

THE VALUATION OF FUTURE CASH FLOWS
AN ACTUARIAL ISSUES PAPER

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1. INTRODUCTION

The value of a set of future cash flows (valuation) can serve many purposes, including the allocation of scarce resources, determination of the price at which a product will be offered (often supplemented with relevant information concerning the market or vice versa), and the estimation of the value of assets, obligations, or companies. The objective of such a valuation is to determine, to the extent possible, the economic value of one or more sets of cash flows or the relative economic differences between the value of alternate sets of estimated cash flows. Such valuations are particularly important in actuarial practice.

The value or worth of a set of cash flows can be determined through an analysis of the cash flows themselves, in addition to reflecting the time value of money and cost of risk. This value can be demonstrated by the actual price at which a transaction occurs; such a transaction can be influenced by additional factors, such as the environment in which a transaction occurs or in some cases the particular buyer or seller involved. Depending on the situation, value may be developed with respect to either a particular decision-maker or a general audience in mind.

Many believe that when attempting to determine the value of a set of cash flows, reliance should be placed only on the price at which two parties are willing to exchange ownership (market price) or the market price of comparable cash flows. However, in many cases in which an assessment of value is needed, the only markets in which comparable cash flows are traded are either thin, undeveloped, or volatile; in other cases there may be no such market at all. In these cases, such market prices may not be readily determinable, available, or reliable.

The objective of this paper is to present and discuss the principles underlying the determination of the value of a set of future cash flows. This topic is central to actuarial practice. Currently, alternative points of view are held on several of the significant issues involved; an attempt is made to present some of them. This paper is intended to serve as a broad overview of this topic and to encourage further discussion of the issues involved.

The original title of this paper was "Present Value of Future Cash Flows". It was changed after I became convinced that the application of present values can best be viewed in a wider valuation context. It is appropriate that the widely used approach of present values, as used by actuaries and other financial professionals, be discussed in terms of its wide applications and in relation to valuations conducted on a market-based approach.

Standards of practice that an actuary should follow in determining such present values will not be explicitly addressed here, although in some cases, some factors that should be taken into account are mentioned. Such standards include the considerations that should be reflected in applying these principles.

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My views on this subject have evolved substantially as I have studied the issues involved. I expect them to continue to evolve. An enhanced version of this paper is possible. As such, I hope that this paper evokes an ongoing dialogue and resolution to some of the substantive and controversial issues discussed here. Many of the issues addressed here require further discussion and rigorous analysis.

2. VALUATION MODELS

2a. Approaches to valuation

Value incorporates both objective and subjective components. It is typically easier to estimate and validate the objective aspects, while in order to determine subjective aspects it may be necessary to observe the results of real experiments. Techniques to quantify both aspects have evolved and improved over time. The most appropriate method, considerations and judgment to be applied in the development of an actuarial assessment of value may vary depending upon the objectives and audience for which the assessment is being conducted.

Value can be viewed in several ways. One approach is to base it on an estimate of the cash flows involved, while another is to base it on some benchmark indicators that can be trusted to fairly demonstrate value. Estimates of values typically involve the application of a model, a representation of reality, generally based on a set of simplifying assumptions. Such a model may be as simple as an observation of comparable market prices or as complicated as an actuary or financial economist can make it. Models are needed both to supplement areas in which insufficient information is available and to enable a decision-maker to better understand the dynamics of financial conditions and effects. In fact, information is a key element, because the less that is known, the greater the uncertainty, the wider the likely range between bid and asked price and in turn risk.

A model is often used to develop estimates of value. A mathematical model is a depiction of reality expressed in mathematical terms. Actuaries study the cash flows reflecting the real world, rather than an idealized world. Models are necessary because the real world rarely permits experimentation that studies the effect of one variable at a time. It is important for actuaries to strive to reflect statistical, probabilistic, behavioral and economic principles. However, although it is necessary for the theory upon which the model is based to be internally consistent, it is more important that it leads to useful results. Sometimes theory, depending on the assumptions used, leads to naïve approximations that need to be adjusted on the basis of experience.

One family of approaches to valuation develops estimates of future cash flows on the basis of a cash flow model from which cash flows are estimated, from which estimates of value are derived, assessed as of a particular point in time (sometimes referred to as a discounted cash flow approach). This value is equal to the expected present value (sometimes referred to as actuarial present values) of a set of future cash flows associated with a particular asset, financial instrument, obligation, product, project, or company. A present value model represents an important method of determining the value of a set of future cash flows. It is often assumed that people generally act in a rational manner anticipated by the concepts underlying present value models, although the risks associated with deviations from this assumption should be recognized.

Other approaches represent variations of present value models, reflecting refinements or amplifications, for example, the use of an option-pricing model for evaluating the cost or benefit associated with embedded options.

The estimation of the value of these future cash flows is central to actuarial practice. Enhanced technology and widespread use of applicable techniques have enabled increased use of the concepts underlying these approaches. Hopefully, this trend should continue.

The combined effect of individual cash flows can be represented in the form of cash or cash equivalents and can be either:

- Positive, representing an asset involving current cash or its equivalent or the expectation of receipt of a future cash flow or
- Negative, representing an obligation to pay cash or its equivalent.

Assets and obligations consist of one or more such cash flows. It is useful to begin the discussion of the valuation of these financial elements in terms of their component cash flows to better understand their combination.

Another family of approaches often used to estimate value relies on a market-based approach. A market is an arrangement for facilitating transactions involving goods by matching buyers and sellers often carried out through the exchange of money or its equivalent. This paper primarily focuses on financial markets, in which the goods traded consist primarily of sets of cash flows (economic goods). An efficient market (one in which perfect competition exists) is one with complete and accurate information available to both buyers and sellers in a voluntary situation (no forced sales or purchases), typically with many buyers and sellers. In such a market, the expected value of market price is uniquely defined as of a particular point in time. In such a market, the opportunities for arbitrage profits are limited.

The relationship between present values and market prices is important to recognize. Several definitions may help:

- *Market price.* The transaction price of a set of future cash flows (possibly combined in the form of a financial instrument, other asset, obligation, or even an entire company) which is traded between two or more parties when sold in a given market. Except in the cases of an efficient market (there are few of these around) or the set of cash flows analyzed is a commodity in which transaction prices are consistent, a market price has to be used with caution to value a similar set of cash flows, as it tends to change over time and between different sets of buyers and sellers. Even in an efficient market, transaction costs such as securitization fees or taxes may be incurred

that would result in the market price as recognized by the buyer and seller being different. By their nature, market prices are marginal in nature, that is, they reflect the valuation of the marginal or last investor prior to valuation. Typically if such prices are used as comparables, a prospective value is desired.

- *Market value.* This is an estimate of the market price, and can serve as a surrogate for the market price if there had been an equivalent transaction in a market. In an efficient market, the expected value of a market price is the market value and is uniquely defined as of a specific point in time. If a market is inefficient, it is the price for a set of cash flows that would have been arrived at if an efficient market had existed.
- *Present value.* This represents an estimate of the underlying value of a set of future cash flows. It is the value of future cash flows taken from a particular point of view at a particular point in time. Some believe that present values serve as a useful surrogate for fair values (while others believe that market values are samples from a population of possible present values).
- *Fair value.* This is defined as the amount for which a set of cash flows could be exchanged or settled in an arm's length transaction between informed and willing parties, other than in a forced sale or liquidation. This amount is derived by applying the principles, which are perceived to underlie the market value of the set of cash flows if it had been currently traded in an efficient market. This concept encompasses an estimate of the market value on a current or prospective basis, whether or not a market for the economic good exists, or a present value representing a fundamental value if not. It could also be viewed as being one of a family of present value approaches, whose assumptions are based on the assessment of a relevant market if it exists and a hypothetical market if it does not. It should reflect expected transaction costs.

Financial economists have generally focused on assets or liabilities that can be easily traded (or securitized). Their theories typically reflect a frictionless world, one that is useful for many purposes, but generally don't exist. Thus, market values are a good starting point and in many cases are sufficient. However, fair value needs to be realistic, as it should take into account such aspects as transaction costs.

In a non-efficient market, differences between market value and present value may arise. These differences may be due to such factors as market imperfections, different risk preferences between buyers and sellers, and variation in perceptions of market risk. Even in relatively "efficient" markets, imperfections exist (although the following quote from the 24 January 24 1998 edition of the *Economist* relates to the volatile situation in East Asia, it can equally apply to any financial market):

“But real financial markets are more complex than the textbook models. Investors are erratic, swayed by waves of excessive optimism followed by waves of excessive pessimism. And it is impossible to eliminate moral hazard, which causes people to take excessive risks in the expectation that a central bank or the IMF will bail them out when things go wrong. These factors can mean that market forces do not allocate capital perfectly around the world.”

Another quote relates to one of the more efficient markets, the New York Stock Exchange. According to Robert Froelich, chief investment strategist for Scudder Kemper Investments on 27 May 1998: “We’re going to have to get used to dramatic swings, because the market’s not trading on fundamentals now. It’s an emotional market.” If this is the case in the situation cited, the determination of value to be recognized must regularly weigh both the relative importance of (1) market determination and (2) underlying fundamentals of a set of cash flows.

This difference can be referred to as a market discount (if the market value is less than the present value, or a market premium if the market value is greater than the present value). Present value models can thus be used to estimate value when an appropriate market value is not readily available, indicating whether to buy or sell a set of cash flows at a given price, or comparing the relative value of two or more sets of cash flows. One approach is then to combine the two general methods, with a present value model used to estimate the “fundamental” value and a market-related approach to estimate the market discount or premium.

One judgment of the difference between fundamental and market-based pricing is revealed by Larry Summers¹ when he noted that empirical tests of the efficient market hypothesis do not generally address whether prices are equal to fundamental values and concludes that frequent divergence of prices and such values by over thirty percent is consistent with available empirical data. This difference is due in part to the fact that most markets respond not only to information, but also to opinion and emotion, both subject to human bias. These can change often and by a significant amount.

Although it appears that, due to a limited number of efficient markets, market value may not be of much use. On the contrary, such values should be derived if possible. In many cases they form the basis of best estimates of value available to a marginal, perhaps typical investor.

Other forms of market-based valuation methods exist. One reflects historical costs (that are based on a previous market price or market value). Various approaches can be used to develop a current value reflecting such a historically based cost, including interest methods (e.g., amortization of a bond purchased at a discount) and depreciation methods

¹ “Does the Stock Market Rationally Reflect Fundamental Values?”, *Journal of Finance*, Vol. XLI, No. 3, July, 1996

(e.g., regular reduction in value based on a rule, typically for properties). These methods are not necessarily related to an assessment of future cash flows.

Another related value estimation method is net realizable value upon settlement. Time becomes important in this approach, because the value at the current time (market value or current settlement value) may be different than if a set of cash flows were settled sometime in the future.

One can also view a market price as a sample of fair values. In the case of an efficient market, the value of all such samples would be identical. To the extent that differences in market prices result only from statistical fluctuations, sampling theory might be used to study these values based on historical observations.

2b. Major components of a present value model

The present value of future cash flows is based on three components: estimates of the amount of the cash flows resulting from the application of a present value model, estimates of their timing, and adjustments made to the estimates by means of one or more discount factors. The estimates made as of a specific valuation date often involve possible future cash flows that are of uncertain amount and timing. As a result, an actuary will consider the range of possible values and not simply a single set of selected cash flows and a single set of discount rates applied to a stream of future cash flows, even if such sets are the most likely ones. The way in which this range is reflected and adjustments made to the set of expectations and uncertainties associated with these cash flows, form the basis of the construction and application of the present value model, applied on the basis of actuarial judgment.

Three fundamental principles of actuarial science underlie the present value model -- expected value of future cash flows, time preference and risk aversion.

- **Expected value of future cash flows.** The components of certain sets of cash flows (or their monetary equivalents) may be analyzed separately in terms of their actuarial risk variables: incidence (or frequency), severity (or average size), and timing. Depending on the types of cash flows involved, it may be desirable to estimate future cash flows based on these separately or in combination. In developing estimates, it is necessary to analyze a range of possible values, along with corresponding probabilities of occurrences likely in the future. These probabilities can either be developed on an objective or subjective basis, the latter typically used if the necessary information or experience is unavailable to develop objective estimates or to supplement experience.

An expected or mean value for the future cash flows would then be developed. Mathematically, an expected value is the probability-weighted measure assigned to

the range of uncertain amounts or scenarios. Alternative measures could include the median (the middle value of a set of possible results) or the mode (most likely value). In some cases, it may be tempting to select what some refer to as the most likely value; this value which represents the mode of a probability distribution, in some cases may be significantly different than the mean value of the distribution. Alternative approaches may be used; for example, the average of a high and low estimate or mean value of each experience assumption that would not necessarily result in the mean value of total value.

In some cases, the mean value may be a hypothetical one; that is, it may be unlikely or even impossible to achieve, as it may be between two possible values. An example is a situation in which a value could be either \$1,000 or \$4,000, with the former being twice as likely as the latter. The mean would be \$2,000 ($\frac{2}{3}$ of \$1,000 + $\frac{1}{3}$ of \$4,000), with a most likely value of \$1,000. Although the \$2,000 value is not a possible result, it is a more appropriate value to assign to the situation.

In other cases, appropriate adjustments are needed (for example, for certain decisions a few extreme outcomes with very low probabilities may be possible; in such a case a result closer to the median or a calculation applying a smaller weight to these extreme values may be appropriate). Judgment is important to determine if these alternatives are preferable. In other cases, the development of an estimate of the mean of a set of cash flows is complicated, for example for long-term contracts such as life insurance.

If all real phenomena were only like the tossing of a coin, and the coin could always be tossed under the same circumstances many times, these values would also be similar; however, experimental evidence most likely will not bear out such a simplifying assumption in which a normal distribution with known parameters. Although most actual distributions are typically not symmetrical, it is usually deemed sufficient to use the mean or expected value. In addition, it can be important to recognize the impact of lack of symmetry and non-linearity of the distributions and risk (see risk preference below) associated with uncertain future cash flows in the other two components of value.

If a set of cash flows is to be bought or sold, it is appropriate to reflect expected associated transaction costs that may be considerable. Since such costs may vary depending on how or when such a transaction occurs, the specific decision being addressed would affect value placed on the cash flows, or alternatively, a range of possible actions.

- **Time preference** (time value of money). Money and time have value because they are scarce resources. This can be seen from the actions of lenders who demand and borrowers who forgo additional money for its use over time. A given amount of

money held now tends² to be valued more than the same amount of money received later because people tend to desire it sooner. This view of time preference is based on axioms involving human preferences.

An alternative interpretation of this component ignores such preferences and is based upon axioms concerning the growth of capital. This indicates that one party is normally willing to compensate another party for the use of money or capital. Presumably, the resulting models should result in similar models with similar answers in many cases. Thus, money may be invested to produce a larger amount at a later date.

Conversely, for a negative cash flow a current obligation tends to be worth less than the same amount due at a future date, based on either of the two views just described.

This seemingly obvious and straightforward concept, which will be referred to as time preference, results in the principle often referred to as the time value of money. The results of its application to a positive cash flow is that the current value of a future positive cash flow is less than if the same amount of cash was available now; conversely, the current value of a future negative cash flow is greater than if the same amount of obligation was payable now. The amount of these differences in value is referred to as a time discount. If equated to an annual percentage reduction, an applicable aggregate rate of annual time discount can be determined. Some work has been done lately in the development of “hyperbolic discounting”, in which individuals are assumed to have a lower discount rate for events far into the future than for closer times (this may explain the lack of savings in early years).

No matter what view is taken, factors in addition to inflation are reflected. In the behavioral model, the residual value of time is sometimes referred to as the real rate of interest, the amount demanded by individuals to lend money. In fact, it is not the current rate of inflation that is reflected, but rather the expected rate of inflation over the period over which time is being viewed. As a result of the future nature of this inflation expectation, the residual value may be difficult to quantify. In the growth of capital model, the residual value represents a real (net of inflation) cost of alternative funds (in some cases referred to as a cost of capital, net of inflation).

The discount represents the financial cost (if a positive cash flow) or benefit (if a negative cash flow) of time; that is, the ability to earn interest on an asset or necessity to pay interest on an obligation. Alternatively in some cases, it may be viewed as an opportunity cost or benefit, or the *price* of time.

² The use of “tend” is used because certain factors or constraints may result in exceptions to this rule which, while not changing the general principles involved, do change their application. The primary exceptions involve situations in which unusual outside influences exist; an example is when a person is unable to claim or use a given amount of money now, but would be able to claim or use it at a future time, or when deflation or devaluation of a currency is anticipated.

As such, although it is sometimes thought of as directly related to inflation, it is not necessarily so. Generally its value is positive, whether in an inflationary or deflationary environment. Whether a person looks at it in a manner similar to the old expression “a bird in hand is worth two in a bush”, it tends to have a value in and of itself. Also inherent in this concept is liquidity preference or desire for flexibility due to the availability of alternative opportunities to use money. This can also be interpreted as the value of the option associated with its current availability. The fact that a positive interest rate could be earned on money held now usually leads to the belief that the value is always positive³.

The significance of time preference may vary considerably among individuals and corporate entities, based on individual circumstances or expectations. Many factors can affect the individual circumstance; for example, different liquidity requirements (e.g., a firm may need to finance additional cash flows in the short-term), availability of alternative sources of investment (e.g., one firm may be a better credit risk than another, resulting in a different cost of debt or capital), or income tax positions. Expectations about the future can also vary; for example, in an alternative use of a cash flow or the instrument or product that generates the cash flow or in different assessments of the probability of future changes in taxation (e.g., expecting to be assessed at a lower rate in a future period). Thus, a time distant cash flow may be viewed as more or less valuable to an individual than its generally assessed economic value that may be a weighted value of many such assessments. Such distinct perspectives may impact personal, political, and public sector decision-making as well.

- **Risk aversion** (risk preference, loss aversion or risk). *Risk* has been defined in many ways. In some cases it has been viewed as synonymous with uncertainty or volatility. Within this paper, it will be defined to as being the probability that a given set of financial objectives is not achieved. Such objectives may involve an inability to repay a loan, insolvency or bankruptcy, or a given probability of significant adverse financial fluctuations. The examples just described have been expressed in terms of an adverse result; it may also represent the cumulative effect of more than one criterion.

³ This in spite of the negative interest rates experienced in Japan in the fall of 1998 and in the U.S. for a period during 1933 and between 1939 and 1941. During these periods, interest rates were quite low. In Japan, banks were in a desperate search for dollars. In the U.S., the negative yields resulted from significant demand for Treasury securities because they were required as collateral for banks to hold U.S. government deposits and that these securities were exempt from personal property taxes in some states while cash was not. Clearly in both cases, there was more involved than the anticipation of deflation in monetary values. These cases show that determining interest rates is more complex than just looking up an inflation rate in a table and adding an arbitrary real rate of return.

