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Examination of actuarial assumptions used for the calculation of the Projected Benefit Obligation (PBO)

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Abstract

The importance of the PBO in corporation-management-strategy has been increasing. In particular, actuarial assumptions underlying the PBO play an important role in the current very dynamic environment.
Economic entities such as corporations face a dynamic environment when they plan to implement M&A for rapid economic growth or plan re-structuring with a reduction in the labor force to achieve lower costs. From the point of view of cash flow projections of the corporate pension system, these dynamics may not be measured appropriately under current methods, as actuarial assumptions for the PBO calculation may not include the impact of these dynamics.

For example, for turnover rate and the rate of salary increases, we currently use a single assumption even though we recognize the effectiveness of using several assumptions according to different conditions. We assume that the assumptions for the PBO calculation would change if conditions change. The following are examples:

- **Turnover rate: It may be distorted by the one-off increase in the number of retirees arising from reductions caused by re-structuring.**
- **Rate of salary increases: Fluctuations of rate of salary increases may be caused by M&A as several corporations are combined into one corporation.**

Thinking of corporations' strategies, PBO calculation methods or the way of estimating assumptions should be developed to reflect the real position of dynamic corporations.

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We set up a model to examine problems and effects by comparing the results using single and multiple turnover rates and rates of salary increases.

The opinion in this paper presents the opinion of the authors. The views in this paper do not represent official views of Deloitte Touch Tohmatsu. All errors remain the authors.

Keywords

Stochastic projection of cash flows; Scenarios; Static pattern; Dynamic pattern

1 Introduction

1－1 Management of pensions

Management of pensions is a crucial matter for corporation-management-strategy because it directly affects the corporate financial statements and it involves the management of financial funds in order to make payments to employees. In addition, corporations face a dynamic environment where they plan M&A for rapid economic growth or plan re-structuring with a reduction of the labor force to achieve lower costs. These make the pension system very influential. In this paper, we use a simple model to investigate how appropriate the current pension management techniques are for a dynamically changing environment. In particular, we consider accounting standards, and Liability Driven Investment (LDI), which has attracted attention in recent years.

A major consideration for pension accounting standards and for LDI is the PBO[2]. The PBO is calculated by discounting future cash flows at the time of evaluation. For the discount rate, Financial Accounting Standards (FAS) recommends use of the spot rate corresponding to the particular duration of the cash flows, and the global rule is heading towards adopting a stochastic way of thinking as in financial engineering. On the other hand, for future cash flows, stochastic projection methods do not seem to have been examined. The reasons may be as follows:

- It is difficult to estimate the future volatility of assumptions such as the turnover rate and the rate of salary increases.

[2] The PBO at a particular date is the value of accrued rights based on service up to that date, but allowing for a projection of the salary on which benefits are based up to the date when benefits are awarded.

- There was a rule that differences between the prediction and the actual amount could be smoothed by not directly reflecting them in the liabilities and to cost in the financial statements (delayed recognition).

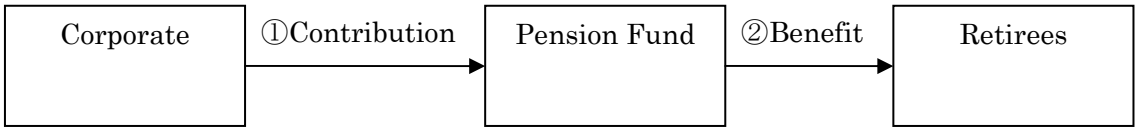
However, the standard is changing towards real time recognition of the differences in both FAS and International Financial Reporting Standards (IFRS). The current situation is different from when the accounting standards rules were first designed.

Therefore, the keyword in this paper is "a stochastic projection of the cash flows", and we consider its effect on LDI and on accounting standards.

1－2 Cash flows

We will focus on the cash flows which the pension fund pays for retirees because the PBO is calculated using them (We pay attention to ②, and not ① in the chart below).

Chart 1



Generally, in the actuarial assumptions to calculate benefit cash flow projections, one has turnover rate, rate of salary increases, mortality rate and proportion selecting a lump-sum. Of these, we will focus on the turnover rate and the rate of salary increases as the assumptions that may be affected by re-structuring and M&A. In general, the mortality rate is rarely distinguished for each corporate unit. The proportion selecting a lump-sum does not tend to be affected by re-structuring and M&A.

2 Model

2－1 Brief overview

We analyze the influence on accounting standards / LDI when we project future volatilities of the turnover rate and the rate of salary increases under re-structuring and M&A (dynamic pattern) or when there is no change (static pattern).

The model pension plan and the model employees to be used for the simulation are set up as simply as possible for the purpose of observing the influence of the difference between the dynamic pattern and the static pattern.

2－2 Word definition

2－2－1 Benefit cash flow projections

The benefit cash flow projection is the projected future benefit for services rendered up to the time of valuation. The PBO is calculated by discounting the benefit cash flow projections.

2－2－2 T_0, T_1 - T_5

T_0 is the starting time of the simulation. T_1 – T_5 represent the end of each year of simulation.

2－3 Model pension plan

2－3－1 Employees （at T_0）

It is assumed that the number of employees is distributed evenly by age from 30 years old to 59 years old at the starting time of the simulation (T_0).

After T_1, the number of employees will have changed in accordance with the turnover rate. In addition, no new employees are assumed to enter.

2－3－2 Salaries （at T_0）

The salaries used to calculate the pension benefit are assumed to be the age x 10,000 yen according to the age of the employee. (Example : The salary of a 30 year old employee is 300,000 yen). After T_1, the salary will increase according to the rate of salary increases.

2－3－3 Payment design

The amount of benefit (lump-sum) = salary at the time of resignation x number of years of service. In addition, it is assumed that all the members select lump-sums.

2－4 Turnover rate

We made three scenarios (one static pattern and two dynamic patterns). The re-structuring assumes redundancies, but not natural wastage. In addition, we do not consider the severance plans that are commonly adopted in the case of redundancies in order to observe only the influence of a difference in timing of the cash flows.

Generally, “the lower age limit” may be set as the condition for the application of the severance plan and the average age limit is about 45 years old. We applied high turnover rates to those older than 45 years old with dynamic patterns for a certain period of time.

How in practice to build a rational dynamic pattern is a practical issue, but we do not

examine it in this paper.

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2-4-1 Static pattern

We apply the same turnover rate in respect of all simulation processes (without considering re-structuring).

2-4-2 Dynamic 1 pattern

Turnover rates for those older than 45 years old are 10% higher than the normal rates for three years from the simulation starting time (The rates after the fourth year are the same as in the static pattern).

2-4-3 Dynamic 2 pattern

Turnover rates for those older than 45 years old are 20% higher than the normal rates for three years from the simulation starting time (The rates after the fourth year are the same as in the static pattern).

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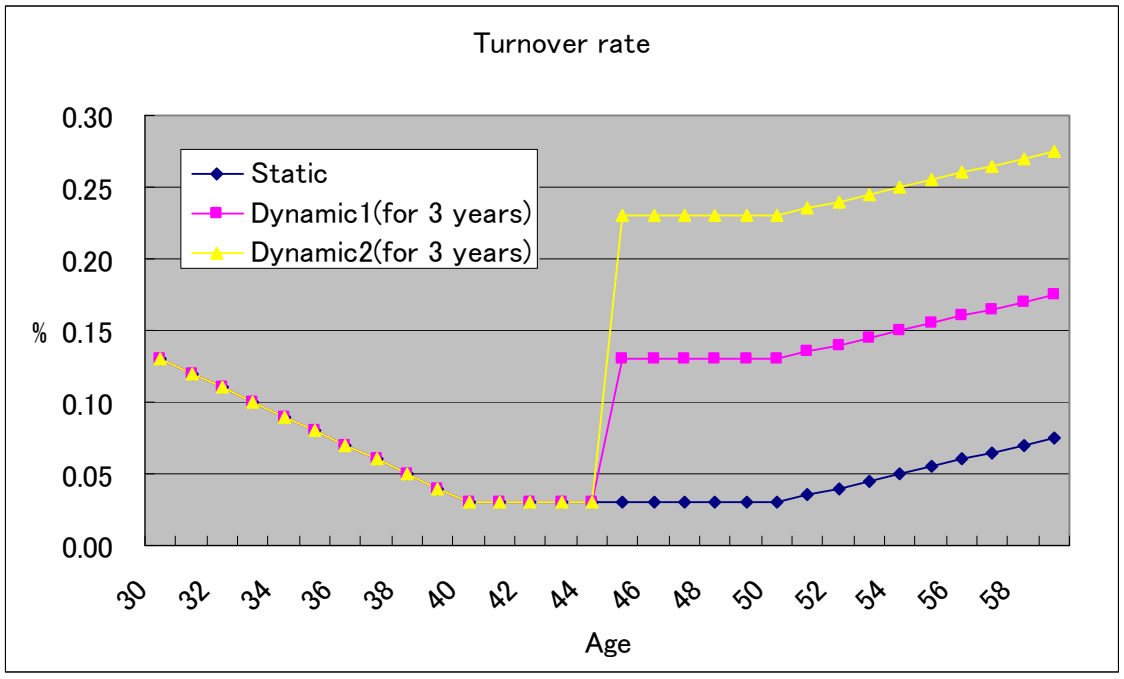
Chart 2



2-5 Rate of salary increases

When corporations are merged, the pension plans are also usually merged. Often the new corporation's pension plan is designed in such a way that vested rights of existing

employees are secured and the amount of benefit does not change suddenly as a result of the merger. Then, when an employee resigns just after the merger, the amount of benefit paid by the new plan is almost the same as under the old plan. The benefit paid gets closer to the standard amount under the new plan and, after a fixed period of time, the benefit will be the standard amount for the new plan. In this simulation, it is assumed that the transitional period is five years and the new benefit level is about 5 % (or 10%) higher than the old level. For increasing the benefit level, we assume that the base amount is increased by 1% a year (or 2% a year) for five years, in addition to the normal rate of salary increases.
In practice, when both corporations have the same multiplicative factor to compute the benefit, salaries might be adjusted to change the benefit level.

2-5-1 Static pattern

We apply only a rate of salary increases for projecting the future increases of salaries (an increasing base amount is not used).

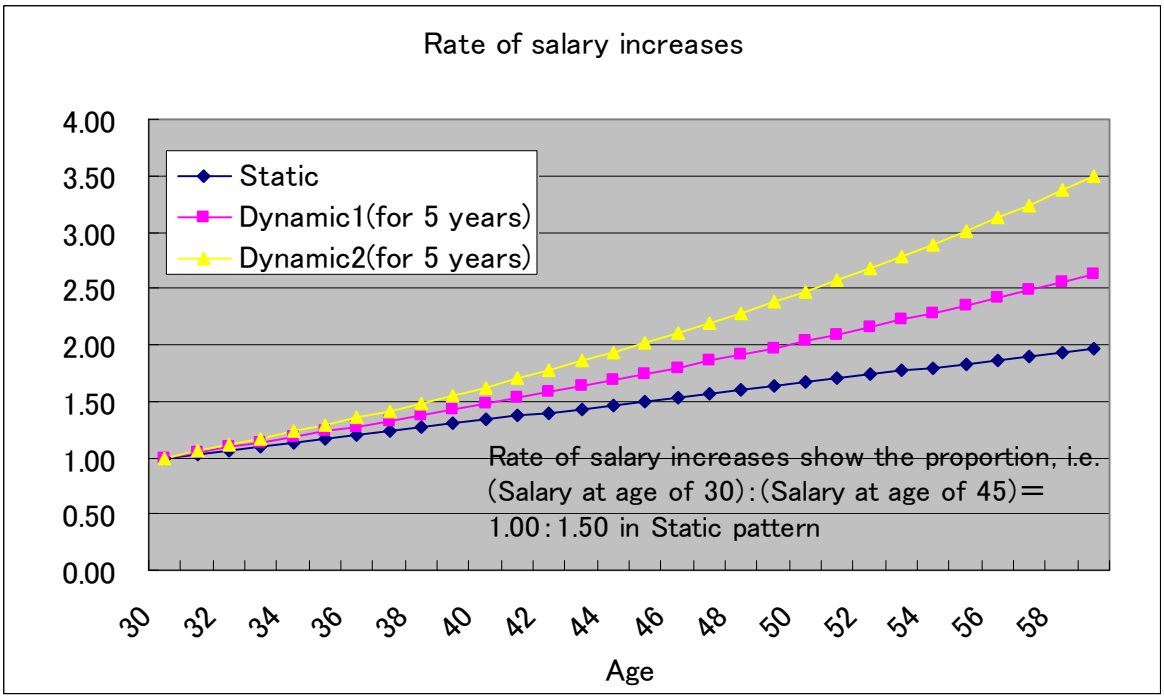
2-5-2 Dynamic 1 pattern

Salaries are raised by increasing the base amount by 1% a year in addition to the rate of salary increases for five years from the simulation starting time（salaries after the sixth year will be increased only by the rate of salary increases).

