



# Joint Colloquium of the IACA, PBSS and IAAHS Sections of the International Actuarial Association

Westin Copley Place Hotel, Boston, U.S.A. – 4-7 May 2008

## Two Factor Approach to Loss Reserving

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# Traditional Loss Reserving Methods

- **Loss Triangles**
  - Paid or Incurred
  - Claim counts
  - Chain Ladder Link Ratios
- **Stochastic Models**
  - Produce a distribution of loss reserves
    - Statistical features of model underlies calibration of loss reserves distribution
      - Mack (1993)
      - Bootstrap (England and Verrall, 1999)
    - Simulation (1992)

# Stochastic Models

- Mack
  - Use statistical features of chain ladder link ratios to calculate process risk and parameter variance
  - Two variance components combined to a standard deviation measure
  - Fits to a lognormal/normal distribution to obtain a distribution of loss reserves

# Stochastic Models

- **Bootstrap**
  - Calculate cumulative fitted values by working backwards
  - Residuals between actual and fitted values randomly rearranged
  - Large number of sampling, combined with statistical characteristics of sampled data, give a distribution of loss reserves

# Stochastic Models

- **Simulation**
  - Calculate mean and standard deviation of link ratios
  - Fitted to lognormal/normal distribution
  - Simulate link ratios to give point estimates
  - Monte Carlo simulation gives distribution of loss reserves
  - Can be performed in many different ways with different distributions

# Disadvantages of One Factor Models

- **Loss Reserve Variability**
  - **Multiple Factors**
    - Calendar Year Trends
    - Accident Year Trends
    - Development Year Trends
  - Trends are not perfectly correlated
- **One Factor Models fail to accurately measure impact of multiple factors individually**

# Disadvantages of One Factor Models

- Calculate loss reserve variability based on historical variability observed in loss development data
  - Limits calculated variability to what occurred during the experience period
  - Many factors impacting loss variability could require longer time periods to demonstrate full range of values
    - Catastrophes
    - Underwriting cycle
    - Inflation

# Need for New Approach

- Shortcomings of one factor models are recognized by actuarial community
  - Increasingly popular use of statistical modeling techniques
    - De-trends loss development until residuals appear to be random
    - Actuarial judgment may distort statistical nature of modeling framework
      - Example – eliminating large losses
    - May limit calculated variability to what occurred during the experience period

# Need for a New Approach

- Call for a modeling approach that can:
  - Accurately extract information from data trends
  - Flexibly introduce variability that is different from what occurred during the experience period
  - Incorporate actuarial judgment
  - Produce a reasonable distribution of loss reserves

