

# Modeling the Underwriting Cycle

By René Schnieper

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# Underwriting Cycles Are a Reality

- US liability crisis mid 1980s.
- Property catastrophe reinsurance crisis early 1990s.
- Significant hardening of the reinsurance market early 2000s.
- ...

# Causes of Underwriting Cycles

- Irrational expectations
- Institutional interventions
- Feedback of profit and losses into surplus
- External factors
- Underwriting profits follow an AR(2) process

# Model Assumptions

- Insurance markets are driven by supply and demand
- Supply is driven by the amount of capital
- Demand is driven by the size of the economy
- Profits are defined by the equilibrium of supply and demand
- Profits feedback into capital

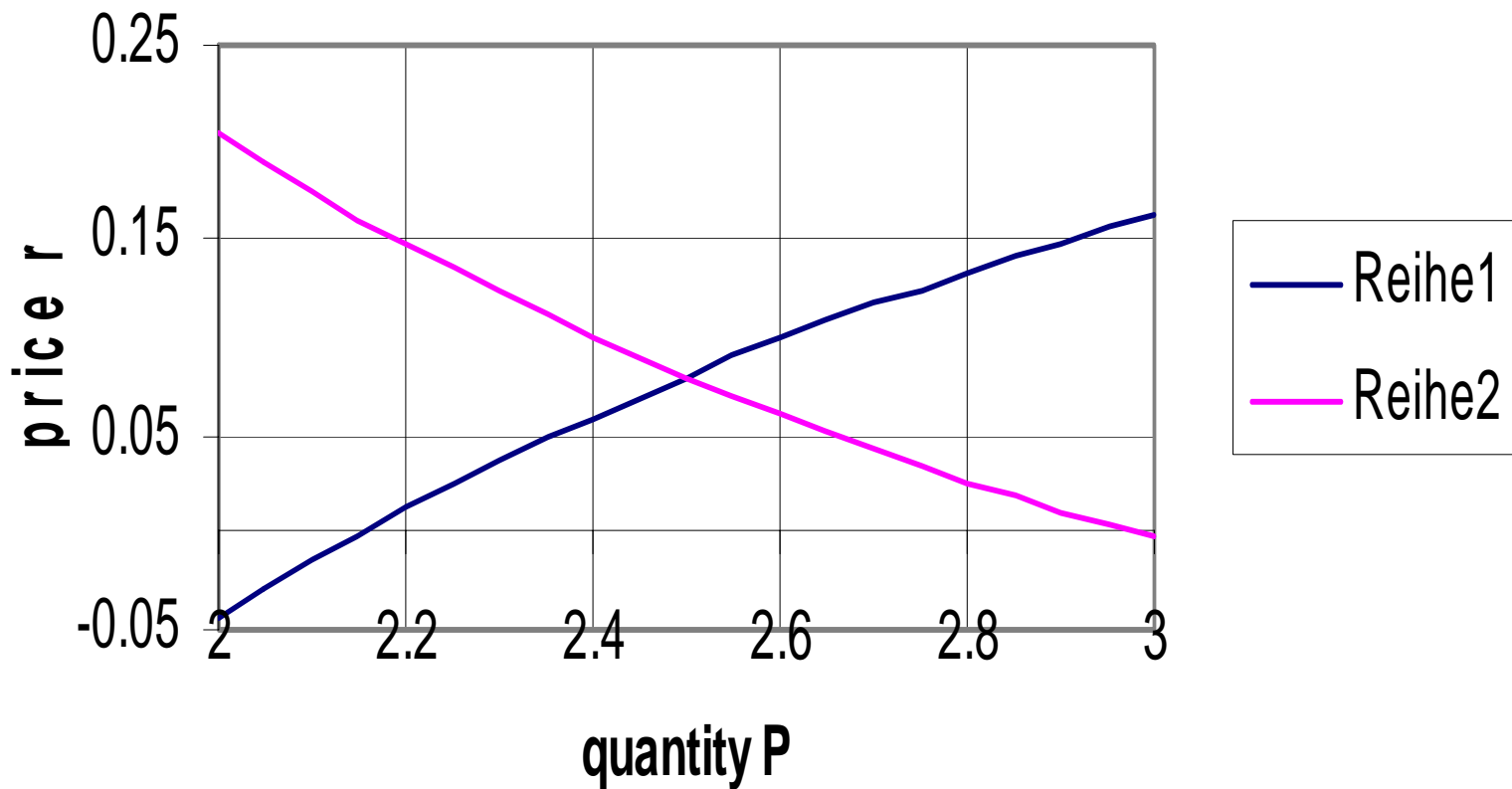
# Model

- Insurance quantity = pure risk premium (P)
- RBC = P/leverage
- Insurance price = rate of return on RBC (r)
- Supply function
- Demand function

$$P - \kappa^s C = \varepsilon^s \left( r \frac{P}{\kappa^s} - r^s \frac{P}{\kappa^s} \right)$$

$$P - P^0 = -\varepsilon^d \left( r \frac{P}{\kappa^s} - r^d \frac{P}{\kappa^d} \right)$$

