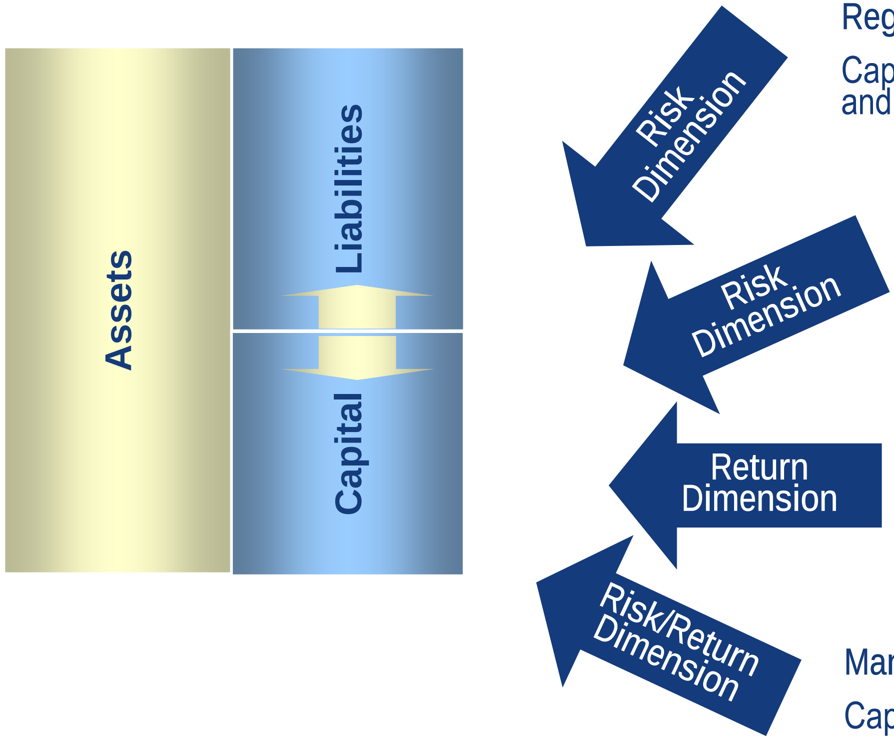
Sharing Risk – An Economic Perspective

36th ASTIN Colloquium, Zurich, 5.9.2005

Andreas Kull, Global Financial Services Risk Management

q

Capital: Shared and competing issue



Regulators
Capital is key for protection of policyholder and for overall financial stability

Policyholders
Capital acts as risk mitigation

Shareholders
Expect a return from invested capital

Management
Capital is the key resource for doing business

Risk transfer strategies should balance risk and return dimension by taking into account

1. Capital constraints (regulators, policyholders)
2. Economic bottom line, i.e. impact of risk transfer on overall return (shareholders, management)