2-5-3 Dynamic 2 pattern

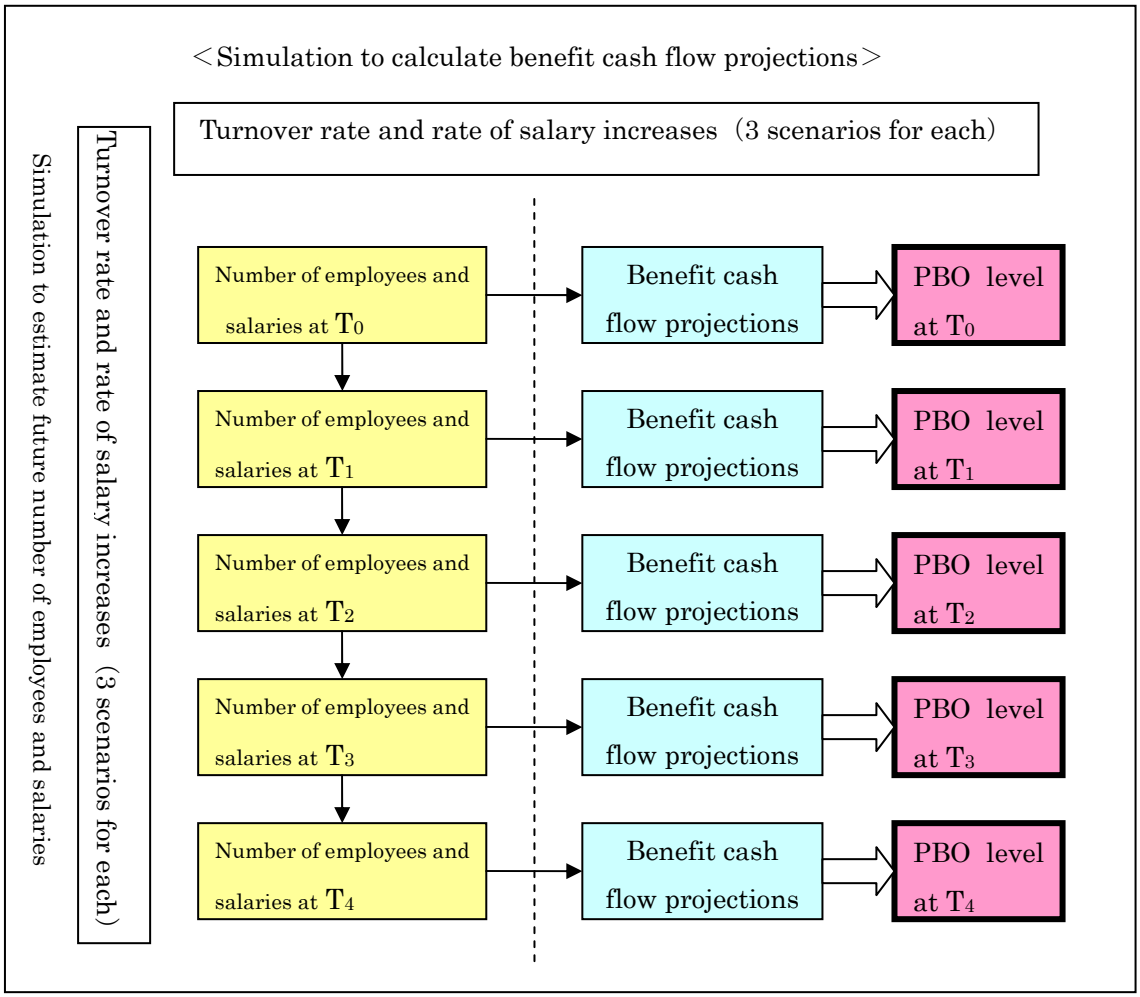
Salaries are raised by increasing the base amount by 2% a year in addition to the rate of salary increases for five years from the simulation starting time（salaries after the sixth year will be increased only by the rate of salary increases).

Chart 3


Rate of salary increases
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
Static
Dynamic1(for 5 years)
Dynamic2(for 5 years)
Rate of salary increases show the proportion, i.e.
(Salary at age of 30):(Salary at age of 45)=
1.00:1.50 in Static pattern
30
32
34
36
38
40
42
44
46
48
50
52
54
56
58
Age

2－6 Image of simulation

Chart 4



Turnover rate and the rate of salary increases are used both for calculating the benefit cash flow projections and for simulating the number of employees and their salaries.
In addition, LDI also has issues after T_1, but we examine only T_0 as the fundamental consideration.

3 Result of analysis

3－1 From the viewpoint of Accounting Standards

In this paper, PBO and actuarial gain or loss are focused and analyzed to investigate influences on Accounting Standards from different assumptions for turnover rates and the rate of salary increases.

The outline of our simulation model used to investigate influences by turnover rate and

rate of salary increases is as follows:

The simulation has two steps. First, we calculate the PBO based on each scenario for the turnover rate and rate of salary increases to describe the differences between the calculation results.

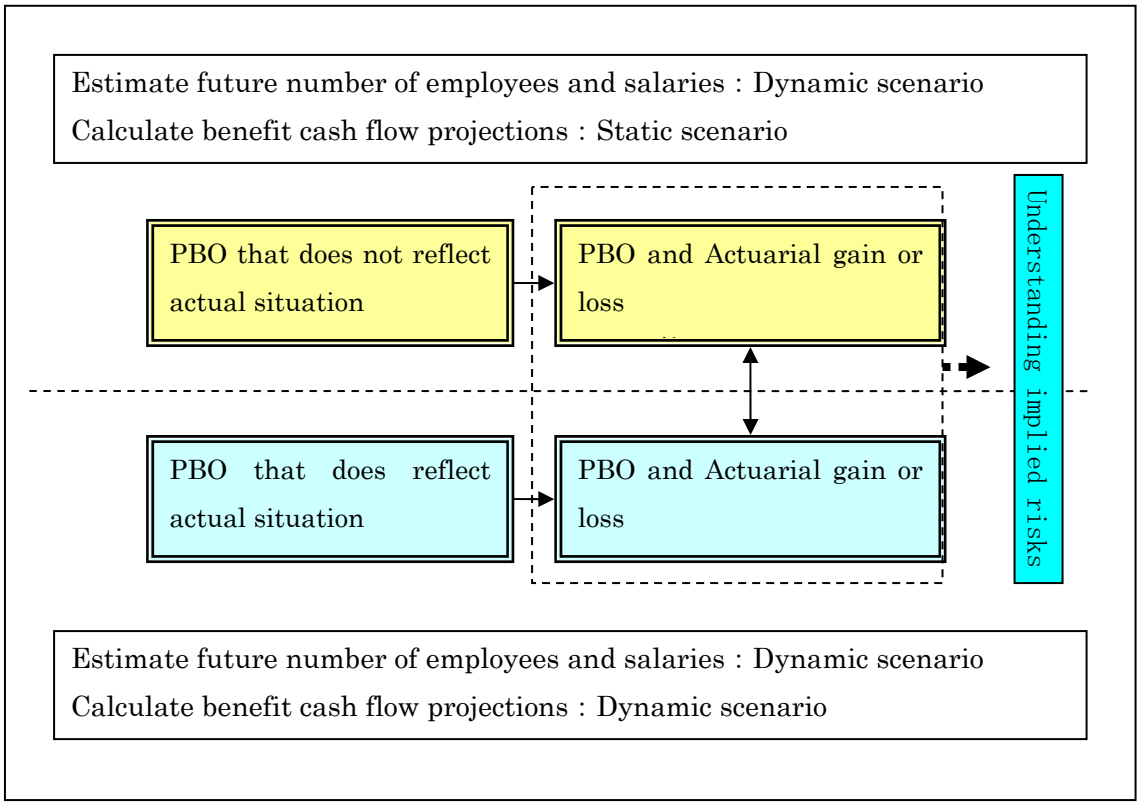
If the differences are significant, we implement a second step to estimate the actuarial gain or loss for investigating the implied risks of the accounting procedure.

In the first step, we set three scenarios for each rate – the turnover rate and the rate of salary increases – and estimate the PBO level at T_0. The discount rate corresponding to the PBO duration is estimated separately by taking it from the yield curve[3], which is estimated from the current Japanese government bond (JGB) data at the end of March, 2006[4].

Chart 5 shows the flows for the second step, a flow to understand the implied risks resulting from the actuarial assumptions. Above the dotted line shows the PBO simulation using the static scenario. Below the dotted line shows the PBO simulation using the dynamic scenario. The implied risks which will influence the accounting procedure under certain actuarial assumptions are considered.

[3] For estimating the yield curve, the Nelson-Siegel model is applied.
[4] JGB market data are used. The upper limit of current maturity is 30 years because there was no JGB over 30 years at the end of March 2006.

Chart 5



The results of the analysis are as follows:

3 - 1 - 1 Turnover rate

Three scenarios of the turnover rate — static, dynamic 1 and dynamic 2 scenarios — are examined. We project the PBOs based on the three scenarios. The discount rates used in these scenarios are different, but in addition we calculate the PBO using the same discount rate in order to compare the PBO levels between the three scenarios. The results are as follows:

Chart 6 shows the changes to the PBO. They show that those PBO levels are almost the same at T_0. The reason is as follows: the total amounts of the benefit cash flow projections are almost the same for the three scenarios, although there are differences of cash flow timing.[5]

The results may be different for a plan which has significant back-loading of the benefit curve.

[5] We calculated PBO based on the scenario that all employees will have been employed for only 5 years. The PBO is almost the same level as the others. The ratio to static pattern is 1.004, and the ratio by using the same discount rate is 0.990. The analysis was enhanced by comments from Mr. Brent Walker.

Chart 6

	Static pattern	Dynamic1 pattern	Dynamic2 pattern
PBO(yen)	221,097,174	221,766,688	222,217,692
Ratio to Static pattern	1.000	1.003	1.005
PBO calculated by using the same discount rate(yen)	221,097,174	220,354,182	219,732,072
Ratio to Static pattern	1.000	0.997	0.994

3 - 1 - 2 Rate of salary increases

Three scenarios of the rate of salary increases — static, dynamic 1 and dynamic 2 scenarios — are examined. We project the PBO based on the three scenarios. The results are as follows: Differences of PBO level at T_0 between these scenarios are significant compared to the case of turnover rates.

The PBO level for the static pattern is lower than for the dynamic patterns. The PBO level of dynamic 1 pattern is located between that of the static pattern and the dynamic 2 pattern.

Chart 7

	Static pattern	Dynamic1 pattern	Dynamic2 pattern
PBO(yen)	221,097,174	229,928,064	239,070,078
Ratio to Static pattern	1.000	1.040	1.081

Next, we estimate the PBOs based on the static pattern and the dynamic pattern for 10 years when actual base salaries are increased in addition to normal salary increases. Chart 8 and Chart 9 show simulations of the PBO. As with Chart 7, the PBO level of static pattern is lower than dynamic patterns. The reason is that the PBOs of dynamic patterns are calculated taking into account future increases in base salaries. The PBO levels are identical after five years, when the transitional period is over.

Chart 8

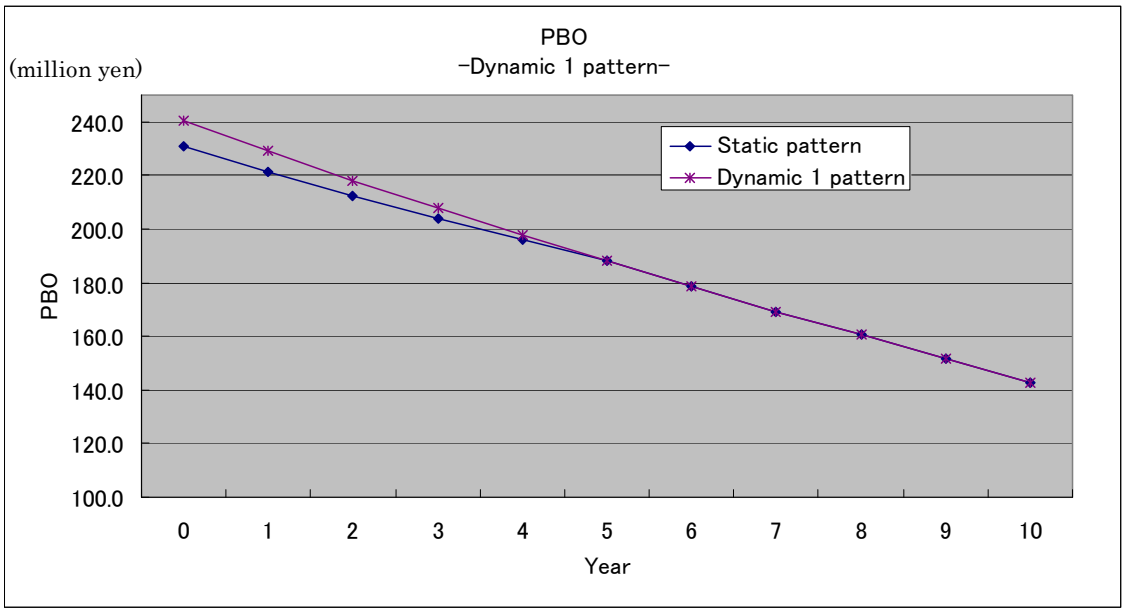


Chart 9

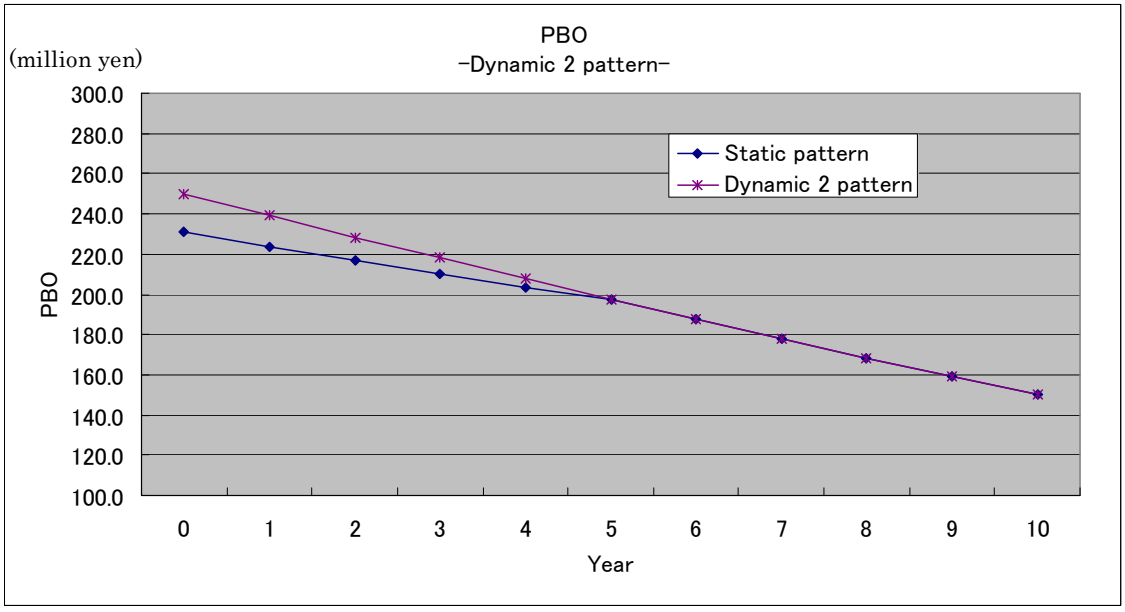


Chart 10 and Chart 11 show ratios of the PBOs based on the static pattern to those based on the dynamic patterns.