# Market Equilibrium



# Market Equilibrium

$$r^m = \frac{(\varepsilon^d C)r^{d'} + (\varepsilon^s \frac{P^0}{K^s})r^s}{\varepsilon^d C + \varepsilon^s \frac{P^0}{K^s}} + \frac{P^0 - K^s C}{\varepsilon^d C + \varepsilon^s \frac{P^0}{K^s}}$$

$$P^m = \frac{\varepsilon^s P^0 + \varepsilon^d (K^s C)}{\varepsilon^s + \varepsilon^d + \varepsilon^s \varepsilon^d \left( \frac{r^s}{K^s} - \frac{r^d}{K^d} \right)}$$

$$\lambda^m = r^m \frac{P^m}{K^s} = \frac{(\varepsilon^d \frac{r^d}{K^d} - 1)K^s C + (\varepsilon^s \frac{r^s}{K^s} + 1)P^0}{\varepsilon^s + \varepsilon^d + \varepsilon^s \varepsilon^d \left( \frac{r^s}{K^s} - \frac{r^d}{K^d} \right)}$$

# Feedback of Profits into Capital

$$C_{t+1} = C_t + \rho(\lambda_t^m + r^f C_t)$$

$$C_{t+1} = C_t + \rho(\lambda_t^m + \tilde{\varepsilon}_t + r^f C_t) + \tilde{\mathcal{G}}_{t+1}$$

$$C_{t+1} = C_t + \rho(\alpha\lambda_t^m + (1-\alpha)\lambda_{t-1}^m + \tilde{\varepsilon}_{t,t} + \tilde{\varepsilon}_{t-1,t} + r^f C_t) + \tilde{\mathcal{G}}_{t+1}$$

$$P_t^0 = \gamma^t P^0$$

*Def :*  $\bar{C}_t = \frac{C_t}{\gamma^t}$  *st'zed process*

*Theorem:*  $\{\bar{C}_t - \mu\}$  *is AR(2) process*

# Simplified Model

*Assume:*  $Var(\tilde{\varepsilon}_{t,s}) = Var(\tilde{\mathcal{G}}_u) = 0$

$$\bar{C}_{t+1} - \frac{1}{\gamma} (1 + \rho r^f + \rho a \alpha) \bar{C}_t - \frac{1}{\gamma^2} \rho a (1 - \alpha) \bar{C}_{t-1} = \left( \frac{\alpha}{\gamma} + \frac{(1 - \alpha)}{\gamma^2} \right) \rho b P^0$$

$$a = \frac{\varepsilon^d r^d - K^s}{\varepsilon^s + \varepsilon^d + \varepsilon^s \varepsilon^d \left( \frac{r^s}{K^s} - \frac{r^d}{K^d} \right)} \quad b = \frac{1 + \varepsilon^s \frac{r^s}{K^s}}{\varepsilon^s + \varepsilon^d + \varepsilon^s \varepsilon^d \left( \frac{r^s}{K^s} - \frac{r^d}{K^d} \right)}$$

*Linear difference equation with cst coefficients*

# Simplified Model (cont'd)

*equilibrium pt of difference equation*

$$\bar{C}^* = \frac{\left(\frac{\alpha}{\gamma} + \frac{(1-\alpha)}{\gamma^2}\right)\rho b}{1 - \frac{1}{\gamma}(1 + \rho r^f + \rho a\alpha) - \frac{1}{\gamma^2}\rho a(1-\alpha)} P^0$$

*characteristic equation*

$$\lambda^2 - \frac{1}{\gamma}(1 + \rho r^f + \rho a\alpha)\lambda - \frac{1}{\gamma^2}\rho a(1-\alpha) = 0$$

*stability*  $\Leftrightarrow |\lambda_i| < 1$  for all char. roots

*instability: reporting lags & capital in/outflows*

# Simplified Model (cont'd)

*market price corresp. to equilibrium pt*

$$r^* = \frac{\frac{1}{r^s} + \frac{\varepsilon^s}{\kappa^s}}{\frac{\alpha}{\gamma} + \frac{(1-\alpha)}{\gamma^2} + \frac{\rho}{(1 + \frac{1}{\gamma} - \frac{\rho}{\gamma} r^f)} + \frac{\varepsilon^s}{\kappa^s}} r^s$$

$$r^* \succ r^s \iff \gamma - 1 \succ \rho \left( r^f + r^s \left( \alpha + \frac{1-\alpha}{\gamma} \right) \right)$$

# Summary of Findings

Underwriting cycles are caused by:

- Supply side factors (retained earnings, capital flows)
- Demand side factors (fluctuations in base demand growth)
- External factors (changes in the value of financial assets, adverse loss developments)
- Interaction of the above factors