Asset allocation : new constraints induced by the Solvency II project

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Frédéric PLANCHET **Pierre THÉROND**

ISFA – Université Lyon 1

Winter & Associés

Context

The aim of this paper is to show how the future solvency system (Solvency II) will affect the risk management of insurance companies and especially the determination of the strategic asset allocation.

These changes will be illustrated with a very simple non-life insurance company which has to compose its financial portfolio among a risky and a riskfree asset.

We choose an asset allocation criterion, use it in the present (Solvency I) framework and then in a Solvency II framework. Thus we observe that Solvency II induces new constraints ; in particular the capital requirement becomes dependent on the asset allocation.

Solvency II

The “Solvency II” project of the European Commission should lead to an harmonized solvency framework for all european insurers.

It leads to introduce explicit quantitative prudence in the balance of the insurers :

- The level of technical provision will be determined with a risk measure (Value-at-Risk or coefficient of variation).
- The equities should be greater than the Solvency Capital Requirement (SCR : amount which controls a ruin probability of the company).

Modelling of the insurance company (1/3)

Let us work with a non-life insurance company with 2 dependent insurance risks S_1 and S_2 :

$$S_1 \sim \mathbf{LN}(\mu_1, \sigma_1) \qquad S_2 \sim \mathbf{LN}(\mu_2, \sigma_2)$$

$$\mathbf{Pr}\left[S_1 \leq s_1, S_2 \leq s_2\right] = C_\alpha\left(F_1(s_1), F_2(s_2)\right)$$

$$C_\alpha(u_1, u_2) = -\frac{1}{\alpha}\mathbf{ln}\left\{1 + \frac{\left(e^{-\alpha u_1} - 1\right)\left(e^{-\alpha u_2} - 1\right)}{e^{-\alpha} - 1}\right\}$$

We will suppose that the exact amount of claims is known and paid at time 1 (no IBNR). For the numerical illustrations we used the parameters :

$$\mu_1 = 5{,}0099 \quad \sigma_1 = 0{,}0377 \quad \alpha = 1$$

$$\mu_2 = 3{,}8421 \quad \sigma_2 = 0{,}3740$$

Modelling of the insurance company (2/3)

Let us suppose that the insurer has to compose its financial portfolio among a risky asset A_1 and a riskless asset A_2.

$$A_1(t) = \mathbf{exp}\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t + \sum_{k=1}^{N_t} U_k\right\}$$

Where $B = (B_t)_{(t \geq 0)}$ is a brownian motion, $N = (N_t)_{(t \geq 0)}$ a Poisson process with intensity λ and $U = (U_k)_{(k \geq 1)}$ a sequence of i.i.d. normally distributed r.v. with mean of 0 and standard-deviation of σ_U (*cf.* Merton [1976] and Ramezani [1998]).

$$A_2(t) = e^{rt}$$

Where the the riskless rate *r* is assumed to be constant over the period.

For the numerical illustrations we used the parameters :

$$\mu = 0{,}6 \qquad \sigma = 0{,}15 \qquad r = 0{,}344$$

$$\lambda = 0{,}5 \qquad \sigma_u = 0{,}2$$

Modelling of the insurance company (3/3)

Let us denote :

- L_0 the amount of technical provisions at time 0,
- E_0 the amount of equities at time 0.

We will suppose that E_0 is equal to the minimum of regulatory capital (SCR in the Solvency 2 framework and solvency margin in the present framework).

The asset allocation will be determined by ω_1 the proportion of risky asset in the initial portfolio.

Asset allocation criterion

Let us denote $$\Lambda_0^{\omega} = \frac{S_1 + S_2}{\omega_1 A_1 + (1-\omega_1) A_2}$$

The r.v. Λ_0^{ω} is the (random) amount of claims to pay at time 1 discounted at the (random) rate of return of the asset of the company.

The criterion that we will use thereafter consists in choosing the allocation ω_1 which maximizes the quantity :

$$\varphi(\omega_1) = \frac{\mathbf{E}\left\lfloor L_0 + E_0 - \Lambda_0^{\omega} \right\rfloor}{E_0}$$

The r.v. $L_0 + E_0 - \Lambda_0^{\omega}$ can be seen as the wealth of the insurer at time 0 if its asset allocation is ω_1.