The ratio at T_0 in dynamic 2 pattern is 92.5%. We consider this difference to be significant and so we move on to the second step of investigating actuarial gain or loss.

Chart 10

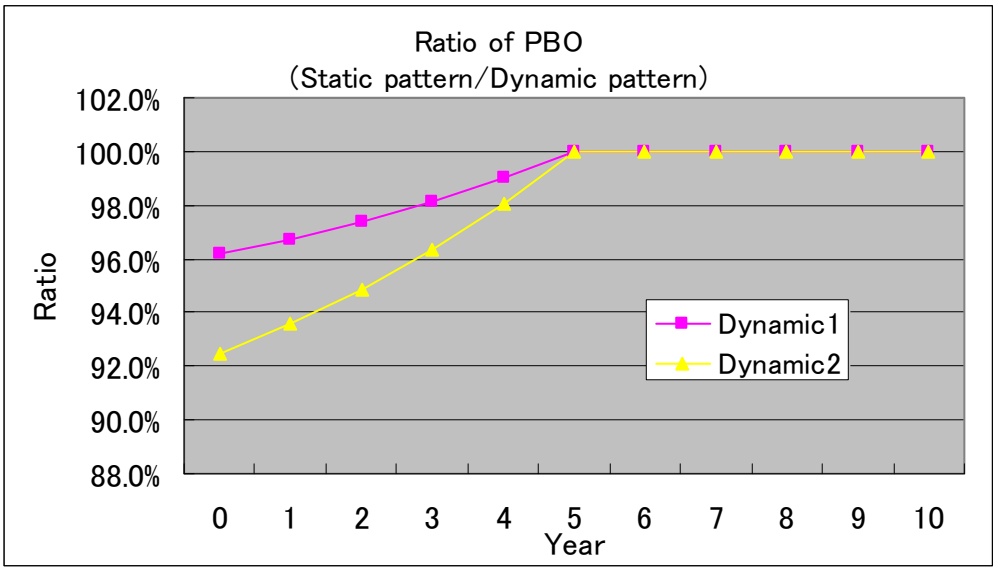


Chart 11

Year	Dynamic1	Dynamic2
0	96.2%	92.5%
1	96.7%	93.6%
2	97.4%	94.9%
3	98.1%	96.3%
4	99.0%	98.0%
5	100.0%	100.0%

Chart 12 to Chart 14 show the simulation of actuarial gain or loss. The reason why the ratio of the loss to the PBO is increasing even after 5 years is that the PBO is decreasing, due to the assumption that no new employees are brought into the simulation.
Actuarial loss at T_5 in the dynamic 2 pattern is more than 10% of the PBO. At this level, it is necessary to amortize according to the corridor rule. Because of this, for calculating the PBO in dynamic environment, the rate of salary increases assumption is a key factor for the accounting procedure, and the assumption should be carefully examined.
In case it is costly to calculate cash flows using dynamic pattern assumptions, ways of estimating the future actuarial gain or loss roughly, and adding it to the PBO, should be examined, though it would be a second best solution.

Chart 12

Year	Actuarial loss(million yen)		Actuarial loss ／ PBO	
	dynamic1	dynamic2	dynamic1	dynamic2
0	-	-	-	-
1	2.3	4.7	1.1%	2.1%
2	4.6	9.2	2.2%	4.3%
3	6.7	13.6	3.3%	6.5%
4	8.8	17.9	4.5%	8.8%
5	10.8	22.0	5.7%	11.2%
6	10.8	22.0	6.1%	11.7%
7	10.8	22.0	6.4%	12.4%
8	10.8	22.0	6.7%	13.1%
9	10.8	22.0	7.1%	13.8%
10	10.8	22.0	7.6%	14.7%

Chart 13

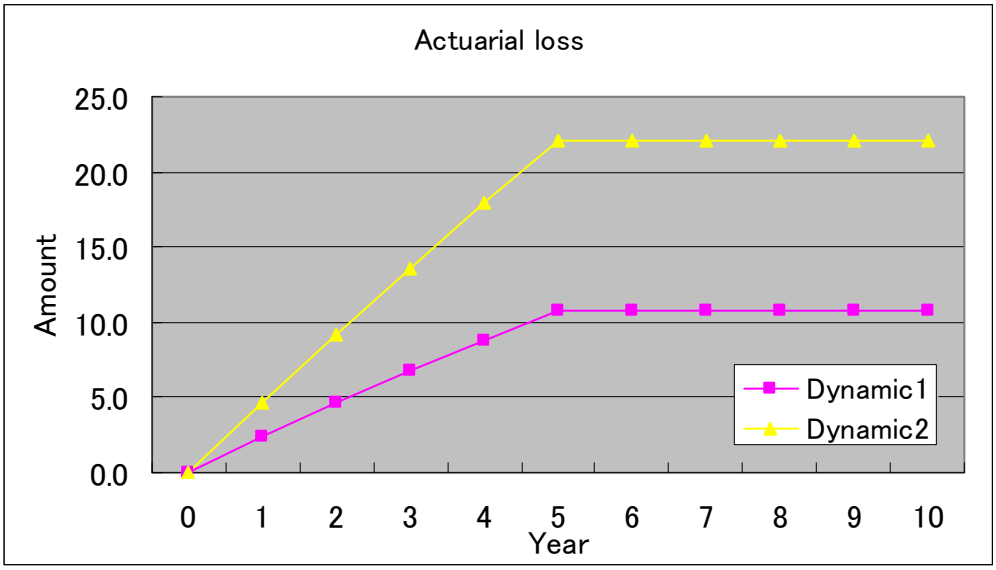
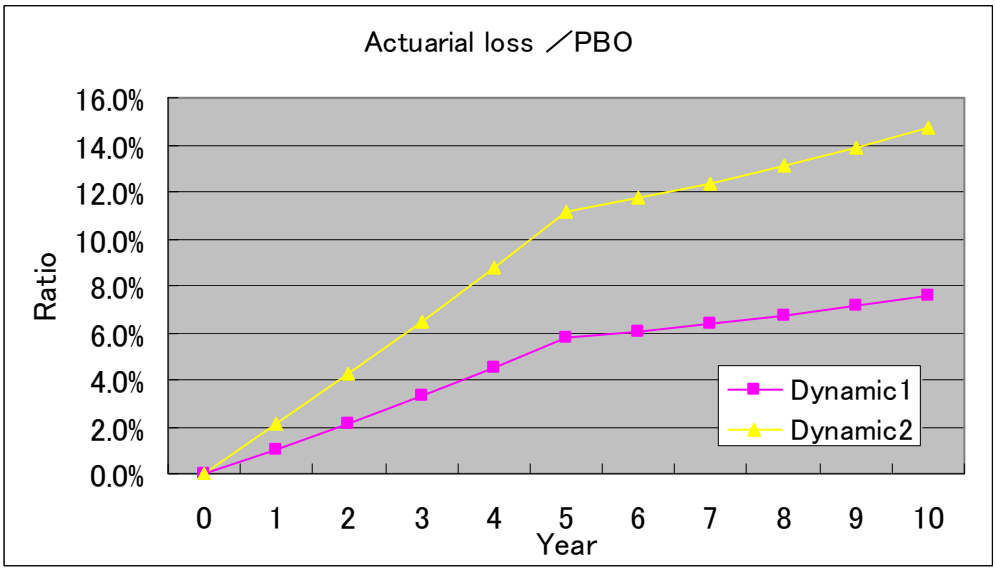


Chart 14



3－2 From the viewpoint of LDI

LDI, Liability Driven Investment, means “Investment based on pension liabilities”. In its initial stages LDI developed mainly in Europe, then spread to the U.S.A., and is now attracting the attention of pension funds in Japan. In this paper, we focus on the PBO, which is the “pension liability in the accounting standard”, and analyze it in order to investigate influences on PBOs and investments by setting turnover rates and rates of salary increases in different ways, despite the wide variety of definitions of LDI.

The “Pension asset portfolio, which has the same variability characteristics of the discounted present value of the future benefit cash flows, and which will be able to pay for the future benefits” is called the “Pension Liability Benchmark Portfolio (the PLB portfolio)”. The PLB portfolio should be identified first to implement LDI. Projections of the levels and timing of future benefit projections are necessary to construct the PLB portfolio and especially, precision and certainty of the projections are key factors for constructing the PLB. The differences in the level and timing of the future benefit cash flows between initial conditions (assumptions made by the pension actuaries when the PLB portfolio was constructed) and revised conditions may cause the PLB portfolio to be revised or restructured. That may influence whether LDI will succeed or not.

The outline of our simulation model for investigating the influence of turnover rate and the rate of salary increases is as follows:
The simulation has three steps. Firstly, we construct PLB portfolios based on each scenario of turnover rate and the rate of salary increases. Secondly, we describe the characteristics of each PLB portfolio, for example, conditional Value at Risk (cVaR). Lastly, we investigate the implied risks of PLB portfolios resulting from the actuarial assumptions. Specifically, we look at how the revised conditions – if the conditions are changed from the initial conditions used to construct the PLB portfolios – would give rise to differences in the future benefit cash flows, cash flow timings and PBO level.

Chart 15 shows the first step, "Flow chart of PLB portfolio construction". The part above the dotted line shows factors on the pension liabilities side, and the part below the dotted line shows factors on the pension assets side. We set three scenarios for each rate — turnover rate and the rate of salary increases— and estimate future benefit cash flows and the PBO level at T_0. The discount rate corresponding to the PBO duration is estimated separately from the yield curve[6], which is estimated from the current Japanese government bond data at the end of March 2006[7].

In constructing the PLB portfolio process, we first selected JGBs whose coupons during the holding period and payments at maturity almost cover the future benefit cash flows mentioned above. Next, we select a portfolio of JGBs which minimizes the difference between the discounted present value of the portfolio of JGBs and the PBO at time T_0. We call such a portfolio of JGBs the "PLB portfolio". The discounted present value of the portfolio of JGBs is estimated using the yield curve mentioned above, for consistency with the logic of the pension actuarial side.

[6] For estimating yield curve, Nelson-Siegel model is applied.
[7] JGB market data are used. Upper limit of current maturity is 30 years because there was no JGB over 30 years at the end of March 2006.

Chart 15　Flow of PLB portfolio construction

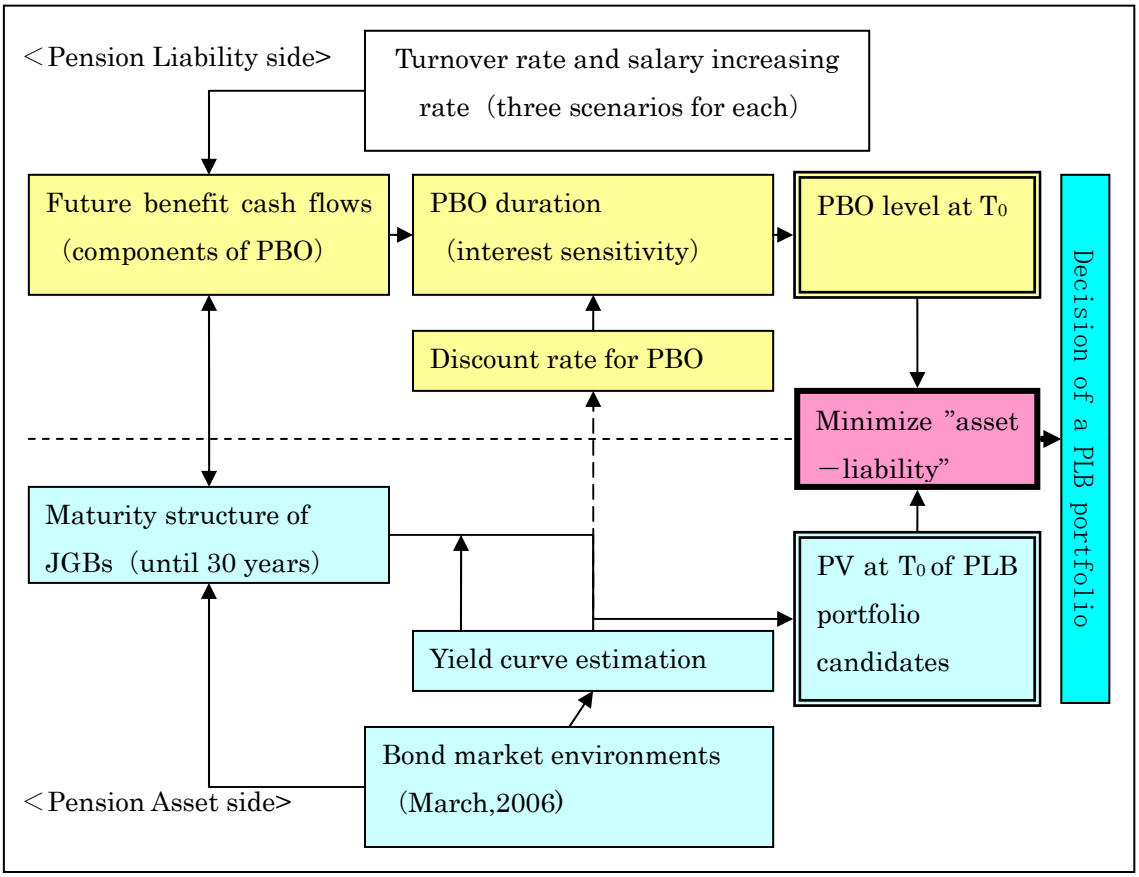
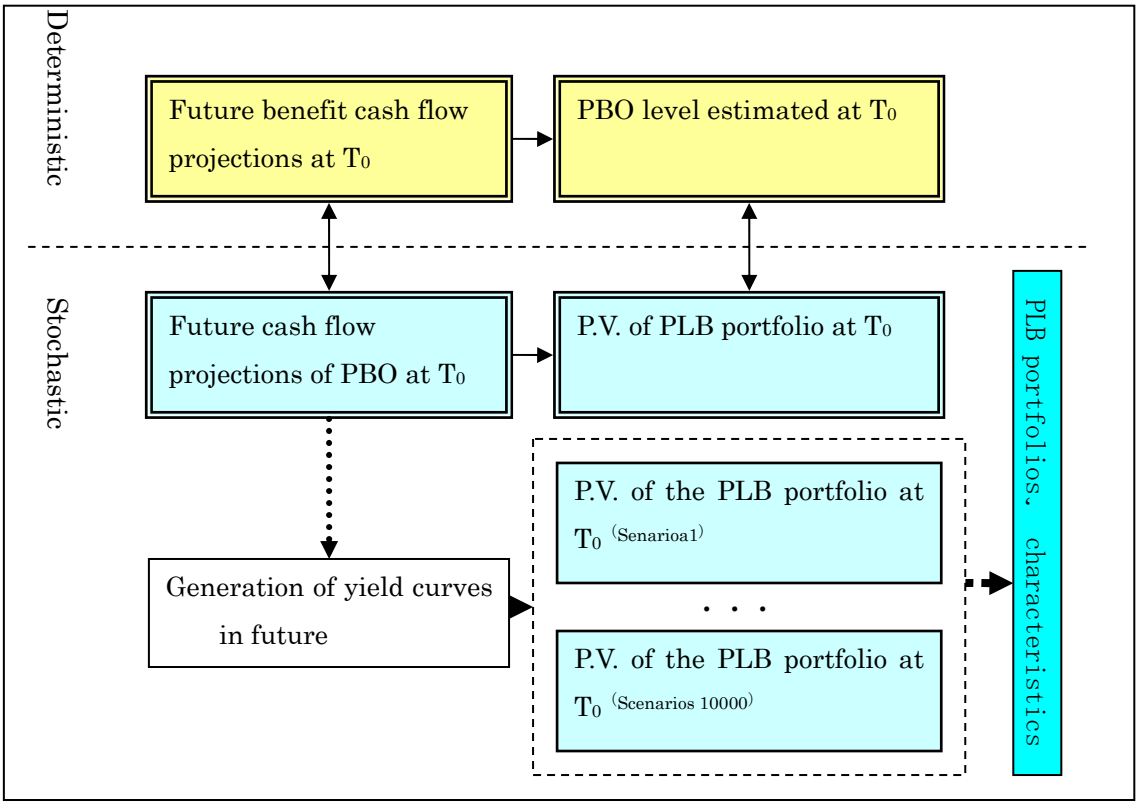