Most people are risk averse⁴. People are usually willing to pay some amount of money to reduce the probability of a reduced amount of wealth or a loss, or reduce the potential uncertainty or variability associated with a future cash flow. Tolerance of risk may also be affected by the decision-maker's current wealth (if an individual) or capital (if a company); such a buffer fund tends to reduce the degree of such risk aversion to the same amount of loss. Both companies and individuals tend to behave in a risk-averse manner in order to avoid the large costs associated with a depletion of their capital resources and the subsequent need to replenish them.

The following is a trivial example applied to a single positive cash flow. A person will prefer to receive (1) a \$10 million cash flow with certainty at a given point in time rather than (2) \$10 million at that same point in time with probability less than 100%, say 80%, and no cash flow with the probability of 100% less that probability, or 20%. This concept can similarly be applied to a negative cash flow. A more problematic choice involves that person who has a 50% probability of receiving \$20 million and a 50% chance of receiving nothing. In this case, the expected value of the combination of these two possible cash flows is the same as in the first case, \$10 million. However, those who are risk-averse would prefer to receive an amount equal to the expected value with certainty, rather than facing a situation in which nothing might be received. Experience with lotteries and other games of chance suggests, however, that the potential for a very large positive cash flow in return for a small outlay may have attraction, even though the expected return is very low and the probability of losing the stake is very high.

Thus, in addition to the expected value, both the variability and range or likelihood of possible outcomes may be relevant to an assessment of choices. The assessment of a person's view toward the risk associated with a future cash flow may be measured in terms of that person's preferences for an alternative cash flow. Theoretically, this may be measured through the use of utility theory, a method of measuring the degree of a person's preference (or utility) for one cash flow compared with that of another cash flow. However, it is difficult to measure, apply and understand such utilities in practice. It is not unusual for different individuals or entities to have different risk preferences.

It should be recognized that markets demand a risk premium, as parties tend to demand a reward for taking risk. However, the degree of risk of an individual or an

⁴ There are some people who are not averse to risk, e.g., some people prefer a risk for the thrill of it. If a person desires higher risk, that person is referred to as being risk-inclined and if a person is indifferent with respect to risk, that person is referred to as being risk-neutral. Practically, almost everyone is risk-averse to at least some degree, with the extent of risk aversion dependent on the expected size and probability of gain and loss. Typically, most people do not mind positive surprises and in fact may pay an additional amount of money for such a possibility. On the other hand, most people do not like negative surprises. Thus, risk is viewed here in terms of a one-sided (unfavorable) level of uncertainty. This may be affected by such factors as the culture in which value is being assessed or the wealth of the decision-maker.

entire market can rise or fall depending on a wide variety of causes. A market's assessment of risk consists of the aggregation of the individual assessments of risk of all of the participants in that market. The perception of risk reflected in market or fair values reflects the assessment made at the time that the valuation is conducted. It is thus may be difficult, if not impossible, to determine the "accuracy" of that assessment because of the different time period involved. This perception may vary between markets and over time. In some cases in which there is an applicable market in several countries, there may arise a question of from which market to gather comparable price information, as different markets may be dissimilar (although, depending on the economic good involved, such arbitrage opportunities should decrease over time as the trend to globalization of many markets continues).

In addition to reflecting the level of risk inherent in an uncertain future cash flow, risk can also be evaluated in relative terms as a comparison of the degree of risk resulting from an alternative source of the cash flow. If provided an opportunity, most people would be favorably disposed toward a cash flow with a lower level of risk, given equivalent expected values. The difference in value of a positive or negative cash flow compared with the same cash flow received or paid at the same time without that risk is referred to as a risk discount. This difference is a relative concept. If equated to an annual percentage difference in value, an aggregate annual rate of risk discount may be determinable. The concept of relative risk will be of importance in section (3d) of this paper when methods of evaluating risk are discussed.

Although useful for analytical purposes to analyze these components separately, in some cases they can be combined. The aggregate effect of time and risk preference is referred to in this paper as the total discount, or more simply, "discount". Although these two factors may be independent, typically they are considered simultaneously because they both affect the value or price of a future cash flow. They may take the form of a discount rate, but may also adjust other parts of a present value model as well. Since the two types of preferences could generally be measured in different ways, conceptually they would be recognized separately, possibly in a two step process. Fortunately, since the resulting rates are typically relatively small (aside from under hyper-inflationary conditions), the technical problems associated with this more refined methodology don't have to be solved, with a focus on the derivation and application of the combined factor. Its derivation may improved if both of these factors are considered.

In addition, two other factors may be incorporated in a present value model: (1) opportunity cost (benefit), if it is desired to consider the non-financial advantages (disadvantages) of not taking an action, and (2) tax effect, appropriate to be taken into account if the effect of taxes resulting from the cash flows is relevant.

Present value models can be categorized in several ways. One distinction that can be made is between those that are deterministically and stochastically (directly reflecting one or more random processes) based. The stochastic approach assigns a distribution to the

possible outcomes. These probabilities are sometimes assigned based on an assumption that only statistical or random fluctuations occur, that is, that the modeler fully recognizes the conditions which the model is being used to evaluate, the conditions don't change, and that no shocks or discontinuities are expected to intervene to change the underlying conditions (possible shocks to the environment may be anticipated in applicable probability distributions). In actuarial literature, the financial implications of the risks associated with such probabilities are referred to as "process risk" and generally reflect the impact of random fluctuations, reflecting the recognition of the uncertainty associated with the incidence, timing and amount of each cash flow. Through a stochastic model, such probabilities can be assigned to reflect random fluctuations, random occurrences or changes.

A stochastic approach may be particularly useful when incidence, severity, and timing affect each cash flow; for example, if an expected cash flow at year ten is greater than at year eight or if it is determined that the likelihood of a cash flow is more likely at year ten than at year eight. Such correlations can be built into this type of model relatively easily.

A stochastic model can also be constructed to recognize the uncertainty associated with an applicable probability distribution (mean and other moments), as well as unlikely catastrophes or outcomes substantively different from the expected value. The financial results from such risks in the actuarial literature are referred to as "parameter risk". The use of a stochastic model may yield insights into the inter-relationships of the factors involved and ultimately may lead to a better fit. One aspect of parameter risk is catastrophe risk, that is one example of the fact that often the expected value is not the only parameter that is important, but the entire distribution of possible values.

An additional type of risk may be present in a situation in which the environment is not well understood, resulting in a significant degree of uncertainty about the appropriateness of the particular present value model selected. This class of risk is referred to as specification or "model risk".

On the other hand, a deterministic model assumes that there is no uncertainty (probability of 100% of a given scenario), or implies a belief that, by focusing on expected values, an adequate approximation to a more refined or realistic stochastic method is obtained. Key aspects of both deterministic and stochastic models may be determined through the application of alternative, deterministically derived scenarios with probability values assigned, the alternatives selected on the basis of a set of representative possible future conditions. Thus, the use of risk-adjusted discount rates could be applied to either type of model, implicitly reflecting risk through its two components – the risk-free component and the risk adjustment. A stochastic model can also be viewed as representing a family of deterministic models, each assigned a certain probability of occurring.

Volatility can be reflected through a model in a number of ways. One approach assumes dynamic reactions, through dynamic models or dynamic systems; for example, through

the retrospective experience rating plans of a casualty insurance program (in which uncertainty in cash flows can be shared by the insurer and the insured) or the anticipation of certain behavior (for example, by management, consumer, or the market) in response to possible alternative outcomes. Another reduces the impact of risk through the use of an appropriate hedge, such as the investment of a matched portfolio of assets and obligations or by purchasing or acting on future options. In any event, it may be difficult or impractical to totally anticipate and completely prepare for the financial impact of future events, particularly those occurring over a longer period of time. The various methods of managing risk and uncertainty may affect the type and form of the present value model applied in a specific circumstance. It may be that more than one type of model could be applied. More recently developed alternatives, such as option pricing models or option adjusted spread models may be refined in the future to model estimated expected values of future cash flows more appropriately.

2c. A set of cash flows

So far, the present value model has primarily been discussed in terms of its application to a single future cash flow or it has been assumed that the individual cash flows can be easily combined. Present value models are of more practical use when the nuances of a combination of future cash flows are recognized, the value of which can be either positive or negative.

If all of the cash flows being examined are positive, the combination is referred to as an asset of the owner of the source of the cash flows, while if they are all negative, they are referred to as an obligation or liability. The categorization of sets of cash flows between various assets and liabilities is often obvious, as for a particular financial instrument such as a bond. In other cases, such as a periodic payment life insurance product with various embedded policyholder options, it is not as obvious, as premiums could be valued separately or aggregated with benefit payments, and the options provided could be independently valued or combined as part of a single contract, with the overall value being either positive or negative.

In part, the decision as to how to combine future cash flows to estimate their value may be based on the substance of the economic good which is the source of the cash flows, the level of correlation among its cash flows, pertinent accounting rules, and their relative size. A set of cash flows studied may include both positive and negative cash flows. Whether this combination is referred to as an asset or a liability will depend upon its nature and timing and if reported in a financial statement on the applicable accounting rules. For example, the value of a product to its producer will generally reflect both the cost to produce it before it is sold and the income stream it is expected to generate through sales; it is viewed as an asset if the value of the income is greater than the associated costs.

The value of a combination of future cash flows may not be additive. Although their expected values will generally be additive, the other components of value may not be. First, the combination itself may add or subtract value (for example, the sum of the assets and liabilities of a company may add enterprise or brand value or control of the entity, among other intangibles, or a portfolio of assets may be able to command a market premium compared with individual sales). In addition, depending on the correlations among the cash flows and the objectives of the decision-maker, the overall risk may be reduced as, for example, the total volatility may be less than that of the separate cash flows.

The determination of the appropriate combination of cash flows to use may be important if different risk adjustments to different components of a set of cash flows. Depending on the methodology used, the combinations of those cash flows can influence their value, both in terms of determining the effect of the uncertainty of the amount or timing of the cash flows or the risk preferences applied, that may vary depending on the degree of the pooling or combination of cash flows. Whether a set of cash flows is categorized as being an asset or liability may not be as significant as how the combination of positive and negative cash flows is made.

Theoretically, in general market values of a set of future cash flows are assumed to be additive (assuming an efficient market), that is, that sum of their values equals the value of their sum and do not affect each other's values. However, as they may not be independent and the number of such economic goods can affect value, in reality the values of many assets and liabilities affect each other. For example, the risk-adjusted value of a portfolio of assets may be different than the sum of the risk-adjusted value of each individual asset. This may also be true for a portfolio of liabilities and in fact entire entities. In some cases the difference is not material. However, since valuation is typically at the portfolio or entity level, care should be placed in valuing sets of cash flows that are non-independent. Generally fair values would be affected as well to the extent there is a difference in the combined value.

The differences in these values can arise as a result of many factors, such as the market effect of trading a large block of assets in a market, due to a lack of perfect efficiency. Other factors may also be involved, as there is a normally a non-zero value that could be assigned to the controlling interest of an entity if an entire entity is being bought (sometimes referred to as good-will), rather than just a non-controlling interest. In sum, in order to determine the value of a set of cash flows, it is appropriate that not only should the interaction between all of the cash flows be evaluated, but also whether the effect of any embedded aspects of the economic goods. The value of a company includes a number of factors, included embedded values, although most are related to their ability to generate future cash flows internally through a distribution system or a brand, or possibly in related firms.

Even if cash flows are generated by an economic good, all of these cash flows may not be available to the owner or investor when they are generated. For example, due to regulatory constraints, a certain portion of total cash flows may not currently be available to owners or their timing may be restricted. Do internally generated cash flows have the same value as distributed funds? For example, the value placed on shareholder dividends (due to time or risk preference) may not be the same as those cash flows not so available. In fact, this is a more general issue, as the timing of shareholder dividend cash flows compared with the internally generated cash flows that was the source of funding for the dividends. In most cases, the difference would be as a result of the application of time preference, in that the time the various cash flows become available would be important.

2d. Audiences

To determine value as defined in this paper, it is necessary to recognize the audience(s) and purpose(s) for which the value-related information will be used. Unfortunately, several audiences could use the same valuation information for their own purposes, a single audience could use such information for different purposes at different times, and individuals within a single class of audience may view such valuations differently based on their own preferences. At least three aspects of value could differ – the risk preference of the audience, the purpose the information is used for, and the influence that the audience has over the size or timing of the cash flows; all this in addition to a possible difference in the estimation of the future cash flows. In order for a valuation to be of value, it should be useful for decision-making.

The following provides an overview of the perspectives and uses for such information by selected types of audiences. Although this brief discussion focuses on the needs for and uses of financial reported information, it could also be applicable to general business or personal financial decision-making.

- **Shareholders, potential shareholders, owners and potential owners.** These will be concerned with both the economic value of their underlying investment and the market price that they could obtain for their investment now or in the future, as well as dividends that may be paid in the interim. Their perspective is influenced by market prices, in that these determine their own financial worth (assuming that their share is eventually sold, although this may differ whether short-term or long-term value is more important). Potential owners are also interested in the financial impact of actions that can be taken to improve future values, prices, and dividends. In addition, whether an entity is for-profit or not-for-profit may affect perspectives toward risk and time factors.
- **Management.** Management is generally interested in optimizing the economic value of the entity. However, since in companies whose shares are traded, their performance (and often their remuneration) is generally evaluated based on a market-

determined share price, they may be even more concerned with its current or future share price. In the instances in which share prices are highly correlated with the underlying economic value of the entity, this is the sign of a relatively efficient market, although as described in this paper other factors influence share price as well. It is well known that a different focus between short-term and long-term financial condition may affect business or individual decision-making; appropriate reflection of present values may reduce somewhat the different perspectives (the discounted value of a future range of probable values is smaller than the corresponding undiscounted values). It is desirable for an accounting system not to introduce incentives to induce management to take actions that are economically adverse to the entity. However, since at least part of the market premium or discount is not related to performance of the particular entity, it may be better that management focuses on fundamental values of the entity, rather than day-to-day fluctuations in share price.

- **Tax authorities.** Tax authorities are interested in having a basis to permit them to implement a method to allocate tax collections among businesses and individuals. In order to accomplish this, a number of other purposes, sometimes reflecting social objectives, may also be considered. In addition, since the payment of taxes is never popular, tax authorities need information (tax bases) that is objective, difficult to manipulate, and easily verifiable, while at the same time allowing them to optimize tax receipts and being fair in their treatment among types of businesses and individuals.
- **Regulators.** If the entity operates within a regulated industry, regulatory authorities are interested in information that will assist them in carrying out their duties to protect the public interest, for such purposes as monitoring solvency of the firm or the fair treatment of users of their services. It is preferable if financial reporting information developed for regulators and other audiences were developed on the same basis, although different levels of acceptable conservatism and specificity (to the company's financial condition) may be appropriate. Regulatory values should not necessarily be related to market-based values. Rather, estimates of value (their definitions of assets and liabilities need not be consistent with those used by general purpose accounting) would typically be based on expected cash flows, based on strategies and uses of assets and liabilities of the current entity's management.
- **Financial analysts and rating agencies.** These serve as information intermediaries and value interpreters, gathering and evaluating financial information. They often massage available information and develop independent analyses, in order to provide their customers with their own assessment of the financial condition of individual entities and individuals. Thus, they are interested in transparent information that is comparable from company to company.
- **Creditors.** Creditors in this context include those who are owed money or services in the future. They are interested in a reasonable assurance that their financial interests

are preserved. This is particularly appropriate in the financial services industry in which long-term obligations are provided (e.g., to policyholders of an insurer). In addition, this also applies to owners of the entity's debt and suppliers of services and products. Company specific information and settlement values would be relevant to creditors.

In addition, the assumptions underlying the calculation of values assigned to a set of cash flows may vary depending on the intended use of the economic goods. In certain instances, these uses may not vary significantly, e.g., in valuing a short-term non-callable government bond. In other instances, often in the case of an entire company, the set of future cash flows may vary considerably based on the actual or intended use.

This variety of audiences and their uses of such information demonstrate that it is important to provide transparent information regarding a set of cash flows so that alternative (but most likely not all) needs for information can be satisfied from a single information source.

2e. Use of valuation models and accounting rules

An accounting system consists of a set of methodologies and constraints imposed by rules for the measurement of the value (balance sheet) and changes in the value (income statement) of assets and obligations in a financial reporting context. Valuations of future cash flows can play a major role in financial reporting. The financial accounting context will be emphasized in this paper. Alternative accounting rules are possible (for example, cash accounting, accrual accounting, fair value based accounting). Several sets of rules can be constructed to define assets and liabilities in an internally consistent manner and meet the criteria described in section 5b of this paper.

Risk can be treated within such a system in several ways. For example, the risk of a possible mismatch of assets and liabilities can be reflected in the value assigned to the liabilities or as an earmarked portion of surplus (risk-based capital), depending on the definition of assets and liabilities used.

Valuations of future cash flows can be used in a number of ways. One categorization of the results of such valuations is between business and personal decision making and input to the financial reporting of an entity's financial operation and condition. The intended audiences for general purpose financial reporting includes investors, potential investors or the general market; if done for a regulatory audience it is generally referred to as regulatory or tax financial reporting, depending on the type of regulator, in the cases cited concerned with solvency or tax bills, respectively. The needs of other audiences can also be addressed with this type of information.

The fact that there are potentially many users of such reporting, each possibly with their own objectives, risk preferences and perspective on the future use of an entity, makes it a difficult to develop a common base of estimates of the financial values for its financial accounts that is of equal value and use to all possible audiences. The potentially wide range of values appropriate for these potential users has to be dealt with in some way. Approaches that could be used to estimate value could be to adopt a least common denominator, average, or focused approach, the latter using estimates relevant to a specified audience and purpose.

Any accounting system should avoid rules that would likely contribute to economically inappropriate or value-destroying decisions because values are determined in an inconsistent or inappropriate manner. Valuation is all about decision-making and allowing better management of the financial aspects of an enterprise. A consistent set of external financial reporting and internal information would be desirable. Although it would be desirable to develop both external financial reporting and internal management information under a consistent set of rules, at a minimum a consistent framework should be established wherever possible.

The basis for a set of accounting rules can be classified as being determined on an historical (retrospective), current, or prospective basis, or a combination of them. These can be viewed in terms of prior, current or expected future fair values, respectively. Historical costs usually represent the applicable market price at some past time; for instance, the valuation of a property is often based on its original purchase price, with annual changes in value reflecting formula-based depreciation. Current values are recognized in a market value system, generally based on the current market prices placed on assets or liabilities or appropriate comparables. In such a system, recognition of a prospectively determined value is generally only made if the currently reported value is impaired, for example, its fair value is less than otherwise valued, and is correspondingly written-down; alternatively, such rules could be applied to the anticipated use or settlement of the assets or liabilities. Actuaries (for property, professional property appraisers) typically approach such valuation on a prospective basis; thus, would update the value of the economic good (property) based on an analysis of the expected future cash flows generated at a suitable discount rate appropriate for the purpose. Both current and prospective approaches could incorporate the present values; the prospective approach is the one many actuaries have more typically been involved with.