This criterion has been examined in Planchet & Thérond [2004].

Asset allocation in the present solvency framework (1/4)

Under the present « Solvency I Directives », for non-life insurers :

- The provisions for claims outstanding are estimated without reference to an explicit level of prudence : the prudence comes from the valuation method and from the absence of discounting.
- The equities of the insurer must be higher than the solvency margin which is calculated by references to the reinsurance ratio, to the level of technical provisions and to the premiums. There is no reference to the asset allocation

Asset allocation in the present solvency framework (2/4)

Balance sheet at time 0

Technical provisions : $L_0 = \mathbf{E}[S_1] + \mathbf{E}[S_2]$

Equities : $E_0^R = 18\% * (1+\gamma)\mathbf{E}(S_1 + S_2)$

where γ is the loading rate of the premiums (15 % in the example).

Balance Sheet at time 0 (present solvency framework)	
$E_0 + L_0 = 241{,}4$	$E_0 = 41{,}4$
	$L_0^1 = 150$ $L_0^2 = 50$

Asset allocation in the present solvency framework (3/4)

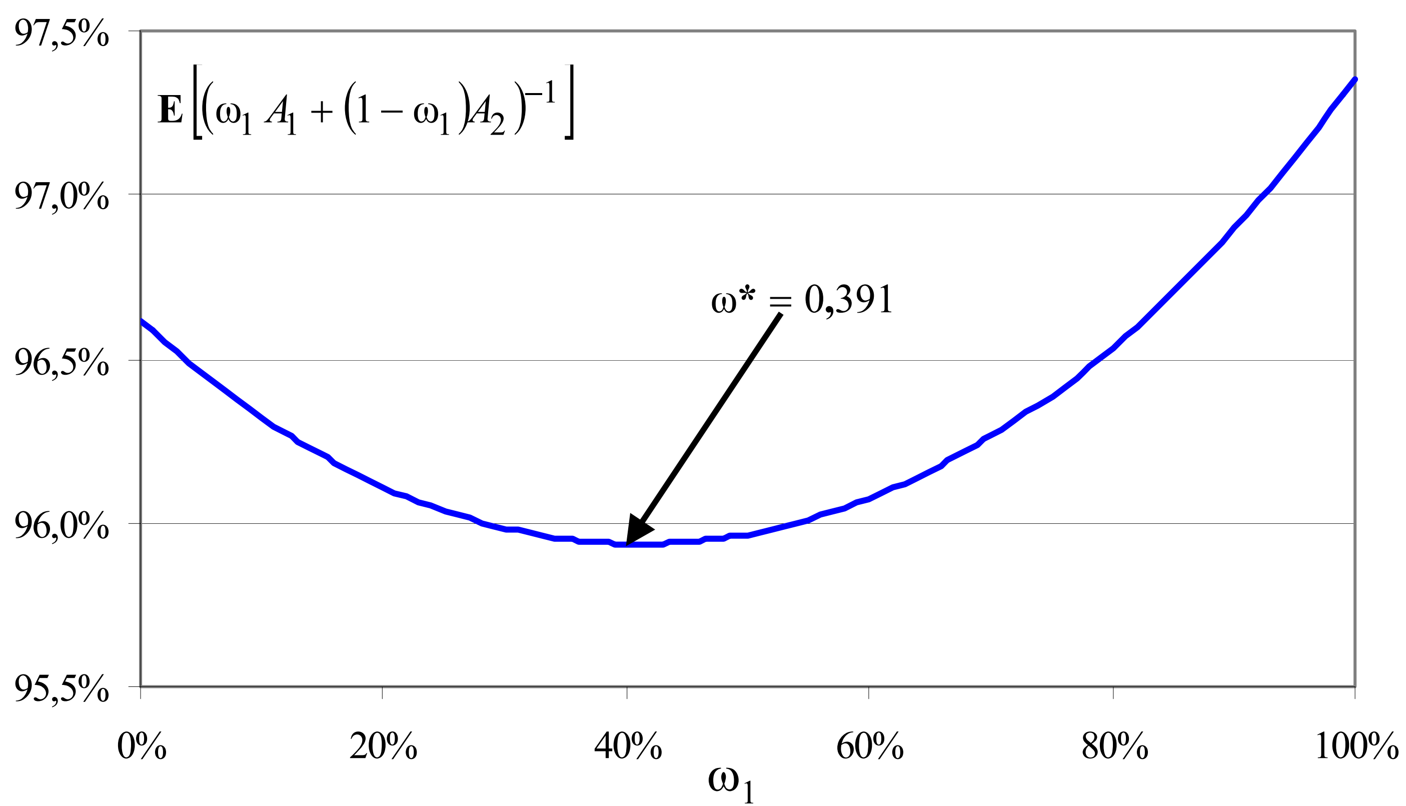
Within the present solvency framework, the asset allocation optimization problem becomes :

