On the financial risk factor in fair valuation of the mathematical provision

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Paper Outlines

- Aim of the paper, Risk drivers and accounting guidelines
- Fair valuation and marking to market
- A deterministic approach
- A stochastic approach and a numerical application
- Conclusions and further remarks
Paper Outlines

- Aim of the paper, Risk drivers and accounting guidelines
Aim and contents of the paper

- It focuses on the financial variable for the provision evaluation
- It analyses the sensitivity of the fair valuation to interest rate parameters

1. In a deterministic scenario, evaluation is studied in a market perspective
2. In a stochastic scenario, sensitivity of cash flows by means of numerical application
Preliminary remarks

- Life insurance risk can be divided in: actuarial and financial risk.

- March 2004, the International Accounting Standards Board introduces the IFRS 4.
Paper Outlines

- Fair valuation and marking to market
Fair and market value

- Fair value of a mathematical provision: "net present value of the residual debt towards the policyholders evaluated at current interest rate (and mortality rates)"

- Marking to market a cultural discrepancy: mathematical vs accounting perspective
Paper Outlines

- A deterministic approach
## A deterministic approach

**The basic model**

\[
K_t = \left[ K_{t-1} + N^x(t-1) \sum_{r=1}^{n-t-1} r p_{x+t-1} e^{-r \delta_{L_{t-1}}} \right] e^{\delta_{A_{t-1}}}
\]

\[
- N^x(t) \left[ 1 + \sum_{r=1}^{n-t} r p_{x+t} e^{-r \delta_{L_{t}}} \right]
\]

<table>
<thead>
<tr>
<th>$n$</th>
<th>Duration of the policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>Initial age insurer</td>
</tr>
<tr>
<td>$N^x(r)$</td>
<td>Number of survivor at age $x+r$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$r p_x$</th>
<th>Survival probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{L_r}$</td>
<td>Int. rate applied to the reserve evaluation</td>
</tr>
<tr>
<td>$\delta_{A_r}$</td>
<td>Ist. rate of return</td>
</tr>
</tbody>
</table>
The evaluation rate risk

The mathematical provision:

\[
R_{t/t<d} = N^x(t) \left[ \sum_{r=d-t}^{n-t} r \, p_{x+t} e^{-r\delta_{Lt}} - P \sum_{r=0}^{d-t} r \, p_{x+t} e^{-r\delta_{Lt}} \right]
\]

The first derivative w.r.t. the evaluation rate:

\[
\frac{\partial R_{t/t<d}}{\partial \delta_{Lt}} = -N^x(t) \left[ \sum_{r=d-t}^{n-t} r \, p_{x+t} e^{-r\delta_{Lt}} + P \sum_{r=0}^{d-t} r \, p_{x+t} e^{-r\delta_{Lt}} \right]
\]

It can be derived that:

\[
\Delta R_t \cong -R_t \cdot D_R \cdot \Delta \delta_{Lt}
\]
A stochastic approach and a numerical application
Model Setup

- $(\Omega,F',P')$, $(\Omega,F'',P'')$ and $(\Omega,F,P)$ are probability spaces

- Assumption on the market:
  - Frictionless
  - Continuous trading
  - No restrictions on borrowing or short sales
  - ZCB and stocks are infinitely divisible
Cash Flows Analysis

Considering a non-homogeneous portfolio:

\[ V_t = E \left[ \sum_{i=1}^{m} \sum_{j>t} X_{i,j} c_i \mathbf{1}_{\{K_{x_i,t} > j\}} \nu(t,j) | \mathcal{F}_t \right] = \]

\[ = \sum_{i=1}^{m} \sum_{j>t} c_i X_{i,j} t_{p_{x_i}} j_{p_{x_i}} + t E \left[ \nu(t,j) | \mathcal{F}_t \right] \]

<table>
<thead>
<tr>
<th>( m )</th>
<th>Sub-портфели</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_i )</td>
<td>n. policies i-th group</td>
</tr>
<tr>
<td>( X_{i,s} )</td>
<td>Cash-flow at s w.r.t. i</td>
</tr>
</tbody>
</table>

| \( r_D_t \) | Survival probability |
| \( \nu(t,j) \) | Stochastic discount factor |
| \( \{F_t\}_{t\in\mathbb{F}} \) | Information filtration |
A numerical application

Let us considering the following portfolio:

- 100 immediate 10-year temporary life annuities, with unitary annual payment, each one issued to a male insured aged 40
- 100 6-year temporary life annuities, 3-years deferred, each one issued to a male aged 40, with periodic level premiums, paid at the beginning of the first three years
- 80 immediate 8-year temporary unitary life annuities, each one issued to a male insured aged 50

Survival probabilities are deduced by using the Lee-Carter method. The term structure is derived from the CIR model.
Lee-Carter Model

A model for human mortality forecasting, main assumption:

\[
\ln(m_{x,t}) = a_x + k_t b_x + e_{x,t}
\]

| \(m_{x,t}\) | Central death rate |
| \(a_x\) | Simple average of \(\ln(m_{x,t})\) |
| \(b_x\) | Sensitivity parameter |
| \(k_t\) | Time mortality index |
| \(e_{x,t}\) | Error term |
Term structure of interest rates

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0$</td>
<td>1.72%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4.5%</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\sigma_\alpha$</td>
<td>0.52%</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.026</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.52%</td>
</tr>
</tbody>
</table>

Years | Standard Deviation | Expected Values
--- | --- | ---
0 | 0.0 | 1.7
1 | 0.2 | 2.4
2 | 0.3 | 3.0
3 | 0.4 | 3.4
4 | 0.4 | 3.7
5 | 0.4 | 3.9
6 | 0.4 | 4.1
7 | 0.4 | 4.2
8 | 0.4 | 4.2
9 | 0.4 | 4.3
10 | 0.4 | 4.4
Sensitivity Analysis of the reserve fair value

The reserve fair value is represented as function of the time $t$ and the interest rate volatility $s$:

\[
V(t, s) = f(t, s) \]

```
Table 3
Non-homogeneous portfolio

<table>
<thead>
<tr>
<th>year</th>
<th>Current values of reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1277,821</td>
</tr>
<tr>
<td>1</td>
<td>1374,9655</td>
</tr>
<tr>
<td>2</td>
<td>1482,6912</td>
</tr>
<tr>
<td>3</td>
<td>1599,154</td>
</tr>
<tr>
<td>4</td>
<td>1405,206</td>
</tr>
<tr>
<td>5</td>
<td>1200,666</td>
</tr>
<tr>
<td>6</td>
<td>985,018</td>
</tr>
<tr>
<td>7</td>
<td>757,719</td>
</tr>
<tr>
<td>8</td>
<td>518,194</td>
</tr>
<tr>
<td>9</td>
<td>190,1108</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>
```
Sensitivity analysis: dependence on $k$

The reserve fair value is represented as a function of the time $t$ and the drift parameter $k$:

![3D graph showing the dependence of the reserve fair value on time $t$ and drift parameter $k$.]
Paper Outlines

- Conclusions and further remarks
Conclusions and further remarks

- The paper propose an assessment methodology based on an integrated evaluation

- The intermediation portfolio is regarded as a set of cash-flows

- The analysis is conducted year by year

- The methodology can be applied to any kind of life contract