In some cases, the historical basis has been used for several practical reasons, due to the fact that its results can often be easily validated, tended to produce stable and understandable results, and methodologies and techniques were not easily available to do otherwise. A prospective basis may be more difficult to validate, as it may incorporate significant judgment as to uncertain future events.

Although the recent trend in accounting is toward fair value based accounting, most sets of accounting rules worldwide for the near future will likely follow a mixed attribute

model, recognizing that in some areas practical considerations may overwhelm the conceptual advantages of measuring value on either a current or prospective basis.

An important accounting rule usually followed in financial reporting is the assumption that the entity reported on is a going concern. If not a going concern, many entries in the entity's balance sheet could be affected by varying degrees of default risk.

One objective of a comprehensive accounting system in setting values in an entity's financial reports is consistency in reporting across types of entities. Reported values should be the same regardless of the type of firm, in which substance over form is more important (otherwise firms could transfer economic goods among subsidiaries. Note that if regulatory constraints result in different cash flows, different values could result.

2f. Whose cash flows?

The first step in determining the value of a set of cash flows is to determine which cash flows will likely be involved and how they will be estimated, whether as a function of a present value or cash flow model, on the basis of comparable market or fair values, or both. How this is done will be influenced by the purposes and audiences for which value will be used.

Sets of cash flows (for example, represented by assets, liabilities or companies) can be categorized in several different ways, including the following that can affect their value:

- The degree to which the entity's operations have or are anticipated to have an effect on the given set of future cash flows. An example of such a set generally not influenced by future operations is a financial instrument such as a bond or stock; an example generally influenced by future operations is a loss reserve of a property/casualty insurance company, in which case the management of the claims process can influence the ultimate amount and timing of payment of insurance benefits.

In any case, inherent characteristics of the set of future cash flows should be recognized; for example, the quality of a loan portfolio or the underwriting of an insured for life insurance. If a given inforce portfolio of loans with low credit quality is being valued, it would be illogical to apply expected experience of an average portfolio of loans. However, the market could reflect in its value an average management of that portfolio going forward. Inherent characteristics will not change for the duration of the portfolio and would be reflected in any valuation.

Two alternative options to recognize the effect on future company operations exist:

- Industry benchmark operations. Since a potential investor or owner of the source of the set of future cash flows may not manage it in the same way as current

- management and the method of operation cannot be predicted, it is reasonable to apply an industry (if the entity is a company) benchmark or average industry performance.
- Entity-specific operations. In order for current management to judge its performance, the current or anticipated operation of the specific entity (referred to by the Financial Accounting Standards Board (FASB - U.S.) as the entity value or entity-specific value⁵ or the Accounting Standards Board (ASB - U.K.) as the “value in use”) is appropriate. For a business decision-maker who intends to change the operations, the estimated impact of those changes would be reflected.

In fact, it may be both valuable and instructive to develop estimates of value on both an “industry” or expected performance basis and on the basis of expected entity performance. The difference in these values would reflect the value-added (or subtracted) that the entity provides.

- The method that an entity uses or intends to use to manage or operate the economic goods (that does not affect its own cash flows). An example of how the economic good can be used differently is a financial instrument that is being used to hedge another set of future cash flows or a financial instrument that is being used by the entity to hedge the interest risk associated with funds generated through the issuance of a life insurance policy.

“Value is based on the eye of the beholder” equally applies to determining the value of future cash flows. For a particular decision-maker, value recognizes the applicable use or management performance relative to the future cash flows, including past and intended operations. For the market as a whole, where there are a number of decision-makers involved, the use of future industry benchmark performance may be the same if not a better measure of value.

Take an example of an insurance company whose entire high quality claims personnel suddenly left the company. Shouldn't the value placed on this loss suddenly decrease the value of the company? What if a company consistently paid its losses at a level consistently and significantly in excess of the rest of the industry? Owners of the firm would certainly reflect these facts in their perceived value or difference in their performance. A case can be made in either case that specific expected future performance should be recognized.

Another example is a non-callable bond, a financial instrument available in a reasonably efficient market. Its underlying characteristics would not be affected by who owns it; as a result, its market value would be based on average assessments of time and risk. However, a particular investor might value it differently depending on whether it was

⁵ The entity-specific value of an asset (liability) is defined as the present value of the future cash flows that the entity expects to realize (pay) through its use (settlement) and eventual disposition over its economic life.

purchased to be held to maturity, held to fund a specific obligation as long as the obligation existed, or traded if another asset was available for the same cost with greater value. For business decision making, it would likely be valued based on its intended use. For external financial reporting, it would likely be valued based on a market assessment.

A difficulty with a system of accounting rules for valuation that reflect the current owners' intended use is that it may result in non-comparable reporting among entities, particularly if not accompanied by adequate disclosure of the effect of the difference.

To make a specific business decision that may be influenced by uncertain cash flows, the decision-maker will determine an estimate of the cash flows based on their intended use. For general purpose external financial reporting, however, future estimated industry benchmarks should be reflected, although the underlying characteristics and past operations of the entity would also be reflected.

If a market is not efficient, the prices at which a specific set of cash flows can be bought or sold may vary, sometimes materially. In addition, the bid and asked prices for a given financial instrument in a market such as a stock exchange may be highly volatile and may vary considerably over time, perhaps from minute to minute or transaction to transaction, even though the expected value of the underlying set of cash flows remains the same. As a result, the market value should be carefully defined in order for such a value to be measurable, e.g., the value at the end of a day or the average of all transactions during the day.

In an inefficient or thinly traded market (this could include situations in which a market is dominated by a small number of players and is not well diversified, with a lack of complete and accurate information, or with different bargaining power of buyers and sellers, and different sizes of transactions), the uniqueness of a price for a set of cash flows may vary due to the different risk or time preferences and intended uses of the applicable buyers and sellers. The range between bid and asked price may be relatively large and the price of similar transactions may vary, sometimes materially, from each other.

The following is an example of an imperfect market. I recently purchased property for a home. I knew that I wanted to tear down the existing structure and build a new house. I was willing to offer a certain price for the property, recognizing my intended use for it, as well as its underlying characteristics, including its size and zoning restrictions. I was competing against other potential buyers, some of who might have been willing to keep the current structure the way it was, modify it, or build a new one. Its future cash flows would clearly be different depending on which bidder was successful. Each of the bidders would place a value on it based on their intended use and the degree that they wanted it, as well as whether a competitive bidding situation existed. If they wanted it badly enough, they would have been willing to pay a market premium for it. Once they had bought it, their value would be based on their intended use. Since the existing

owners didn't have a stake in its future use, its market value to them would have recognized the estimated price they expected to receive on the sale, based on comparable properties with a wide number of potential uses; they set an asking price based at a level that they believed would match the anticipated demand. In such a market, the value placed depends on its intended or expected use.

In some cases, no market exists at all, in which case a fair value of the economic good (such as a life insurance contract in the eye of the issuer) or sets of cash flows would have to be estimated based on its underlying value. Since the conditions of an efficient market do not exist, a fair value would have to be based on the present value of expected future cash flows. If no comparable transaction occurs from which to estimate an appropriate market value, there may be limited alternatives available to an entity-specific value.

In summary, appropriate fair values for a set of cash flows for financial reporting purposes will depend on the degree of efficiency of the market involved, in the following manner:

- In an efficient market, a transaction or market price of a set of cash flows (similar in this case to a commodity) reflects market average values as assessed by the participants in the market.
- In a market that is not efficient, individual assessments, influenced more by entity-specific values, become more important.
- In a non-existent or very thin market, it may be appropriate to assess fair values based on entity-specific values, since any transactions would tend to be uniquely determined based on value perceived by the buyer or seller.
- For external financial reporting, fair value (based on industry performance benchmarks) should be used if available.

If the effect of the operations of an entity affects the estimated future cash flows, the value for a particular decision-maker will vary depending on how the decision-maker will operate and the decision-maker's estimate of the affected cash flows.

2g. Recognition of present values

The discount due to present values should be recognized in any case in which its probable effect is material to the matter at hand. Most actuaries are comfortable with this position, as long as an appropriate level of risk is reflected. This overall approach is consistent with any valuation relying on market-based principles (without a market discount or premium).

The value placed on a set of future cash flows depends on the type of model used. Where a market value is unavailable or is deemed to be unsuitable for a particular application, a significant factor to consider in determining which present value model to apply is the degree of materiality of any resulting discount relative to the purpose and application of the valuation. In many cases in which an actuary is involved, the future cash flows in question are relatively long-term and/or uncertain. As a result, the impact of discounting will generally be material to the determination of the value. However, such a determination should be made on an individual case basis and according to the purpose for which the valuation is conducted (e.g., according to a given set of accounting rules or to meet given regulatory requirements).

In some current accounting literature, guidance indicates that when the amount or incidence of the set of cash flows is not measurable (no reliable estimate is available), no estimate should be reflected in a financial statement (e.g., Statement of the FASB No. (SFAS) 5); rather, disclosure should be made of the material uncertainty. Differences in opinion regarding the reliability of an estimate may arise in the application of this accounting rule. An actuary should be able to develop an unbiased (no reason to believe that the estimate is either too high or too low) estimate for the expected value, based on a probabilistic assessment, of most situations in which future cash flows are involved or develop financial risk management approaches to minimize any such bias and risk resulting from such a bias, such as through hedging or risk transfer, e.g., the use of derivatives or insurance. For example, if an obligation is fully insured, the consideration payable for this insurance can then be substituted for the more uncertain values.

Such a determination is typically made on a fact and circumstance basis; as for example, if the level of uncertainty associated with the estimation of future cash flows is greater than the otherwise stated amount of surplus of an entity. Thus, for financial reporting purposes, an accountant may restrict the direct application of a present value model to cases in which an expected value of the amount or timing of the future cash flows can be estimated within a financially acceptable range. If no value is available or if an unacceptably large range of probable values can be deduced, then other approaches (disclosure only) or possibly surrogate values will be used, depending on the application, e.g., if a new tax law is implemented with no interpretations or regulations available or if a reliable surrogate for a market value is not available. Disclosure only may be unsatisfactory when a business or financial decision has to be made.

In summary, most situations call for the application of present values as long as their impact is material in comparison with the use of and context of the valuation. This is consistent with market-based valuations. Partly because of the relative complexity of determining a present value relative to its impact, the values of many current assets and obligations have been expressed without such an adjustment. In some applications, e.g., provisions for property/casualty exposures in many jurisdictions, present values are not currently used, even though the effect would be material. In this case, the absence of discounting has traditionally been rationalized as representing an acceptable approach to

provide for adverse deviations, i.e., an implicit allowance for risk. However, now that technologies exist to do so, the significant set of advantages of recognizing such risk or uncertainty in an explicit manner overwhelms the rationale for implicit allowance.

3. RISK

3a. The concept

The term risk has been used in many ways. I believe that the most appropriate meaning is the estimated probability that a given set of objectives will not be achieved. Such objectives can take many forms. Those that are primarily related to financial risk in the context of this paper relate to a set of future cash flows, typically representing the avoidance of failure (for example, loss or insolvency) or the achievement of a given degree of estimability or avoidance of uncertainty that an adverse event will occur. Sometimes its application relates to the cost (or value) of the uncertainty associated with not achieving such objectives. It is this latter meaning that will be used in the remainder of this paper. Uncertainty⁶ is expressed in terms of the range of possible outcomes. Its importance may be significant in determining the present value of a set of future cash flows, as the price of all rational economic transactions will reflect an anticipated degree of risk.

Risk analysis consists of the study of the set of all or the most significant probable outcomes and the assignment of applicable probabilities to those outcomes. Several types of risk are likely to make up the total degree of risk⁷ associated with a set of future cash flows. In fact, the types of risks faced will depend on whose perspective is being taken, the particular purpose for which the results of the valuation are to be used, and the environment in which the set of cash flows is placed (e.g., affected by the risk management techniques applied or the relative risks in that environment). This is another reason why discounts may vary, in some cases considerably, depending on for whom and for what purpose the value is being developed. For example, in a pension plan the perceived risks associated with receiving pension plan benefits may differ between those already retired, those who intend to retire in more than ten years time, and the plan sponsor.

Risk has sometimes been measured by the degree of volatility of the value or price of a set of cash flows. Although volatility (or rather avoidance of volatility) can form a significant factor in overall objectives from which risk is determined, it may not be the only or even most important one. Although volatility can destroy value (through reduction in market assessment of an entity or in requiring additional capital), it can also be of benefit assuming that a sufficient risk premium can be obtained to compensation for the volatility. In fact, volatility reflects both favorable and unfavorable deviations. Since

⁶ With respect to the future result of a particular experiment, the degree of possibility that a particular result occurs is measured by its estimated probability. Often this includes subjective opinion regarding the possibility of the event, i.e., personal belief.

⁷ For example, the reason for buying life insurance include the avoidance of a number of risks, such as the risks of not regularly saving and bad personal investments, as well as certain costs associated with premature death and disability.

risk only involves adverse results, it is a one-sided, not a two-sided test. Having said this, measures of volatility may serve as a surrogate or provide sufficiently accurate measurement of a significant portion of many of the risks analyzed.

Risk can be managed in several ways. Obvious methods are to limit the risk assumed in the first place, or to sell or securitize some or all of the future cash flows elsewhere in the market. If it is an efficient market, the market price represents an equilibrium value related to the outcomes of a joint determination of the parties involved in the market, with a narrow spread between bid and asked price. However, in markets that are not completely efficient, the bid and asked prices may be relatively far apart, if for no other reason than the buyers and sellers may perceive risk differently and have different plans for managing the set of future cash flows. If no market exists at all, the results of a present value model can be applied to simulate what some call the fair value of the set of cash flows, assuming no market discount or premium exists (or reflecting an estimated premium or discount).

Since the “market” cannot determine the specific environment in which the set of cash flows will exist after a sale, the market assessment of risk, by its nature, reflects the consensus assessment of many players reflecting their own allowance for risk. The market’s assessment, in a sense, corresponds to an average or expected set of assumptions. The more efficient the market, the better the risks can be managed through various risk management techniques, such as diversification, pooling, insurance, etc. In such a case, the market will assume that such risks are managed through market mechanisms.

The use of such market mechanisms costs money (at the minimum, transaction costs) and thus an implicit assumption with regards to the average cost of handling such risk situations will be called for. There may not be a market which can manage such risks (if indeed there exists a market for the set of cash flows in the first place), so that two sets of assumptions may be required – one based on an “average” or expected set of circumstances and market-based risk assessment, including the cost of offsetting the risk associated with these circumstances and the other risks associated with the specific circumstances of the entity within which the set of cash flows will occur and its assessment of risk, which may or may not be identified with comparables into premiums. This uncertainty contributes to the variation in market prices and the difficulty of quantifying risk in many cases. Thus, it is common to assess the cost of risk through the use of a present value model. In fact, a fair value can be viewed as being the aggregation of many such present value models, the average of which is reflected in the prices associated with the market, if one exists, or otherwise that is estimated as if such a market existed.

A set of cash flows may be subject to one or more types of risk. As a result of the potentially large number of sources of risk, it may be useful to take a holistic or entity

(enterprise)-wide view of risk, although in most situations, only the most significant aspects would typically be focused upon.

A key first step in risk assessment is the identification of the risks that need to be addressed. The following is one such categorization of some of the major risks that may impact a set of future cash flows (not all apply in each case):

- **Credit or counter-party risk.** This includes the risk of default, usually associated with not receiving a positive cash flow (either all or a part of that expected) from another party. Although seemingly a simple hazard, it can be difficult to define, as default is often not complete. On the other hand, once in default, a company's values (and reputation) can change many of the values on its balance sheet and an entirely new set of risks can arise.
- **Market risk.** This could include interest rate, call, reinvestment or prepayment risk, depending on the asset or liability being assessed. It also could reflect changes in the market's attitude to risk, expectations about the future, or a change in the mix of investors.
- **Pricing risk.** Particularly relevant for long-term contracts or those in which considerable volatility in cash flows is expected, this risk may take the form of not meeting the original pricing assumptions and expectations, that require estimating expected risks in advance.
- **Liability risk.** This risk also reflects the likelihood that estimates of future cash flow obligations are inappropriate (such as those cash flows for property/casualty insurance loss obligations).
- **Information risk.** Inappropriate or inaccurate information may be used or relied upon to develop estimates of future cash flows. Such reliance can result in inaccurate estimates that may result in inappropriate decisions. Other risks may result from the existence of asymmetric information, that is, one party having more or better information than another party.
- **Adverse selection risk.** This risk results from choice of another party, whether due to an option that can be exercised unilaterally by the other party to the potential detriment of the first party, or due to asymmetry of information.
- **Moral hazard.** The existence of a contract defining the cash flows may alter incentives and change the probability distribution of future cash flows. This is another example of a risk resulting from asymmetric information. For example, managers of a financial institution with guaranteed deposit insurance might behave differently than managers without such guarantees.

- **Asset / liability mismatch risk.** This will depend upon the degree of correlation between the set of future cash flows associated with assets providing for obligations of the entity. The significance of this risk will vary depending upon whether or how assets are allocated or segmented. This risk is in large part due to reinvestment risk, that is, the possible lower return on reinvested assets than that expected to be needed to match the future cash flows associated with the liabilities. A combination risk (combined with foreign currency risk) is when an obligation is expressed in one currency with its funding is provided in another currency.
- **Liquidity risk.** This risk can take several forms -- for example, liquidity needs of the owner of an asset or the party that needs to settle an obligation, or the entire market could become less liquid. The existence of a liquid market is a frequent assumption in financial economics. It is commonly ignored and it may be difficult to quantify this risk. If liquid assets or liabilities are involved and can be easily sold to others or positions easily unwound, such as is generally the case in banks, it is common to ignore this risk, although examples such as Long Term Capital Management which was unable to unwind its large positions in 1998 indicate that there may be limits to liquidity in any market. If non-liquid assets or liabilities are involved, this may be a very real risk. This can occur as a result of over-leveraging.
- **Industry risk.** This risk is associated with the possible overall deterioration of the industry in which the entity is involved, caused by such factors as lack of technological advances or market disfavor within the industry..
- **Contagion risk.** The adverse condition or reputation of another entity or industry may “rub off” on another. It doesn’t matter whose “fault” it is, as in some cases just the appearance of similarity will be sufficient to cause difficulties.
- **Reputation risk.** This risk is associated with the reduction in the value of an entity if its reputation deteriorates. In the extreme case, this could result in a “run on the bank” scenario.
- **Foreign currency risk.** This arises if multiple currencies are involved or if the future cash flows are likely to be transacted in a different currency than the one in which the risk is being assessed. The likelihood of relative currency fluctuations may change the value as expressed in a particular currency.
- **Country (sovereign) risk.** This risk is associated with operating in a foreign country, including expropriation or the inability to export profits.
- **Business and operational risks.** These risks reflect the operation of an entity, such as in its systems, production or human resource risks; for insurance the pricing or policy termination risks. In addition, this could involve the risk of fraud and dishonesty on the part of its employees or agents. For a married person, the divorce

risk could result in, among other things, a reduction in financial resources by a factor of, say, fifty percent.