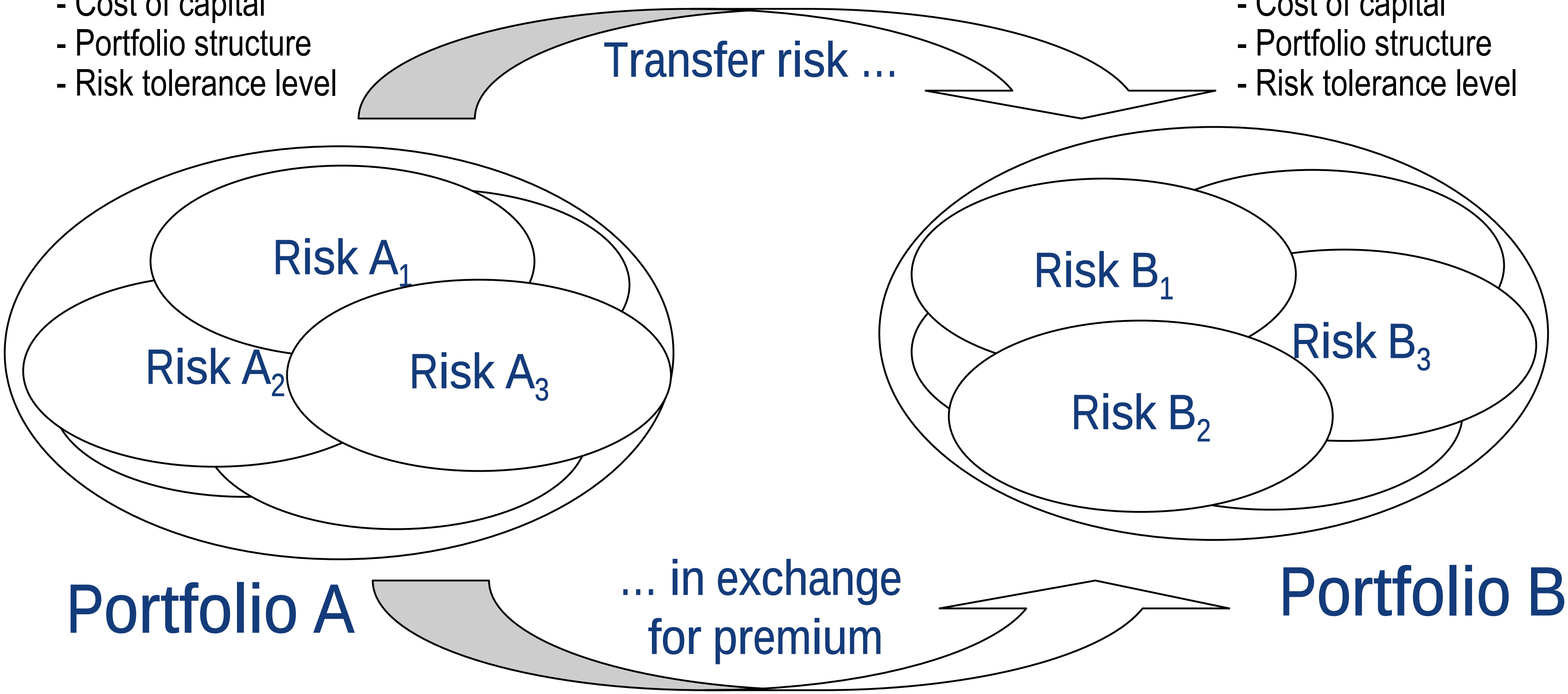
Sharing and Transferring of Risk

- Risk transfer should balance risk/return dimensions by taking into account
  - Capital constraints (regulators, policyholders)
  - Economic bottom line, ie. impact of risk transfer on overall return (shareholders, management)
- A sound model of risk transfer and risk sharing should
  - Allow for relevant constraints (e.g. available capital, risk tolerance)
  - Focus on economic value
  - Include effects of portfolio diversification and dependency issues
  - Allocate risk based capital consistently
- What if risk exchanging parties employ similar principles?
  - Consistently model economics of risk transfer (premium principle)
  - Take into account portfolio structure of both risk ceding and receiving party
- Key issue: Sharing risk ‘optimally’ in this context

What risks should be shared/transferred?

Key parameter Party A
- Cost of capital
- Portfolio structure
- Risk tolerance level

Key parameter Party B
- Cost of capital
- Portfolio structure
- Risk tolerance level



Objective Party A
- Transfer risk
- Meet capital constraint
- Maximize overall return

What risk for which premium?

Objective Party B
- Take over risk in exchange for premium
- Attain a profitability hurdle

A Simple Model (I)

Net underwriting profit (risk class *i*)	$U_i = P_i - (L_i - R_i) - P_i^*$	*P:* Premium, *L:* Loss, *P*:* Risk transfer premium, *R:* Loss covered by risk transfer.
Economic result (risk class *i*)	$Z_i = U_i - \lambda \cdot \mathrm{RBC}[Z_i]$	RBC: Risk based capital, λ: cost of capital
Risk based capital (risk class *i*)	$\mathrm{RBC}[Z_i] = -\mathrm{E}[Z_i \mid Z \le F_Z^{-1}(\alpha)]$	α: Risk tolerance level, RBC: Risk based capital (defined as conditional expected shortfall relative to portfolio result)
Premium for risk transfer (risk class *i*) quantities with * refer to risk receiving party	$P_i^* = (1-\lambda^*) \cdot \mathrm{E}[R_i] + \mathrm{E}[R_i \mid R_i \le F_{Z^*}^{-1}(\alpha^*)]$	*P*:* Premium for risk transfer, *R:* loss covered by risk transfer, λ*: Cost of capital, α*: Risk tolerance level
Portfolio distribution, Total RBC	$Z = \sum Z_i, \qquad \mathrm{RBC}[Z] = \sum \mathrm{RBC}[Z_i]$	Note additivity of RBC
Objective and constraint	$\mathrm{E}[Z(r)] = \max!, \quad \mathrm{RBC}[Z(r)] = \mathrm{const}$	*r*: Risk transfer control parameter(s), eg XL retentions

