

# Simplified estimation of structure parameters in hierarchical credibility

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# Agenda

- Hierarchical credibility
- Pseudo-estimators of structure parameters
- New unbiased estimators
- Numerical example
- Conclusions

# Hierarchical credibility structure

Sector level  $j$



Cell level  $k$



Exposure unit  $t$

# Application

Sector: Car brand (E.g.  $j = \text{Aston Martin}$ )



Cell: Car model (E.g.  $k = \text{Aston Martin DB5}$ )



Unit: Individual car (E.g.  $t = \text{Switzerland LU 6789}$ )

# Structure parameters (Variance components)

$Y_{jkt}$  observed risk premium, claim frequency or severity

$w_{jkt}$  exposure weight       $\text{Var}(Y_{jkt}) = b + a + \frac{\sigma^2}{w_{jkt}}$

Sector:  $b =$  sector level variance



Cell:  $a =$  cell level variance



Unit:  $\sigma^2 =$  unit level variance

## Credibility factors

- Cell level:  $z_{jk} \doteq \frac{w_{jk\cdot}}{w_{jk\cdot} + \sigma^2/a}$
- Sector level:  $q_j \doteq \frac{z_{j\cdot}}{z_{j\cdot} + a/b}$
- Used in credibility estimators (details omitted here)

The problem:

Estimate the structure parameters  $a$ ,  $b$  and  $\sigma^2$

Parameter	Standard	New
$\sigma^2$	Unbiased	= Unbiased
$a$	Pseudo	Unbiased
$b$	Pseudo	Unbiased

Usual unbiased estimator of  $\sigma^2$

$$\hat{\sigma}^2 = \frac{1}{\sum_j \sum_k (T_{jk} - 1)} \sum_{j=1}^J \sum_{k=1}^{K_j} \sum_{t=1}^{T_{jk}} w_{jkt} (Y_{jkt} - \bar{Y}_{jk.}^w)^2$$

where

$$\bar{Y}_{jk.}^w = \frac{\sum_t w_{jkt} Y_{jkt}}{\sum_t w_{jkt}}$$

## Pseudo-estimator of $a$

$$a = E \left[ \frac{1}{\sum_j (K_j - 1)} \sum_{j=1}^J \sum_{k=1}^{K_j} z_{jk} (\bar{Y}_{jk.}^w - \bar{Y}_{j..}^{zw})^2 \right]$$

where

$$z_{jk} \doteq \frac{w_{jk.}}{w_{jk.} + \sigma^2/a}$$

$$\bar{Y}_{j..}^{zw} = \frac{\sum_k z_{jk} \bar{Y}_{jk.}^w}{\sum_k z_{jk}}$$

Omit  $E[\cdot]$  and solve iteratively for  $a$ .

## Unbiased estimator of $a$

$$\hat{a} = \frac{\sum_j \sum_k w_{jk.} (\bar{Y}_{jk.}^w - \bar{Y}_{j..}^{ww})^2 - \hat{\sigma}^2 \sum_j (K_j - 1)}{w_{...} - \sum_j (\sum_k w_{jk.}^2) / w_{j..}}$$

where

$$\bar{Y}_{j..}^{ww} = \frac{\sum_k w_{jk.} \bar{Y}_{jk.}^w}{\sum_k w_{jk.}}$$

Cf. Bühlmann-Straub estimator (non-hierarchical)

## Pseudo-estimator of $b$

$$b = E \left[ \frac{1}{J-1} \sum_{j=1}^J q_j (\bar{Y}_{j..}^{zw} - \bar{Y}_{...}^{qzw})^2 \right]$$

where

$$q_j \doteq \frac{z_{j.}}{z_{j.} + a/b}$$

$$\bar{Y}_{...}^{qzw} = \frac{\sum_j q_j \bar{Y}_{j..}^{zw}}{\sum_j q_j}$$

Omit  $E[\cdot]$  and solve iteratively for  $b$ .

## Unbiased type estimator of $b$

$$\hat{b} = \frac{\sum_j z_j \cdot (\bar{Y}_{j..}^{zw} - \bar{Y}_{...}^{zzw})^2 - \hat{a}(J-1)}{z_{..} - \sum_j z_j^2 / z_{..}}$$

where

$$\bar{Y}_{...}^{zzw} = \frac{\sum_j z_j \cdot \bar{Y}_{j..}^{zw}}{\sum_j z_j}$$

$z_{j.}$  assumed known here (depend on  $a$  and  $\sigma^2$ )

# Comparison

Dubey & Gisler (1981) found in the non-hierarchical Bühlmann-Straub case

- “neither of the two estimators is universally better than the other” (in terms of variance)
- *Pseudo-estimator* has one and only one positive solution iff *unbiased estimator* is strictly positive
- Else *pseudo-estimators* converges to zero

## Numerical example

Artificial population from Dannenburg, Kaas & Goovaerts (1996),  
Table 4.1

(Equal weight case in parenthesis.)

	<i>True value</i>	<i>Pseudo-est.</i>	<i>Unbiased est.</i>
a	1.000	1.152 (1.093)	1.209 (1.093)
b	25.000	25.309 (25.259)	25.300 (25.259)

## Conclusion – Pseudo- vs. New Unbiased

- Both are reasonable
- Presumably none is unilaterally best in MSE
- Unbiased is easier to compute
- Pseudo-estimator is conceptually more difficult

## References

Dannenburg, D.R., Kaas, R. & Goovaerts, M.J. (1996): *Practical actuarial credibility models*. IAE, Amsterdam.

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