On the financial risk factor in fair valuation of the mathematical provision

Rosa Cocozza

Università di Napoli Federico II
Dipartimento di Economia Aziendale

Donato De Feo

MAS04 Universität Zürich/ETHZ

Emilia Di Lorenzo

Università di Napoli Federico II
Dipartimento di Matematica e Statistica

Marilena Sibillo

Università di Salerno
Dipartimento di Scienze Economiche e Statistiche

Paper Outlines

- Aim of the paper, Risk drivers and accounting guidelines
- Fair valuation and marking to market
- A deterministic approach
- A stochastic approach and a numerical application
- Conclusions and further remarks

Paper Outlines

- Aim of the paper, Risk drivers and accounting guidelines

Aim and contents of the paper

- It focuses on the financial variable for the provision evaluation

- It analyses the sensitivity of the fair valuation to interest rate parameters

  1. In a deterministic scenario, evaluation is studied in a market perspective

  2. In a stochastic scenario, sensitivity of cash flows by means of numerical application

Preliminary remarks

- Life insurance risk can be divided in: actuarial and financial risk

- March 2004, the International Accounting Standards Board introduces the IFRS 4

Paper Outlines

- Fair valuation and marking to market

Fair and market value

- Fair value of a mathematical provision: *"net present value of the residual debt towards the policyholders evaluated at current interest rate (and mortality rates)"*
- Marking to market a cultural discrepancy: mathematical vs accounting perspective

Paper Outlines

- A deterministic approach

A deterministic approach

- The basic model

$$K_t = \left[K_{t-1} + N^x(t-1) \sum_{r=1}^{n-t-1} {}_r p_{x+t-1} e^{-r\delta_{L_{t-1}}} \right] e^{\delta_{A_{t-1}}} - N^x(t) \left[1 + \sum_{r=1}^{n-t} {}_r p_{x+t} e^{-r\delta_{L_t}} \right]$$

n	Duration of the policy
X	Initial age insurer
$N^x(r)$	Number of survivor at age x+r

${}_r p_x$	Survival probability
δ_{L_r}	Int. rate applied to the reserve evaluation
δ_{A_r}	Ist. rate of return

The evaluation rate risk

The mathematical provision:

$$R_{t/t<d} = N^{x}(t)\left[\sum_{r=d-t}^{n-t} {}_{r}p_{x+t}e^{-r\delta_{L_t}} - P\sum_{r=0}^{d-t} {}_{r}p_{x+t}e^{-r\delta_{L_t}}\right]$$

The first derivative w.r.t. the evaluation rate:

$$\frac{\partial R_{t/t<d}}{\partial \delta_{L_t}} = -N^{x}(t)\left[\sum_{r=d-t}^{n-t} r\,{}_{r}p_{x+t}e^{-r\delta_{L_t}} + P\sum_{r=0}^{d-t} r\,{}_{r}p_{x+t}e^{-r\delta_{L_t}}\right]$$

It can be derived that:

$$\Delta R_t \cong -R_t \cdot D_{R_t} \cdot \Delta\delta_{L_t}$$

Paper Outlines

- A stochastic approach and a numerical application

Model Setup

- (Ω,**F‘**,**P‘**) , (Ω,**F“**,**P“**) and (Ω,**F**,**P**) are probability spaces

- Assumption on the market:
  - Frictionless
  - Continuous trading
  - No restrictions on borrowing or short sales
  - ZCB and stocks are infinitely divisible

Cash Flows Analysis

Considering a non-homogeneous portfolio:

$$V_t = E\left[\sum_{i=1}^{m}\sum_{j>t} X_{i,j} c_i 1_{\{K_{x_i,t} > j\}} v(t,j) \middle| \Im_t \right] =$$

$$= \sum_{i=1}^{m}\sum_{j>t} c_i X_{i,j} \; {}_t p_{x_i} \; {}_j p_{x_i+t} \, E\left[v(t,j) \middle| \Im_t\right]$$

m	Sub-portfolios
c_i	n. policies i-th group
$X_{i,s}$	Cash-flow at s w.r.t. i

${}_r p_t$	Survival probability
$v(t,j)$	Stoshastic discount factor
$\{F_t\} \subset F$	Information filtration