- **Capital risk.** The risk of not being able to continue as a going concern, that is, running out of adequate capital resources. In some areas, this is referred to as insolvency risk.
- **Expense / inflation risk.** There exists a risk that expenses will not be as predicted, whether due to external factors such as inflation or internal factors, as a result of mis-estimation or changes in operation or sales relative to original expectations.
- **Tax or legal risk.** Decisions about the future are often made based on the assumption that current laws or judicial opinions, or tax rules or their interpretation will continue. There is a risk that these may change. This is one example of the larger category that include political or regulatory risk, in which the environment can change based on a government decision.

A set of cash flows can be affected by a number of types of risks. For example, a bond combines both credit risk and interest-rate risk. There has recently been an increased tendency to unbundle certain risks through the use of a market, whether through the selling off of tranches from mortgages, through the use of swaps or credit derivatives, or catastrophe bonds.

Almost any set of cash flows involves risk, depending on one's financial condition and objectives. For example, the acquisition of government securities can be risky if the objective is to protect against inflation risk or with respect to interest-rate risk, or depending on the derivative involved one set of credit or default risks may be traded against another set.

The methods used to apply an adjustment for applicable risks may vary depending on the types and incidence of risk (reflecting efforts to mitigate these risks, including risk management techniques) associated with a particular set of cash flows and their relative significance. For example, in many cases risk may not occur uniformly over time. The creditworthiness of a bond at the time of its purchase may be quite good (based on a just-completed financial analysis of a company during a period of strong economic growth); however, over a ten year period it is increasingly likely that the finances of the company will deteriorate, the economy will turn sour or interest rates will vary so as to lead to a call of the bond. If the bond is held or could be purchased by investors in a number of countries, currency, inflation, and tax risks to its owners can vary considerably. Additional sources of possible risk can depend on individual circumstances, including the need for liquidity, access to risk management tools, appropriate professional staff to manage such risks, and ownership of similar assets or obligations permitting internal diversity or pooling of risks. Such potential variability also can contribute to fluctuations

in market price of this bond as the mix of potential purchasers or investors varies and deviations from an efficient market occur.

As can be seen through this simple example, the analysis of risk can be complicated. As a result, to make it practical, it is common to compare the level of risk to a high quality benchmark value determined by the market, for example, to a U.S. government security of a comparable duration. Moreover, aversion to and recognition of certain risks may vary by the economic condition and the individual situation of the parties involved.

Uncertainty, and thus risk, tends to increase with time and size of the relevant adverse probability. As a result, often a discount will tend to increase over a longer period, generally consistent with risk preferences, which in part contributes to the common upward sloping yield curve. This can be seen from the difficulty in finding long-term hedges as compared to the availability of short-term hedges. Also, risk charges can be greater for a low quality loan or asset portfolio.

Risk can be classified as being **diversifiable or non-diversifiable** (systematic). Diversifiability refers to the possibility that the owner of the set of cash flows can effect one or more purchases or transactions that enable the owner to eliminate or reduce that portion of the risk, such as the risk from random or chance year-to-year fluctuations. This often is effected through the operations of the law of large numbers, in which relatively homogenous risks are combined over space or time to reduce the overall expected fluctuations. Examples of diversifiable risk include the risk of random fluctuations in a situation with a smaller number of exposures or the risk of local economic or meteorological adverse conditions. Techniques used include diversification through adding or spreading exposures, insurance or reinsurance, or securitization.

The value of diversification varies by situation. Correlations between segments (whether characterized in terms of geography, industry or other risk characteristic) of a market are not always stable, so that a seemingly high degree of correlation one day may prove very different at a later time. An increase in the number of similar exposures reduces the risk premium for many insurance exposures, in some cases due in part to a non-diversified portfolio or due to the lack of an efficient market in which to trade these exposures (in which arbitrageurs could reduce this premium). On the other hand, recently the simple existence of diversified companies within a conglomerate recently has generally not been valued highly by the market. Many other factors, such as in the conglomerate example, including the ability to manage the diverse units effectively, may be involved. In most cases, diversification is of some and in many cases of significant value, even though it rarely is completely effective. Although typically viewed in terms of spatial diversification, this concept can also be applied to time diversification, in which case the risk of volatility in the return of equities over time (assuming a long-run higher rate of return greater than that of fixed securities) is not as large as would otherwise be the case.

The notion of non-diversifiability arises when the amount of risk cannot be reduced by increasing the number of independent units covered. Financial economics indicates that the expected return for financial risk should only provide for non-diversifiable financial risk (and risk associated with imperfect or non-existent markets and diversification risk premium, if any), although it is uncertain that certain markets actually behave in such a manner. It must be remembered that what appears to be correlated in one scenario may not be in another. Even according to theory, the conclusion is only valid if an efficient market exists; however, many markets, such as in the case of most insurance obligations or highly risky loans in tight market conditions, or in situations in which markets do not exist at all, are arguably less than efficient.

Financial economics typically assumes that there is no cost to diversify, i.e., no “excess profits” available to the risk taker. However this is not always the case (e.g., transaction costs, risk premiums, inefficient market costs). Excess profits may exist in an inefficient market. It is doubtful that many people will take risk without expecting any reward in excess of an expected cost of the risk. A price would be expected to be paid for this benefit (whether or not it includes excess profits), even if only restricted to an opportunity cost for forgone profits for keeping the risk. Where there are diversification costs, they should be reflected in the risk adjustment process. An additional problem concerns whether a market or portfolio actually is diversified, either initially or as time goes on. An underlying hypothesis of financial economics is that holding a diversified portfolio of assets reduces risk, which while correct in theory, may be quite difficult to implement in practice. How best to reflect any residual diversifiable risk may be a difficult issue to deal with and should be considered on a case-by-case basis.

The non-diversifiable risk is sometimes referred to as the risk of mis-estimation of the expected value of future cash flows (parameter or model risk). Sensitivity testing of alternative scenarios or stochastic analysis may provide insight that can be useful in the determination of the cost of such risk. This may be used to develop estimates for both the expected value and also the expected distribution of possible value, which may result in various degrees of confidence in the accuracy of the estimates. Although some feel this is not practical, actuaries prepare such estimates frequently in the case of insurance-related products and the market “prices” such risks every day. It can be dangerous to ignore such risks, which in effect would be the same as assuming that there are no such risks. In such cases, the same discount may not be appropriate for all purposes and all users of this information. Such analysis may also be helpful in determining an acceptable estimate of the impact of any market imperfections.

An example of non-diversifiable risk is the price of a put and call in the money for an identical underlying financial instrument. If there was no risk premium, the expected value would be expected to be zero. Since there is a positive cost or value associated with both of these, it can be seen that risk is reflected by a market. In this case, it is not diversifiable risk that is being reflected, as (other than such factors as transaction costs and size of the overall market) assuming the market is relatively efficient, the non-

diversification premium should not be large; this risk as described is a non-diversifiable risk, in this case relating to expected volatility (depending on whether you own the put or call, the risk is one side or the other).

Not only do investors demand a risk premium for non-diversifiable risk, but they will also normally demand a risk premium for keeping diversifiable risk, although it may be smaller than that associated with non-diversifiable risk. The size of these risk premiums will vary by economic condition (e.g., the 1998 Asian financial crisis) or shareholder opinion (e.g., flight to quality that contributed to liquidity problems at Long Term Capital Management), degree of market imperfection, lack of complete information, transaction costs, size of risk and the investor's and the market's overall current perception of risk. Further research is needed to better quantify the amount of such risk premiums.

Another aspect of the overall risk premium involves emotion or market perceptions unrelated to overall risk. One aspect of this perception relates to what I refer to as the lemming (or bubble) aspect of risk. As an example, from the Outlook section of the October 5, 1998 *Wall Street Journal* is: "Risk taking is out. Flight to safety is in. That has been gospel in the U.S. financial markets over the past two months or so, a dizzying and swift change in sentiment" that has resulted in a significant increase in risk premium because of the massive change in investor behavior. An example of the cyclical nature of market behavior that is difficult to predict:

"Much of banking history consists of one speculative bubble after another, from Dutch tulip bulbs in the 17th century to property in the 1980s – and now emerging markets and hedge funds in the 1990s. Each tends to be fuelled by an explosion of credit, a wave of unwarranted optimism and a subsequent mispricing of risk. Low American interest rates in the 1990s encouraged investors, looking for higher returns and buoyed by dreams of new paradigms, to pour money into emerging markets, domestic equities and hedge funds. As a result, the current crisis is in some part an overdue correction in overvalued, over-leveraged markets."

Bubbles can be difficult to predict, as can the timing of their bursting. History is full of financial bubbles. For a number of years, market pessimists have predicted that the U.S. stockmarket cannot continue to grow in value forever; will this result in the same type of asset bubble that occurred in Japan ten years ago?

Such lemming actions can be caused by common perceptions spread through a variety of methods, such as word-of-mouth, the financial media, or by competitive processes (e.g., the insurance underwriting cycle). It may be exacerbated through common use of the same models leading to many people taking similar action at the same time. This can lead to financial contagion or a domino effect, such as in a run-on-the-bank risk; this can be based on actual or perceived similarities in different sources of cash flows. It doesn't have to be a contagious risk (or risk that is common in many affected circumstances); it can simply be a concentration (for example, of poor quality of loans) risk. Even if one

source of risk is of poor quality, a similar risk of not-such-poor quality can be sucked in at the same time as part of a chain reaction, some of what may have occurred during the Asian financial crisis of mid-1998; on the other hand, that not-such-poor risk may look good in comparison – it depends on the situation and the intensity of beliefs involved.

For a complicated set of cash flows, dependent on many sources and types of constituent interdependent cash flows, as exists in many insurance products, it may be necessary to construct a relatively complicated model (actuaries have increasingly preferred stochastic analysis or sensitivity analysis through alternative scenario testing) to assess these risks over the range of their probable outcomes. In other circumstances, possibly as a function of the size and degree of risk in comparison with other funds of the user of a valuation, rules of thumb or simple reliance on the market's assessment of risk may be deemed preferable. In any case, a comprehensive assessment and explicit recognition of the aggregate levels of risk should be considered in any business decision. It will certainly be reflected in the working market.

3b. Whose viewpoint?

If future cash flows were entirely predictable, then there would exist no risk that the amount and timing of those cash flows would not be achieved. As uncertainty increases with respect to their amount and timing, the need increases to adjust values to reflect valuation realities.

In addition, the exposure to, assessment of, and aversion to risk and uncertainty may vary among entities and individuals. This variation can be large, depending on the set of cash flows in question, the particular risk that the entity or individual is subject to, the degree that the set of cash flows in question are correlated with the other cash flows of the entity, and the purpose of the valuation. Thus, the risk premium, the expected cost of the risk taken on, associated with the valuation of a particular set of future cash flows can also vary by the decision-maker and its purpose. The less efficient the market, the greater the weight that should be placed on entity-specific assessment. Possible approaches that could be taken include the use of a more prudent value, the entity's view of risk, or a weighted value of measurement of a comparable set of cash flows for which there is a market, if one exists.

The market also assesses risk. However, since it is not a individual person, this assessment must be an aggregate assessment. The only sensible approach to measure the market's assessment is to measure the difference between market prices and expected values of future cash flows, that is, comparing the risk as inherent in market values, if there is a market, to a relatively risk-free value. This would recognize the assessment of risk as indicated in the aggregate activity of the market.

Since the external market relies on reported balance sheet values, it may be most appropriate to reflect an aggregate assessment of risk in financial reporting prepared for external use, since such values can't reflect individual assessments. This is inconsistent with the conclusion that management should reflect its own assessment of risk in the development of values used in its decision-making. In most cases, management should attempt to understand the market's assessment as input to its business decisions; however it would be folly to only consider average hypothetical assessments when relevant individual assessment and preferences are available. In fact, as discussed earlier, risk preference is a personal characteristic. In many cases, market and individual assessments of risk (except for the fact that insider information may be superior) will be similar, so this discussion may not result in different values, even though different purposes will often result in different values (e.g., regulators' greater emphasis on solvency related risks).

In determining the price of or whether to participate in a market for a set of cash flows, be it a product, asset, or a firm, the risk associated with intended use of the set of cash flows would have to be considered. For a specific decision, it makes no sense to recognize an average assessment of risk or a typical application of the set of cash flows. However, if the intention is then in turn to trade the set of cash flows, the benchmark market's assessment may form a consideration in determining value to the firm.

Since a market-measured risk premium is not available or of limited reliability in a non-existent or limited market, alternative approaches to measurement are needed. One reason why the difference between bid and asked price tends to increase in a relatively inefficient market is that individual risk and time preferences, as well as their application to the perceived risks, are a more significant element in these prices. These attitudes and assessments are affected by the decision-maker's wealth, value attributed to risk taking, and returns demanded in order to be an active player in the market.

There may be differences between the value that owners/management assign to an entity as it is currently being operated, reflecting its own assessment of risk and risk preferences (entity-specific use and risk assessment), and the value which a market assigns (market value or fair value if an inefficient or no market exists). The assessment of a potential buyer may also differ significantly if the entity is to be operated in a different manner after purchase. This difference is one cause for the range between bid and asked prices in a market, whether or not an efficient market exists. An additional reason for this difference is that buyers and sellers have different risk and time preferences and different intentions for use of the economic good. This may result in different values being assigned to the set of cash flows. This difference can often be observed in the case in which the price of an entity when sold differs significantly from that assigned by the market immediately prior to the purchase.

It may be difficult to determine the overall risk preference of a firm because of the variety of decisions to be made and the personalities involved. For a particular decision, a

number of individuals may be involved, not just the chief executive officer. Decisions are made more consistently within a firm if risk preference guidelines are well communicated, possibly through the use of a set of corporate hurdle rates to analyze business decision-making. Regarding effective internal management reporting, meaningful capital allocation and firm specific assessment of risk would be most significant contributing elements, but benchmarks reflecting market perceptions may be a valuable supplement.

3c. Application of risk adjustment

Although inherent in both present value and market value models, explicit recognition of risk in present value calculations is sometimes overlooked in the determination of values. This is most often due to lack of time, insight, or educational experience. Explicit recognition, as compared with implicit recognition, of risk premiums is to be preferred. This encourages explicit assessment of the future cash flows, as well as forces the decision-maker to identify the risks involved and to attempt to quantify them. The approach that permits such an explicit recognition, improved information for a decision-maker, and easier validation of expectations would generally be preferred to one that does not.

There are several major approaches that can be taken to adjust for risk. Risk adjustment can be applied to the expected cash flows directly, their timing, or through the discount rate(s) applied. The general approaches include:

- Application of risk free (or certainty-equivalent) discount rate to risk adjusted expected cash flows,
- Application of risk-adjusted discount rate to expected cash flows, and
- Adjustment of both discount rate and expected cash flows for part of the risk.

Conceptually, the form of adjustment for risk should be consistent with the type of risk involved. This would permit more consistent winding down of this adjustment over time as the remaining level of risk diminishes in size. If certain aspects of risk are directly proportional to time and the duration of the cash flow, it may be appropriate to reflect that risk as an adjustment to the discount rate(s). However, if, for example, it is more heavily concentrated in the early or late portion of an arrangement, the risk adjustment might be more appropriately made to the expected value of cash flows, with no adjustment to the discount rate. More often, risk associated with a set of future cash flows consists of a combination of types of risk, each with its own expected incidence, in which case, a combination of the two approaches would be appropriate. Theoretically, the results of the approaches could be identical; however in practice due to the many factors involved, such equivalence would more likely be due to luck unless solved for directly.

Although it is tempting to adjust all three to obtain the best answer appropriate at the time of initial acquisition and subsequent revaluation, doing so may not be desirable for practical reasons, including the possibility of double-counting or overlooking a significant element of risk and contributing to the difficulty in measuring performance. Therefore, it may be more practical to combine all such risks in their application to either the cash flows or the discount rate.

In some cases, it can be difficult to translate the expected uncertainty into a risk margin or risk premium, whether reflected as an adjustment to the discount rate or to expected cash flows and their timing. The amount of risk to participants in a transaction depends not only on their expected variability and the uncertainty associated with the cash flows, but also the risks they are exposed to and their risk tolerance. A transaction serves as a hedge against a participant's other risks; it effectively may have a negative marginal risk associated with it.

In cases in which it is uncertain how the existence of risk should be communicated and analyzed, it could be argued that among transparency, practicality, and simplicity, transparency might be the most important characteristics considered. In this way, monitoring and understanding will be facilitated. In addition, as different audiences may be involved, it would be more appropriate if the facility was available to be able to make adjustments to both, both for market related values and for their own assessment of the risk involved, based on their own preferences.

It may be difficult to determine a consistent provision for risk by adjusting the discount rate and in discounting expected cash flows. By applying risk adjustments to expected cash flows, explicit application to the various types of risk may be applied to different categories and sources of cash flows. As time unfolds, risk margins may not be properly released if allowance for risk is concentrated in the discount rate.

In developing stochastic analyses, it is generally advisable, if practical, to reflect present values for each set of generated cash flows, rather than just discounting the mean value and desired percentile results. This is due to the potential importance of difference in the timing of the cash flows under the scenarios generated.

Particularly if the assets in question are complex, it may be more reliable to adjust for default risk by determining the default discount rate inherent in market values of similar assets than estimating the appropriate adjustment to be applied to estimated cash flows. Similar review of other risks as they apply to alternative types of cash flows could be made. More extensive discussion as to the types of situations in which the three approaches would be more appropriate, may develop a set of rules or criteria to apply for a given type of circumstance.

Mechanically, in general a single equivalent discount rate can be determined (unless too many non-offsetting positive and negative cash flows are anticipated – see section 5d for a further discussion of this issue) such that a provision for risk in any set of cash flows can be computed on an equivalent basis. However, it is best to determine the most appropriate components underlying the present value of a set of expected cash flows; if practical expedients are needed, so much the better. In many cases, a single interest rate may not be appropriate.

The type of adjustment (expected cash flows, discount rate) also may be influenced by the intended use of the resulting values. If it is necessary to determine intermediate values, such as the year-by-year value of the future cash flows or the undiscounted value of the future cash flows, it may be more convenient only to adjust the discount rate, thus applying a risk-adjusted discount rate to the unadjusted-for-risk expected cash flows. The choice may also depend on the decision-maker's (if not the market's) comfort level in dealing with estimates derived from the evaluation. If the decision-maker is more comfortable in comparing values based on a common discount rate, a common risk-free discount rate may be preferable.