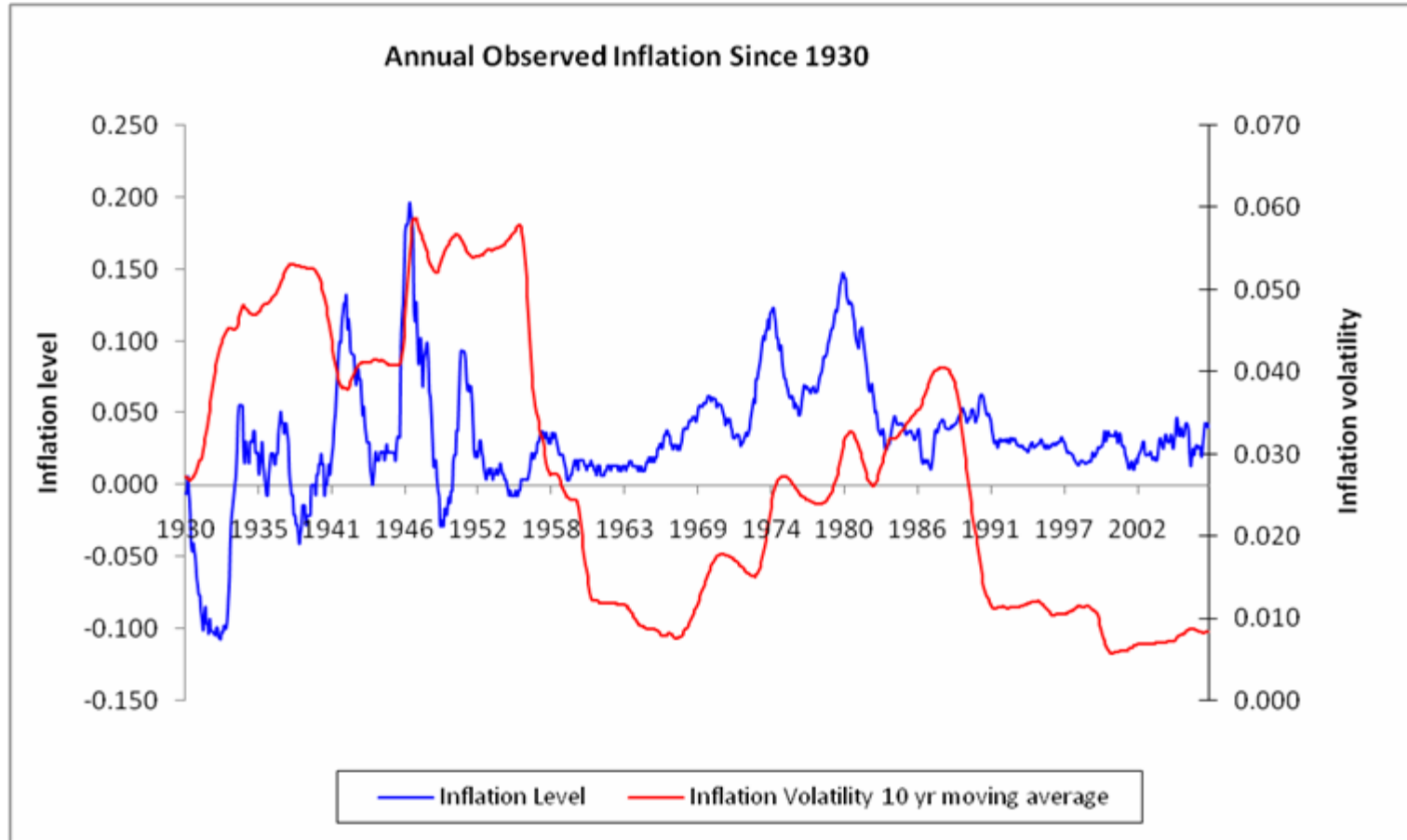
## One Factor -> Two Factor

- Similar to interest rate models, a properly calibrated two factor model can provide a more accurate depiction of loss reserve development than a one factor model
- Which two factors?

# Inflation

- Inflation is widely recognized to have a significant impact of loss reserve development
  - Ratemaking
  - Loss reserving