Chart 16 shows the flow of the second step, “a flow to understand the characteristics of the PLB portfolio”. Above the dotted line shows the pension liabilities side in the same way as Chart 15. Factors on the pension liabilities side have not been considered as stochastic variables, but are considered as deterministic scenarios for simplicity. Below the dotted line shows the pension assets side as in Chart 15. However, the difference is that factors below the dotted line have been treated as stochastic. The reason that we assume only the assets side factors to be stochastic is that we focus on the implied investment risks resulting from matching future cash flows from pension assets with future cash flows in the PBO. Default risk is assumed to be ignored in these implied risks since the assumption is that only JGBs are available to be invested in. For simplicity, we assume that interest rates are the only stochastic variables because they are the main cause of fluctuations of the value of the portfolio of JGBs. Specifically; we generated 10,000 interest rate scenarios, and then calculated the characteristics of the PLB portfolios (e.g. averages, standard deviations and cVaRs of returns of the portfolios).

The interest rate scenarios are based on data at the end of December 2006, and cover yield curves from 1-year maturity to 30-year maturity. For simplicity, we adopted a mean reversion process model that the yield curve at the end of December 2006 was set to the means of the model.

Chart 16　　Flow of understanding the characteristics of the PLB portfolio



As the third step, implied risks which will influence PLB portfolios under certain actuarial assumptions are considered. Especially, as shown on Chart 17 below, we examine the differences between PLB portfolios' cash flows (which will be designed under initial actuarial conditions) and future benefit cash flow projections (which will be derived from revised actuarial conditions).

Chart 17 Comparison analysis matrix of implied risks derived from actuarial assumptions

Pension liabilities side / Pension assets side		Future benefit cash flow projections and PBO level at T_0			
		Turnover rate& dynamic1	Turnover rate& dynamic2	Rate of salary increases & dynamic1	Rate of salary increases & dynamic2
PLB portfolio	Initial turnover rate & static	Comparison analysis of cash flow mismatches and surpluses derived by changes of actuarial assumptions			
	Initial rate of salary increases & static				

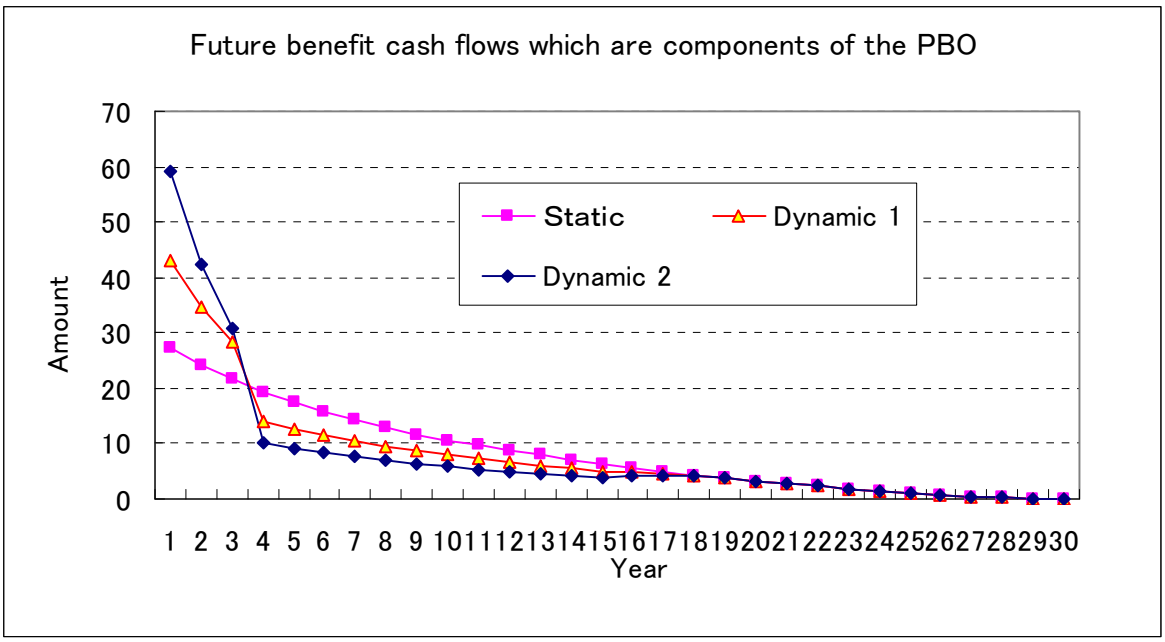
The results of the analysis are as follows:

3-2-1 Turnover rate

Three scenarios of turnover rate, static, dynamic 1, and dynamic 2, are examined. We projected future benefit cash flows based on the three scenarios. The results are as follows:

The trend line of dynamic 2 pattern is quite different from that of the static pattern due to the projection of a large number of retirees in the first three years. The trend line of dynamic 1 pattern is located between that of the static pattern and that of the dynamic 2 pattern.

Chart 18 Future benefit cash flows which are components of the PBO



As shown in Chart 19 to Chart 21 below, we construct a PLB portfolio corresponding to each scenario, and compare cash flows of the pension assets with those for the pension liabilities. Each graph shows that future cash flows from the pension assets are almost able to replicate cash flows from the pension liabilities. However, cash flows from the pension assets exceed cash flows from the pension liabilities at 20-years and 28-years maturity. The reason is as follows; in Japan, 20-year JGBs and 30-year JGB were issued at the end of March 2006 (No JGB exists with maturity between 20-years and 30-years), and their volume in both the Japanese primary and secondary market is small. Due to the lack of JGBs to correspond to each maturity, JGBs of the other maturities should be held to cover the gaps. [8]

[8] The results of analysis in this section (3-2-1) and 3-2-2 section are under assumptions of the PLB portfolios constructed in JGBs. The characteristics of the PLB portfolios might be influenced by anomalies at 20 and 28 years in Japanese bond market, and then due to the anomalies, the analysis and conclusions in 3-2-1 and 3-2-2 might be different. We examined these assumptions and analyzed each case by smoothing anomalies in the Japanese bond market. In other words, we assumed fully matching the PLB portfolios. Definitely, the conclusions that "turnover rate assumption, which influences the rate of change of surplus, would be a key factor for implementing LDI" remained unchanged. Main charts of the results of analyzing these are shown in Appendices. The analysis in the Appendices has been enhanced by comments from Mr. Brent Walker.

Chart 19　Future cash flows of benefits and JGBs（Static pattern）

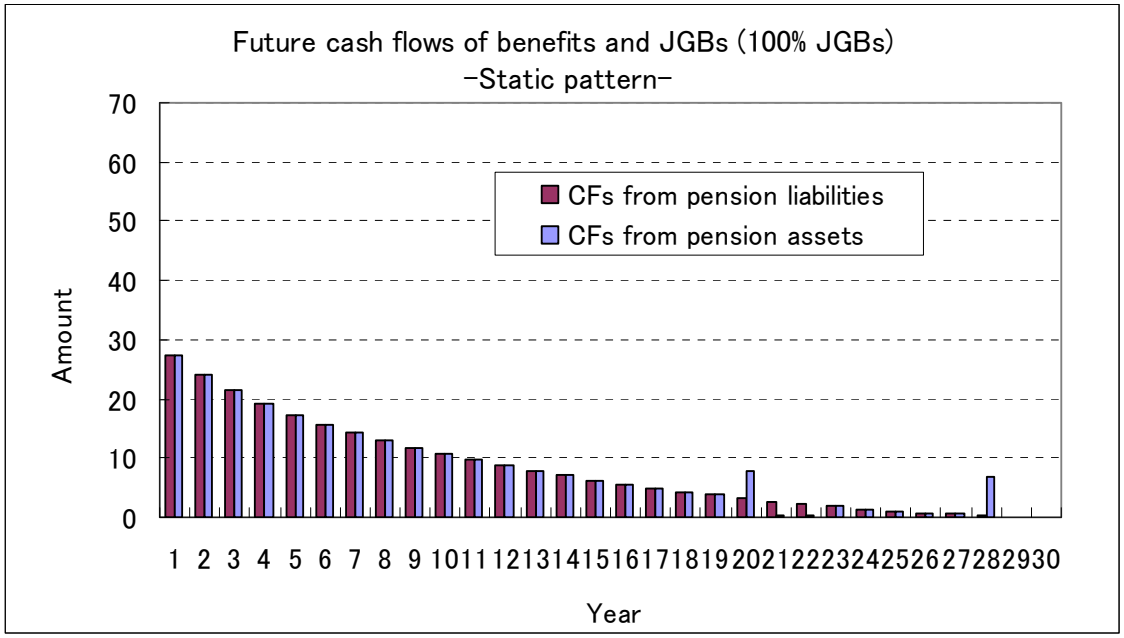


Chart 20　Future cash flows of benefits and JGBs（Dynamic 1 pattern）

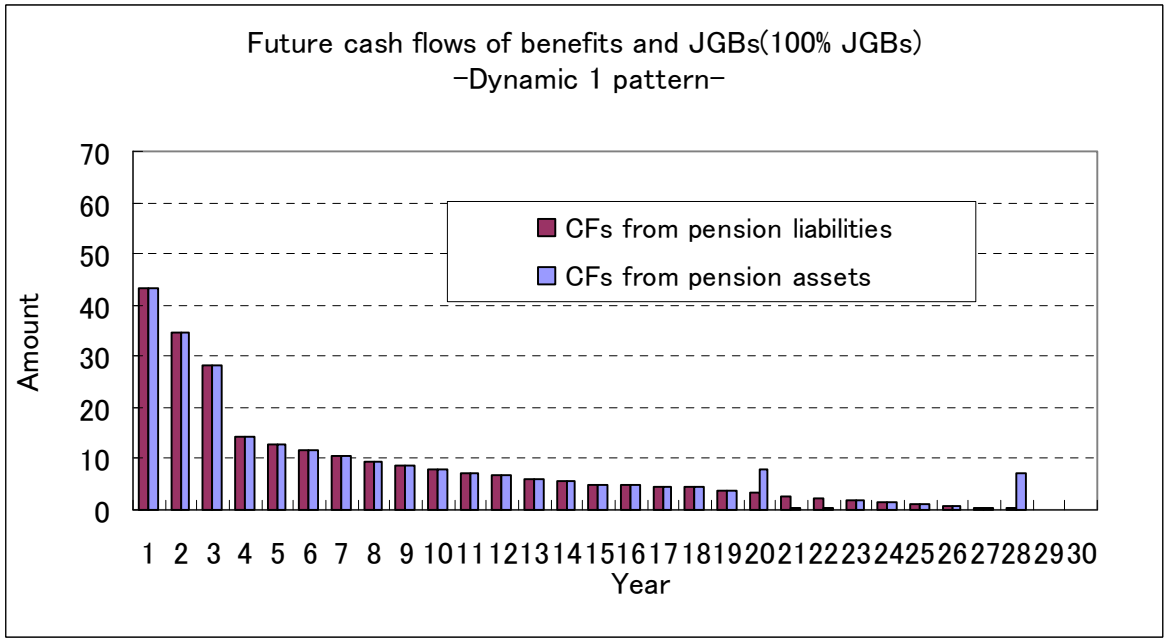
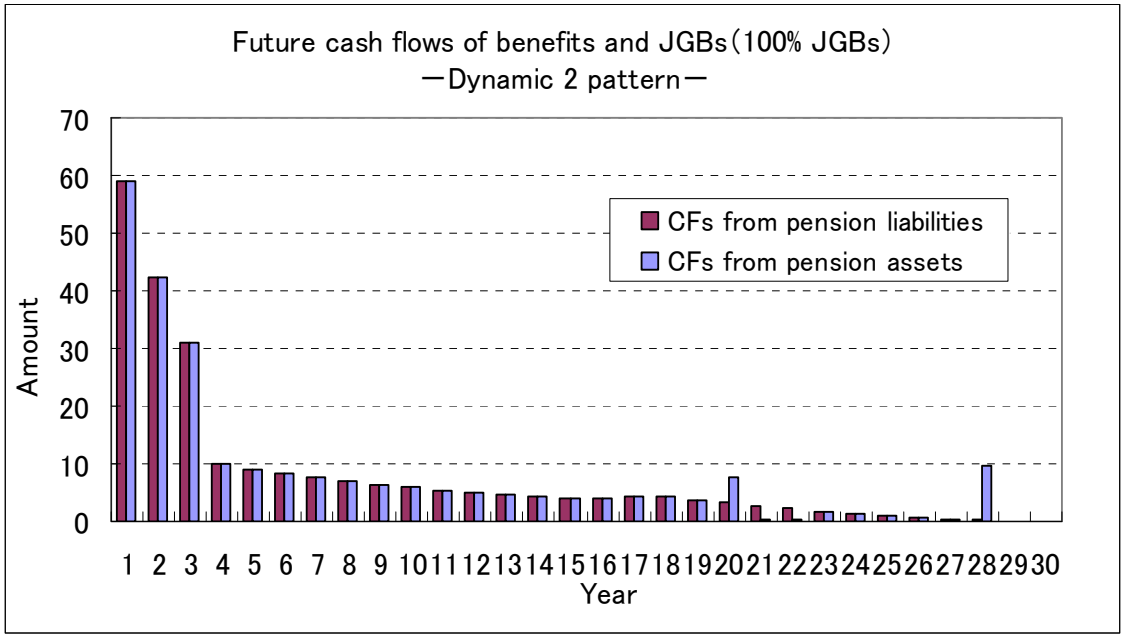