A Simple Model (II)

- Typical constraint extreme value problem
- Solved using Lagrangian multipliers κ, i.e. by maximizing expression

$$\varphi(\vec{r}) = \mathrm{E}[Z(\vec{r})] + \kappa \cdot \mathrm{RBC}[Z(\vec{r})]$$

- This leads to condition

$$\nabla_{\vec{r}}[(\lambda^* - \lambda + \kappa \cdot (1-\lambda^*)) \cdot \mathrm{E}[R] -$$

$$\lambda^* \cdot (1-\kappa) \cdot \mathrm{E}[R \mid R \le F_{Z^*}^{-1}(\alpha^*)] +$$

$$(\lambda - \kappa) \cdot \mathrm{E}[R \mid R \le F_{Z}^{-1}(\alpha)]] \overset{!}{=} 0$$

Risk receiving portfolio

Risk ceding portfolio

- Other conditions include concavity of E[*Z(r)*] and convexity of κ·E[*Z(r)*].

A Simple Model (IV)

- Solution $\vec{r} = \vec{r}_0$
  - is a vector of optimal risk transfer control parameters (e.g. XL retentions)
  - characterizes optimal risk transfer per risk class *i*
  - maximizes overall economic result on portfolio level
  - for a given risk based capital RBC_0.

- Main parameters
  - Structure of risk transferring and receiving portfolios (Z and Z^*)
  - Risk tolerance levels (α and α^*)
  - Capital costs (λ and λ^*)
  - available risk based capital (RBC_0, related to Lagrangian parameter κ)

- Numerical solution necessary

Example

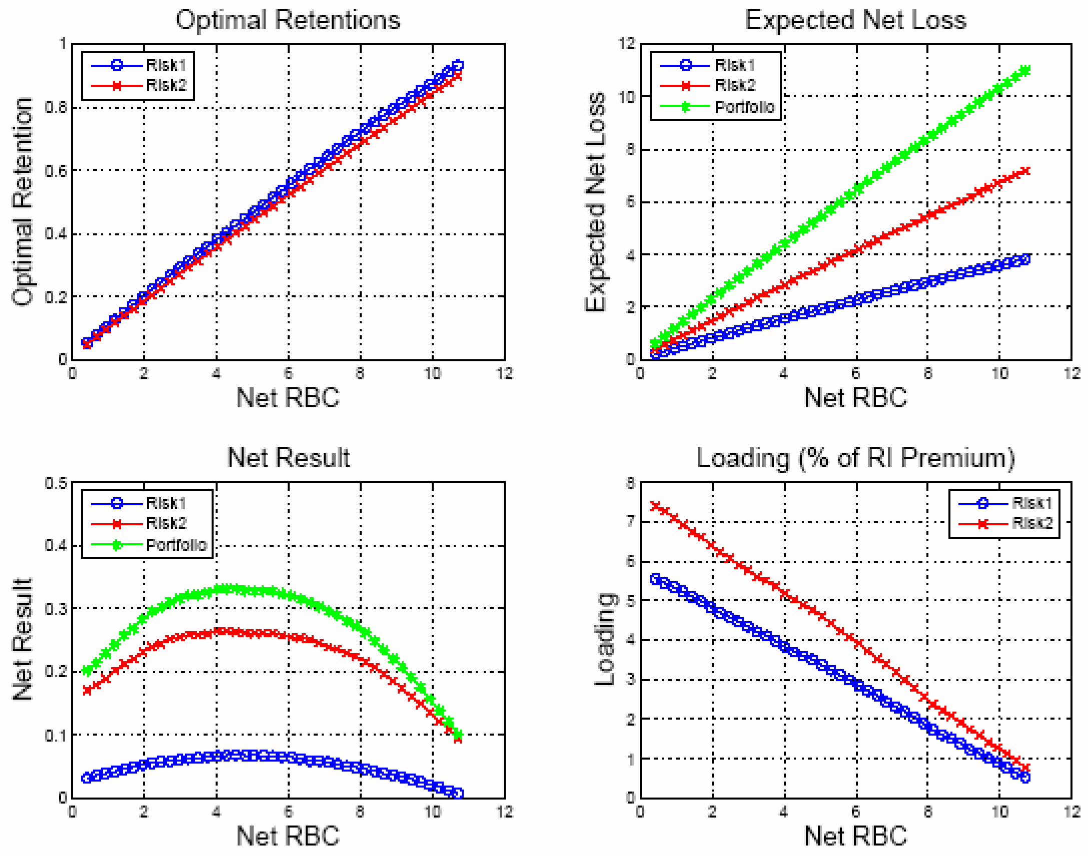
- Illustration for simple proportional and non-proportional reinsurance structures
- Portfolio structures (simple compound Poisson processes)