# Trends in Inflation

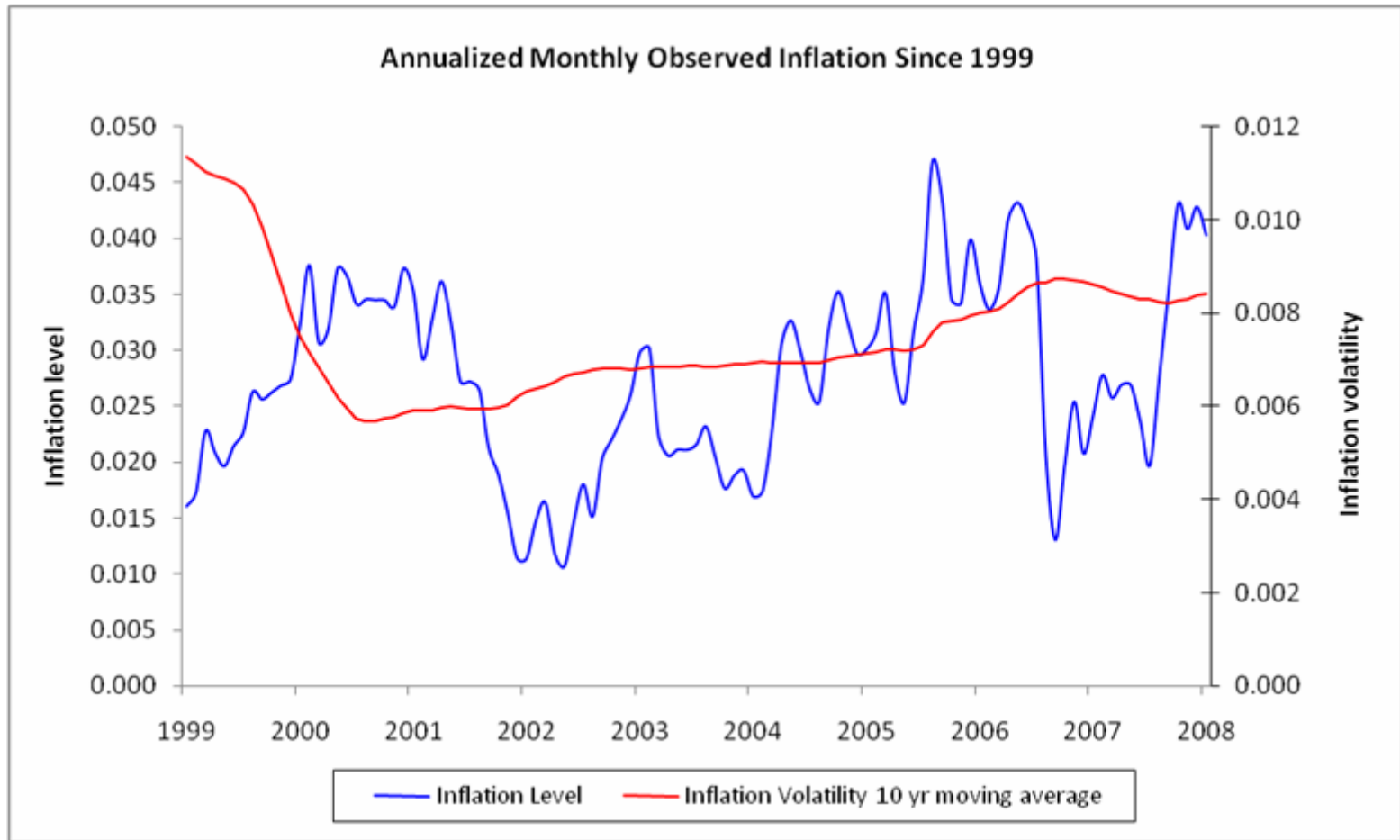


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# Trends in Inflation

- **WSJ Headline April 10, 2008**
  - “Inflation, Spanning Globe, Is Set to Reach Decade High”
- **Reasons inflation is increasing worldwide**
  - Increasing food prices
    - Demand driven
  - Increasing oil prices
  - Increasing risk premiums on debt
  - Within US additional factors
    - Depreciation of the dollar
    - Fed has lowered discount rate to deal with credit crisis

# Trends in Inflation



# Two Factor Approach

- Inflation-driven variability
- Residual variability
- 3-step process

# Two Factor Approach

- **Inflation-driven variability**
  - **Calendar year trend**
    - Use Masterson Claim Cost Index of the line of business, of which the data belongs to, to calculate claim cost inflation
    - Deflate losses
    - Obtain “real” loss development

# Two Factor Approach

- **Residual variability**
  - Calculate link ratios based on “real” loss development
  - Simulate link ratios to calculate ultimate losses

