GRADUATING THE SAUDI CRUDE MORTALITY RATES AND CONSTRUCTING THEIR MONETARY TABLES

By

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Abstract: The actuarial method to develop a law of mortality or a mathematical expression to graduate the Saudi crude mortality rates have been applied, also monetary tables have been constructed to assist calculating the actuarial present value for the Saudi life annuities (SPVA).

This paper contains:-

1. Introduction

2. Graduation Methods

3. The preliminary test for a mathematical model

4. Fitting the Model

5. Monetary Saudi Tables

6. Conclusion

1. Introduction

Graduation may be regarded as the principles and methods by which a set of observed (or crude) probabilities are adjusted in order to provide a suitable basis for inferences to be drawn and further practical computations to be made.

The observed data may be regarded as a sample from a larger population, so that the observed probabilities derived there from, are subject to sampling errors. Providing these errors are random in nature, they may be reduced by increasing the size of the sample and thereby extending the scope of the investigation. A simpler, cheaper and more practicable alternative is often to use graduation to remove these random errors.

2. Graduation Methods

The actuarial methods of graduation can be described broadly as:

- a) graphical method,
- b) summation method,
- c) Kernel's method,
- d) the method of osculatory interpolation,
- e) the spline method,
- f) the curve fitting or parametric method,
- g) graduation by reference to a standard table,
- h) difference equation method and
- j) linear programming method

3. The preliminary test for a mathematical model

The first important contribution towards finding a mathematical model was made by Benjamin Gompertz (1825), who argued that the force of mortality μ_x can be represented by the formula BC^x.

A development of Gompertz's law was subsequently made by Makeham (1860), who adapted the formula

$$\mu_x \quad \text{or} \quad q_x = A + BC^x \qquad (1)$$

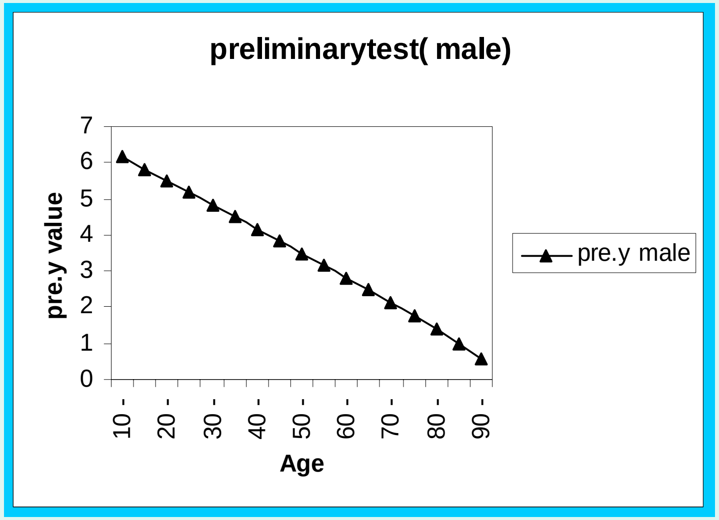
when Gompertz's law holds the graph of the following equation is a straight line [3],