	Severitity			Frequency
	Type	μ	σ	λ
Risk class A_1	Normal	2.0	1.0	2
Risk class A_2	LogNormal	2.0	1.5	4
Risk class B_1	Normal	2.0	1.0	4
Risk class B_2	LogNormal	2.0	1.5	8

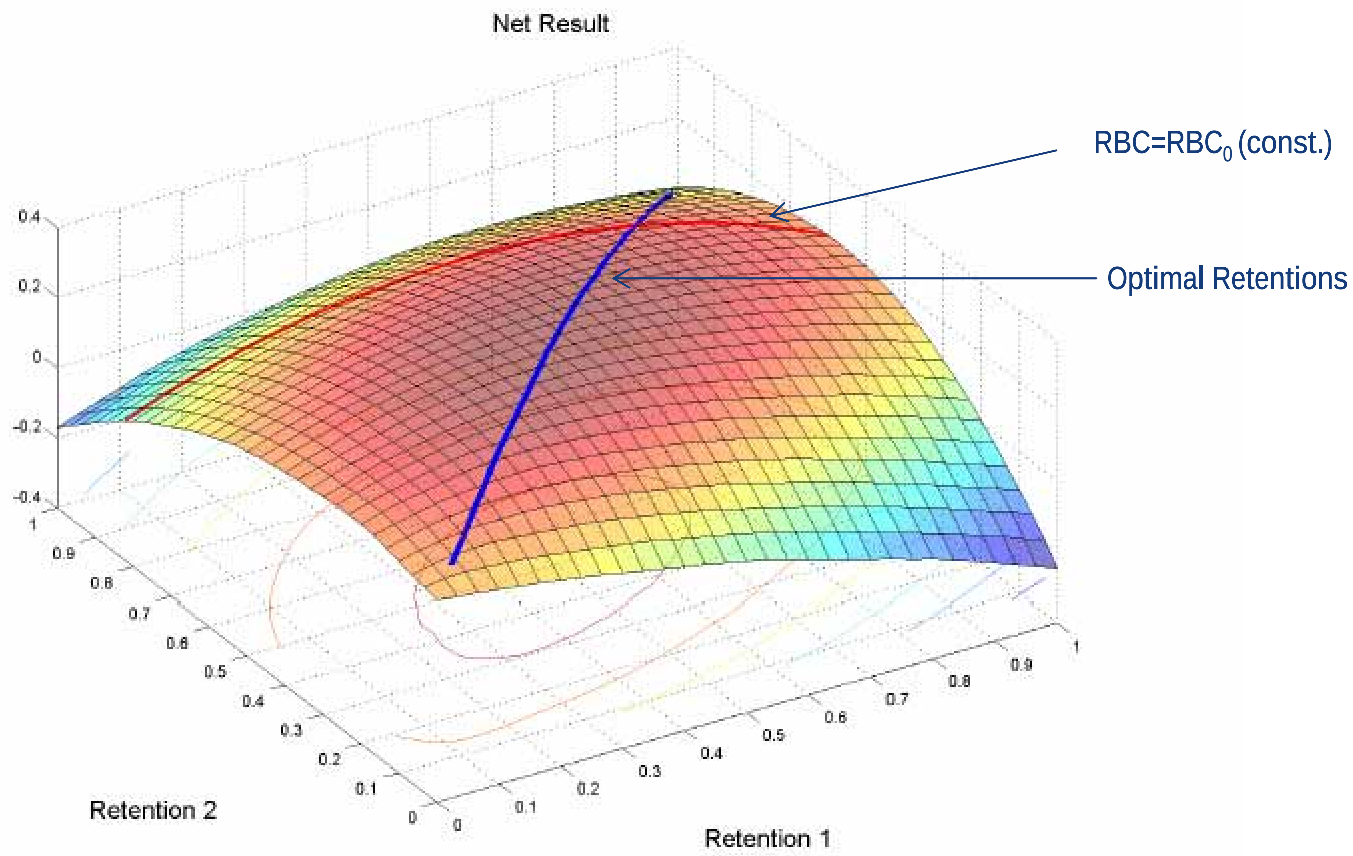
- Risk tolerance levels and capital costs