Chart 21　Future cash flows of benefits and JGBs（Dynamic 2 pattern）



Each chart from Chart 22 to Chart 24 below shows the key statistics of the PLB portfolio, the PBO and the surplus for each scenario. A rate of change in pension assets means a return on the PLB portfolio, and a rate of change in pension liabilities means a rate of change of the PBO at time T_0. The averages of rates of change in pension liabilities in these charts are negative as they correspond to the rates of changes in pension assets. In other words, the rate of change in pension assets (which is an average) plus the rate of change in pension liabilities (which is also an average) equals the rate of change in surplus (which is also an average). Because the cash-flow matching takes precedence over duration matching in designing PLB portfolios, we will find differences of rates of change and duration between pension assets and pension liabilities in each chart. However, we will not consider the optimality of PLB portfolios further hereinafter, since in this paper we intend to understand the influences of pension actuarial assumptions on PBOs and PLB portfolios, by examining differences in the characteristics of initial PLB portfolios and PBOs based on revised actuarial assumptions.

As shown in Chart 22 to Chart 24, changes in duration seem to be characteristic. The duration of the dynamic 1 pattern is shorter than that of the static pattern on both the pension assets side and the pension liabilities side, due to the increasing weight of JGBs or benefit-cash-flows for the next three years compared to the total period. Compared to

a dynamic 1 pattern, the duration of a dynamic 2 pattern is even shorter. The standard deviation of the dynamic 1 pattern is lower than that of the static pattern, and then that of the dynamic 2 pattern is even lower than that of the dynamic 1 pattern, due to shorter durations. However, the standard deviation of the surplus of the dynamic 1 pattern is higher than that of the static pattern, and that of the dynamic 2 pattern is higher than that of the dynamic 1 pattern. The mismatch between the pension assets and the pension liabilities seem to be widening. Differences of cVaRs(95%) of changes of surplus between the static pattern and the other 2 patterns are shown in these charts, however, there is no significant difference of cVaR(95%) between the two scenarios.

Chart 22　Key statistics of the static pattern

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	7.158 year	7.676year	
Average	1.95%	−1.39%	0.56%
Standard deviation	5.60%	5.67%	0.95%
Median	2.20%	−1.61%	0.57%
Mode	14.03%	−11.36%	2.67%
Kurtosis	8.92%	−0.55%	16.35%
Skewness	−35.34%	35.88%	−10.53%
cVaR（95%）			−1.48%

Chart 23　Key statistics of the dynamic 1 pattern

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	6.199year	6.758year	
Average	1.40%	−1.27%	0.13%
Standard deviation	4.77%	4.80%	1.11%
Median	1.60%	−1.40%	0.15%
Mode	12.06%	−9.34%	2.73%
Kurtosis	6.04%	−7.32%	9.51%
Skewness	−31.38%	35.43%	−9.28%
cVaR（95%）			−2.22%

Chart 24　Key statistics of the dynamic 2 pattern

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	5.914year	5.401year	
Average	1.79%	−1.18%	0.61%
Standard deviation	4.18%	4.07%	1.29%
Median	1.95%	−1.27%	0.63%
Mode	11.61%	−7.68%	3.93%
Kurtosis	4.06%	−12.34%	16.44%
Skewness	−26.51%	36.63%	−8.01%
cVaR（95%）			−2.13%

Now, we examine how differences between the future cash flows of pension assets and pension liabilities will be shown and how surplus changes, by comparing some cases focusing on the turnover rate. The cases are as follows:

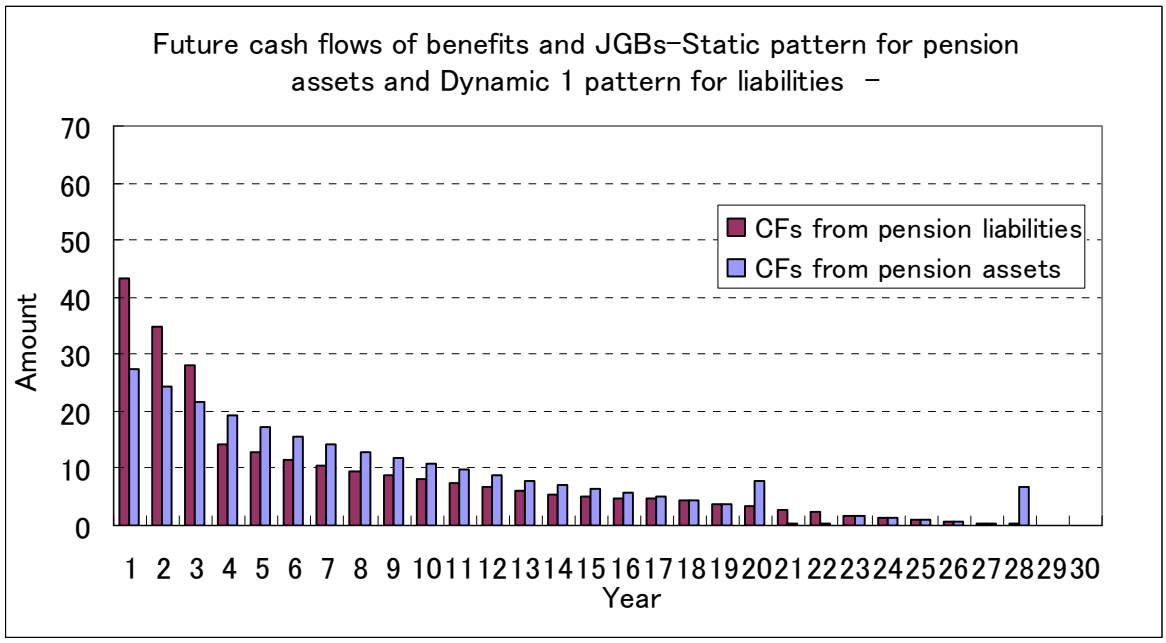
Case 1: The PLB portfolio was designed under initial actuarial assumptions (the static pattern) and then actuarial assumptions (= turnover rate) are changed to the dynamic 1 pattern. Future benefit cash flows are recalculated under the dynamic 1 pattern.
Case 2: The PLB portfolio was designed under initial actuarial assumptions (the static pattern) and then actuarial assumptions (= turnover rate) are changed to the dynamic 2 pattern. Future benefit cash flows are recalculated under the dynamic 2 pattern.

First, we examine Case 1. Chart 25 shows the future cash flows from the pension assets and liabilities, and differences between them. Chart 25 shows a cash flow pattern which is quite different from Chart 19. The differences of cash flows between pension assets and pension liabilities are negative in the first three years, which reflects the increase in retirees, and then they turn positive. Over 20-years of maturity, the differences fluctuated from positive to negative.

Chart 26 shows the influence of the revised turnover rate assumption on surplus. The right column shows how the rate of surplus changes in case 1. For comparison, we prepared a base case in which the PLB portfolio is designed under actual actuarial assumption (dynamic 1 pattern) at time T_0. The base case is shown in the left column. The average, median, and mode in the chart change positively compared to the base case. However, the standard deviation increases, and the kurtosis and skewness differ

greatly from those of the base case. These statistics show that the riskiness of case 1 is larger than that of the base case. cVaR(95%) in Case 1 is -2.21%, and is at the same level as in the base case which is -2.22%. Shortfall risk does not seem to show significant change.

Chart 25　Future cash flows of benefits and JGBs（Static pattern for pension assets and Dynamic 1 pattern for liabilities）



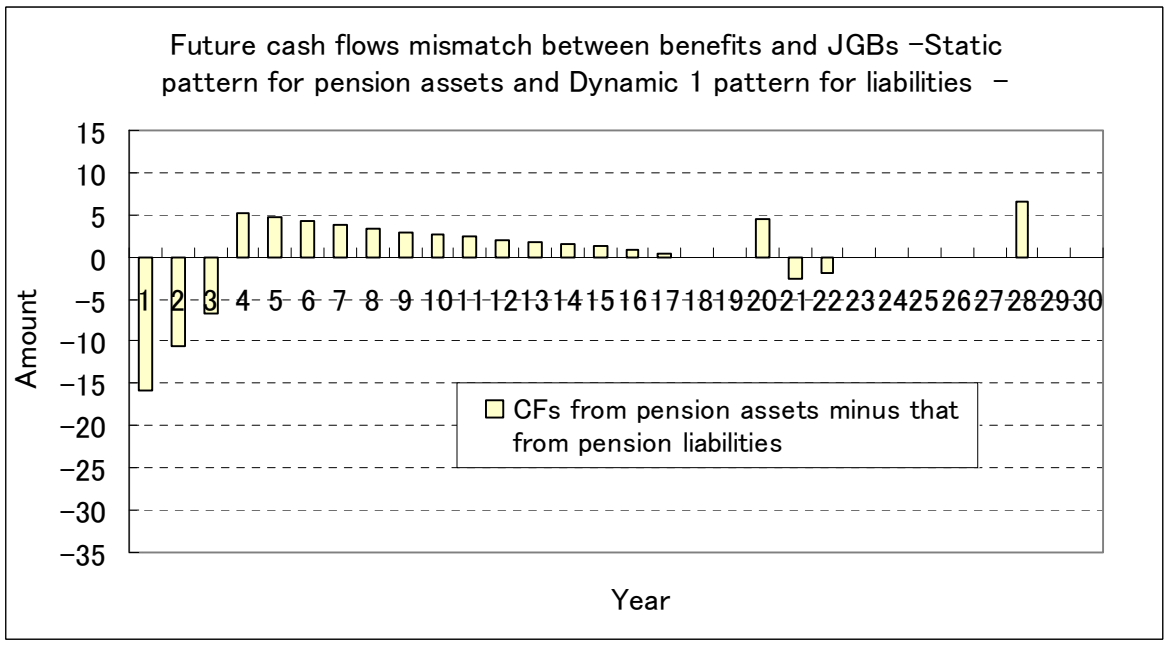
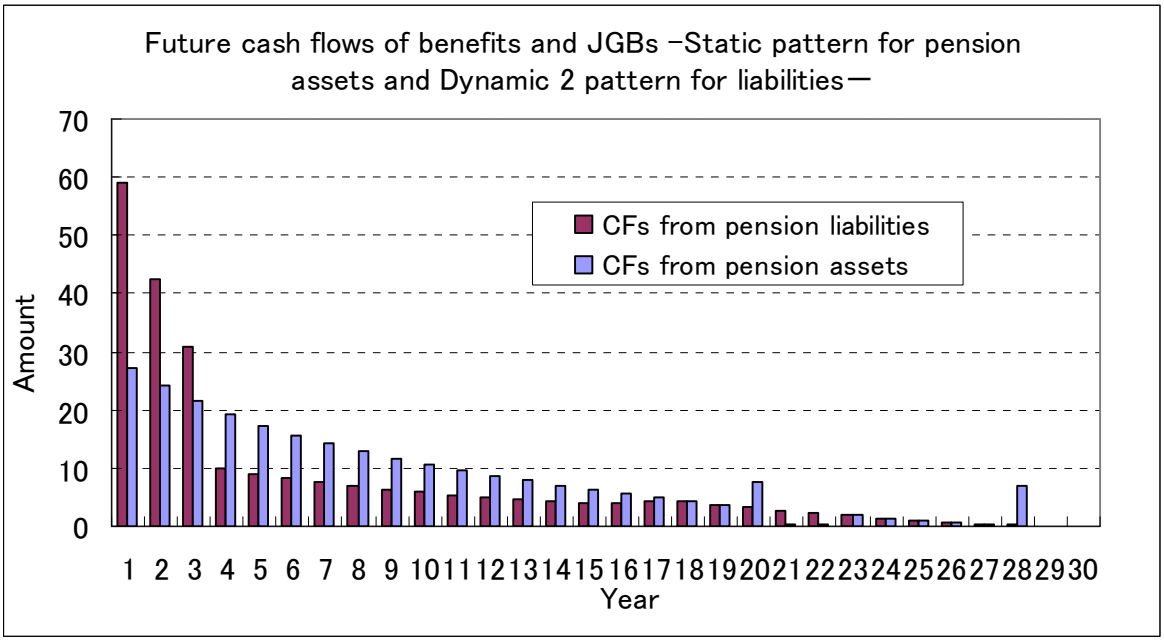


Chart 26 Key statistics of two surpluses, one of which is a static pattern for both assets and liabilities, and the other is a static pattern for assets and dynamic 1 pattern for liabilities

	Rate of change of surplus (dynamic 1 pattern for both)	Rate of change of surplus (static pattern for assets and dynamic 1 pattern for liabilities)
Average	0.13%	0.68%
Standard deviation	1.11%	1.35%
Median	0.15%	0.66%
Mode	2.73%	4.70%
Kurtosis	9.51%	36.68%
Skewness	−9.28%	5.98%
cVaR（95%）	−2.22%	−2.21%

Similarly, the influence of the revised turnover rate (dynamic 2 pattern) assumption on surplus are examined here. Chart 27 shows future cash flows from assets and liabilities, and the mismatch between them. It is clearly different from Chart 20, and the mismatch is increased compared to Chart 25. Chart 28 shows the influences on rates of surplus. The column to the left of the chart shows key statistics when designing a PLB portfolio under dynamic 2 pattern actuarial assumptions initially, and the right column shows key statistics when actuarial assumptions are revised after designing the PLB portfolio. Average, median, and mode on the right side also change positively, however, standard deviation additionally increases, and both kurtosis and skewness show deviations from the initial statistics. We see that riskiness is increased. In additon, cVaR(95%) has also decreased from -2.13% to -3.65%, and we find that shortfall risk has increased.

