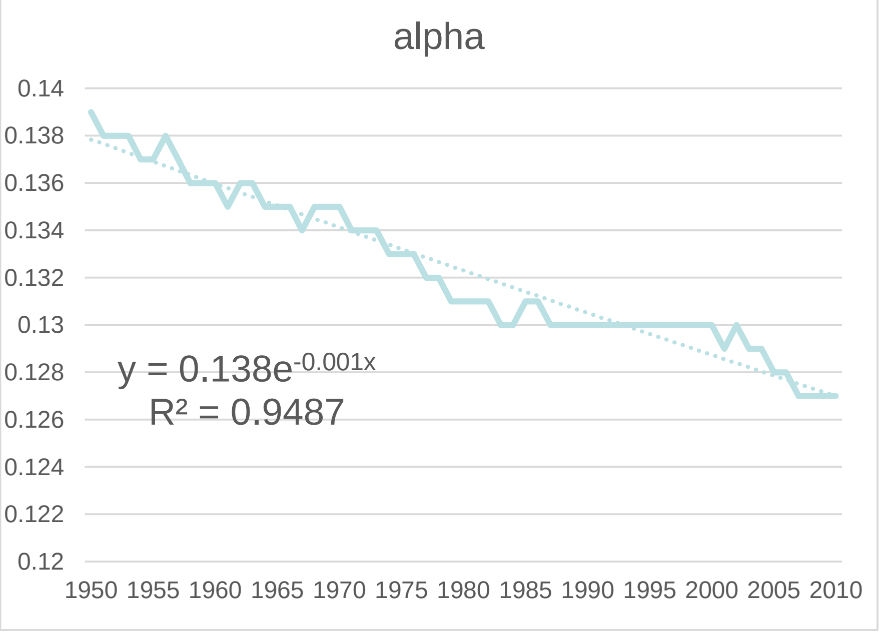


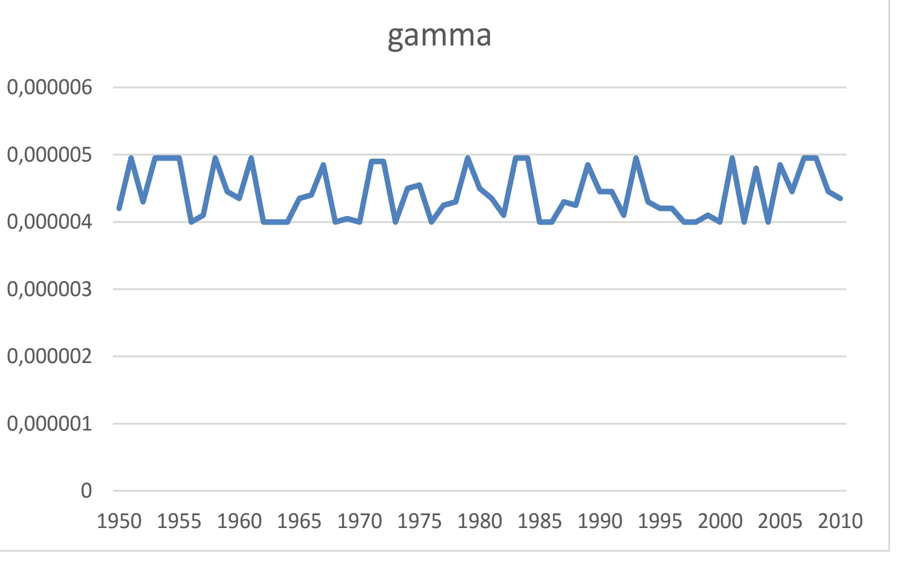
Numerical results

$$\frac{dD}{dx} = alpha(t) * D - beta(t) * D^2 + gamma(t) * D^3$$

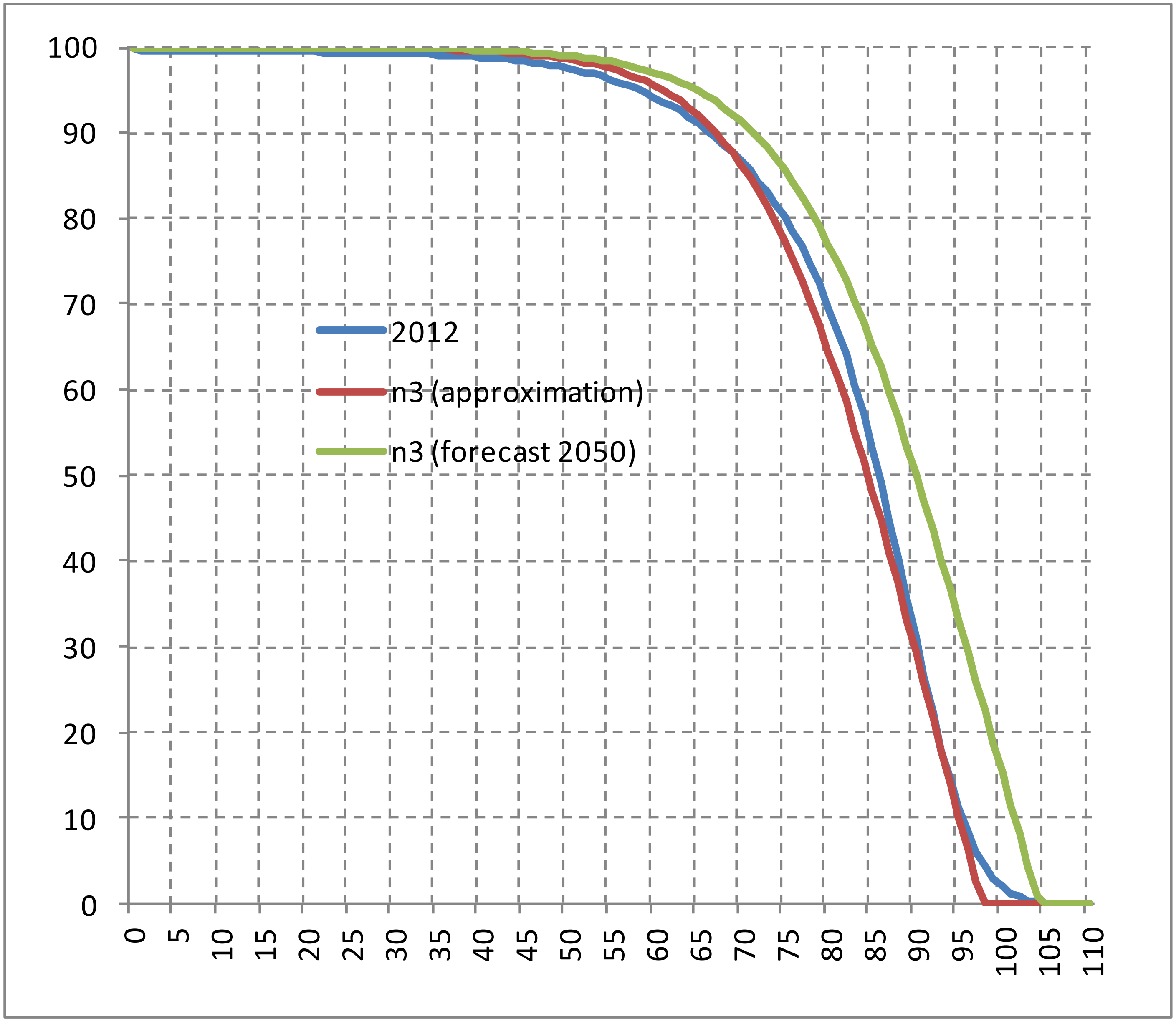
$$\sum_{x=0}^{w} (e_x - e'_x) = min$$



