

## A Two-Factor Approach to Loss Reserve Variability

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## Abstract

Traditional loss reserving techniques calculate loss reserve variability as if it were based on a single factor. These approaches all measure the variability in historical loss development patterns and use the derived measures to determine a distribution of loss development factors and to calculate loss reserve ranges, in essence limiting the calculated variability to what occurred during the experience period. However, there are multiple factors that impact the variability of loss development. These factors impact loss development in different directions – by calendar year, by accident year and by development year. A small trend change in one direction is likely to produce loss development patterns in aggregate that are significantly different from the historical patterns. A one-factor approach fails to model the impact of the multiple factors individually, potentially giving highly inaccurate loss reserve range estimates. The increasingly popular use of statistical modeling techniques in loss reserving provides tools to analyze and project loss development patterns in these three directions more accurately. However, in order to calculate loss reserve variability, the statistical features of the modeling framework have to be preserved, which in turns limits the calculated variability to what occurred during the experience period to a certain degree.

Inflation is a key element in loss reserve development. The traditional approach for determining loss reserve variability is reasonable as long as inflation is relatively constant. If inflation, or inflation volatility, were to increase, actual loss reserve variability would turn out to be higher than expected based on the traditional approaches. This paper introduces a two-factor approach to loss reserve variability that models inflation variability and residual loss reserve variability separately. This approach will provide more accurate estimates of loss reserve ranges, especially when the level or variability of inflation shifts from the levels inherent in the loss reserve experience period.

## I. Introduction

The primary traditional loss reserving techniques used to determine loss reserve variability are based on the inherent assumption that this variability arises from a single factor. The Mack (1993), bootstrap (England and Verrall, 1999) and simulation (1992) approaches all measure the variability in historical loss development patterns and use the derived measures to determine a distribution of loss development factors and to calculate loss reserve ranges. The Mack method uses chain ladder link ratios to obtain a mean value of the loss reserve. The statistical features of chain ladder link ratios are used to derive formulae for the process risk and parameter variance of the loss reserve. These two variance components are combined to obtain a standard deviation measure. A lognormal or normal distribution is then fit to the mean and standard deviation to obtain a distribution of loss reserves.

The bootstrap method calculates a triangle of cumulative fitted values by working backwards on the data triangle using chain ladder link ratios. The residuals between the actual and fitted values are randomly arranged to obtain a new triangle of data that has the same statistical characteristics as the actual data. New link ratios are obtained from the sampled triangle to calculate a point estimate of loss reserve. The sampling of residuals is performed a sufficiently large number of times to obtain a distribution of the loss reserve. The statistical characteristics of the sampled data are then used to derive formulas for the parameter variance and the total standard deviation measure of the loss reserve.

The simulation method calculates the mean and standard deviation of the link ratios, which are then fitted to a lognormal or normal distribution. Link ratios are simulated based on those distributions to obtain a point estimate of the loss reserve. Then Monte Carlo simulation is performed to obtain a distribution of the loss reserve.

Although the Mack, bootstrap and simulation approaches are able to calculate loss reserve variability based on a single factor, there are many situations in which these methods would fail to produce an “accurate” and meaningful distribution. First, there are indeed multiple factors that impact the variability of loss development. A change in inflation can affect loss payment patterns, impacting loss development. Factors within a company, such as claim department staffing, selected year end cut-off dates, loss settlement practices and the experience of claim department personnel, will all affect loss payment patterns and loss development factors. In addition, random factors that affect when claims occur during a year and, more importantly, when the outlier large claims occur, will impact the variability of loss development factors. These factors impact loss development in different directions – by calendar year, by accident year and by development year, and are often highly uncorrelated. For example, inflation is a calendar year effect, impacting all claim amounts that are settled in a calendar year. A high inflation rate in one calendar year would affect the loss development factors along the diagonal on an accident year basis. A small trend change in either direction is likely to produce loss development patterns in aggregate that are significantly different from the historical patterns. The traditional approaches aggregate all of these effects into a single variability measure, and this one-factor approach fails to model the impact of the multiple factors individually, potentially giving highly inaccurate loss reserve estimates.