	Risk Tolerance	Cost of Capital
Portfolio A	10%	9%
Portfolio B	10%	14%

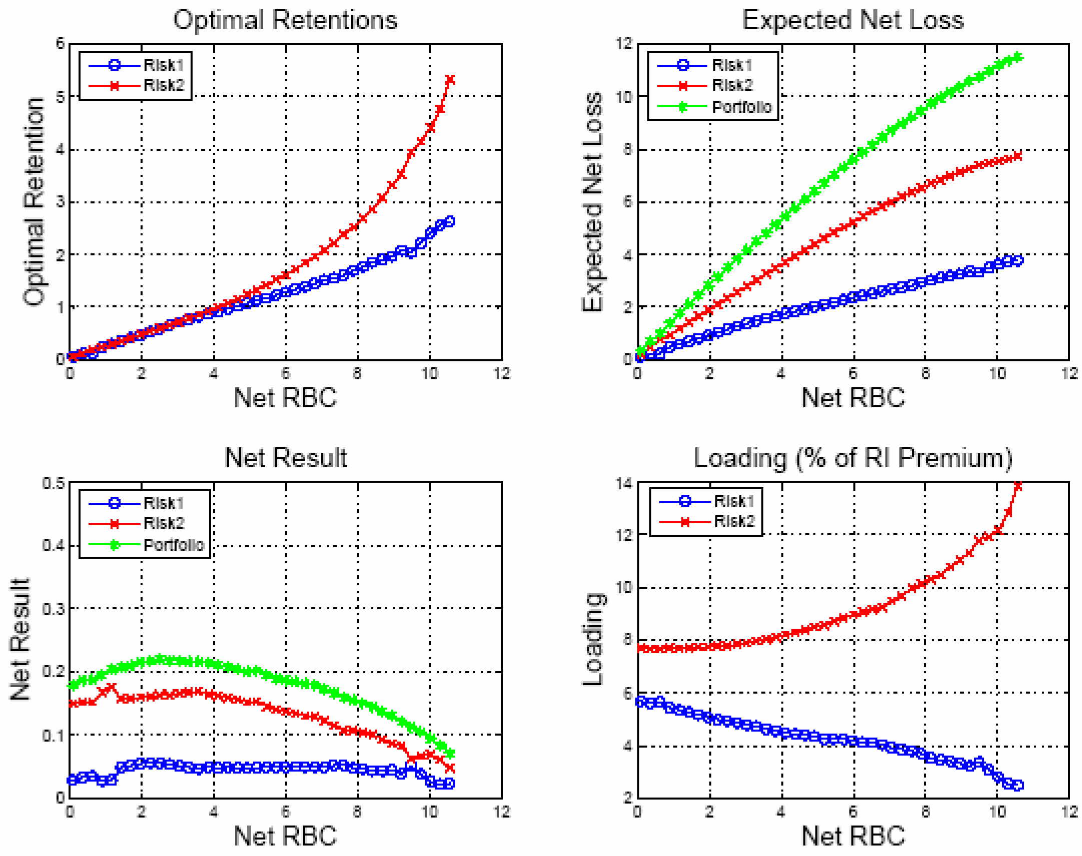
Example – Proportional Reinsurance (I)