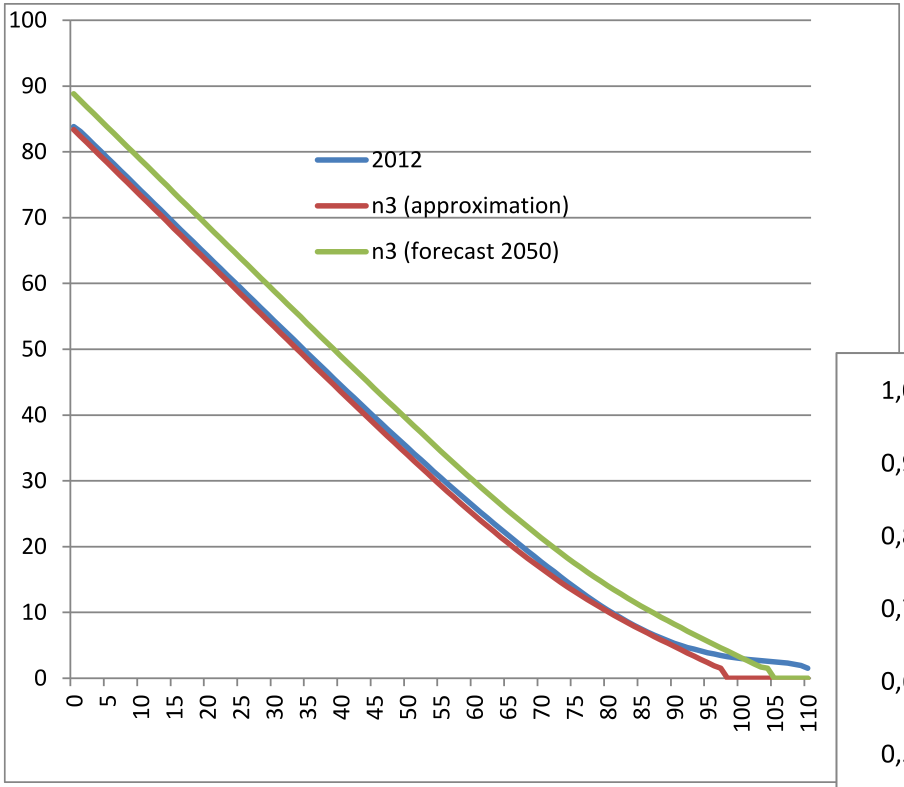




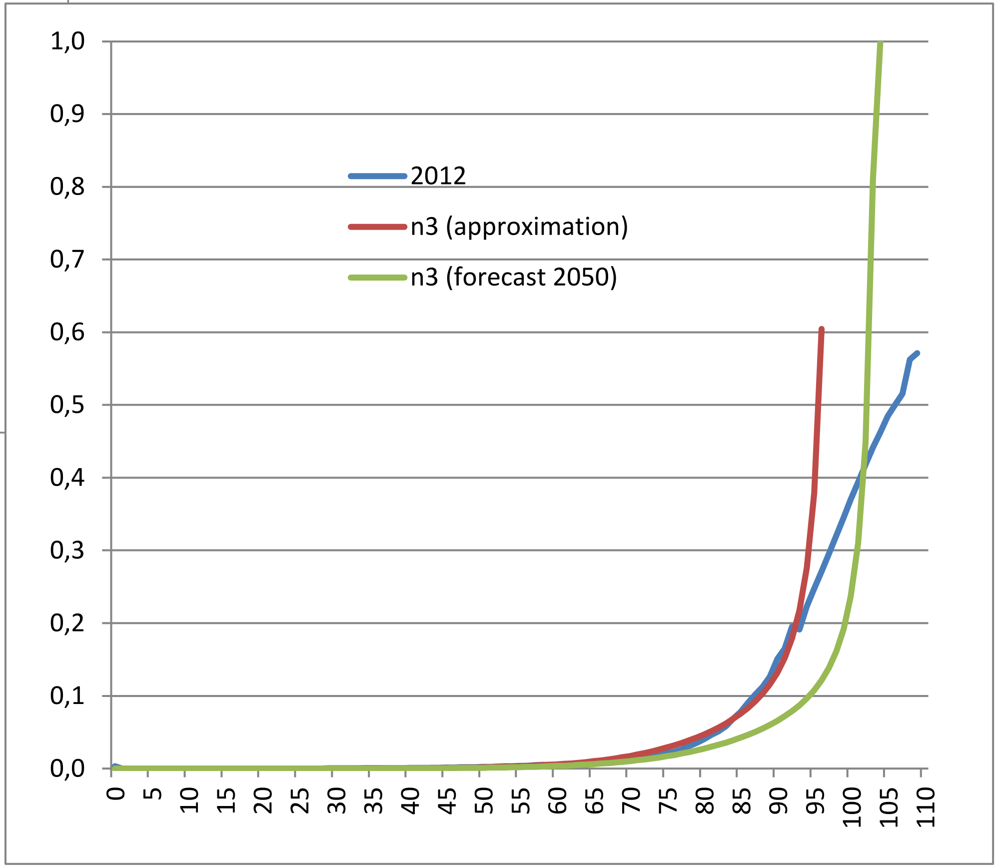
Result of forecasting (Lx)