Second, the traditional approaches limit the calculated variability to what occurred during the experience period. Both the Mack and bootstrap methods use only information from the historical loss development patterns and assume future development would follow those patterns. The simulation method allows customized inputs for simulating link ratios, but an increase or decrease in the mean or the standard deviation compared to that obtained from the historical data is difficult to justify, or properly quantify, on a one-factor basis.

The greater predictive power in calculating loss reserve variability by using multiple highly uncorrelated factors has been recognized by the increasingly popular use of statistical modeling techniques in loss reserving. The statistical nature of the modeling framework also allows separation of parameter uncertainty and process variability (Barnett and Zehnirith, 1999). However, sound actuarial judgment has always played an important role in loss reserving analysis and the introduction of this input, which cannot be explicitly derived from the data trends, would distort the statistical nature of the modeling framework, thus failing to produce a “correct” distribution of loss reserves. In other words, statistical modeling techniques also limit the calculated variability to what occurred during the experience period to a certain degree. Such conditions suggest the need for a modeling approach that can encompass the following: 1) the ability to accurately extract information from the data trends, 2) the flexibility in introducing variability that is different from what occurred during the experience period, 3) the input of sound actuarial judgment and, 4) the ability to produce a reasonable distribution of loss reserves. The model developed here meets all of these objectives.

## II. Inflation

Inflation is a key element in loss reserve development. The significance of the impact of inflation on loss development has long been recognized. The Masterson Claim Cost Index, which provides claim costs for different lines of business, was first developed in 1968 by Norton Masterson (1968) and has since been updated annually. These indices are a weighted-average inflation measure of specific CPI indices based on how each factor contributes to a claim in a particular line of business, providing an excellent insight into the impact of inflation on loss development. Claim cost inflation has generally exceeded the general Consumer Price Index during the years 1936-2004, especially in lines of business affected by medical costs which have consistently exceeded the general inflation rate.

The traditional approach for determining loss reserve variability is reasonable as long as inflation is relatively constant. If inflation were to increase, or inflation volatility were to increase, actual loss reserve variability would turn out to be higher than expected based on the traditional approaches. Modern quantitative financial techniques provide advanced methods for modeling inflation and interest rates. In addition, Barnett and Zehnwirth (1999) discussed the methods of separating data trends into calendar year, accident year and development year trends. The independence of accident year and development year trends means that any loss development pattern can be expressed in terms of any two trends. Since inflation has a significant impact on loss development, these approaches can be applied to loss reserving calculations to separate the variability in loss development into two components – inflation-driven variability and residual variability.

By deflating historical loss experience, the residual variability generated by company practices and claim timing factors can be determined based on the simulation method. Stochastic simulation can be used to incorporate the loss reserve variability dependent on future inflation based on inflation parameters – the current inflation rate, the long term mean inflation rate, the mean reversion speed and a volatility factor. The total loss reserve variability can then be determined by combining the separate impacts of these two factors. Although the residual variability could be further separated into components reflecting company practices (staffing, cutoff dates, claim negotiation policy, etc.) and the statistical variation (claim distribution and timing patterns) this is not done here due to the complexity and randomness of the variability of these components and the lack of meaningful data. The input of sound actuarial judgment is therefore essential in situations where the actuary has information about loss development trends that cannot be readily observed in the historical loss patterns. This two-factor approach forms a basis for a modeling framework that can separate the loss reserve variability into components in order to accurately introduce variability that is different from what occurred during the experience period. An extension of this approach can include additional factors that are deemed to have significant impact on loss reserve variability and can be modeled with reasonable accuracy.

## III. Economic Value

As discussed in D'Arcy, Au and Zhang (2007), the appropriate method to value loss reserves is by calculating the economic value of loss reserves instead of the nominal value. By incorporating a real interest rate model into the two-factor loss reserve modeling framework, the economic value of loss reserves can be easily calculated by discounting the cash flows in each

year by the corresponding risk-adjusted interest rate. The results can be compared to the nominal values to observe the true impact of loss development under different economic scenarios.