A numerical application

- Let us considering the following portfolio:
  - 100 immediate 10-year temporary life annuities, with unitary annual payment, each one issued to a male insured aged 40
  - 100 6-year temporary life annuities, 3-years deferred, each one issued to a male aged 40, with periodic level premiums, paid at the beginning of the first three years
  - 80 immediate 8-year temporary unitary life annuities, each one issued to a male insured aged 50
- Survival probabilities are deduced by using the Lee-Carter method. The term structure is derived from the CIR model

Lee-Carter Model

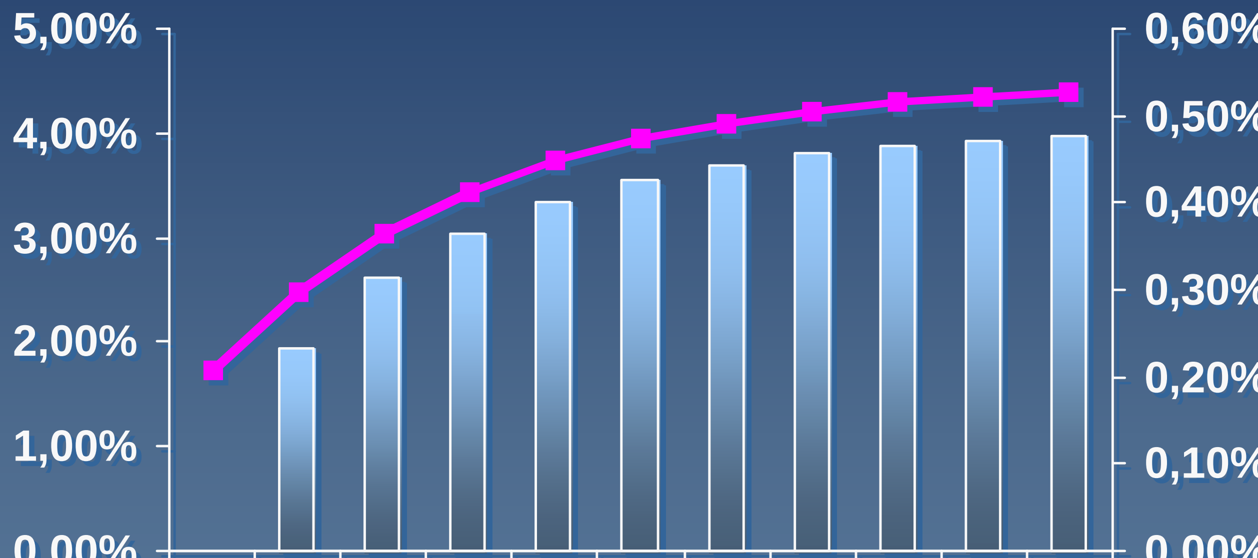
- A model for human mortality forecasting, main assumption:

$$\ln(m_{x,t}) = a_x + k_t b_x + e_{x,t}$$

$m_{x,t}$	Central death rate
a_x	Simple average of *ln($m_{x,t}$)*
b_x	Sensitivity parameter
k_t	Time mortality index
$e_{x,t}$	Error term

Term structure of interest rates

r_0	1,72%
γ	4,5%
ϕ	0,97
σ_α	0,52%
κ	0,026
σ	0,52%



	0	1	2	3	4	5	6	7	8	9	10
Standard Deviation	0,0	0,2	0,3	0,3	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Expected Values	1,7	2,4	3,0	3,4	3,7	3,9	4,1	4,2	4,2	4,3	4,4