# Two Factor Approach

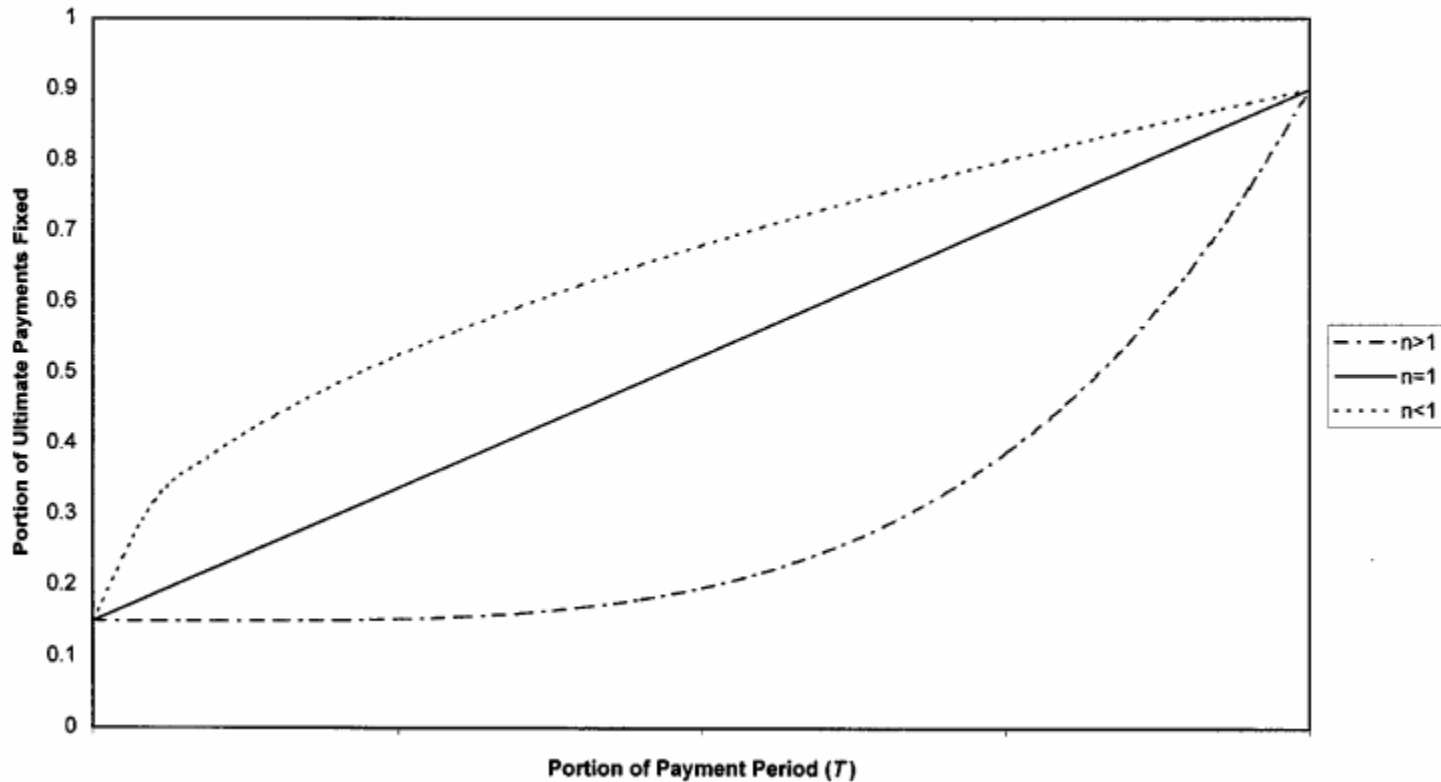
- Inflation-driven variability
  - Simulate CPI inflation rates
  - Regression on CPI inflation and claim cost inflation for the data period
  - Apply simulated rates to calculated regression formula to obtain simulated claim cost inflation
  - Inflate ultimate “real” losses to ultimate nominal losses

# Two Factor Approach

- Impact of inflation on loss development
- Fixed Claim Model (D'Arcy and Gorvett, 2001)

# Two Factor Approach

FORMULA FOR "FIXED" COSTS  
 $f(t) = k + [(1 - k - m)(t/T)^n]$



# Results

- **Loss Data**
  - Auto Bodily Injury
  - 1974-1991 (high inflation rates and high inflation volatility)

# Results

Percentiles	Two-Factor Approach	Mack	Bootstrap	Simulation
Mean Value	326,386	358,453	360,955	357,593
1 <sup>st</sup>	62.13%	75.88%	73.93%	61.02%
5 <sup>th</sup>	70.98%	82.11%	80.54%	70.15%
25 <sup>th</sup>	86.16%	91.87%	91.79%	85.68%
50 <sup>th</sup>	98.07%	99.33%	99.07%	98.15%
75 <sup>th</sup>	111.61%	107.40%	107.14%	112.27%
95 <sup>th</sup>	135.46%	120.17%	121.64%	135.77%
99 <sup>th</sup>	154.68%	130.04%	141.50%	156.11%

# Results

- **Lowest mean value**
- **Range similar to simulation method, greater than Mack and Bootstrap**
- **Model parameters based on variability occurred during experience period**
  - **Expected to produce similar results as Mack, Bootstrap and Simulation**
  - **Can introduce different variability**

# Economic Value

- Appropriate method to value loss reserves

Percentiles	Nominal Values	Economic Values
Mean Value	326386	283647
Standard Deviation	64415	53336
90% C.I. Range Ratio	82.95%	
1 <sup>st</sup>	62.13%	63.44%
5 <sup>th</sup>	70.98%	72.12%
25 <sup>th</sup>	86.16%	86.81%
50 <sup>th</sup>	98.07%	98.52%
75 <sup>th</sup>	111.61%	111.31%
95 <sup>th</sup>	135.46%	133.66%
99 <sup>th</sup>	154.68%	150.74%

# Summary

- Two Factor Approach can more accurately calculate loss reserve variability when future variability is expected to be significantly different from observed variability
  - Introduce greater flexibility in measuring loss reserve variability
  - Produce reasonable ranges
  - Allow economic valuation of loss reserves