#### IV. The Model

A model has been developed to illustrate the two-factor approach as discussed in the previous sections. This model is available at <http://www.business.uiuc.edu/~s-darcy/papers/TwoFactorReserveVariabilityModel.xls>. The model is comprised of two modules to calculate the inflation-driven variability and the residual variability respectively. A triangle of cumulative loss data has to first be deflated before the two variability components can be calculated; the historical Masterson Claim Cost Index of the particular line of business is used to deflate the loss data. The mechanisms of deflating and inflating the losses (when calculating the inflation-driven variability) are similar to each other but complex, so a detailed explanation of this process follows.

##### IV-A. Deflating the Losses

A triangle of cumulative paid loss data is deflated using the Masterson Claim Cost Index of the particular line of business. For example, for a 10 year triangle of Auto Bodily Injury loss data from 1980 – 1989, the Masterson Claim Cost Index for Auto Bodily Injury from years 1980 – 1989 will be needed to deflate the losses appropriately. The triangle of cumulative loss data is first adjusted to obtain a triangle of incremental paid loss data. The incremental losses are then deflated to the corresponding Accident Year (AY) dollar amount. Incremental losses of Development Year (DY) 1 – 10 of AY1, which is 1980, are deflated to a constant 1980 dollar value. Incremental Losses of DY 1 – 9 of AY2, which is 1981, are deflated to a constant 1981 dollar amount and so on, until finally the only loss data of DY1 from AY10, which is 1989, is deflated to a constant 1989 dollar value. Each point of incremental loss in the triangle is deflated by a claim cost index that corresponds to the cost of a claim settled in that particular development year for that particular accident year. This approach eliminates the inflation generated loss development, leaving only the residual, or underlying, loss development factor. The claim cost of an AY1 claim settled in DY1 is different from that of an AY1 claim settled in DY2 due to the increase in claim cost from DY1 to DY2 (1980 – 1981). This is analogous to general inflation. In addition, the claim cost of an AY1 claim settled in DY1 is also different from that of an AY2 claim settled in DY1 since the former includes the claim cost of year 1980 only and the latter includes the claim cost of year 1981 only. In essence, imagine a triangle of claim cost indexes where each cell contains a deflation multiplier that is multiplied to the incremental loss amount in the same cell of the triangle of incremental loss data. The deflation multipliers are developed using the Masterson Claim Cost Index under either the Taylor method or the D'Arcy and Gorvett method.

Cash flows from unpaid claims are sensitive to inflation rate changes. Under the Taylor (1977) model, inflation in a given year fully affects all claims that have not been settled. D'Arcy and Gorvett (2000) propose a model that reflects the relationship between unpaid losses and inflation. This model separates a portion of unpaid claims that are “fixed” in value from those which are not. This fixed claim component, once determined, will not be subject to future inflation even though the claim has not been settled; the remaining unfixed component continues to be exposed to inflation. To give an example, assume an auto accident in which the insurer of

the at fault party will pay the claim under bodily injury and property damage liability. The damage to the car can be quickly determined and its nominal value becomes fixed from that point onwards, even though the claim has not been paid. Similarly, medical treatment already received is fixed in value when the service is provided. Any pain and suffering compensation, on the other hand, is generally determined at a later date. This portion of the claim will continue to be affected by inflation until settled. As a result of only exposing partial segments to inflation, inflation's impact on the loss is greatly reduced. A representative function that displays these attributes is:

$$f(t) = k + \{(1 - k - m)(t/T)^n\}$$

where  $f(t)$  represents the proportion of ultimate paid claims "fixed" at time  $t$ ,  $k$  = the proportion of the claim that is fixed immediately,  $m$  = the proportion of the claim that will be fixed only when the claim is settled, and  $T$  = the time at which the claim is fully and completely settled. The exponent,  $n$ , can be divided into three cases: the linear case  $n = 1$ , when the remaining claim value is fixed uniformly up to its ultimate settlement; the convex case  $n > 1$ , when the rate of fixing the value of a claim increases over time, and the concave case  $n < 1$ , when the rate of fixing the value of a claim increases quickly initially but slows down as time approaches the ultimate settlement date. The greater the  $n$ , the more closely the fixed claim model will resemble the Taylor model.