$$-\ln(-\ln(1-q_x)) = \ln\left(\frac{\ln C}{B(C-1)}\right) - x \ln C$$

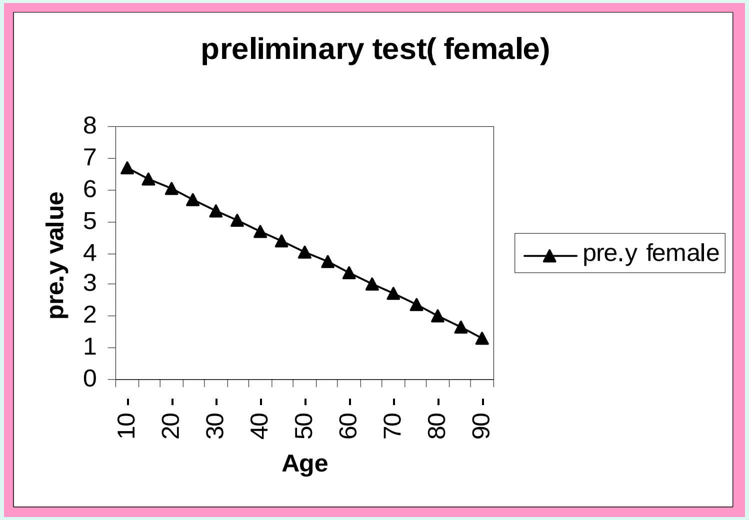
Table(1)
Saudi Crude Mortality Rates

Age Interval	q_x^0 **Male**	q_x^0 **Female**
10 -	**0.00209**	**0.00126**
15 -	**0.00292**	**0.00175**
20 -	**0.00408**	**0.00243**
25 -	**0.00569**	**0.00337**
30 -	**0.00794**	**0.00468**
35 -	**0.01106**	**0.00651**
40 -	**0.01541**	**0.00904**
45 -	**0.02151**	**0.01255**
50 -	**0.02998**	**0.01744**
55 -	**0.04183**	**0.02423**
60 -	**0.05834**	**0.03365**
65 -	**0.08136**	**0.04675**
70 -	**0.11463**	**0.06494**
75 -	**0.15825**	**0.09021**
80 -	**0.22068**	**0.12531**
85 -	**0.30778**	**0.17407**
90 -	**0.42926**	**0.24180**

Figure(1)



Figure(2)



4. Fitting the Model

Various curve fitting have been tried [7], [8], which failed to represent the Saudi data. Heligman and Pollard mathematical formula (eq.(5)), could represent the Saudi rates:-

$$q_x = A^{(x+B)^C} + D\exp\left(-E\left(\ln\frac{x}{F}\right)^2\right) + \frac{GH^x}{1+GH^x} \qquad (5)$$

Where q_x is the probability of someone exact age x dying before exact age $x+1$, and A, B,…, H are parameters to be estimated.

the central mortality rates were transformed to q_x^0 values by the classical formula

$$q_x^0 = \frac{2m_x}{2 + m_x} \qquad (6)$$

The parameters of the curve were estimated by least squares Gauss-Newton iteration [9]. The function minimized was

$$S^2 = \sum_{x=0}^{85} \left(\frac{q_x}{q_x^0} - 1.0\right)^2 \qquad (7)$$

Using the procedure UNABR from The United Nations Software Package for Mortality Measurement (MortPak-Lite), The graduated life tables for both sex are presented in Appendix(I), also their estimated parameters are presented in the following table,

Table(2)

Graduation parameters for Saudi mortality experience 1990-1993

Parameter	Males	Females
A	0.02779	0.01776
B	0.56113	0.82686
C	0.43809	0.37970
D	0.00119	0.00012
E	0.86895	1.83524
F	63.37608	23.35901
G	0.00012	0.00007
H	1.08145	1.08175

The mathematical formula in equation (5) contains three terms each representing a distinct component of mortality, [4]:

The first term representing a rapidly declining exponentially, reflects the fall in mortality during the early childhood years as the child adapts to his or her new environment and gains immunity from the diseases of the outside world, this term has three parameters:

- A : which is nearly equal to q_i
- B: is an age displacement, it measures the location of infant mortality.
- C: measures the rate of mortality decline in childhood, the higher the value of C, the faster mortality declines with increasing age.