Some believe that risk adjustment of the cash flows is superior, for reasons including:

- it is difficult to objectively construct risk loadings by varying the discount rate;
- the sign of the adjustment to a rate may vary depending on whether positive or negative cash flows are involved;
- the discounting may complicate the aggregation of estimates of cash flows if they involve different discount rates, with the alternative of using average discount rates not being easily understandable;
- several sources of risk are not proportionate to time; and
- any discount rate related risk margin might at least theoretically be replicated by adjusting the cash flows (the reverse is true but may be more complicated).

Perhaps the reason financial analysts have tended to equate risk with discount rate adjustments is that this approach has traditionally been used in evaluating the risks associated with the bond market. For credit and default risk evaluation for bonds, it may be desirable to risk-adjust via the discount rate. The bond market tends to use the discount rate to price this risk. The reflection of risk together with risk and time preferences can explain a great deal of the differences in bond yields. If the expected cash flows are risk-adjusted (whether or not a stochastic model is used to reflect asymmetry in the expected cash flow distribution and transaction costs), the use of a risk free discount rate may result in a higher value than otherwise derived market values. This

is due to the existence of a risk premium applied for managing a credit portfolio. A comparison of risk adjusted bond yields may improve investment decisions.

In certain cases, the risk adjustment, if applied to the discount rate, could result in a negative discount rate. This is more likely in a low interest rate environment or when dealing with negative real interest rates in determining the value of negative cash flows, such as the value of a provision for property/casualty losses. Although there is nothing conceptually wrong with this, it can be difficult to understand and then explain. In such cases, risk adjustment may be better made to the cash flows themselves.

In addition, different components of the cash flows may carry with them a different degree of risk (or desired return). If so, it may be easier to discount them all at a common interest rate, which would be the risk-free rate. For example, a set of cash flows consisting of several currencies may be involved. Approaches that can be used in such conditions include averaging risk factors, separately discounting each appropriate grouping or combination of cash flows, or through reference to similar types of cash flows traded in a market in which such information can be obtained.

3d. Methodologies

The estimation of future cash flows usually is the first step in the analysis of risk. Actuaries typically focus on the distribution of such expected values, sometimes selecting point estimates as expected values, sometimes focusing on a range of reasonable values. Historical experience of similar sets of cash flows can be useful, but normally are not the only source of the basis for estimating an applicable range of possible values. In some cases, the identification of the reasons for the distribution of possible values can be useful in identifying sources of possible risks involved. Possibly stochastic methods, a study of alternative probable scenarios, or use of dynamic methods may be used. If it is expected that a non-symmetric distribution is likely, either a degree of prudence is reflected (selection of a higher than 50% percentile) or a more refined approach is used.

Actuaries have commonly referred to the adjustment for risk (risk margin or risk premium) as a provision for adverse deviations (PAD). Though the way that such a provision may be applied may differ (adjustment to the cash flows, their timing, or to the discount rate), the method used within a given entity should be derived in a consistent manner. However, due to the variety of types of cash flows involved and differences in relative marketability of the economic good involved, different techniques and approaches have been applied.

Differences also may arise according to whether application of entity-specific or market surrogate measures serve as a basis for these estimates. Actuaries have historically relied on entity-specific measures, in part due to their concern that the entity being analyzed remains a going concern and a focus on insolvency risk. For example, professional

guidance has been developed in Canada to provide actuaries with a range of generally acceptable level of PADs by type of cash flow or experience characteristic (such as interest earning, expense level or mortality rates), with the actuary applying professional judgment as to where within that range the PAD should be selected. In most other countries, such specific guidance has not been provided; rather, education and training from which to base such judgments is available.

No single methodology for risk adjustment has achieved universal consensus. Further investigation is warranted in this area, although it may turn out that no single method will ever achieve universal acceptance or be universally applicable. Currently, the methods used do not necessarily come up with equivalent answers, due to differences in underlying assumptions and personal application of them. Each decision-maker should make applicable point (or ranges of) estimates according to the decision-maker's personal risk assessment and preferences.

A common measurement approach used, where practical, in the market valuation of risk is direct observation. A risk premium can take the form of the margin a third party would require to purchase a set of cash flows if positive over the price it would take to purchase a risk-free set of cash flows with otherwise similar characteristics (or sell a set of cash flows if negative). As defined, such a risk premium typically reflects the relative risk involved. One technique sometimes used relies on the concepts underlying the capital asset pricing method (CAPM), which assumes that expected return is related to expected variability as measured by its standard deviation. An issue in the application of this method is whether the risk measurement criteria reflect the market's views or the risk preferences of the current "owner" (entity-specific value). In any event, its risk profile is compared with the corresponding risk profile of an aggregation of similar risks in a comparable market. The comparison results in the inferral of an equivalent discount rate.

Additional methods that have been used, often in combination, include the following.

- Discount rate and/or the underlying cash flows adjusted to reflect **prudent margins**. If the risk is related to time, an adjustment of the discount rates (risk-adjusted discount rates) to reflect prudent margins have been used. For risks not associated with time, this approach would often be combined with a PAD related to the uncertainties and risks of the cash flows. For pricing a product, the level of prudence may be set at a level appropriate to maintain a given competitive position. In other cases, the level of prudence is set at a judgmental level or at approximately the same level as provided through the application of more rigorously derived methods.
- **Option-adjusted spread**. As applied to assets, this method is relatively straightforward, particularly for marketable financial instruments. For an instrument without an observable price, a matrix of similar marketable instruments can be constructed, based on relevant characteristics. For each combination of these characteristics, the option-adjusted spread is computed. For liabilities, it is not as

straightforward and may be difficult to apply, such as in the cases involving offsetting assets (for example, future premiums for insurance) and in the selection of similar liabilities to use as comparables.

- **Utility theory.** This method reflects the economic theory of human preferences, sometimes using refined stochastic models. As such, it can be viewed as a comprehensive theory in which all of the methods can be characterized as special cases. This theory has been criticized because of the difficulty in determining appropriate utility functions for individuals and groups. Ruin theory can be viewed as a subset of utility theory, with a cliff utility function when surplus equals zero.
- **Option theory.** A combination of decision theory and option pricing mathematics can be applied to determine the value of a set of uncertain future cash flows. This approach can add value by recognizing that reality is rarely a choice of all or nothing. It is often used to price hedges, useful for asset / liability management and modeling. It is increasingly being used to analyze the cash flows for capital budgeting decisions and capital project analysis.

Business strategy and decision making is viewed more in terms of a series of options rather than as a one-time decision, reflecting new opportunities as they develop and old possibilities as they disappear. As active decision-making regarding timing of decisions and amounts (since such strategies as sharing of risk and application of risk management techniques are possible), option pricing methodologies have become increasingly popular. This is particularly appropriate when comparing alternative sets of cash flows. This approach values these real, not necessarily financial options. Because of options that are available, dynamic modeling and decision-making can be applied to reflect a changing environment and the impact of the decision-makers actions.

- **Scenario tests.** Either a discrete number of separately run alternative scenarios or a large number of stochastically generated scenarios are run, based on assumed statistical distributions or professional judgments as to probable, but not highly unlikely cash flow scenarios.
- **Value-at-risk.** This method, sometimes referred to as surplus at risk and commonly used at banks, was originally developed for trading portfolios. It attempts to estimate the amount of money an entity will be likely to lose during a certain period. This is done through certain stress-test assumptions and has primarily focused on the analysis of market risks, lately in an increasing number of contexts. Although the assumptions used are not worst-case, they typically reflect significant adverse results, such as through adverse trading conditions. These models often look at the risks associated with an entire company (enterprise risk). Banks typically apply this approach over a several day or week period. One advantage is that the results are often summarized in one or a small number of values that various levels of management can understand.

Actuaries, who have applied similar risk-theoretical approaches over a longer period of time, are attempting to apply such summarization concepts over a longer time horizon.

It is becoming more common to attempt to evaluate and manage enterprise risk, the net effect of the risks associated with an entire entity. One reason for this trend is the realization that risk is not necessarily additive, since various risks are not independent of one another and other risks not associated with individual assets or liabilities can be important. Such analysis enables a focus on embedded values and firm value, to let management more easily see the effect of diversification and various risk management strategies. It also encourages an examination of all types of exposure to risk, prioritizing them and permitting contingency planning and risk management to be conducted in a better prioritized manner.

The first step in determining the market's perspective of an economic good's risks is to determine the type (and form) of risk recognition. If a set of market prices is used as a surrogate for market or fair value, then a discount rate may in most cases be determined implicitly by solving for a certainty-equivalent. Second, market risk for similar economic goods would have to be estimated. If an efficient market exists, this may not be too difficult – subtract the total expected market value from comparable market prices. As an example of an insurance application, a price quoted in the reinsurance market may be used to estimate a market-based risk discount rate.

Several issues need to be answered in order to adjust for the impact of risk in the estimation of the future cash flows, including the following:

- Does prudence, sometimes equated with **conservatism**, have a role to play in the estimation of the value of a set of cash flows? What standards of prudence are appropriate - should they be case-specific? What level of provision for adverse deviation should be provided for in a risk discount? If the use of expected values means that an entity, such as one providing an important financial security role in the economy (such as a bank, insurance company or pension fund) whose future cash flows are material and uncertain will become insolvent or bankrupt half the time, reported values may not provide sufficient or appropriate information regarding that company's financial accounts. Value does not include an excess of prudence. These related issues may best be addressed through applicable actuarial standards to provide for the availability of sufficient guidance for a professional to apply. Such guidance may be limited to a discussion of the considerations that should be reflected in any such calculations.
- **Random fluctuations.** If the risk desired to be reflected includes the risk of random fluctuations (usually considered a diversifiable risk for financial reporting purposes), an adjustment relating to statistical variance or standard deviation may be appropriate. Depending on the objective of the discounting exercise, it may be deemed

inappropriate and provided for through other means, whether through capital or through risk management techniques.

- Should such **risks** as currency, default, liquidity or tax risks be reflected? If a market based risk adjustment is being made, such risks should implicitly be included in available market prices.
- Should the risk considered be **one-sided or two-sided**? Since risk usually involves only adverse deviations from expectations, it would be natural to restrict analysis to such adverse outcomes. However, with so many options and possibilities during intermediate periods, it is not always easy to identify the adverse results. For example, an increased amount of policy terminations of an insurance product may be more or less desirable, depending on the characteristics of the products and insurers involved.
- Should risk vary depending on the **degree of efficiency of the market**? The impact of market imperfections is an appropriate risk to consider, although some believe that it is impractical to reflect quantitatively the effect of such imperfections. The less efficient the market, the more valuable the use of fundamental analysis or application of present values becomes.

The more efficient the market and the more risk management techniques are used, the less need there is for adjustment for certain types or aspects of risk. A market price may be presumed to have been adjusted for risk, as assessed through an efficient or an inefficient market. The larger the risk, the more important the practice of risk management becomes. In addition, the larger the impact of risk, the more significant the decision as to how best to adjust for risk.

3e. Risk management

Use of effective risk management can reduce risk premiums, encourage better decision making, and better recognition and understanding of risk, and allow tighter pricing of products. The effect of risk management in reduction of risk should be reflected in a manner consistent with the reflection of risk. The market tends to reward reduction in risk, and thus should reward effective application of risk management techniques. If risk is applied to expected cash flows, then the effect of risk management should also be reflected in expected cash flows; in a corresponding manner, if applied through discount rates, risk management would be reflected in discount rates.

Risk management has taken many forms within different industries, entities, and individuals. Within an insurance company, for example, the treasury and investment departments have traditionally managed an entity's financial risks, including currency, market, and equity risks. Business risks within an insurance company have typically been

managed by actuaries, sometimes separately for each business unit. Asset / liability management risks are increasingly managed jointly by the actuarial and finance or investment staffs. Operational risks, such as those associated with systems, production and human resources, have been managed through applicable departments or through a traditional risk management department, and have focused on the identification, mitigation, and control of risks and insurance coverages. Event risks, covering possible catastrophes, legal risk, and changes in public or tax policies, are also managed by traditional risk management areas and through traditional risk management techniques where available at an acceptable price. They can also be coordinated in a more comprehensive manner as part of enterprise risk assessment and management, a method more frequently applied.

The correlation or interdependence of cash flows influences the level of uncertainty and risk associated with the present value of their combination. Even when the expected value of a combination of cash flows is stable, variations in their value resulting from changes in certain conditions (such as changes in the economy or characteristic mix) may be influenced by the degree of their correlation. If the correlation is non-zero, then it would be appropriate to model (particularly if the range of their probable values or distribution is reflected) their value as a function of these conditions and their changes. Because the resulting impact of a combination may be difficult to quantify, care is needed to appropriately reflect the correlation of cash flows when determining present values.

A balance between the theoretical and practical is most desirable. Based on the experience of the fall of 1998, according to the *Economist* (page 85 in its November 14, 1998 edition) “the boss of one big (investment bank) firm calls super-sophisticated risk managers ‘high-IQ morons’: quite simply, they relied too much on theory and not enough on market nous”. The key is to recognize risks, quantify their effects (through some of the methods mentioned in previous section), and either be willing to manage them effectively or be willing to live with the cost of possible losses. This is one reason why it is useful to estimate the range of losses (unwinding positions, selling off assets or liabilities in a stressful situation) and situations that could lead to them, whether or not the extreme values would be recognized in a financial statement.

To reduce the effect of such uncertainties or, in this case, to reduce risks associated with a given set of cash flows, a variety of risk management techniques may be applied, including hedging such risks. If a complete (efficient) or partial hedge has been used, it would be appropriate to reflect the increase in certainty associated with those cash flows, by applying a different discount rate to that set of cash flows (or adjusting the expected cash flows, if a risk adjustment is applied in this manner. If risk is adjusted through the discount rate, the overall discount rate would be equal to: discount rate (hedged) x %effective hedge + discount rate (unhedged) x (1 - % effective hedge). Thus, the effect of an increase in a partially effective hedge would be to use a discount rate between the hedged and unhedged rate.

Types of such hedges include:

- A matched set of cash flows, with a set of obligations and a corresponding set of assets (such as through the use of duration matching or immunization);
- A financial instrument (such as through the use of derivatives, insurance, credit enhancements, dynamic experience adjustments (including dividends, retrospective experience adjustments or other guaranteed elements of an insurance program)) that reduces the fluctuations or uncertainty in aggregate future cash flows with such a set of cash flows negatively correlated with the first;
- External guarantees, such as through U.S. state guarantee funds for insurance or the U.S. Pension Benefit Guarantee Corporation for pensions, even though such guarantees tends to increase moral hazard and thus encourage overall riskier behavior by the financial institution benefiting from the guarantees (actually the consumer in the case of insurance and banks and the employee in the case of pensions);
- Offsetting risks (such as selling both life insurance and annuities with offsetting mortality experience trends); and
- Although not a hedge in the strict sense, moving risk to a third party or back to the first party, such as a reinsurer or to a customer through dividends or limited or no interest guarantees.

Financial economics, popularized by Harry Markowitz, Merton Miller, Fisher Black and Myron Scholes, among others, has made the pricing of derivatives a science and greatly increased their value. It has enabled those who do not want to bear certain risks to shift them to others who do. However, the theory underlying this science often includes important assumptions that have to be carefully monitored. In addition, advocates do not always agree on their conclusions. For example, efficient markets, maintained through the use of arbitrage to reduce the difference between prices and their fundamental values and the benefits of diversification, sometimes exist. However, their application to particular situations must be examined, the assumptions of which may constitute another risk that varies in size by circumstance. For example, in some cases, when the need for arbitrage is the greatest, credit may be most difficult for potential arbitrageurs to obtain.

Two approaches commonly used by actuaries to reduce risk are worth noting. One is reinsurance, many types of which can reduce the range of possible fluctuations through the elimination of specified types of extreme swings in retained experience or sharing a risk in a proportional manner. The other is the use of various dynamic control processes, which actuaries have applied in a wide variety of risk situations by techniques to reduce the impact of positive correlations or systemic risks influencing the size or timing of cash flows. Examples of applications include the insurance of non-independent events such as mortality and earthquake hazards or interest rate movements, situations in which a high

level of uncertainty can exist, increasing the level of responsibility and incentives and aligning the motivations of insureds, with as well as for other purposes). Examples of techniques in use include the use of dynamic analysis or dynamic control systems, including feedback loops, experience adjustments, deductibles and coinsurance, and dividends (bonuses). Pooling and diversification techniques may also be applied. Depending on the degree of effectiveness of these methods, adjustments in PADs may be appropriate. These are referred to as applications of the actuarial control cycle.

There exist a wide variety of situations in which the risk management of a set of cash flows is desirable. Variable (unit-linked) insurances or annuities are on one end of the spectrum, in which the obligation is explicitly expressed in terms of the asset – the policyholder bears the interest rate risk (a financial intermediary always bears some of the risk) and the two are by necessity (by contract) tied together, a perfect “hedge” with respect to that risk, with management charges dependent on fund performance; this matching reflects the fact that interest rate risk has been transferred to the policyholder. At the other end of the spectrum of risk retention is a situation in which there are no assets (e.g., governmentally provided insurance with no pre-funding) or the assets are managed in a completely independent fashion. While different discount rates, or at least different risk adjustment, would be appropriate for the two extreme situations, how to reflect the differences in the middle of the spectrum of hedging or risk management is more problematic.

An alternative to risk management is the allocation of additional risk capital to provide for a reduction in the overall impact of risk to an institution. However, often the dead-weight costs associated with such capital can prove more costly than the use of many risk management techniques.

One reason for basing a **discount rate** on a matched (or replicated) set of future cash flows is to reduce overall risk and as a result to minimize the size of the risk adjustment needed and thus risk capital needed to invest in the business.

In determining the fair value of a set of obligations, it is important to decide whether a discount rate should serve as a function of the expected earned rate of assets held backing the obligations (if serving as a hedge), or be independent of such assets. Other questions that may be applicable include:

- If the future cash flows are negative, should it matter if assets are explicitly allocated to the obligation?
- What if the assets are illiquid or could become illiquid (and under what circumstances could this occur?)
- What if there are no current assets backing the obligation?

Conceptually, when an entity has negative cash flows, as in the case of the obligations of an insurance company, pension fund, or bank, funds from some source must be available to enable the entity to settle its obligations. In such a case, if called on to attest to the appropriateness of a given level of provision, an actuary may provide an asset adequacy opinion (regarding the adequacy of the size of the assets underlying corresponding insurance obligations as of a valuation date), i.e., the actuary will indicate whether the available sources of funds, that is, current or future positive cash flows including those of currently allocated assets, will be sufficient to provide for the relevant future obligations. In order to do so, the actuary will typically reflect entity-specific characteristics of the obligations and risks, and may use a lower discount rate than expected applied to negative cash flows to reflect interest-related risks and risk-adjusted cash flows to reflect non-interested related risks.