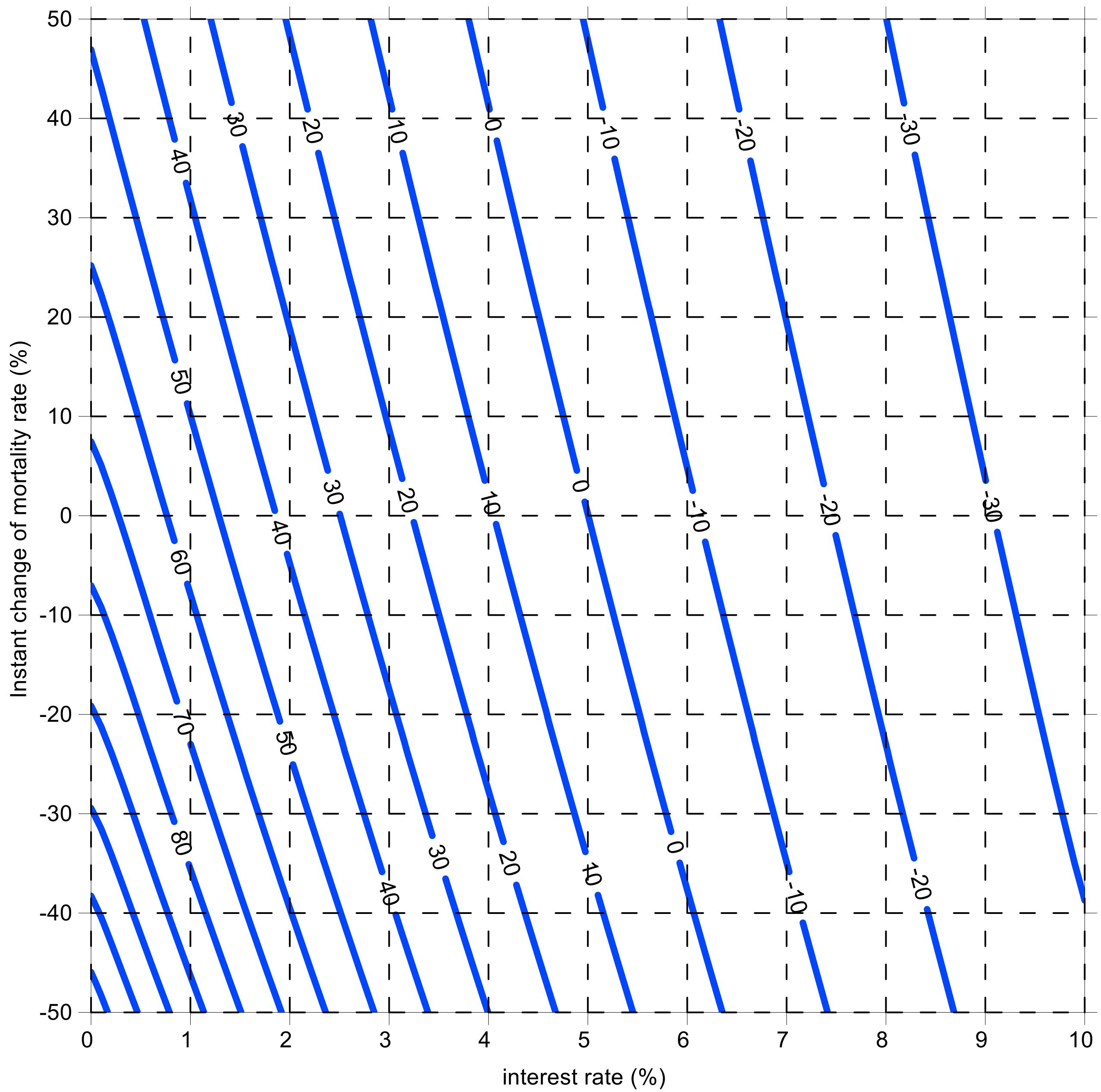
Result of forecasting (ex)



Result of forecasting (qx)