$$\inf_{\omega\in[0,1]} \mathbf{E}\left[\left(\omega_1 A_1 + (1-\omega_1) A_2\right)^{-1}\right]$$

On can show that this optimization problem has a non-trivial solution ω* iff :

$$r < \mu < r + 2\sigma^2 + \lambda\left[\mathbf{exp}\left(2\sigma_u^2\right) - \mathbf{exp}\left(\sigma_u^2/2\right)\right]$$

Asset allocation in the present solvency framework (4/4)



Asset allocation in a Solvency II framework (1/5)

Let us suppose that :

- The level of technical provisions is a Value-at-Risk at 75 % of the distribution of the claims outstanding discounted at the riskfree rate *r*.

  The technical provisions are calculated per book of contracts.

- The Solvency Capital Requirement (SCR) is the amount of equities which controls the ruin probability at a time horizon of 1 year with a probability of 99,5 %.

  The probability of ruin is calculated on the global risk of the company. This global risk includes particularly the risks related to the assets on the one hand and the dependence between the insurance risks on the other hand.

Asset allocation in a Solvency II framework (2/5)

Balance sheet at time 0

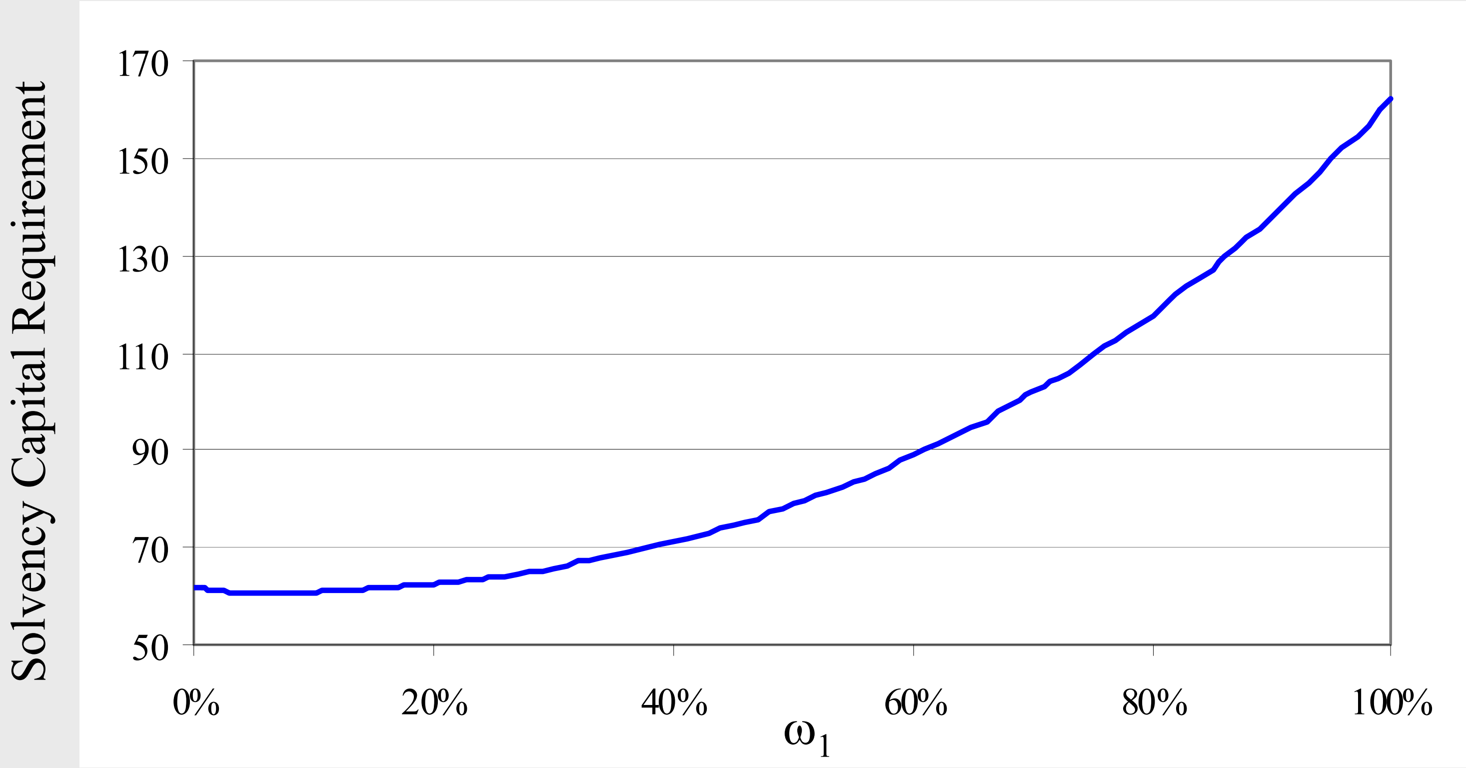
Technical provisions : $L_0 = L_0^1 + L_0^2 = \mathbf{VaR}(S_1, 75\%)e^{-r} + \mathbf{VaR}(S_2, 75\%)e^{-r}$

Equities : $\mathbf{Pr}\left[\left(L_0 + E_0^R\right)\left(\omega_1 A_1 + (1-\omega_1) A_2\right) \geq S_1 + S_2\right] \geq 99{,}5\%$

➔ The SCR depends on the asset allocation ω_1.

Liability at time 0 (solvency 2 framework)
$E_0 = f(\omega_1)$
$L_0 = 206{,}52$

Asset allocation in a Solvency II framework (3/5)



Asset allocation in a Solvency II framework (4/5)

Under the Solvency II framework, the asset allocation optimization problem becomes :