If the cash flows underlying the assets and obligations are completely matched, thus resulting in an efficient hedge against asset / liability mismatch risk, the resulting value would be independent of possible interest rate market movements and certain other risks. One example of an efficient hedge as mentioned above, is the case of a well-run separate account, unit trust, or mutual fund, in which the obligations are explicitly a function of the allocated assets (although these could be looked at as never containing such risk in the first place). In a more general situation, it may be possible to identify some of the positive and negative cash flows that can be matched, leaving only a residual set of cash flows. In such a case, an adjustment for such a risk is not appropriate, and the risk adjustment need only be addressed with respect to the residual cash flows or other risks.

In many cases, an efficient hedge cannot be obtained or is judged to be unaffordable. If the correlation between a set of assets and obligations is high, it may be appropriate, especially if a combination of assets or obligations is not marketable (e.g., can not be securitized), to reflect a lower level of risk. The lower the correlation, a higher degree of risk (reflecting the level of remaining risks, such as credit, currency, and mismatch risk) should be reflected, rather than fair values of the component parts. Such an approach would be appropriate if an allocated set of assets backed the obligations. Another way of looking at the market discount is to estimate the current value of the set of future cash flows as if the entity would settle or sell them.

If it is determined that the discount rate should be a function of the expected earned rate of a set of assets backing the obligations, then it would be appropriate for applicable investment expense, a form of transaction costs, to also be reflected. Typically, the investment expense would be included with transaction costs, i.e., the marginal cost associated with the acquisition, maintenance and eventual termination of those assets (e.g., acquisition or management fees). In addition, it may also be appropriate to reflect the indirect cost of management of those assets. It is most common for this expense to reduce the interest discount rate, as the investment expenses should be related to the level of net expected interest to be earned.

For some obligations, it may be determined that the rate of discount should not be based on the portfolio of assets actually held. An example could include situations in which no designated assets are allocated to back the obligations or if there is an inefficient market for the obligation (that does not include any corresponding asset). In such cases, adjustments should therefore be made for risk and the discount rate(s) would be determined on the basis of an ideal portfolio of assets which minimizes the risk that the asset cash flows will be insufficient to cover the cash flows associated with the obligation.

4. THE DISCOUNT RATE

4a. Time value of money

As described earlier in this paper, in order to analyze underlying values associated with a set of future cash flows, it is necessary to develop a present value model. Even though there are many variations in the basic model, at its heart is the selection of an appropriate discount rate. Although risk, risk preference and time preference can all be applied through the discount rate, alternative models exist, that is, to reflect these to the expected cash flows instead. As a result, in the following discussion these basic elements are discussed together, regardless of the form of their application. Also, the discussion presents two approaches to the selection of such a rate.

Discounting a series of cash flows at an arbitrary benchmark discount rate will produce just that, an arbitrary present value. In order to select the most appropriate present value model, together with its accompanying assumptions, an actuary typically evaluates the objective of the valuation, relevant aspects of the cash flows and the environment that could affect their size and timing, and the risks associated with them. Since, at least conceptually, risk and time preference can vary by the individual user of value-related information or decision-maker (not to mention the risk criteria), there most likely will not be a unique value for all such users.

Such an approach can be viewed as being inconsistent with the assignment of a common value for all owners or investors of an asset or liability (or any economic good). However, such a common value is important to produce publicly available, reasonably comparable information that can be of value to many users through financial reporting. Thus, for this purpose, consensus or aggregated assumptions are necessary as assessed by the market. The lack of an efficient market requires estimates of value that would have been developed by such a market.

One approach that could be take is to apply a present value model by using a discount factor based on a market-based approach. This develops values as if each element in a financial statement is independent of each other, i.e., developed in a marginal manner. Although assuring consistency in reporting, this may not capture the value of the enterprise as a whole. It also has significant limitations, as it may be difficult to consistently determine the effect of the interaction of all assets and liabilities on the aggregate risk and in turn value of, for example, a financial institution.

Disclosure of risk-based capital would be of value, reflecting estimates of the effect of interactions of these aspects of the financial condition and operations of the entity, as well as the effect of interaction of assets and liabilities and internally generated goodwill. In any event, sufficient disclosure of information is needed for others to determine their own estimates of value. Such market-based values, while implicitly recognizing average

market-based assessment and risk, do not require supplemental information developed through present value models. Problems arise for those economic goods for which efficient markets do not exist. Nor does it completely resolve the needs of decision-makers related to whether to participate in the market.

Some argue that there is no sufficiently practical method to form an objective basis for an accounting standard for the general application of present values to all assets and liabilities. Of course, other than with due consideration to materiality and reliability of estimates, this should no longer be the case. In general, determining present values with today's technology is not that difficult. For particularly complex calculations, such as those applied to insurance and benefit obligations, actuarial standards of practice are available to specify the considerations that should be reflected in any such calculations. In addition, point in time estimates of market value are sometimes not reliable or stable, not relate to the future transaction prices, and in order to be of direct use to the individual user, should reflect their intended use. For all these reasons, the choice of discount rate is important.

In market-based terms, the discount rate(s) should represent threshold rate(s) of return considered necessary to attract a willing buyer "the objective (of which) is to approximate the rate(s) which would have resulted if an independent lender had negotiated a similar transaction under comparable terms and conditions with the option to pay the cash price upon purchase or to give a note for the amount of the purchase which bears the prevailing rate of interest to maturity"⁸. A similar but shorter definition is the yield that would make an investor indifferent between receiving a single cash payment today or a larger single cash payment sometime in the future.

There are several possible bases for selection of a discount rate, including:

1. Current market rate. This is the basis for a market-approach. The rate selected should vary by duration, possibly be based on spot rates, and be consistent with its intended application.
2. Current asset earned (coupon) rate. For a fixed income security, this could be based on historical market prices, reflecting the original coupon rate; for equity, it would likely include an equity premium. If the asset reflects a market-based value, the earned rate would actually reflect the current market rate for the remaining duration of the security.
3. Fact and circumstance rate. A different rate would be selected based on the particular business decision being addressed. It could also reflect both individual time and risk preferences.

⁸ Accounting Principles Board (APB) 21

4. Current borrowing rate. This rate would be one that is available to a low-credit risk borrower.
5. Current settlement rate. This rate represents the equivalent yield from the viewpoint of a seller. A spread would generally exist between the borrowing and settlement rate.
6. For liabilities, corresponding expected earned or credited rate. If the corresponding assets are valued on a market based value, the rate would not be the portfolio or amortized rate (if fixed income), but rather the current interest rate based on the expected maturity, corresponding to the type and asset quality of the investments which provide funding available or designed to provide for the future obligations. For equities, an expected equity premium is often reflected. Although there would still be an asset / liability mismatch risk, this would typically not be reflected directly through the discount rate applied. If the liabilities represent funds held for a third party, the rate could reflect the interest rate credited to those funds.
7. The entity's cost of capital or hurdle rate for making investment decisions. This would generally include a premium for risk-taking. In the former case, it represents the average rate that the entity could borrow at in order to raise sufficient funds to make an investment. The hurdle rate, which may be related to the former, represents the minimum rate at which an entity should be able to earn on its investment in order to make an investment be worthwhile.
8. Regulatory or required rate. The rate dictated by regulatory authorities (e.g., a maximum reserve rate for insurers) is required in order to achieve some public good, such as solvency of a financial institution.
9. Real interest rate. This would only be applied to non-inflation adjusted cash flows and is the nominal rate adjusted for expected inflation for the applicable duration.

In addition, the rate used could consist of risk-free, risk-adjusted or option-adjusted rates. Smoothed or point-in-time values could be applied, depending on the needs of the situation. It could also reflect the entity's default risk (see section 4e for a discussion of this possibility). If an investment is being made, it may be appropriate to reflect the amount of risk capital required in the determination of the discount rate.

Several fundamental issues that underlie the determination of the basis of discount rates, in some cases mentioned elsewhere in this paper, are discussed below. In some cases, more issues are raised than answered.

- **Time value of money.** Should one of the types of rates listed above be used, based on for example, the current yield curve, current asset yield rates, weighted average cost of capital, current incremental borrowing rate, smoothed historical yield, or one

that represents a longer term view expectation of interest rates that will be effective over the period during which the obligation will be paid? The answer depends on the nature of the cash flows. If they are short term in nature in an efficient market, then the current yield curve should be utilized. However, in applying risk management techniques of financial security systems, such as ones that provide long-term insurance protection or retirement benefits, the current yield curve only gives a snapshot or transitory look at the market that will be relevant over the relevant period and thus does not necessarily relate to the entire program. Historical or even current interest rates may be of only limited relevance, since the rate should represent the accumulation of moneys invested and reinvested for years to come, and thus in such cases expected future rates should be considered in the selection of discount rates. Otherwise, short-term fluctuations may unduly influence expected long-term value and decisions. Although yields on current long-term assets may be useful to base a discount rate in certain circumstances, such assets may not relate to either the assets being held or, if they back up a set of obligations, they may not relate to the future size of the future obligations.

Should the risk adjustment be applied through option pricing methods or simple observation of the level of adjustment implied by the market or similar sets of cash flows as assessed by the market? In any event, time preference would be reflected through the application of yield curve techniques. If similar cash flows are not traded in a market, such a determination may be difficult and cash flows not as closely related to the economic good being valued may have to be relied upon.

Should the current market comparable rate (such as is represented by a high quality corporate bond) be used, as required in the recently revised IAS 19 regarding treatment of retirement benefits for financial reporting purposes, or should the rate used be associated with the type of assets underlying obligations for insurance companies or retirement plans (reflecting an equity premium, assuming that the expected long-range salary increases will be correlated with expected long-term equity earned rates for those invested in equities)? Current opinion is divided on this issue, due principally to differences in background and experience of those involved. Assuming that an effective asset / liability management system is in place and that an efficient market does not exist for such programs, it could be appropriate for the rates to reflect the assets underlying the obligations. Some actuaries believe that rates should not automatically reflect yields on assets actually held, but rather on the set of assets that would optimally reduce risk in relation to their corresponding liabilities.

- **Entity-specific cash flows and time value of money.** Should the discount rate depend on the specific entity's operations or perspective, be entity-neutral, or vary by application or by the type of cash flow involved? In general, for a particular user of such information, it would be appropriate to reflect the decision-maker's situation and intended use of the cash flows (if able to influence the cash flows), including time and risk preference. For financial reporting, the most appropriate value is not as obvious,

as there is more than one user of this information. The owner of the asset or liability would be interested in both the entity-specific value (reflecting the environment in which it is operated) and in the market value (the estimated price for the firm currently offered in the marketplace). In fact, the value and associated risks need to reflect the company's expected use of an asset or settlement of a liability and the impact of the entity's proprietary skills in that use or settlement. At the same time, the owner or investor may be interested in what the market perceives the value to be, as fair value may be a more appropriate indicator of current or future market prices. If there is no market for the asset or obligation, the expected value based on entity-specific values may be the only valid value determinable.

“Entity-specific measurement, by its nature, brings something else with it” (Wayne Upton, staff to FASB on September 30, 1998); that is, the entity has what could be classified as an intangible asset or liability, representing the difference between the entity's performance and perceptions as compared with corresponding values representing the “average” value represented by the “market”. Is it so bad to reflect the contributors to value?

It has been argued that for financial reporting purposes the discount rate should not vary by situation, as it is more important that the resulting values be verifiable, objective, and consistent with the overall assessment of the value of time as placed on any cash flows by the market and under all circumstances. However, it is argued here that for business decision-making, the preferences of the decision-maker should be reflected.

In many instances, particularly for marketable financial instruments, there are established market rates for similar instruments, portfolios or transactions. Where such market rates exist and are relevant to the expected future cash flows, it may be more efficient and relevant to look to those rates. If not available or relevant, alternative methods would be called for.

- **Relationship between obligations and supporting assets.** This is a controversial issue, in that in many types of situations actuaries have often reflected the mix of current and anticipated assets in their liability discounting for many years, while there are some that now contend that this approach is inappropriate for financial reporting purposes. Reasons for this approach include concern with insolvency risk and protection of policyholders and other stakeholders against credit risk relating to financial security systems that actuaries have historically been involved with. In addition, distributable capital or surplus is based on actual interest earnings and not those of a hypothetical portfolio.

If the discount rate applied in the measurement of a liability is being considered, such as in the case of the obligation to provide certain insurance benefits, should the interest rates of assets actually held backing up those obligations be reflected in the

determination of the discount rate? If a current market-based approach is applied, the mix of assets involved would be reflected, rather than just the timing of their expected cash flows. This mix and timing of cash flows are necessary to analyze current asset / liability management and risks. Of course, if liquid and if there is a market, these elements could be traded. However, it seems to be naïve to assess value of an entity independent of the current characteristics of the entity. It also has to be noted that it would be inappropriate to reflect the total risk premium inherent in the return of an asset portfolio, since otherwise a high risk investment strategy (e.g., in junk bonds) could be used to justify a lower value of liabilities than a risk-free strategy.

In the case of many obligations, such as for insurance or retirement benefits, expected market-based rates corresponding to the duration of the assets should be reflected in determinations of value. This should be supplemented by a risk-adjusted capital valuation that would, among other things, reflect any asset / liability mismatch risk. These rates more appropriately relate to the expected benefit cash flows that the assets are being used to fund, whether through a trust, union contract, or as part of a legal requirement such as ERISA in the U.S. In such a case, the value that the obligations would take in settlement with a third party would not be relevant. If based on historical (including amortized) cost, it would be appropriate to reflect the underlying assets. The only situation in which the yield rate of the underlying assets would be ignored is if the assets are based on a fair or market value based accounting system, reflecting tradable assets.

Although it may be appropriate that the credit risk of the assets underlying the liabilities not be reflected in the valuation of the liabilities (being reflected in the value of the assets), it can be important to reflect the changing duration mix, asset mix and nature of the cash flows involved. It is relevant to reflect the changing risk profile of a set of assets, the discount rate being kept up-to-date on a frequent basis to reflect any change in the nature and timing of expected liability related cash flows.

For financial reporting purposes, a desirable characteristic for the value of a liability is to be independent of the value of the assets held. This conclusion assumes that any asset / liability mismatch risk is reflected as part of risk capital that is entity-specific. Although for reporting purposes, such an approach reduces the necessity to directly allocate assets to the liabilities, this type of allocation permits more effective management of a significant risk of a financial intermediary and other firms to a lesser extent.

Relationship between discount rates and cash flows. Significant characteristics associated with future cash flows should be reflected in the discount rate(s) selected. In the examples of pension plans or insurance, if an efficient market existed and the consensus of that market was to fund these benefits by a particular type of asset (whether or not that type was actually used), it would seem logical to reflect current or expected return on those assets in the discount rate selected. If they operated in an

inefficient or non-existent market, a decision-maker would want to reflect more entity specific values, by discounting future benefits at rates commensurate with the assets funding the benefits.

If for example, the amount of future cash flows are expected to be related to the return on equities (such as in many pension plans) or mortgages (such as cash flows for insurance benefits whose expected amount is related to returns on mortgage that serve as their funding vehicle), and funded in way to minimize the risk of not providing such benefits, the discount rates used should correspond to the expected returns of the underlying assets. Then the level of PAD would depend on the degree of correlation with other available expected cash flows (reflecting the degree of hedging applied) or for the purpose for which the information is to be used. A measure, such as an applicable government bond (a stand-in for current risk-free discount) rate, unrelated to those benefits, would not be appropriate.

Should the discount rate applied to future cash flows be related to factors that would be expected to influence the level of the benefits to be paid? If the benefits are expected to be adjusted by changes in such factors or indices as the cost of living, payroll or equity performance, then it would be appropriate to reflect expected yields from equities (assuming that the program is so invested). If the benefits are fixed in nature, then interest rates related to fixed income assets might be more appropriate. In some cases, the composition of the actual portfolio may deviate from a theoretically desirable relationship, perhaps by choice or by regulatory requirements. In such a case, it may be more appropriate to reflect the actual portfolio, as that is what will influence the amount of ultimate shortfall and will have to be contributed by either the sponsor of the plan, the entity owning the fund, or from future contributions in excess of normal.

An appropriate question that should be addressed is the degree of expected correlation between expected benefit cash flows and corresponding assets. Of course, it would desirable to achieve a near-perfect correlation, such as in the case of the use of cost of living-indexed bonds where available as a good match for cost of living linked benefits. The lower the degree of expected correlation (over the period of the expected cash flows involved), the smaller the extent that such rates should be reflected. Asset / liability investigations using stochastic models can be applied to search for optimal or semi-optimal investment allocations for particular sets of liabilities.

An argument has been made that since assets are fungible, that is, can be traded for other assets with significantly different characteristics, a risk-free discount rate should be used, not necessarily related to cash flows relating to given obligations. Such reasoning does not appear as relevant as the characteristics of the cash flows and overall investment characteristics that the market would expect to be applied in portfolio selection. On the other hand, just because credit-risky assets are used,

doesn't mean that that a higher discount rate should be used, as this could result in insufficient and inappropriate benefit provisions; however, it is expected that if a return greater than the risk margin is anticipated, a higher provision may be appropriate.

A significant issue in pension actuarial work in the U.S. in the 1970s was whether explicit assumptions should be made regarding both wage inflation and interest rates and whether it was acceptable for them to offset each other. The American Academy of Actuaries recently decided that separate and explicit assumptions concerning these factors should be made. Not only are they not perfectly correlated, but also examining the impact of both assumptions will generally lead to a more thorough and meaningful analysis.

4b. Application to negative and positive cash flows

The financial effect of the application of a discount rate may depend on how negative and positive cash flows are combined. That is, if one component is included with cash flows of an opposite sign, the present value may be affected (assuming that the discount rate is risk-adjusted). For instance, the positive expected cash flows arising from the considerations or contributions to an insurance contract or retirement plan are usually combined with corresponding benefit and expense outflows; if the present value of these positive cash flows were considered separately, different treatment for discounting or risk would result in different net values.

If different components of a set of cash flows bear significantly different risk levels or if some are positive while others are negative, the decision as to whether to discount the components of that set of cash flows separately may become significant. This is a powerful argument to adjust for risk and risk preference through adjustment of expected cash flows, rather than through the discount rate. However, significant time preference differences as applied to positive and negative cash flows could still be reflected. To the extent that positive and negative cash flows are combined or aggregated (e.g., in a hedged asset portfolio, in a product such as whole life insurance with its future premiums and benefits, or in a retirement plan with both contributions and benefits), and risk, risk preference or time preference is reflected, several approaches could be taken, including:

- Positive and negative cash flows could be separated and discounted separately, depending on their nature and
- Positive and negative cash flows could be combined to the extent that they are expected to be highly correlated, indexed by contract, and / or managed together on the basis of a board approved asset / liability management plan.

The determination of whether or how to offset negative and positive cash flows may be somewhat arbitrary. Particularly if present values are used, value may vary depending on the allocation of cash flows, particularly if expected future cash flows are offset against each other before a discount rate(s) is applied. It may also depend on the purpose for which the valuation is performed or applicable accounting rules.