We continue with the previous example to illustrate the difference between the Taylor method and the D'Arcy and Gorvett method. Suppose the inflation figures of Auto Bodily Injury claim cost from 1987-1989 were 10%, 8% and 6% respectively. Under the Taylor method, the deflation multiplier of AY8-DY1 will be  $1/1.1 = 0.9091$ ; that of AY8-DY2 will be  $1/(1.1*1.08) = 0.8418$ ; and that of AY8-DY3 will be  $1/(1.1*1.08*1.06) = 0.7941$ . Under the D'Arcy and Gorvett method, the deflation multipliers are larger due to the "fixed" nature of claim settlement over time. The deflation multipliers are the results of complex integration calculations and any reader who is interested in the algorithm is encouraged to study the relevant formulas and calculations in the original paper describing this model. In the case with  $k = 0.15$ ,  $m = 0.5$  and  $n = 1$ , the deflation multipliers of DY1 – DY3 of AY1 will be 0.9368, 0.8893 and 0.8540 respectively.

#### IV-B. Residual Variability

The incremental loss triangle is deflated using the above algorithm and adjusted to obtain a deflated cumulative loss triangle. Loss development factors (LDFs), also known as link ratios, are calculated using the deflated cumulative loss data based on both straight average factors and weighted-average factors. The standard deviations of LDFs for each DY period are then calculated. In the case where there are minimal observations, i.e. DY9 – DY10 LDF (only 1 observation), the standard deviations of previous periods can be used instead. As described in the previous section, the residual variability in this model is calculated using the simulation approach. The mean and standard deviation of the simulated LDFs are selected inputs that can differ from the historical loss patterns if the actuary feels that adjustments would be appropriate. The simulated LDFs follow lognormal distributions with the assigned mean and standard deviation that correspond to the respective DY period. They are simulated individually, which means the simulated LDFs for the same DY period for different AY years are different, as opposed to simulating one LDF that will be applied to the same DY period across all AY years.

In essence, this is analogous to completing the upper observed LDFs triangle with a lower triangle of simulated LDFs.

The simulated LDFs are applied to the deflated cumulative loss triangle to complete the triangle. The incremental future loss amounts for each DY period are calculated to proceed with the calculation of inflation-driven variability.

#### IV-C. Inflation-Driven Variability

The inflation-driven variability is calculated by multiplying the incremental future loss amounts by the corresponding inflation multipliers that are developed using an algorithm similar to the derivation of the deflation multipliers.

A one-factor Ornstein-Uhlenbeck Inflation model is used to generate general inflation (Consumer Price Index) paths. The Ornstein-Uhlenbeck model uses a mean-reverting process with the current short-term inflation reverting to the long term mean

$$dr_t = \kappa_r (\mu_r - r_t)dt + \sigma_r \sqrt{dt} dz_r$$

where  $t$  is the time,  $r$  is the current inflation,  $\kappa$  is the mean reversion speed,  $\mu$  is the long-term inflation mean,  $dt$  is the time step,  $\sigma$  is the volatility and  $dz$  is a Weiner process. Regressions performed on historical data of the CPI and Masterson Claim Cost Indexes of the applicable line of business determined the coefficients to use in this model. The generated inflation path from the model would then be run through the regression formula to obtain a path of claim cost indexes.

Since all claims that we are concerned of in loss reserving have already occurred, they have been exposed to historical claim cost inflation. As a result, the inflation multiplier is a function of the past claim cost indexes and the generated claim cost indexes, which is calculated under either the Taylor method or the D'Arcy and Gorvett method, to be consistent with the deflation multipliers. We can continue with the previous example to illustrate the difference between the Taylor method and the D'Arcy and Gorvett method.