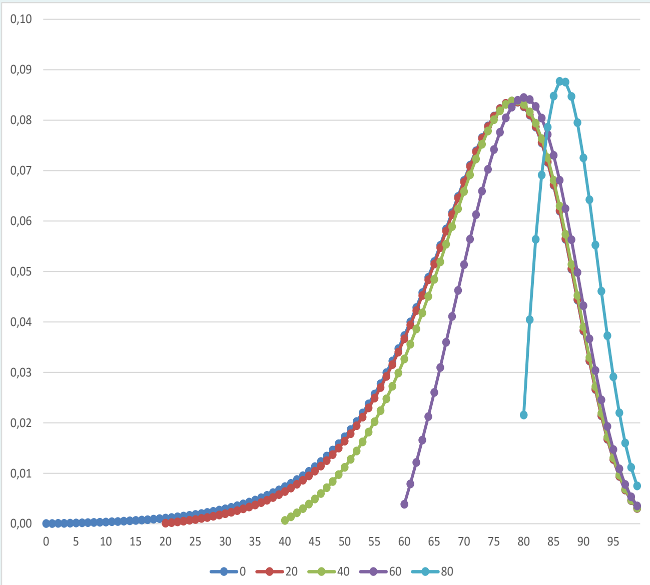


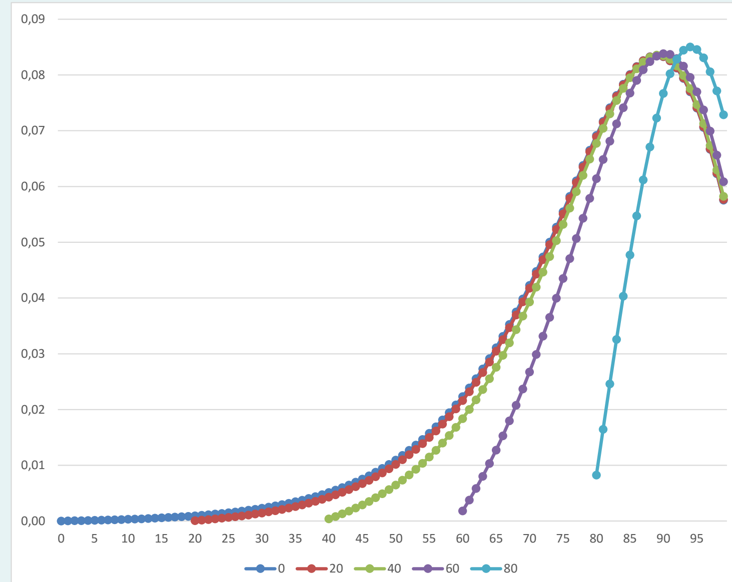
Sensitivity analysis (change of annuity at age 60)



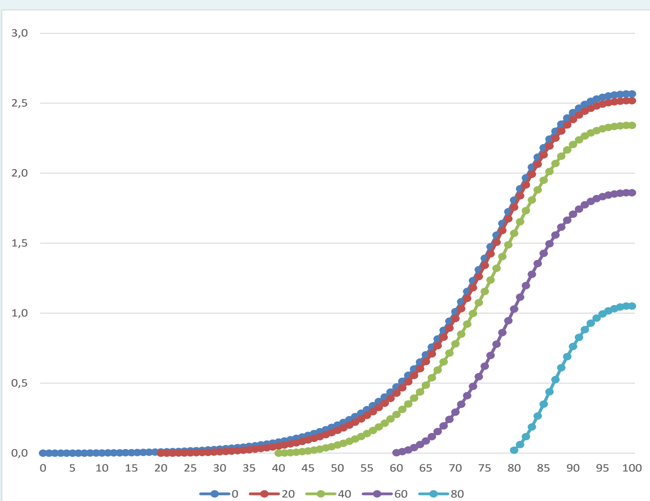
Instant change of mortality, contribution of individual age

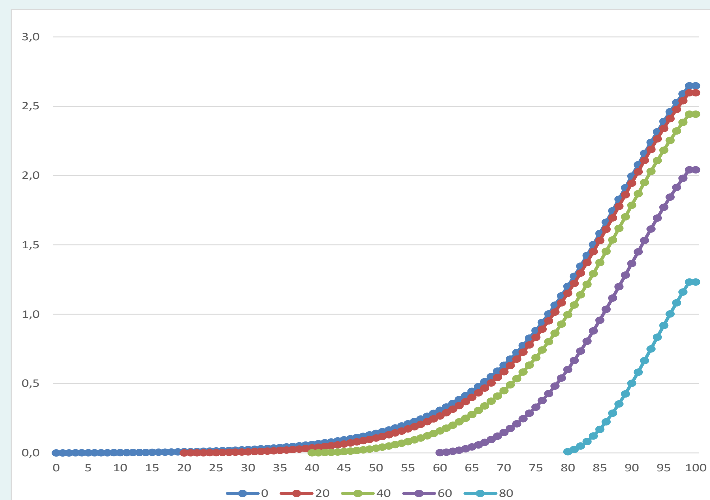
Difference of contingent probabilities (p_x'-p_x), e_0=70,7, e_0'=73,3 delta q_x=-20% for each age	Difference of contingent probabilities (p_x'-p_x), e_0=80,5, e_0'=83,1 delta q_x=-20% for each age





Longevity growth (∑(p_x'-p_x))





Annual changes of mortality
Contribution of individual age

Difference of contingent probabilities (p_x'-p_x), e_0=16, e_0'=18,3 annual q_x=-2% for each age	Difference of contingent probabilities (p_x'-p_x), e_0=24, e_0'=27,2 annual q_x=-2% for each age