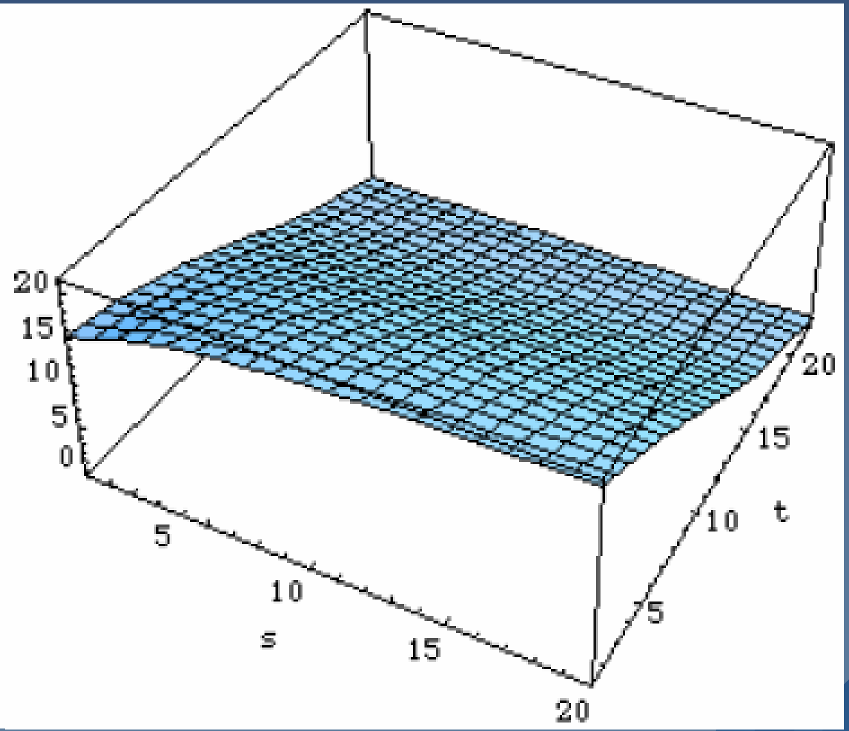
Years

Sensitivity Analysis of the reserve fair value

The reserve fair value is represented as function of the time t and the interest rate volatility s:

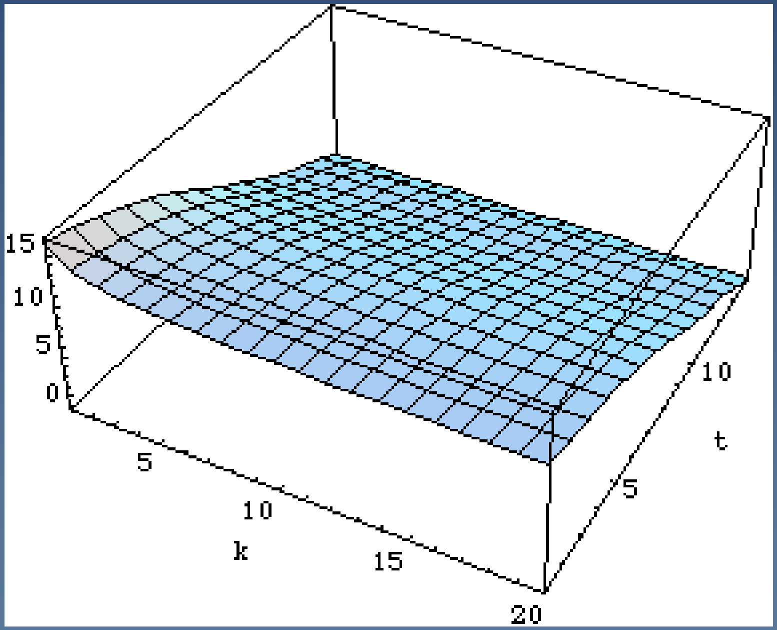
Table 3
Non-homogeneous portfolio

year	Current values of reserves
0	1277,821
1	1374,9655
2	1482,6912
3	1599,154
4	1405,206
5	1200,666
6	985,018
7	757,719
8	518,194
9	190,1108
10	0



Sensitivity analysis: dependence on k

The reserve fair value is represented as function of the time t and the drift parameter k:



Paper Outlines

- Conclusions and further remarks

Conclusions and further remarks

- The paper propose an assessment methodology based on an integrated evaluation
- The intermediation portfolio is regarded as a set of cash-flows
- The analysis is conducted year by year
- The methodology can be applied to any kind of life contract