We will use the same inflation figures of Auto Bodily Injury claim cost from 1987 – 1989, i.e. 10%, 8% and 6% respectively. Suppose the generated inflation figures of Auto Bodily Injury claim cost from 1990 – 1992, after putting the generated CPI figures through the regression formula, for one simulation, are 5%, 4% and 3% respectively. Under the Taylor method, the inflation multiplier of AY8-DY4 will be  $1.1 * 1.08 * 1.06 * 1.05 = 1.3222$ ; that of AY8-DY5 will be  $1.322 * 1.04 = 1.3751$ ; and that of AY8-DY6 will be  $1.3751 * 1.03 = 1.4164$ . Similarly, the values calculated under the D'Arcy and Gorvett method are the results of complex integrations and in the case of  $k = 0.15$ ,  $m = 0.5$  and  $n = 1$ , the inflation multipliers from 1990 – 1992 will be 1.2126, 1.2481 and 1.2766 respectively.

Under this approach hundreds of inflation paths are generated. A lower triangle of inflation multipliers is calculated for every generated inflation path and each cell in the triangle is multiplied by the corresponding cell in the simulated incremental deflated loss triangle to obtain a final cumulative loss triangle. The current cumulative loss amounts are deducted from the ultimate loss amounts to obtain individual years' loss reserve figures and a total loss reserve figure. Finally, a distribution of loss reserves calculated using the two-factor approach is obtained by performing Monte Carlo simulation and generating new values for Step II and Step III in each iteration to calculate the residual variability and the inflation-driven variability.

#### IV-D. Economic Value

We use the risk-free interest rate to discount the cash flows to calculate the economic value of loss reserves. The risk-free interest rate is modeled as the nominal interest rate, which is a function of the simulated general inflation rate and real interest rate. From Irving Fisher's formula:

$$i = (1 + r) * (1 + \pi) - 1$$

where  $i$  = nominal interest rate,  $r$  = real interest rate,  $\pi$  = inflation.

After performing Step III to calculate the inflation-driven variability, an incremental loss triangle is calibrated using the final cumulative loss triangle. A real interest rate path is generated using a two-factor Hull-White model to calculate the nominal interest rate path using the above formula, which is then applied to discount the corresponding calendar year cash flows. The Hull-White model uses a mean-reverting process with the short-term real interest rate reverting to a long-term mean, which is itself stochastic and reverting to a long-term mean.

$$dr_t = \kappa_r (l_t - r_t) dt + \sigma_r \sqrt{dt} dz_r$$

$$dl_t = \kappa_l (\mu_r - l_t) dt + \sigma_l \sqrt{dt} dz_l$$

where  $t$  is the time,  $r$  is the short-term rate,  $l$  is the long-term mean,  $\kappa$  is the mean reversion speed,  $\mu$  is the ultimate long term mean reversion level,  $dt$  is the time step,  $\sigma$  is the volatility and  $dz$  is a Weiner process.

To continue with the previous example, suppose the generated nominal interest rates are both 5% for 1990 and 1991 and 7% for 1992. Cash flows in AY10-DY2, AY9-DY3, ... and AY2-DY10 are discounted by 1.05, using the 5% interest rate; cash flows in AY10-DY3, AY9-DY4, ... and AY3-DY10 are discounted by  $(1.05)^2 = 1.1025$ ; cash flows in AY10-DY4, AY9-DY5, ... and AY4-DY10 are discounted by  $1.1025 * 1.07 = 1.1797$ . In essence, all cash flows along a diagonal are discounted using the same number. An incremental economic loss triangle is then obtained to calculate the individual year's economic loss reserve figures and a total economic loss reserve figure. Finally, Monte Carlos simulation produces a distribution of nominal loss reserves along with a distribution of economic loss reserves. The 5<sup>th</sup> and 95<sup>th</sup> percentiles of each distribution are used to calculate the range of the 90% confidence intervals. The economic range is then divided by the nominal range to obtain a 90% confidence interval range ratio. This measure indicates how more or less volatile the economic values are compared to the nominal values. A value of 1 or above shows that economic values are more volatile than the nominal values, while a value below 1 shows that the economic values produce smaller and less volatile ranges than the nominal values.