Chart 27 Future cash flows of benefits and JGBs (Static pattern for pension assets and Dynamic 2 pattern for liabilities)



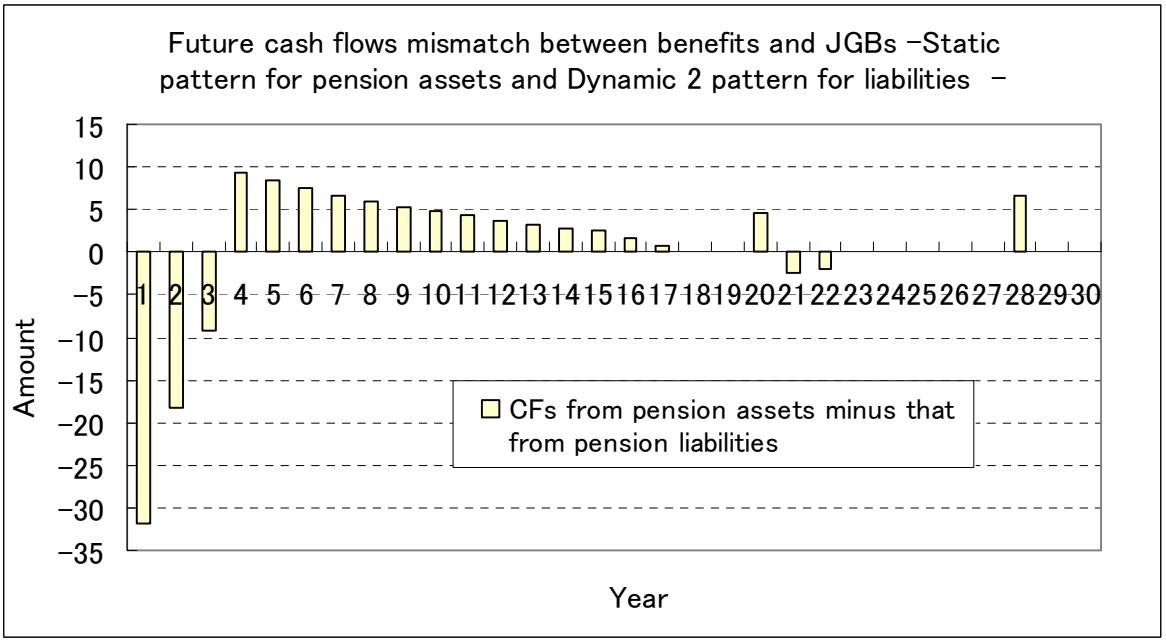
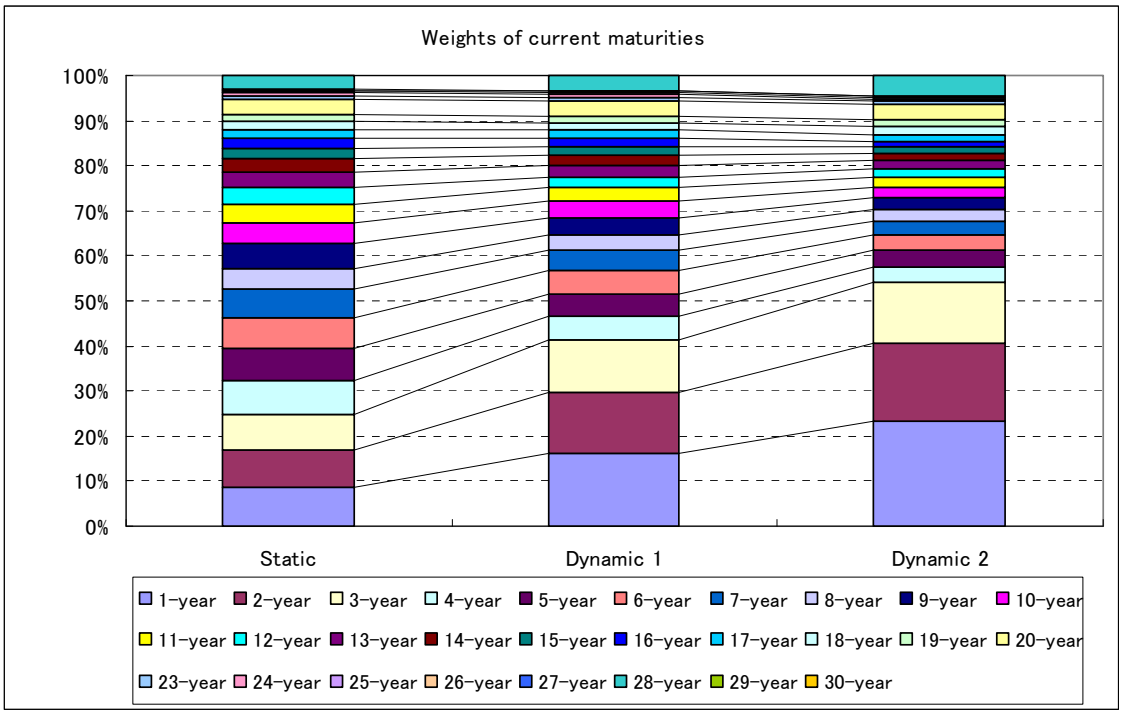


Chart 28 Key statistics of two surpluses, one of which is a static pattern for both assets and liabilities, and the other is a static pattern for assets and dynamic 2 pattern for liabilities

	Rate of change of surplus (dynamic 2 pattern for both)	Rate of change of surplus (static pattern for assets and dynamic 2 pattern for liabilities)
Average	0.61%	0.76%
Standard deviation	1.29%	1.97%
Median	0.63%	0.81%
Mode	3.93%	6.35%
Kurtosis	16.44%	51.30%
Skewness	−8.01%	−12.33%
cVaR（95%）	−2.13%	−3.65%

Chart 29 shows how the weights of maturity in each PLB portfolio differ from each other, especially in short term bonds. The weight of short term bonds in the static pattern, whose current maturity dates are under three years, is under 25%. However, the weight in the dynamic 1 pattern is under 40%, and that in the dynamic 2 pattern is over 50%. This means that PLB portfolio rebalance will occur and rebalancing costs will also be incurred, if the PLB portfolio is redesigned due to changes of characteristics in the pension liabilities after designing the initial PLB portfolio. For these points, the turnover rate assumption at the time of designing the initial PLB portfolio is a key factor for implementing LDI, and the assumption should be carefully examined.

Chart 29　Comparison among weights of current maturity dates of each PLB portfolio

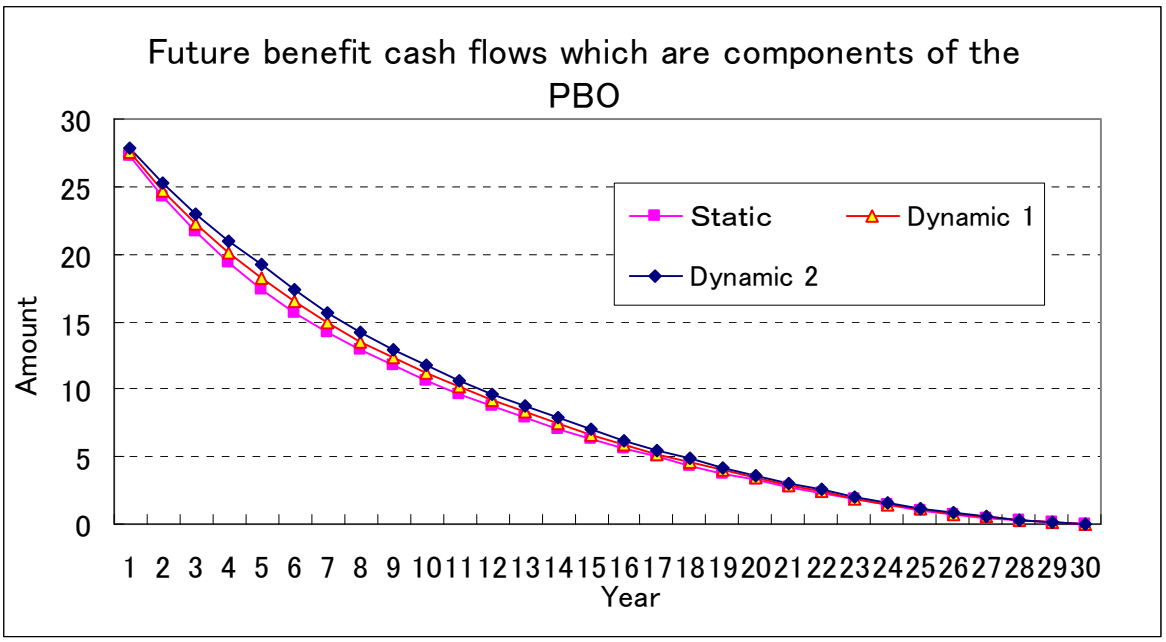


3 - 2 - 2　Rate of salary increases

Three scenarios of rate of salary increases, which are static, dynamic 1, and dynamic 2 scenarios, are examined in the same way as in the case of turnover rate.　We projected future benefit cash flows based on the three scenarios. The results are as follows:

Chart 30 shows no significant differences among the three patterns of future benefit cash flows compared with the patterns of turnover rate. Focusing on cash flow levels, the dynamic 2 pattern is the highest, followed by the dynamic 1 pattern, then the static pattern.

Chart 30 Future benefit cash flows which are components of the PBO



As shown in Chart 31 to Chart 33 below, we construct a PLB portfolio corresponding to each scenario, and compare the flows of pension assets in each with those of the pension liabilities. We found no significant differences between their characteristics.

Chart 31 Future cash flows of benefits and JGBs (Static pattern)

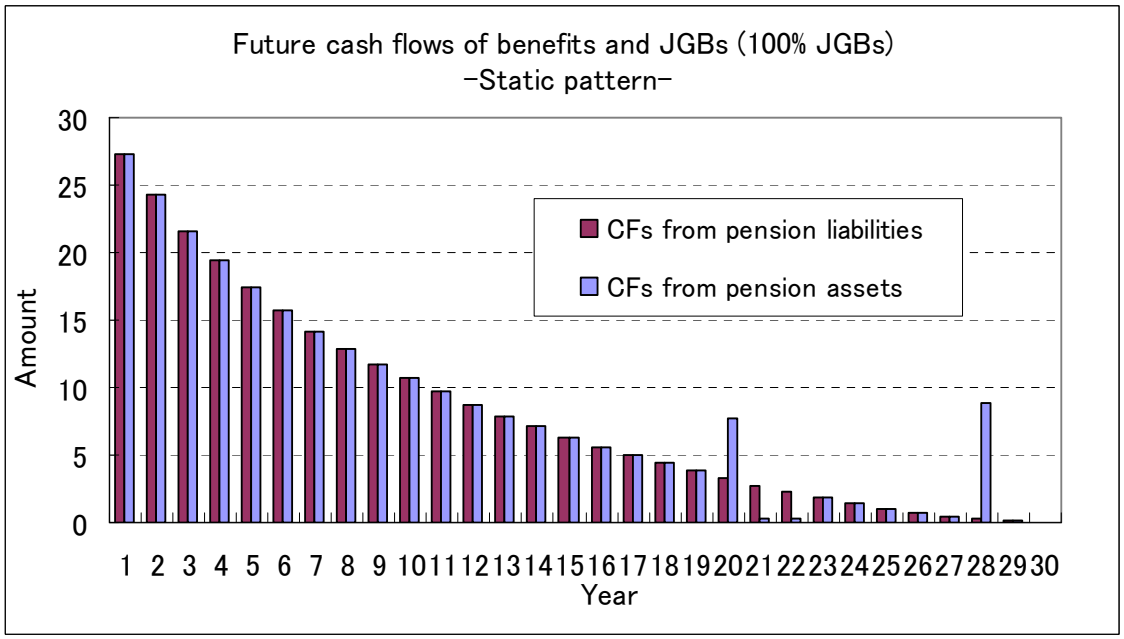


Chart 32 Future cash flows of benefits and JGBs (Dynamic 1 pattern)