A given change in a discount rate will have a different effect if applied to a negative as compared with a positive cash flow. That is, although a higher discount rate will result in a smaller present value if applied to a positive cash flow, it will result in a larger present value if applied to a negative cash flow. As a result, the degree of conservatism introduced through the use of a given adjustment in a discount rate may be different depending on the direction of the cash flow. This may affect the type of risk adjustment deemed appropriate. In fact, a given decrease in a discount rate that is generally associated with the introduction of a PAD provides a non-conservative value if applied to a negative cash flow.

4c. Discount rate structure

A discount rate can take one of several forms. The simplest is a single compound rate of interest, independent of the time at which a cash flow is expected. More complex forms include separate rates that apply to the cash flows for separate years or to each separate cash flow associated with the risk associated with each source or type of cash flow. If the amount of discount warrants, as discussed earlier in this paper, it is necessary to determine how the risk adjustment should be allocated between the cash flows and the discount rate(s).

For many sets of cash flows, it is sufficient to apply the discount to the middle of the period, reflecting average timing of the cash flows during that period. If the difference is material, the discount rate would be applied by more accurate assumptions.

The value of time and in turn the discount rate need not be constant over time, but it should be proportional to the length of time being considered. This implies that the term structure of interest rates should be considered, possibly using the spot rate applicable for each duration of cash flows. Conceptually, it may be appropriate to apply a separate discount factor to cash flows depending on when they are expected to be received, with a separate discount rate applied for each set of cash flows expected within a given year. However, current practice varies – for analysis of the rate of return from business projects, it is currently common practice to discount all future cash flows at a single rate, while for other purposes, discount rates associated with current yield curves are applied. If a set of cash flows is affected by inflation, it is appropriate to reflect expected inflation; most importantly, it is important to be consistent between cash flow projections and the discount to be applied, also mentioned in the next section of this paper. For cash flows involving complex longer-term uncertainties, such as for certain insurance products or

retirement or health care arrangements, both time and risk preference may be reflected in a larger discount rate (assuming negative future cash flows).

A discount rate may also be implied by the relevant market price and a single rate solved for, even if multiple rates would be more appropriate. A practical problem arises with complex cash flow streams in which case it may not be possible to solve for a single interest rate. Another difficulty with such a simplified approach is that typically it assumes that the cash flows are certain with respect to incidence, size and timing. In reality, this is not the case. In addition, particularly if a stochastic model is applied, it may be more appropriate if a set of duration-based discount rates is applied.

In principle, different cash flows in a particular application should be discounted on the basis of a consistent methodology. For example, if values of a set of cash flows are determined on the basis of a set of alternative scenarios, either deterministically or stochastically developed, it would also be appropriate to reflect different discount rates in a manner consistent with the scenario being analyzed.

A single discount rate or multiple discount rates can be applied to a set of cash flows expected to occur at more than one point in time. Appropriate discount rates are generally based on the specific duration of each cash flow. Sometimes a single discount rate is used to represent a weighted average of applicable discount rates.

In any event, if a risk free rate serves as the basis for a discount rate, it should be of the same nature and duration as that of the cash flow to which it is applied. If the risk is adjusted for in the cash flows, then the discount rate should not be adjusted for the same type of risk. Often the risk free rate will take the form of the equivalent government bond or note (assuming that the appropriate governmental unit represents extreme safety, i.e., no credit risk, as it usually has the ability to raise funds through alternative approaches, including taxation).

4d. Nominal or real discount basis

The determination of whether to discount at a nominal or real rate will depend upon the basis for the determination of the cash flows to which the discount rate will be applied. It is important to be consistent. If the future cash flow is expressed in terms of today's monetary values, it would be appropriate to apply a real discount rate, while if expressed in actual or nominal monetary terms, it would be appropriate to apply a nominal discount rate.

For example, if the set of future cash flows to which a discount rate is to be applied has been estimated at its actual size at the future point in time, then a nominal (according to most economic theories, this consist of real interest rate plus expected inflation, plus appropriate risk adjustments) discount rate should be applied. If the cash flows are

sensitive to future inflation (e.g., a series of cash flows indexed to a cost of living index) and are expressed in terms of today's currency, then a discount factor should be expressed in terms of a real interest rate, plus an appropriate risk adjustment.

4e. Entity's own default risk

A significant accounting issue involves the credit risk associated with the discount rate applicable to an entity's obligations (in the form of a bond or insurance obligations). For example, if a debt obligation of the entity is traded in a market, its credit risk would be reflected. Why shouldn't such an obligation reflect a similar credit risk if owned by the entity itself? This is a significant issue, for example, for a financial intermediary with significant obligations involving future cash flows.

Arguments in favor of this practice include the fact that the values of such obligations should be the same, independent of who owns them. In addition, it could obtain an immediate profit if it chose to sell the obligation on an open market, so there is no reason why they shouldn't recognize this difference in value on its balance sheet. This treatment is consistent with the obligation taking the form of equity (owning its own stock), the value placed would recognize the value of the equity, implicitly recognizing the risk associated with future dividend payments.

A significant argument against this practice relates to its inherent inconsistency, in that the worse the credit position of the entity, the smaller the value of the liability, and the better the reported financial condition. In addition, the whole basis of presentation of financial statements is that the firm is assumed to be a going concern. If it is not, then many other values on the balance sheet should also be discounted for credit risk. Such credit risk should only be recognized to the extent it is not reflected elsewhere in the entity's balance sheet. In the case of an insurer, should the present value of an insurer's benefit liability, representing a significant amount of its total liabilities be reduced because of a deteriorated credit position?

It is useful to think about the reason for this apparent inconsistency. It is generally caused by the inability of most accounting systems to reflect internally generated goodwill (the exception being purchase accounting rules when explicit goodwill is recognized). Alternatively, the amount that it would take to obtain complete default-free financing of its overall balance sheet should be reflected in its risk-based capital. It seems inconsistent to recognize the effect of a credit risk in obligations of an entity while not reflecting that credit risk elsewhere in its balance sheet. If that reduction in value were recognized, the inconsistency would most likely be eliminated (except to the extent that internally generated accounts were not reflective of fair values in the market).

Recognition of an entity's own credit risk would be equivalent to issuing a default-free bond along with a default put option. The value of the obligation would be reduced by

the value of the put (although if offered as a separate financial instrument, it would be recognized as an asset in financial reporting).

To the extent that an efficient market in the entity's obligations (e.g., insurance obligations) does not exist, the arguments against such reflection would seem to be strong.

The opposing arguments lead to the conclusion that, based on current accounting rules, inconsistencies arise no matter which answer is given. Further discussion is needed before a resolution of this issue. In evaluating the value of an entity, the entity's credit risk and its effect on its entire financial condition would obviously be evaluated and reflected. For the purpose of financial reporting, it would seem appropriate to disclose the impact of such credit risk.

4f. Locked-in or dynamically adjusted

It is appropriate to review an initial assessment of discount rates on a regular basis to assure that the assumptions made remain valid. Of course if historical or hedge accounting rules are used, then it may be appropriate to maintain internal consistency not to update the discount rate. Although an argument could be raised that a reduction in value should only be recognized in the case of impairment of value, this does not seem to provide a realistic assessment of value, which is the objective of financial reports or valuations. A corridor in which changes in value are not recognized, either for materiality reasons or to avoid undue fluctuations in value when uncertainty of estimates may overwhelm periodic changes in assessed value, has been used in certain circumstances.

If a fair value approach is used, which is prospective in nature, it would be appropriate to reassess all significant assumptions used in the development of the present value of future cash flows at the time of each valuation, including the estimates of amount and timing of future cash flows, risk adjustments, and discount factors used. Thus, the discount applied would generally change. All current available information that is relevant should be used. If a hypothetical set of assets matched by duration was used in the prior valuation of a liability, that set needs to be re-evaluated periodically if the expected liability durations have changed.

All assumptions should be reviewed periodically in view of current and expected future experience. This includes the discount rate. If the discount rate has been determined on the basis of a matched set of assets or liabilities, then the financial structure of the arrangement should be reevaluated and the discount rate may have to be adjusted as appropriate. If a revision of any of the key factors that influenced the selection of the discount rate(s) occurs, such as a change in market rates (and the market's attitude to risk), or expected cash flows should be revised accordingly. Consistency in methods used

in determining assumptions would generally be followed, but the same assumptions may not be used.

Appropriate reflection of a change in discount rates would be determined. In addition, disclosure of the effect of changes in discount rate would also be expected to be provided to the user, but this is no different than a change in other significant assumptions.

4g. Taxation

In most cases, the party whose financial values are being impacted by the results of a present value model is subject to taxation. In addition, the market and any third party will evaluate cash flows on the basis of the net financial impact of the cash flows, which would reflect relevant taxes. While the impact of taxes may not change the conclusions reached, sometimes it will, particularly when long-term assets or obligations are considered. After-tax results represent the most appropriate basis to evaluate financial decision-making because these results affect future cash flows as well. This should be reflected in cash flows, risk adjustment, and the resulting risk discount rate in an internally consistent manner. It is not uncommon to have taxes be ignored or not thought through in business decision-making, which in some cases may lead to sub-optimal decisions.

It is typical in financial reporting that taxes of the enterprise in which the cash flows are expected to be paid are recognized, but they are gross of the taxes of the investor in the enterprise. Although this is appropriate from the enterprise's point of view, if the investor was making a decision for herself or himself, the impact of the investor's taxation would also be reflected. In determining market values, partly because it is usually difficult to determine an average tax rate for all possible investors and it isn't particularly relevant to many decisions, the investors' tax is usually ignored.

As there are several methods that can be applied to adjust for risk, there are also several ways to introduce tax into a present value model. The two primary approaches are:

- **Net/net method.** This method explicitly reflects the amount of taxation as a cash flow. If used, it would be appropriate to apply an after-tax discount rate. However, if the taxation is derived as a result of interest either earned or payable, then if the interest income is not reflected in these cash flows, the introduction of such taxes in this manner may be questionable. In addition, the value would then have to be entity-specific, as different parties may be subject to different tax rates.
- **Gross/gross method.** This alternative method does not reflect taxes in the set of cash flows and the discount rate applied would be grossed-up by the applicable tax rate. Because taxation should be reflected in some manner, the effect of taxation on the set

of tax cash flows would have to be determined separately, discounted at the appropriate discount rate.

Although the results derived from the application of these two methods may be identical, such convergence may only be accidental. In fact, a great deal of analysis may have to be expended to derive the same results, reflecting the tax rate and situation (tax timing, tax loss carry-forwards, etc.), the discount rate and the amount and timing of the cash flows involved. The determination of whether pre- or post-tax values should be used may depend on the relative reliability of the two types of estimates and on the particular situation, as well as the type of information that would serve to be useful to the audience. In either case, it is important not to either count tax twice or to ignore it completely. Clear communication and disclosure where needed are appropriate.

In financial reporting, whether to report a tax provision separately from the valuation of the cash flows will depend on the particular accounting standard in place. However, to reflect such a provision in a manner inconsistent with the valuation of other than tax cash flows would at best lead to inconsistent results. It should be kept in mind that the enterprise is taxed and not the particular cash flows. Thus, the entity's overall tax rate should be reflected. Even though this argument might lead to a separate tax provision, it also could lead to a misleading conclusion regarding the set of cash flows being valued.

In practice, estimates of many kinds may have to be made. For example, it may be difficult for a multi-national organization, subject to many different tax rates and approaches to taxation, to determine a single appropriate tax rate. In such case, a gross/gross method may be preferable, or at least a separately calculated tax effect by country, even though either approach could be made equivalent. In the derivation of such values, it is appropriate if material to reflect the different timing of tax payments.

Changes in taxation, if already enacted, are generally also reflected in the evaluation of future cash flows. Other than through sensitivity testing, it is not common to explicitly anticipate future changes in tax law in such calculations. On the other hand, the market certainly anticipates future tax laws and regulations, even if a significant change in tax rules would immediately change such values; in this case, the market is adjusting to more up-to-date information. Business decision making has to reflect expected tax effects, even when uncertainty exists with respect to the tax rate to be in effect at the time of the expected cash flows – some assumption has to be made, the default assumption of using the current rate would not be appropriate.

Almost all taxes other than taxes related to income are commonly treated as adjustments to cash flows and do not affect the discount rate, although they would be discounted. Relying on this analogy, income taxation would be treated in a similar manner, although such an approach is uncommon.

5. OTHER NECESSARY ITEMS

5a. Applications

A few examples of application of some of the concepts covered in this paper follow, focusing on the characteristics that differentiate them from the more general situation and therefore do not constitute a complete description of discounting as applicable:

1. **Retirement programs.** Many pension actuaries reflect a fair value of liabilities independent of the assets actually held, appropriate to maintain consistency between externally and internally funded plans. This reason may not be as relevant for funding purposes, since the amount of assets needed to meet future pension obligations will depend on the expected return on them and the risk of under or over funding may not be an issue for the plan sponsor. However, it may be helpful to have similar approaches for accounting and funding.

The argument could be made that the type of assets the fund chooses to invest in does not affect the nature or amount of the obligation. Although true, this does not reflect the nature of the future cash flows promised. It is more appropriate to match the change in retirement benefits with expected returns and discount rates to class of asset. In addition, a fair value of such cash flows would recognize a discount rate derived from the optimal matching portfolio and not determined in a manner unrelated to the cash flows. It is also appropriate to reflect such rates of returns and discounts denominated in the same currency as the benefit payments, while in considering the investment of future contributions, other approaches than the current market rates (such as average historical rates) may be reflected.

The discount rate should be a market-determined rate(s) that reflects a current assessment of the time value of money and the financial risks specific to the obligations. It should be noted that risks associated with demographic changes over time are better reflected as adjustments to expected cash flows, as they are not correlated with market returns.

The discount rate used represents the expected return from assets of appropriate nature and term, related to expected cash flows. Different discount rates should be applied to different types of benefits when they differ in a significant manner. In particular, fixed-money liabilities should be discounted at rates on fixed bonds of appropriate design, while price-related liabilities should be discounted at rates on price-related bonds of appropriate duration. Many liabilities are salary-related, while a significant portion of funds is invested in equities. Based on several studies, the two are positively correlated over long periods, although other studies have questioned this relationship. It is generally assumed, based on historical experience, that the long-term return on equities will exceed the long-term return on bonds by a

significant margin and that long-term corporate earnings (that ultimately provide for the funding of such benefits) and payroll trends both tend to be positively correlated with equity returns. In addition, pensionable pay increases may vary by economic conditions, which also drive equity returns. Therefore, many actuaries involved in retirement programs generally believe that equities may serve as a reasonable proxy for risk-free assets for salary-related liabilities and that it is appropriate to reflect the net expected return on equities. The use of lower expected bond interest rates in present value calculations would tend to overstate pension costs. Alternatively, if the assets used to fund payroll-related benefits were bonds, such a plan would face material risks or likely additional funding requirements due to the lack of correlation between the type of assets invested and the benefit cash flows that will occur.

2. **Large scale capital investment projects.** In a traditional net present value calculation, the present value of expected profits is compared with the present value of anticipated expenditures. The discount rate used is based on the company's hurdle rate, which is typically set to allocate a given set of investment resources for projects returning a minimum return on investment, or rate at which investments must yield for it to be given the go-ahead. Recently, an option-pricing approach has been increasingly used, in cases in which the entire investment is reversible or there is an ability to delay on irreversible investment. Option-pricing methods apply a higher discount rate to that portion of the investment that is non-discretionary, while the discretionary portion is discounted at a rate closer to a risk-free rate. The correlation of the investment's cash flows and those of all other investments are also examined in order to recognize the net addition to the investment portfolio to the risk of the firm or the public entity if an infrastructure project.
3. **Life insurance.** Since life insurance contracts are currently not traded in a marketplace (although recently, viatical companies have provided a limited market for older insureds), a market value cannot be estimated. As a result, the fair value can be represented by the application of a discounted cash flow model. As indicated earlier in this report, there is currently some debate as to whether the discount rate to be applied to insurance liabilities should reflect corresponding assets (reflecting the hedging effect of asset / liability management policies adopted by the company) and the relative interest-sensitivity of the product. If dynamic adjustment provisions are included in the policy, through such means as policyholder dividends or experience refunds, the risk level is also reduced, with a possible reduction in the discount rate. There is currently a difference of opinion as to whether all cash flows or just cash flows that are distributable should be the basis for the cash flows to be discounted.

One common pricing method (Anderson's method as first described in James Anderson's profit-testing paper in the *Transactions of the Society of Actuaries*, issue 11, pages 357-420) is based on the use of a discounted value of annual book profits, with discount rates significantly higher than estimated investment earnings rates. This higher discount rate reflects the relative riskiness of the block of business, but it

also generally includes provision for profit over and above the earned interest rate, with the return on investment (generally defined as the regulatory accounting initial loss or including any attributable risk-based capital).

A question that has often been raised by people within the insurance industry is whether the business of insurance (life or property/casualty), with its complex exposures, policyholder options, and long-term uncertain cash flows (even some so-called short-duration contracts are exposed to risk over a long period) is similar enough to other business to require the treatment of risk assessments in a similar manner. This is a particularly important question because there is a limited market for insurance obligations, with significant inefficiencies.

4. **Property/casualty loss provisions.** Traditionally, property/casualty loss provisions (sometimes referred to as loss reserves) have not been discounted. This approach has been used for several reasons, including the large amount of uncertainty associated with future claim payments, particularly for cash flows related to outcomes from the U.S. court system. Some actuaries believe that a substantial risk margin would be appropriate, if a discounting approach were followed. In fact, in a few cases, the resulting discount rate could be negative, particularly in a low interest rate environment. It is most important to be able to explicitly allow for risk and to be able to measure such allowance for risk. This permits better performance measurement. Although it may be feasible to adjust for risk in the discount rate, it is currently more common to reflect it in the level of expected future cash flows.

5b. Criteria to judge usefulness of valuation results

Several criteria can be applied to a valuation model to determine its appropriateness for the development of present values or fair values. Valuations should provide credible results that meet the need of the decision-maker or market-based applications in a financial reporting context. The following is a brief review of some of the factors to consider as part of a validation process in the derivation of such values:

- **Comparability.** One source of comparability is the value assigned by an efficient market. Unfortunately, such a market rarely exists. The results of a given market value in an inefficient market should be used with caution, since the conditions that contribute to one value or transaction may not apply to another. Nevertheless, such indications should not be ignored and their relevance should be evaluated. Another basis to assure comparability of values is a sound methodology that follows an internally consistent set of accounting rules; if actuarial assessments are relied upon, such assessments should be based on applicable actuarial standards of practice.
- **Transparency.** In many applications, transparency is important. The availability of transparent information and appropriate explicit disclosures of assumptions can

promote comparability and, if necessary, can be used by many users to adjust values to a more common base. Results of a valuation and associated disclosures should be prepared to satisfy the needs of the users and be prepared in a manner in which the users can understand.