#### V. Running the Model

The two-factor loss reserve model, which is designed in Microsoft Excel, begins with an input worksheet. The model can accommodate cumulative loss triangles of any dimension up to 18 x 18. The user would need the Masterson Claim Cost Index figures for the line of business that the loss data belongs to, which are available for reference in the MCCI worksheet. Parameters for the Ornstein-Uhlenbeck inflation model, the claim cost regression formula, the D'Arcy and Gorvett model (the case of  $k = 0$ ,  $m = 1$ ,  $n = 1$  is essentially the Taylor method) and the two-factor Hull-White real interest rate model have to be entered before running the specified number of simulations. The user can also choose to produce a histogram of the simulated loss

reserve distribution. The model deflates the losses with the calculated deflation multipliers and for each iteration, simulates LDFs and inflation multipliers to calculate the residual variability and the inflation-driven variability respectively, and finally discount the losses to obtain nominal and economic loss reserve values. An output worksheet collects the total nominal and economic loss reserve values from each iteration and calculates the mean, standard deviation, minimum, maximum, as well as the 5, 25, 75 and 95 percentile of the simulated loss reserve distribution. The 50% and 90% confidence interval range ratios are calculated to compare economic ranges to nominal ranges.

## VI. Parameterization

We used a set of 18 years of Auto Bodily Injury loss data from 1974 – 1991 to demonstrate how the model works. We performed straight OLS regressions on annual inflation data in Consumer Price Index (CPI All Items) from 1951-1991 to determine the parameters for the Ornstein-Uhlenbeck inflation model. We chose this timeframe because the adoption of Keynesian monetary policies in the late 1940s had a significant impact on inflation and therefore data before 1950 would not reflect variability that was relevant to subsequent periods. The current short-term rate is 2.6% (the CPI inflation in 1991), the long-term mean is 3.90%, the volatility is 2.20% and the mean-reversion speed is 0.25. The claim cost regression formula has a slope of 1.28 and an intercept of 0, i.e. the claim cost inflation is a direct multiple of the generated inflation. The slope is obtained by performing regression analysis on the Auto Bodily Injury Liability MCCI inflation versus the CPI inflation for the period 1951-1991. We believe that a regression formula with no constant term would be more reasonable, giving the appropriate variation to the simulated claim cost. We also performed regression analyses for different time periods: 20, 25, 30 and 35-year period up to 1991. The results showed a declining slope for shorter periods. The absolute differences between CPI inflation and Auto Bodily Injury Liability MCCI inflation in later years are larger than those in earlier years. However, these larger differences actually represent a smaller multiple of the high CPI inflation from mid-1970s to late-1970s, thus producing smaller slopes. We believe a slope of 1.28 calculated from the period 1951-1991 is more appropriate since it encompasses periods of both high and low inflation.

For the fixed claim model, we used the linear case ( $n=1$ ), with  $k$  (portion of claim fixed at inception of claim) set at 0.15 and  $m$  (portion of the claim fixed at settlement) set at 0.5 (as used in D’Arcy, Au and Zhang (2007)). For the parameterization of the two-factor Hull-White real interest rate model, we used the parameters chosen in the Ahlgrim, D’Arcy and Gorvett (2005) Modeling Financial Scenarios paper. The parameters are summarized in the table below:

Ornstein-Uhlenbeck inflation model			Two-factor Hull-White real interest rate model				D’Arcy and Gorvett fixed claim model		
$\kappa$	$\mu$	$\sigma$	$\kappa_r$	$\mu_r$	$\sigma_r$	$\kappa_l$	$\sigma_l$	$k$	$m$
0.25	3.90%	2.2%	1	2.8%	1.00%	0.1	1.65%	0.15	0.5

The users of the model should be aware of potential problems that can arise from the use of what seems to be appropriate parameterization based on historical volatility. One issue is the frequency of getting negative inflation values, and thus the ‘magnification’ of the deflation in the claim cost index. If the frequency of negative values is uncomfortably high, a suggested solution

is to limit the general inflation or the claim cost inflation to positive values only. Similarly, the simulation approach used to generate residual loss variability can produce LDFs below 1. Cumulative paid loss development can, in fact, decline due to salvage, subrogation or error corrections. However, in practice paid LDF below 1 are relatively rare and generally only slightly below 1. If the simulations generate an excessive proportion below 1, or unrealistic values, then adjustments to the model could be made. One possible solution would be to set a floor for the value of the simulated LDFs; another would be to adjust the standard deviation.