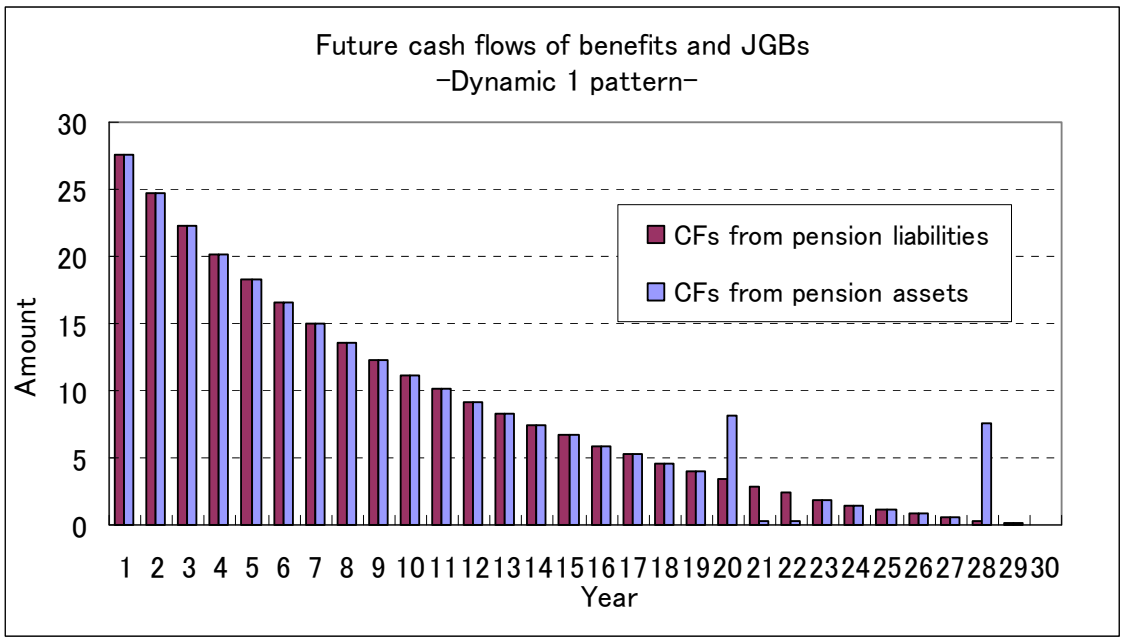
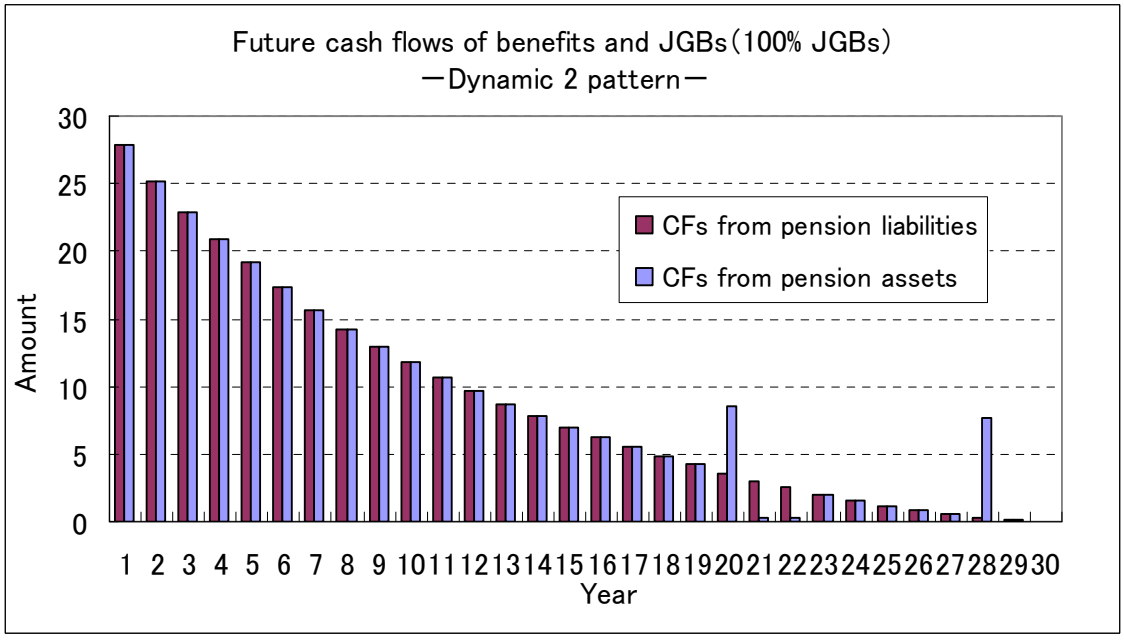


Chart 33 Future cash flows of benefits and JGBs (Dynamic 2 pattern)



Each chart from Chart 34 to Chart 36 below shows key statistics of the PLB portfolio, the PBO and the surplus of each scenario of rate of salary increases in the same way as with the turnover rate. We find small differences of rate of change and durations between the pension assets and the pension liabilities in each chart for the same reasons mentioned in the section on turnover rate, and also we will not discuss further

the optimality of PLB portfolios hereinafter.

In the cases of the rate of salary increases, we found no significant differences between those charts, unlike the case of turnover rate.

Chart 34 Key statistics of the static pattern

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	7.580year	7.676year	
Average	1.95%	−1.39%	0.56%
Standard deviation	5.68%	5.67%	1.00%
Median	2.21%	−1.61%	0.57%
Mode	14.34%	−11.36%	2.98%
Kurtosis	8.77%	−0.55%	15.26%
Skewness	−34.80%	35.88%	−3.46%
cVaR（95%）			−1.54%

Chart 35 Key statistics of the dynamic 1 pattern

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	7.216year	7.735year	
Average	1.95%	−1.38%	0.57%
Standard deviation	5.66%	5.73%	0.97%
Median	2.21%	−1.61%	0.58%
Mode	14.20%	−11.45%	2.76%
Kurtosis	9.04%	−0.37%	15.75%
Skewness	−35.33%	36.00%	−9.02%
cVaR（95%）			−1.50%

Chart 36 Key statistics of the dynamic 2 pattern

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	7.272year	7.793year	
Average	1.95%	−1.38%	0.57%
Standard deviation	5.71%	5.78%	0.98%
Median	2.21%	−1.61%	0.58%
Mode	14.28%	−11.53%	2.75%
Kurtosis	9.21%	−0.20%	15.65%
Skewness	−35.49%	36.11%	−9.66%
cVaR（95%）			−1.51%

Now, we examine the influence on the rate of salary increases assumptions, in the same as was done with the turnover rate. The cases are as follows:

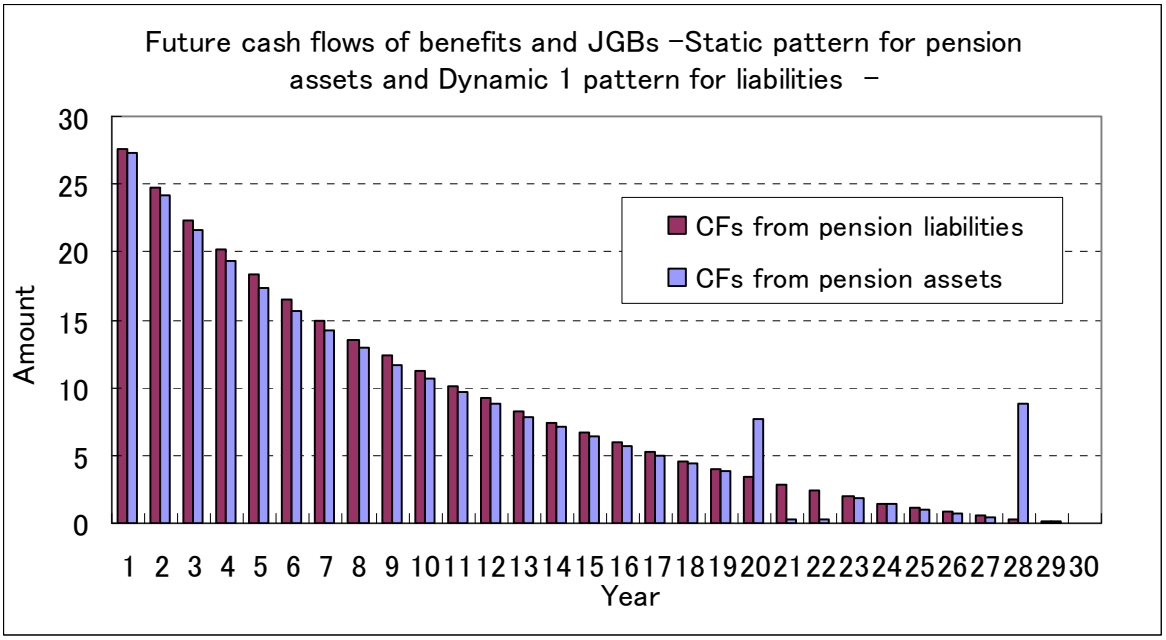
Case 1: The PLB portfolio was designed under initial actuarial assumptions (the static pattern) and then the actuarial assumptions (= rate of salary increases) are changed to the dynamic 1 pattern. Future benefit cash flows are recalculated under the dynamic 1 pattern.

Case 2: The PLB portfolio was designed under initial actuarial assumptions (the static pattern) and then the actuarial assumptions (=rate of salary increases) are changed to the dynamic 2 pattern. Future benefit cash flows are recalculated under the dynamic 2 pattern.

First, we examine Case 1. Chart 37 shows the future cash flows from pension assets and liabilities, and differences between them. Chart 37 shows a slightly different cash flow pattern from Chart 31 compared with case 1 of turnover rate.

Chart 38 shows the influence of revised rate of salary increases assumptions on surplus. The column on the right shows the rate of surplus change for case 1. For comparison, we prepared the base case that the PLB portfolio is designed under actual actuarial assumptions (dynamic 1 pattern) at time T_0. The base case is shown in the left column. Only kurtosis on this chart changes positively compared to the base case. However, the other key statistics do not seem to differ significantly from those of the base case, the cVaR(95%) in Case 1 is -1.50%, and is at the same level as in the base case which is -1.56%. Shortfall risk does not show any significant change.

Chart 37 Future cash flows of benefits and JGBs (Static pattern for pension assets and Dynamic 1 pattern for liabilities)



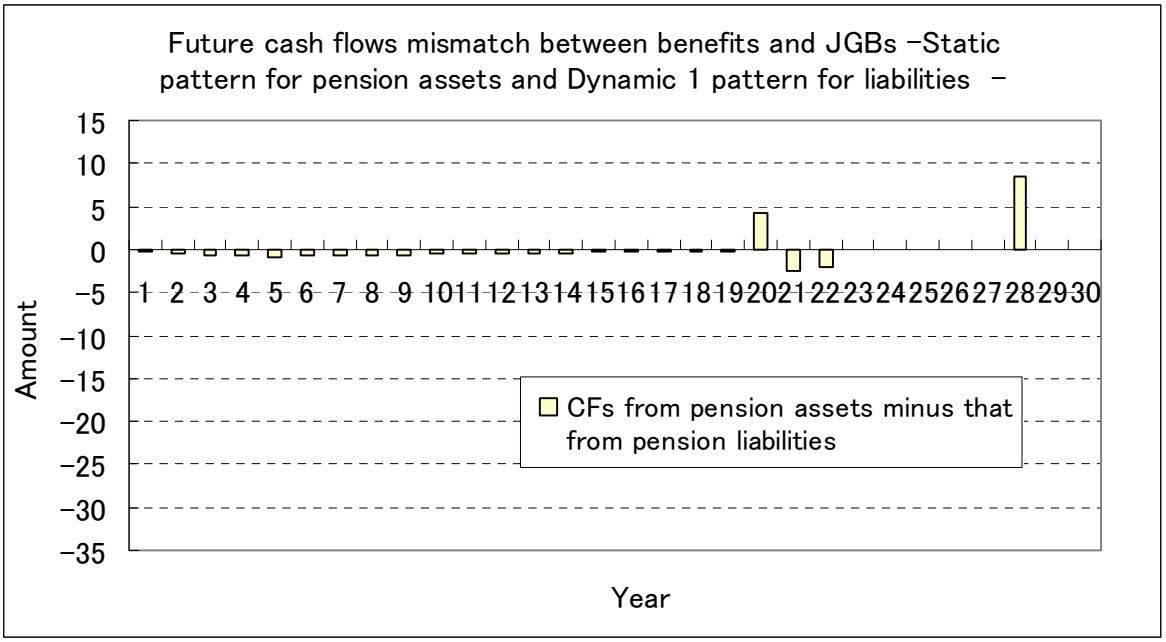
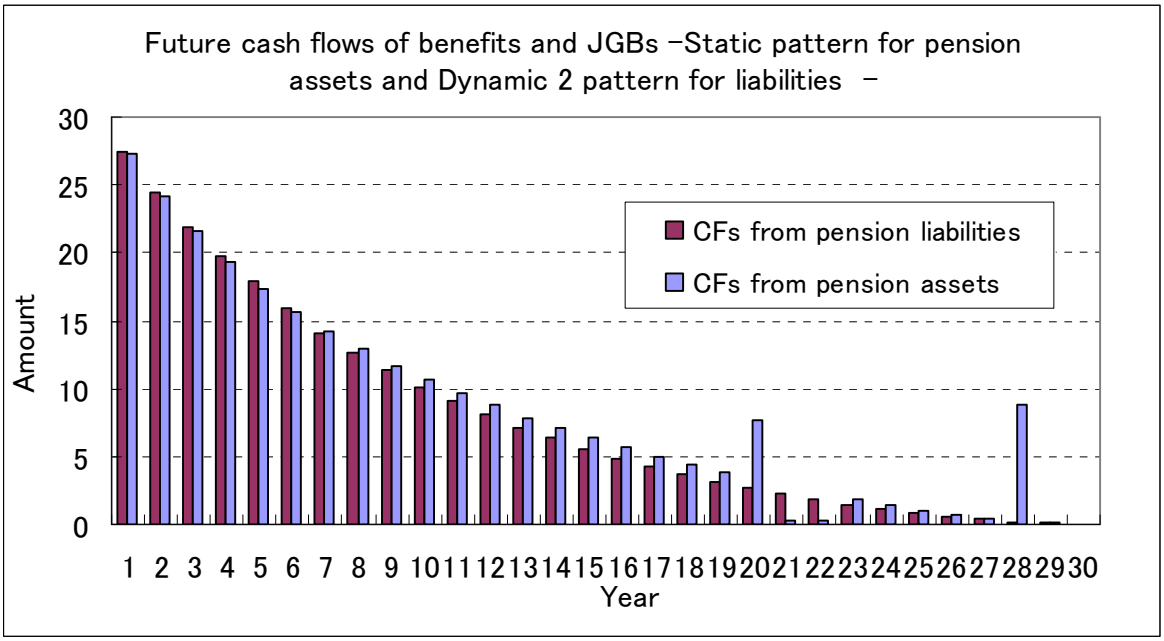


Chart 38 Key statistics of two surpluses, one of which is a static pattern for both assets and liabilities, and the other is a static pattern for assets and dynamic 1 pattern for liabilities

	Rate of change of surplus (dynamic 1 pattern for both)	Rate of change of surplus (static pattern for assets and dynamic 1 pattern for liabilities)
Average	0.57%	0.57%
Standard deviation	0.97%	1.00%
Median	0.58%	0.57%
Mode	2.76%	2.89%
Kurtosis	15.75%	15.35%
Skewness	−9.02%	−7.23%
cVaR（95%）	−1.50%	−1.56%

Similarly, the influence of revised rate of salary increases (dynamic 2 pattern) assumption on surplus are examined here. Chart 39 shows the future cash flows from assets and liabilities, and the mismatch between them. It is not significantly different from Chart 32, and their mismatch is slightly increased from Chart 38 but not significantly. Chart 40 shows the influences on rates of surpluses. The column on the left of the chart shows key statistics when designing a PLB portfolio under dynamic 2 pattern actuarial assumptions initially, and then the column to the right shows key statistics when actuarial assumptions are revised after designing the PLB portfolio. Only skewness is of this chart on the right side seems to differ from that on the left, and the other key statistics do not change significantly. In addition, cVaR(95%) is also slightly decreased from -1.51 to -1.60%, and we also find that shortfall risk does not change significantly.