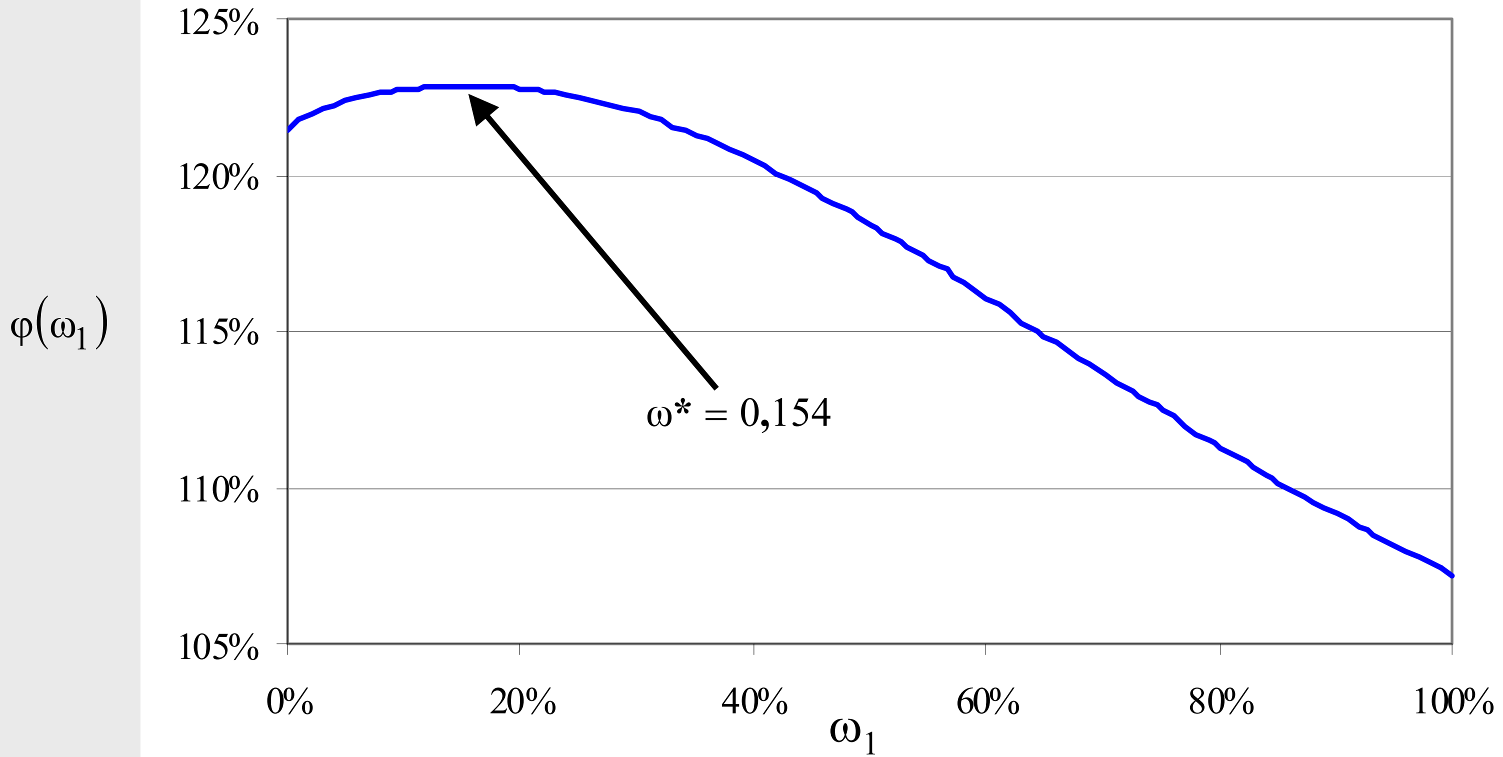
$$\sup_{\omega_1 \in [0,1]} \{\varphi(\omega_1)\}$$

Where :

$$\varphi(\omega_1) = \frac{\sum_{i=1}^{2} \left(\mathbf{exp}\left\{\mu_i - r + \sigma_i \, \boldsymbol{\Phi}^{-1}(0{,}75)\right\} - \mathbf{exp}\left\{\mu_i + \sigma_i^2/2\right\} \mathbf{E}\left[\left(\omega_1 \, A_1 + (1-\omega_1)\, e^r\right)^{-1} \right] \right)}{E_0^R(\omega_1)}$$

We solve this problem using Monte Carlo simulations.

Asset allocation in a Solvency II framework (5/5)



Conclusion (1/2)

Within the present solvency framework, the capital requirement (solvency margin) does not depend on the asset allocation.

➔ The Asset-Liability Manager has to determine an asset allocation given the level of capital requirement.

Within the Solvency II framework, Capital requirement (SCR) and asset allocation are dependent.

➔ The Asset-Liability Manager has to determine a couple « asset allocation / capital requirement » under the link constraint.

Conclusion (2/2)

The asset allocation criterion studied depends explicitly on the level of the capital

➔ Obtaining the optimal couple « asset allocation / capital requirement » requires the resolution of a single optimization problem.

The resolution of this problem will generally be carried out using Monte Carlo methods.

Contacts

Frédéric PLANCHET

fplanchet@jwa.fr

Pierre THEROND

ptherond@jwa.fr

Winter & Associés

http://www.jwa.fr

9, rue Beaujon
F-75008 Paris

18, avenue Félix Faure
F-69007 Lyon

ISFA

http://www.isfa.info

50, avenue Tony Garnier
F-69366 Lyon Cedex 07