## VII. Results

We ran 10,000 simulations to obtain a distribution of nominal and economic loss reserve values. We also performed loss reserve analyses using the Mack method, the bootstrap method and the simulation method. Since the Mack method and the bootstrap method require the use of weighted-average link ratios to perform loss development calculations, in order to be consistent, both the two-factor approach and the simulation method used weighted-average link ratios to perform simulations. We compared the results obtained under different methods, which are briefly summarized in the table below:

Percentiles	Two-Factor Approach	Mack	Bootstrap	Simulation
Mean Value	326,386	358,453	360,955	357,593
1 <sup>st</sup>	62.13%	75.88%	73.93%	61.02%
5 <sup>th</sup>	70.98%	82.11%	80.54%	70.15%
25 <sup>th</sup>	86.16%	91.87%	91.79%	85.68%
50 <sup>th</sup>	98.07%	99.33%	99.07%	98.15%
75 <sup>th</sup>	111.61%	107.40%	107.14%	112.27%
95 <sup>th</sup>	135.46%	120.17%	121.64%	135.77%
99 <sup>th</sup>	154.68%	130.04%	141.50%	156.11%

The two-factor approach produced the lowest mean value for the total loss reserve of all four methods. Since the experience period of the data, 1974-1991, is a period of relatively high inflation, the parameterization of the inflation model in the two-factor approach based on the period 1951-1991 is likely to generate inflation values that are lower than those observed in the experience period. The Mack, bootstrap and simulation methods can only calculate variability based on what is observed in the experience period, therefore giving higher mean values than the two-factor approach. The values listed on the table are percentages of the corresponding mean. For example, the 95<sup>th</sup> percentile of the two-factor approach is 135.46% of its mean, which is equal to 442,114. The summary shows that, under this particular scenario, the two-factor approach produced reserve ranges similar to those of the simulation method, while the Mack method produced results comparable to those of the bootstrap method.

The results presented above are nominal values to be in line with the current statutory rules regarding loss reserves valuation. However, as discussed in prior research by D'Arcy, Au and Zhang (2007), the appropriate method to value loss reserves is to measure the economic value of loss reserves. The two-factor approach facilitates the calculation of economic values by incorporating the use of financial models and recognizing the timing of cash flows (losses). This

capacity is not readily available in most common loss reserving methods such as the Mack, bootstrap or the simulation method. The table below compares the nominal values to the economic values of the loss reserves regarding this data set:

Percentiles	Nominal Values	Economic Values
Mean Value	326386	283647
Standard Deviation	64415	53336
90% C.I. Range Ratio	82.95%	
1 <sup>st</sup>	62.13%	63.44%
5 <sup>th</sup>	70.98%	72.12%
25 <sup>th</sup>	86.16%	86.81%
50 <sup>th</sup>	98.07%	98.52%
75 <sup>th</sup>	111.61%	111.31%
95 <sup>th</sup>	135.46%	133.66%
99 <sup>th</sup>	154.68%	150.74%

The results show that the economic values at each percentile are lower than the nominal values. The 90% confidence interval range ratio is 82.95%, which means that the difference between the 5<sup>th</sup> and 95<sup>th</sup> percentiles of the economic values is roughly 20% less than that of the nominal values. Economic value is a more accurate measure of the true impact of loss reserves on the insurer, especially in periods of highly volatile inflation as discussed in the prior research. Further research should therefore focus on the economic value of loss reserves.

## VIII. Conclusion

Loss reserve development can vary from year to year as a result of many causes. All the effects that can lead to variations in results tend to be lumped together in most approaches to calculating loss reserve ranges. A two-factor approach to measuring loss reserve variability allows actuaries to consider the impact of inflation separately from the remaining stochastic factors affecting loss development. Whenever future inflation is expected to vary from the levels inherent in the experience period, this approach will produce more appropriate reserve ranges than other methods. An added enhancement of the model developed for this research is the ability to determine economic values for loss reserves. As fair value accounting standards become more widely applied for insurance liabilities, this feature should become an important part of any loss reserving determination.

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