Chart 39 Future cash flows of benefits and JGBs (Static pattern for pension assets and Dynamic 2 pattern for the liabilities)



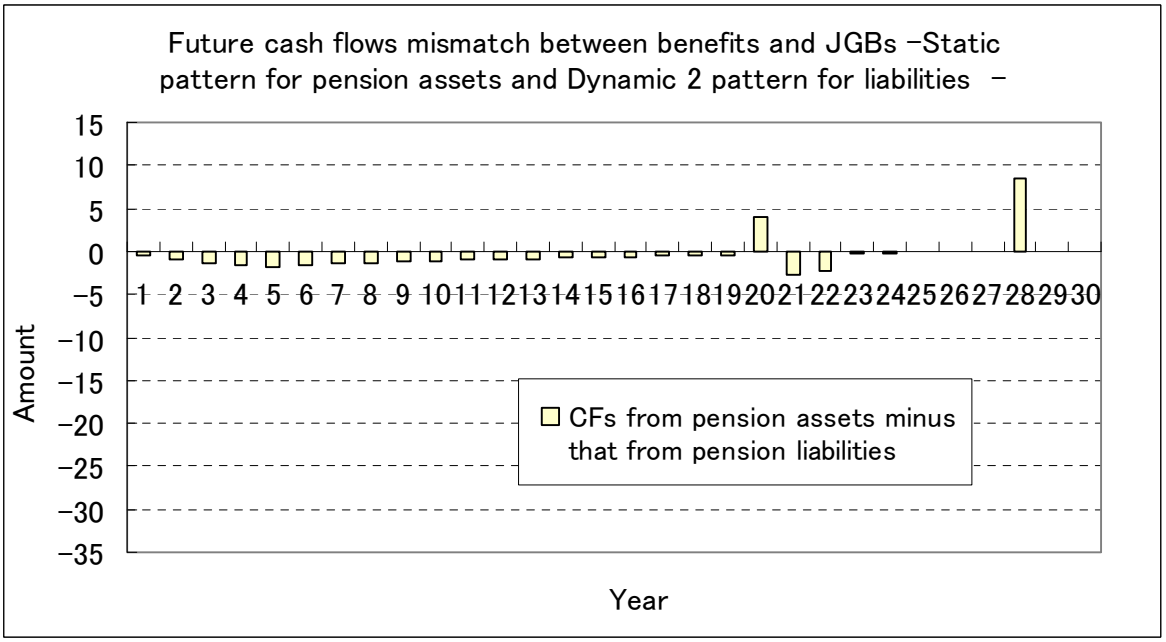
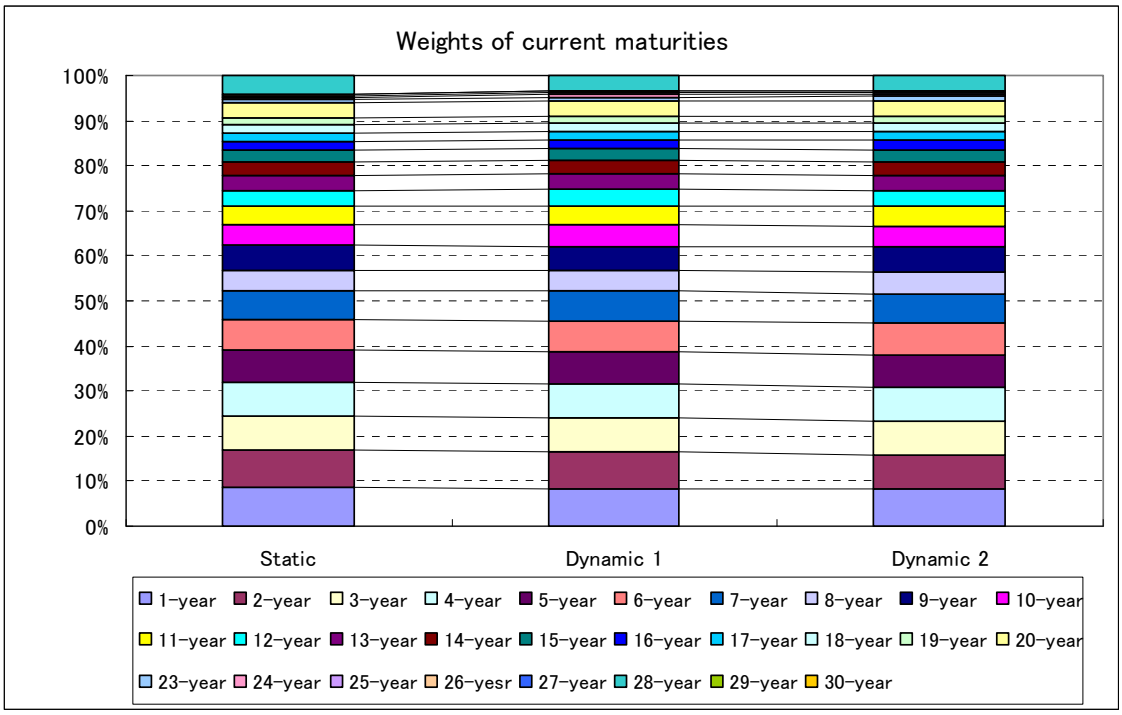


Chart 40 Key statistics of two surpluses, one of which is a static pattern for both assets and liabilities, and the other is a static pattern for assets and dynamic 2 pattern for liabilities

	Rate of change of surplus (dynamic 1 pattern for both)	Rate of change of surplus (static pattern for assets and dynamic 1 pattern for liabilities)
Average	0.57%	0.57%
Standard deviation	0.98%	1.01%
Median	0.58%	0.58%
Mode	2.75%	2.81%
Kurtosis	15.65%	15.49%
Skewness	−9.66%	−10.70%
cVaR（95%）	−1.51%	−1.60%

The weights of maturity dates in each PLB portfolio are examined here, in the same way as was done in the section about turnover rate. We find that there are no significant differences between the weights of maturity dates in each PLB portfolio due to the similar patterns of three future benefit cash flows (Chart 41). The short term bonds weight of the static pattern,whose current maturities are under three years, is under 25%, whereas that of the dynamic 1 pattern is almost 24%, and that of the dynamic 2 pattern is almost 23%. This means that PLB portfolio rebalancing will not be needed frequently, even if the PLB portfolio is redesigned due to changes of characteristics in pension liabilities after designing the initial PLB portfolio. From these points, the rate of salary increases assumption at the time of designing an initial PLB portfolio is a less important factor for implementing LDI than the turnover rate assumption.

Chart 41 Comparison among weights of current maturities in each PLB portfolio



The results of the above analysis show that the turnover rate assumption has a more significant impact on future benefit cash flows than the rate of salary increases assumption. The turnover rate assumption, which influences the rate of change of surplus, is a key factor for implementing LDI.

4 Appendices

We examined each case in section 3-2-1 and 3-2-2 with smoothing anomalies in the Japanese bond market. The following section 4-1 corresponds to 3-2-1, and 4-2 corresponds to 3-2-2. The results in this section are similar to the results in 3-2-1 and 3-2-2, excluding the characteristics of the surplus in Chart 47, Chart 48, and Chart 49, especially “kurtosis” and “skewness”. Despite higher “kurtosis” and lower “skewness”, the conclusions that “turnover rate assumption, which influences the rate of change of surplus, would be a key factor for implementing LDI” remain unchanged as we expected. The main charts of the analysis are following:

4－1 Turnover rate

Chart 42 Key statistics of the static pattern under the smoothing PLB portfolio

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	7.676year	7.676year	
Average	1.69%	－1.39%	0.30%
Standard deviation	5.33%	5.67%	0.87%
Median	1.94%	－1.61%	0.34%
Mode	12.77%	－11.36%	1.41%
Kurtosis	9.55%	－0.55%	27.94%
Skewness	－37.16%	35.88%	－31.50%
cVaR（95%）			－1.66%

Chart 43 Key statistics of the dynamic 1 pattern under the smoothing PLB portfolio

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	6.758year	6.758year	
Average	1.50%	－1.27%	0.23%
Standard deviation	4.46%	4.80%	1.00%
Median	1.69%	－1.40%	0.27%
Mode	11.05%	－9.34%	1.71%
Kurtosis	6.72%	－7.32%	10.88%
Skewness	－33.85%	35.43%	－23.99%
cVaR（95%）			－1.95%

Chart 44 Key statistics of the dynamic 2 pattern under the smoothing PLB portfolio

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	5.401year	5.401year	

Average	1.32%	−1.18%	0.14%
Standard deviation	3.76%	4.07%	1.09%
Median	1.46%	−1.27%	0.17%
Mode	9.60%	−7.68%	1.92%
Kurtosis	4.81%	−12.34%	5.14%
Skewness	−30.55%	36.63%	−21.75%
cVaR（95%）			−2.22%

Chart 45 Key statistics of two surpluses, one of which is a static pattern for both assets and liabilities, and the other is a static pattern for assets and dynamic 1 pattern for liabilities (under the smoothing PLB portfolio)

	Rate of change of surplus （dynamic 1 pattern for both）	Rate of change of surplus （static pattern for assets and dynamic 1 pattern for liabilities）
Average	0.23%	0.42%
Standard deviation	1.00%	1.08%
Median	0.27%	0.41%
Mode	1.71%	3.43%
Kurtosis	10.88%	28.26%
Skewness	−23.99%	3.31%
cVaR（95%）	−1.95%	−2.21%

Chart 46 Key statistics of two surpluses, one of which is a static pattern for both assets and liabilities, and the other is a static pattern for assets and dynamic 2 pattern for liabilities (under the smoothing PLB portfolio)

	Rate of change of surplus （dynamic 2 pattern for both）	Rate of change of surplus （static pattern for assets and dynamic 2 pattern for liabilities）
Average	0.14%	0.51%
Standard deviation	1.09%	1.67%
Median	0.17%	0.54%
Mode	1.92%	5.09%
Kurtosis	5.14%	55.37%

Skewness	−21.75%	−15.34%
cVaR（95%）	−2.22%	−3.28%

4－2 Rate of salary increases

Chart 47 Key statistics of the static pattern under the smoothing PLB portfolio

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	7.676year	7.676year	
Average	1.69%	−1.39%	0.30%
Standard deviation	5.33%	5.67%	0.87%
Median	1.94%	−1.61%	0.34%
Mode	12.77%	−11.36%	1.41%
Kurtosis	9.55%	−0.55%	27.94%
Skewness	−37.16%	35.88%	−31.50%
cVaR（95%）			−1.66%

Chart 48 Key statistics of the dynamic 1 pattern under the smoothing PLB portfolio

	Rate of change in pension assets	Rate of change in pension liabilities	Rate of change in surplus
Duration	7.735year	7.735year	
Average	1.98%	−1.38%	0.60%
Standard deviation	5.38%	5.73%	0.88%
Median	2.23%	−1.61%	0.64%
Mode	13.17%	−11.45%	1.72%
Kurtosis	9.70%	−0.37%	27.62%
Skewness	−37.24%	36.00%	−31.18%
cVaR（95%）			−1.38%

Chart 49 Key statistics of the dynamic 2 pattern under the smoothing PLB portfolio

	Rate of change in	Rate of change in	Rate of change in

	pension assets	pension liabilities	surplus
Duration	7.793year	7.793year	
Average	1.96%	−1.38%	0.57%
Standard deviation	5.43%	5.78%	0.88%
Median	2.21%	−1.61%	0.61%
Mode	13.25%	−11.53%	1.72%
Kurtosis	9.84%	−0.20%	27.31%
Skewness	−37.33%	36.11%	−30.86%
cVaR（95%）			−1.42%

Chart 50 Key statistics of two surpluses, one of which is a static pattern for both assets and liabilities, and the other is a static pattern for assets and dynamic 1 pattern for liabilities (under the smoothing PLB portfolio)

	Rate of change of surplus （dynamic 1 pattern for both）	Rate of change of surplus （static pattern for assets and dynamic 1 pattern for liabilities）
Average	0.60%	0.30%
Standard deviation	0.88%	0.89%
Median	0.64%	0.35%
Mode	1.72%	1.32%
Kurtosis	27.62%	27.38%
Skewness	−31.18%	−31.90%
cVaR（95%）	−1.38%	−1.72%

Chart 51 Key statistics of two surpluses, one of which is a static pattern for both assets and liabilities, and the other is a static pattern for assets and dynamic 2 pattern for liabilities (under smoothing PLB portfolio)

	Rate of change of surplus （dynamic 1 pattern for both）	Rate of change of surplus （static pattern for assets and dynamic 1 pattern for liabilities）
Average	0.57%	0.31%
Standard deviation	0.88%	0.92%
Median	0.61%	0.36%

Mode	1.72%	1.24%
Kurtosis	27.31%	26.39%
Skewness	−30.86%	−31.60%
cVaR（95%）	−1.42%	−1.78%

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