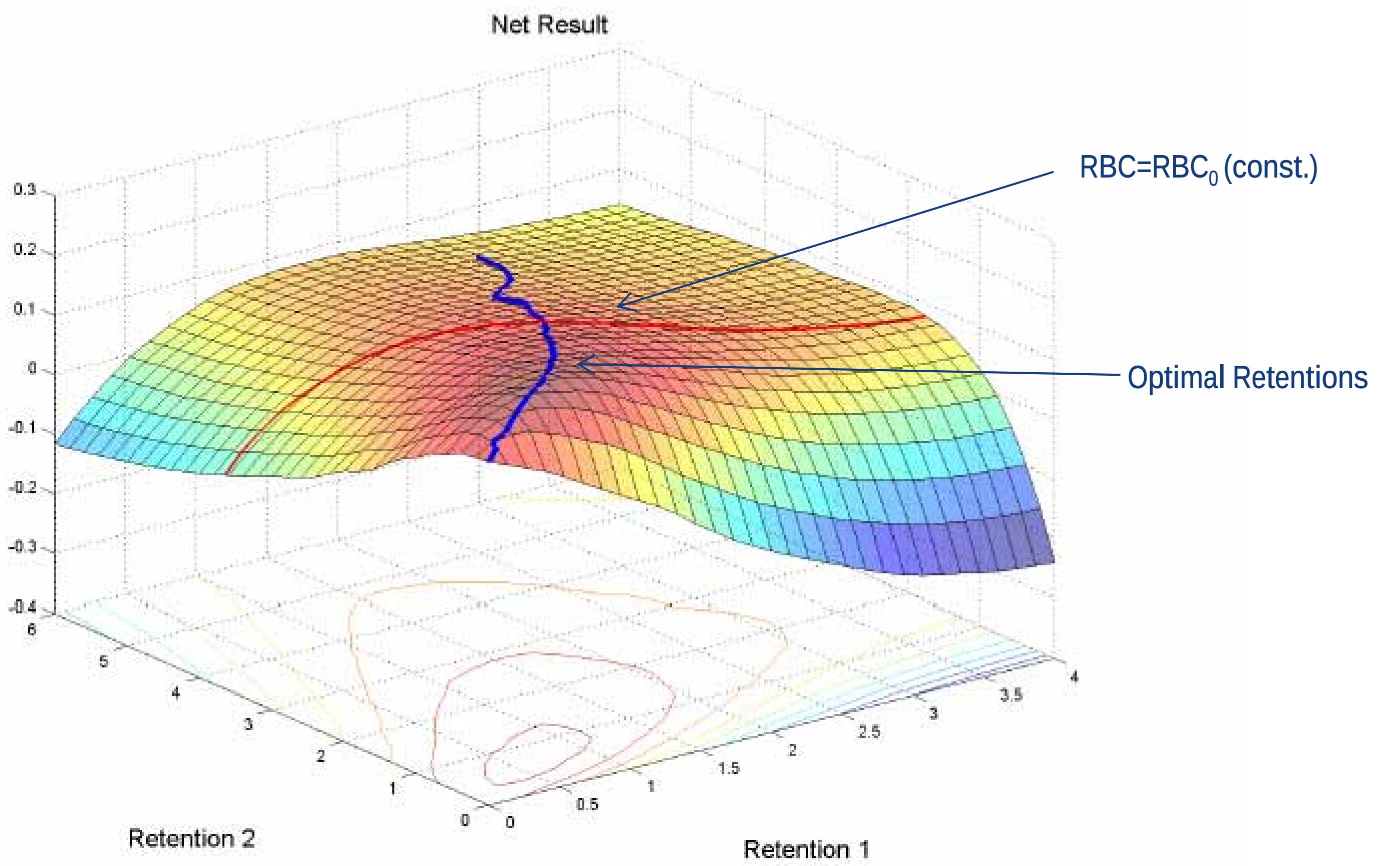
Example – Proportional Reinsurance (II)



Example – Non-Proportional Reinsurance (I)



Example – Non-Proportional Reinsurance (II)



Conclusions

- Sharing risk and diversification is crucial
- Workable scheme for companies that have internal models in place
- Optimal also for risk receiving party? → Extend the framework to *n* risk sharing parties

Contact

Dr. Andreas Kull
Ernst & Young Ltd
Global Financial Services Risk Management
Brandschenkestrasse 100
CH-8002 Zürich

Tel: +41 58 286 3537
Email: andreas.kull@ch.ey.com