- **Relevance.** The development of values should not include factors unrelated to the cash flows being evaluated. Values should be responsive to underlying changes in the characteristics of the set of cash flows or to applicable markets. Although responsive assumptions may result in significant volatility, if they reflect underlying causes, then reflection of such volatility would be appropriate. In certain cases, intention to dispose (or not to dispose) of specific assets or liabilities should be reflected. There are differences of opinion regarding the extent to which intention is relevant. For example, if it is the intention to hold on to an asset until maturity, this intention should be reflected in a valuation. If used to hedge another set of cash flows, it would be appropriate to be consistent in the basis for valuation for the two sets of cash flows, no matter what the accounting rules for the asset or liability is. Overall, emphasis should be placed on the generation and disclosure of useful information to the user.
- **Reliability.** The more difficult it is to estimate the amount or incidence of future cash flows, the less reliable the resulting present values will likely be. The more soundly based and objective the estimation process and basis for the estimates, the resulting estimates will tend to be more reliable and acceptable.

It is important to validate any valuation model used. In many cases, a professional standard or set of rules may exist, which if followed would increase the reliability (and comparability) of the resultant estimates. Several alternatives exist to assure such validation, involving one or more risk management techniques, including hedging, insurance, diversification, and pooling. If the set of cash flows is perfectly correlated in a negative fashion, then this problem is solved; for example, if the set of cash flows are fully insured, then a relatively certain cash flows results or if the set of cash flows is hedged by use of a derivative that is closely correlated to the set of cash flows, then a significant improvement in the reliability of and conversely a reduction in uncertainty associated with the estimation of the resulting cash flows will likely occur. This assumes that the insurance or derivative product is placed with a firm with sound capitalization and that no custody disputes arise. It may be that the most reliable benchmark would be derived from information gathered from an efficient market, if it existed for the set of cash flows in question.

It is usually appropriate to perform a retrospective review (validation) of any experience assumption, so that future projections can be developed in an improved manner, i.e., an information feedback loop. However, if the assumptions are implicitly adjusted for risk, this may be difficult without pre-adjusted values.

- **Completeness.** It is important that all sources of variation and options inherent in the source of the cash flows be considered; however, they may be ignored if not material. In practice, it is likely that more obscure adverse scenarios will be ignored and a small aggregate margin in the discount rate may be appropriate to cover this. If a set of possible future cash flows is not measurable, possibly due to lack of relevant information, but possibly of material size, it is important to disclose this fact. If at all possible, an estimate, even if crude, should be attempted.
- **Realism.** How important this criterion is depends on the application. For some users of valuations, it is preferable to be prudent, particularly regulators primarily concerned with solvency risks. According to IAS 37 (paragraph 43), “caution is needed in making judgements under conditions of uncertainty, so that income or assets are not overstated and expenses or liabilities are not understated. However, uncertainty does not justify the creation of excessive provisions or a deliberate overstatement of liabilities.” The message here appears to be, when in doubt, err on the side of conservatism, but don’t create any intentionally excessive margins.
- **Symmetry.** It would be desirable if the approach applied worked with both negative and positive cash flows.
- **Simplicity.** Using Ockham’s razor as a guide, if there is a choice between two methods producing similar values, the simpler one should be relied upon. This will contribute to the practicality of implementing an approach to valuation. However, it should be remembered that the continuing validity of such a choice should be reviewed regularly.
- **Objectivity.** It is desirable if two actuaries with the same attitude to risk, faced with the same conditions, would derive similar values and would apply the same or similar discount rate. By following a consistent set of actuarial standards of practice, the possibility of significant deviations will be reduced. Potential manipulation of assumptions and moral hazard should be minimized. However, it must be accepted that absolute consistency when uncertain future cash flows are involved is unlikely to be achieved. In the development of valuation estimates, the objectivity of the modeler is important; in some cases, this may mean independence of the modeler from the decision-maker. It is also important to attempt to avoid incentives that could lead to risk of manipulation, moral hazard, or fraud.

In order to assure achievement of these objectives, it is important for actuaries to reflect relevant accounting rules and actuarial standards that should be applied in the particular situation in which uncertain cash flows are being valued. Such rules and standards may provide valuable guidance to assure that the basis for the development of present values is conducted in a reasonable and objective fashion.

Practical considerations may influence a valuation. Timeliness is important and short cuts may be necessary, as long as it is likely that they will not lead to materially incorrect estimates. Cost-benefit trade-offs regarding the degree of refinement in models used will often have to be made; judgment should apply, although materiality also will determine how much refinement is called for.

5c. Disclosure

In developing standards of financial reporting, a significant objective should be to provide the user with sufficient information to make an informed decision from the viewpoint of that user. In terms of the values reported on, it is appropriate to provide the basis for discounting. As mentioned earlier, particularly in view of the many purposes, viewpoints or audiences for a valuation, it is important that each user has the information needed to derive relevant and personal conclusions regarding significant adjustments made, if any, in assessing value. The difficulty is exacerbated when combining many different types and sizes of cash flows over time and information regarding these cash flows. For example, without sufficiently detailed and clearly presented information, when combined with changes in the risks exposed, it may be difficult to interpret reported financial results as they are affected by the unwinding of discount over time.

To the extent practical, it is important for the type and level of disclosure to be appropriate for the expected audience. If sufficiently detailed information is not provided, where that information can be obtained should be included in the write-up supporting the valuation or financial reports in which the values are included. In actuarial appraisals, it is common for the decision-maker to be provided a range of estimated appraisals based on alternative discount rates, so that the decision-maker can make the final decision based on the decision-maker's preferences. Although actuaries may select a range of reasonable discount rates from which a price can be determined, it is up to the decision-maker to decide on what discount rate is most relevant to the decision being made.

Many in business believe that confidential information may be provided if too much information is disclosed. Others believe that too much information will simply contribute to information overload. This tension in the type and amount of information provided is one reason why financial analysts or information intermediaries / consultants can serve a valuable role in interpreting such information or adding perspective to that information. As in any case, disclosure must appropriately reflect the need for different levels of detail for the likely audiences for the information provided.

In financial statements in which the impact of present values is material, significant assumptions should be disclosed, including the range of discount rates used and the type of risk adjustment applied underlying such present values. In addition, for users of present value information, it would be appropriate to communicate the basis and the

effect of revising discount rates and their unwinding (reversal) of discounts and their effect separately from the impact of the actual cash flows themselves.

Accounting issues arise in determining how to reflect interest expense and the unwinding of discount rates. The treatment will generally depend on the type of item being measured and how risk adjustment has been handled. For example, for property/casualty loss reserves, it may be appropriate to reflect such unwinding as a part of claims expense, rather than as interest costs. Other alternatives exist, including separate disclosure of interest unwinding or as a reduction in investment income.

5d. Certain technical issues

The following summarizes a few significant technical issues associated with this topic.

1. **No expected value.** The uncertainty associated with a set of cash flows may be so great that an expected value may not be able to be reliably quantified. A related problem is that the range of probable values may be so wide that, although an expected value could be derived, there is little certainty that that value is reliable. The implications of such a situation, arising for example when available data is of questionable quality, there is a significant unhedged or uninsured risk of a catastrophic event, or if the distribution of probable outcomes is extremely wide.

Various approaches have been taken in determining or reporting the value of such a set of possible future cash flows. They include: (1) assigning no value and adding disclosure (this is least preferable, as this is equivalent to assigned no value), (2) determining the most likely value and disclosing the source of the material uncertainty, (3) assigning a conservative value considering the range of possible values, (4) and implementing a risk management technique to reduce the level of uncertainty.

2. **Definition of discount rate.** It is important to recognize alternative definitions that have been given to a discount rate in determining present values. Mathematics of finance courses point out a distinction between:
 - A **discount rate**. This is the mathematically determined annual percentage rate at which a monetary value at the end of a year is reduced. The present value is expressed in terms of $(1 - d)$, where d is the discount rate, and
 - An **interest rate** at which money will accumulate, i.e., for the same one year period, the value at the end of the period is $1 / (1 + i)$, where i is the yield or interest rate.

- The **force of interest** (or force of discount) reflects the instantaneous measure of the time value of money. Although theoretically most appropriate, it is not commonly used, as its impact may not be significant. However, in cases of rapidly changing underlying values (or continuous compounding), it is more accurate.

Using this terminology, the present value of \$1 payable in one year, at a 10% discount rate is about \$0.89 ($1 / (1.1)$), while the accumulated value of \$0.89 at a 10% interest rate is about \$0.98. In most non-actuarial literature it is generally the “*i*” that is referred to when discussing a discount rate. Practices vary considerably in different situations and markets may apply different approaches, such as the frequency of compounding. Care should be taken to assure that the application is consistent with the value derived and the intent underlying the application. It is common for comparison purposes to convert such yields to annual rates of interest.

Adjustments are made if the frequency of the receipt of the cash flows is different from the implied compounding frequency of the rates. The discount rate is typically expressed in terms consistent with the underlying cash flows, but less frequently than annual. Clarity in such communication can be surprisingly important.

3. **Whether to discount interest earnings.** In some situations, actuaries have accumulated a set of assets and cash flows and then discounted the end-value. This may produce misleading results, in that different values could be obtained depending on the time horizon selected. As an example, if the same set of cash flows are accumulated with interest over two different periods, and then are discounted at the same interest rate, different values are obtained (assuming that the accumulating interest rate is not equal to the discount rate). If the interest rate equals the discount rate, no such problem arises, as the answer does not vary by the time horizon selected.
4. **Multiple equivalent discount rates.** In some cases, it may be desired to determine a level equivalent discount rate for a set of cash flows. If there are both positive and negative cash flows, it may develop that there are multiple solutions to an equivalent discount rate. One way of dealing with this situation is to separately value the positive and the negative cash flows before combining them.

6. EXECUTIVE SUMMARY

Understanding and quantification of the value of a set of future cash flows is central to actuarial science and the valuation of uncertain or contingent future cash flows in a wide number of decision-making situations and in the financial reporting of an entity's financial condition.

There are two distinct, but not totally mutually exclusive families of valuation approaches to future cash flows – a market-based approach and a present value approach. Market-based valuation can be viewed as consisting of fundamental values (generally produced by a present value model) or based on observed values demonstrated by transactions involving comparable sets of future cash flows in an efficient market. Because in most cases an efficient market does not exist, such observations need to be supplemented (or even replaced) by the results of present value models, in which case it may form the basis of a fair valuation. They both reflect risk and time preferences of the users of the valuations. In evaluating future cash flows, expected cash flows must be estimated, supplemented by the time value of money and risk preferences. In the case of inefficient markets or markets that respond to many factors in addition to the fundamentals of the particular economic good, there tends to exist a market premium or discount.

Value is a function of the audience and is influenced by not only fundamental values but also by opinions of other parties to a particular transaction and to those similar. Its components are made up of estimates of future cash flows, adjusted for risk, risk preference, and time preference.

Business decision-makers tend to rely on values appropriate to the specific operations of an entity, reflecting the intended use of the economic good, rather than the aggregate consensus of a market, resulting from the many participants in a market, although the market value can substantially influence the value associated with the economic good. Market values tend to reflect consensus view of a number of buyers and sellers, reflecting consensus risk margins or premium. For the valuation of many liabilities, for which fewer efficient markets exist, value in use may be more appropriately reflected. Although it may not be appropriate to reflect the actual earned rate of assets corresponding to these liabilities, it is appropriate to reflect the type of benefits provided and asset classes and durations associated with the expected settlement of the liabilities.

Adjustment for risk through both a discount rate or through expected cash flows may be appropriate in different situations, although there may be more practical reasons to favor adjustment for provisions for adverse deviations through expected cash flows.

This paper does not reach definitive conclusions in all areas affecting the valuation of future cash flows. For example, there are significant reasons for and against the reflection

of an entity's credit risk in the value of its obligations. Further work addressing the issues raised is encouraged.

DEFINITIONS

- **Accounting system.** Consists of the methodology and constraints imposed by a set of rules on the measurement of the value of assets and obligations in a financial reporting context.
- **Asset.** A set of positive cash flows, or a positive net result of a set of positive and negative cash flows. In accounting literature for financial reporting purposes, alternative definitions can be applied; for example, the International Accounting Standards Committee defines it as “a resource controlled by an enterprise as a result of past events and from which future economic benefits are expected to flow to the enterprise”.
- **Cash flow model.** A model that estimates future cash flows, generally from a given asset or liability. Generally a distribution of probable values is generated, from which a mean or other statistical measures can be derived.
- **Diversifiable risk.** A risk that can be reduced through the addition of similar sets of cash flows. Examples in which certain risks have been diversified include an asset portfolio including companies in multiple industries, a million insured lives covered by life insurance, or properties spread over a wide geographic areas covered by property insurance or provided loans. Although such risks can be reduced, they can rarely be completely be eliminated, because portfolios of such sets of cash flows typically cannot reduce all fluctuations, due either to finite available resources or clumpiness in the portfolio.
- **Diversification.** An approach to volatility reduction in which several independent or partially independent sets of cash flows (e.g., from financial instruments or debts) are combined.
- **Economic good.** A good that consists of a set of one or more future cash flows, usually uncertain in value at the current time.
- **Entity-specific value (value-in-use).** An interested party’s assessment of the present value of a set of future cash flows based on the specific use intended of the source(s) of those cash flows.
- **Efficient market.** A financial market in which perfect competition exists, characterized by the availability of complete information to all parties, typically involving many buyers and sellers in a voluntary situation (no forced purchases or sales).

- **Expected value.** A probability weighted measure assigned to a range of uncertain amounts or scenarios.
- **Fair value.** The amount for which a set of cash flows could be exchanged or settled in an arm's length transaction between informed and willing parties, other than in a forced sale or liquidation. In situations in which there is an efficient market, this value would be equivalent to a market value. In situation in which such a market does not exist, an estimation of such a value such as one developed through a present value model.
- **Hedge.** A set of cash flows (economic good) that is used to reduce the uncertainty associated with a different set of future cash flows. Typically, the two sets are related in either a highly positive or negative manner (correlation). A complete hedge exists when the two sets are identically matched; an incomplete hedge exists when a substantial, but not complete hedge exists.
- **Liability.** A set of negative cash flows, or a negative net result of a set of positive and negative cash flows. In accounting literature for financial reporting purposes, alternative definitions are applied; for example, the International Accounting Standards Committee defines it as "a present obligation of the enterprise arising from past events, the settlement of which is expected to result in an outflow from the enterprise of resources embodying economic benefits".
- **Market.** A market for a class of goods is an arrangement for facilitating transactions involving such goods by matching buyers and sellers often carried out through the exchange of money or its equivalent. It may occur at a given location. This paper generally addresses financial markets, in which the goods traded consist primarily of sets of cash flows (economic goods).
- **Market price.** The transaction price of a set of future cash flows (possibly combined in the form of a financial instrument, other asset, obligation, or even an entire company is actually traded between two or more parties) when sold in a given market. Except in the case of an efficient market (there are few of these around) or where the set of cash flows analyzed is a commodity in which transaction prices are consistent, this has to be used with caution to value a similar set of cash flows, as they tend to change over time and between different sets of buyers and sellers.
- **Market value.** This is an estimate of the market price, and can serve as a surrogate for the market price if there had been a reasonably equivalent transaction in an efficient market. It is the price for a set of cash flows that would have been arrived at in a market due to the interaction of many parties with many points of view that can be expected to bring if sold in a given market.

- **Model risk.** The risk associated with the mis-identification of the model applied to a particular situation.
- **Non-diversifiable (systemic) risk.** The risk that occurs when no addition of similar sets of cash flows can reduce overall risk, due to the existence of risks that affect all such sets of cash flows, including changes in the overall economy, overall changes in population mortality, or the probability of a large earthquake.
- **Obligation.** A duty that one is bound to fulfil by contract, promise or moral obligation, which in many cases can be satisfied with one or more future cash flows or their equivalent.
- **Parameter risk.** The risk associated with mis-estimation of experience that affects the timing or amount of future cash flows.
- **Present value.** An estimate of the underlying value of a set of future cash flows. The value of a set of future cash flows is taken from a particular point of view at a particular point in time. It represents an estimate of the underlying value of the set of future cash flows.
- **Present value model.** A mathematical model that reflects the time value of money and applicable risk margins to reflect the impact of uncertainty associated with a set of future cash flows.
- **Probability.** The likelihood or chance that a given event (incidence, timing or amount) occurs.
- **Process risk.** The risk associated with statistical fluctuations.
- **Provision for adverse deviation (PAD or PfAD).** The risk margin used to reflect the level of uncertainty in the amount and timing of uncertain future cash flows. This reflects one-sided risk, that is, adverse risk. If related to what would be reflected by a market, it is also referred to as an “adjustment for risk” or “market value margin”.
- **Risk.** The probability that a given set of objectives will not be achieved. It typically is objectively measured and does not reflect personal or subjective beliefs. It sometimes is used to refer to the cost or value associated with not achieving that set of objectives, measured as the expected stakeholder deficit (in an insurance context is often referred to an expected policyholder deficit). This term has been associated with expected volatility; however, typically an event is not considered “risky” if a favorable outcome is achieved. Rather, in statistical terms it is a one-sided deviation. It is a “loss” relative to that set of objectives, rather than a deviation from a specific objective (although in a particular case, this deviation could be the objective). In other cases, it is used to identify a situation (or the loss resulting from the situation) in

which an economic loss will occur; this is a typical example of the more general concept of risk.

- **Risk discount rate.** The annual rate equivalent to the reduction in value over a given period of time associated with the risk of a future set of cash flows.
- **Risk-free rate.** For a specific period of investment, the interest rate for which it is essentially certain that the cash flows associated with the investment will be received. The most commonly used measure is the U.S. Treasury bond or bill rate (depending on duration). This is more accurately characterized as a default-free rate, as it is thought to be the most credit-worthy of financial instruments, but is still subject to other types of risk, such as market risk, foreign currency risk (assuming an outside the U.S. perspective), and asset / liability mismatch risk (in which the liability cash flows are of a different duration of the bond or bill, with reinvestment required).
- **Risk management.** A family of methods used to recognize, reduce or manage risk. Included are derivatives, dynamic systems, insurance, and pooling.
- **Time discount rate.** The annual rate equivalent to the reduction in value of a set of cash flows associated with the time value of money.
- **Time value of money.** The value or worth of a future cash flow at the time of valuation.
- **Total discount rate.** The sum of the risk discount rate and the time discount rate.
- **Uncertainty.** The absence of certainty. With respect to the future result of a particular experiment, the degree of possibility that a particular result occurs is measured by its estimated probability. This may include subjective opinion regarding the possibility of the event, i.e., personal belief. Some use it with the same meaning as risk, but that is incorrect.

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