The second term is a function similar to the lognormal, reflects accident mortality for males and accident plus maternal mortality for the female population. it has three parameters:

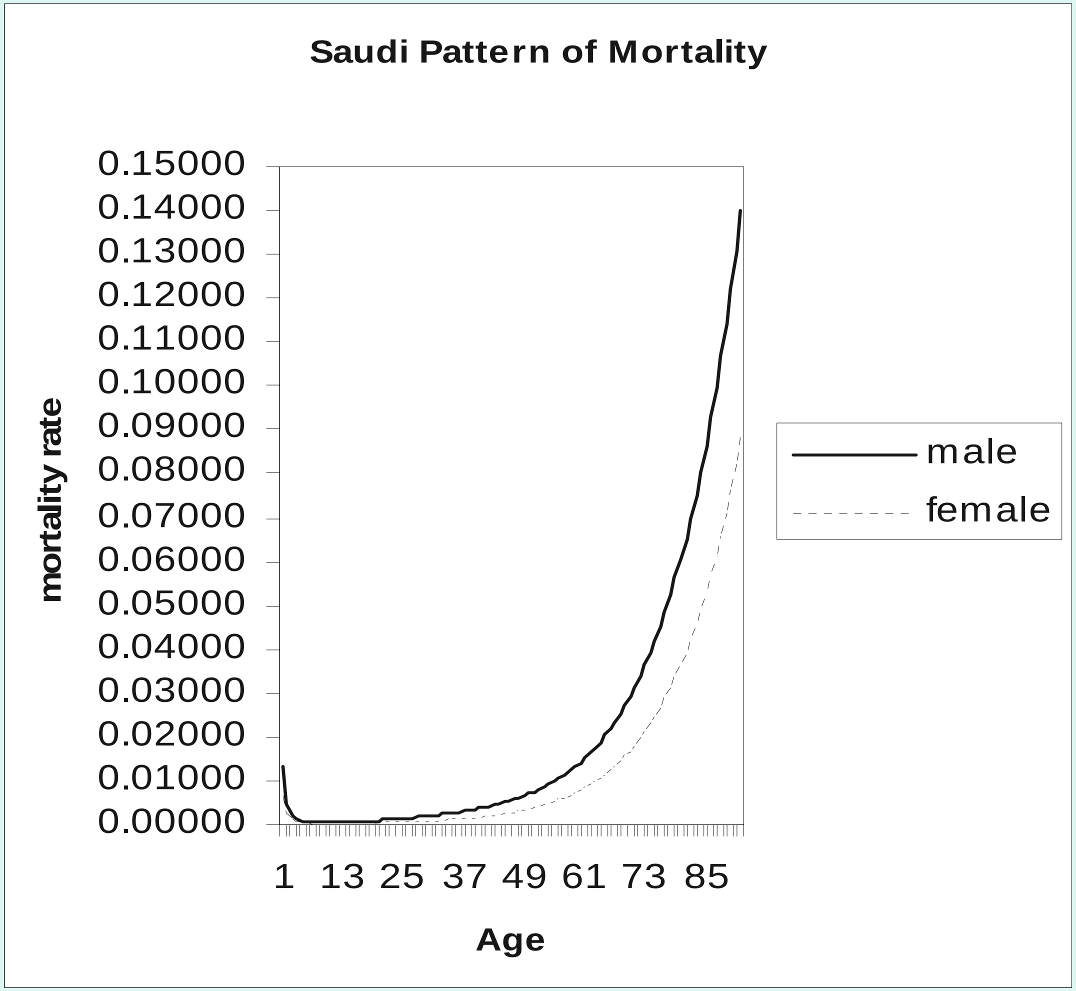
- D: measuring the severity.
- E: representing spread.
- F: indicating location.

The third term in the formula, the well-known Gompertz exponential, reflects the near geometric rise in mortality and it is generally considered the ageing of the body, i.e. senescent mortality and it has two parameters:

- G: represents the base level of senescent mortality.
- H: represent the rate of increase of that mortality.

Comparing between the Saudi parameters of males and females in table(2), males have experienced higher child mortality than females (156.48%), which appears from the values of parameter A. Also, the higher B value for females indicate that they have relatively lower infant mortality, in the same time males have higher value of C, i.e. their rate of mortality decline with age has been faster. Parameter D for male is higher than the female, may be that return to some accident mortality, and the following figure(Figure(3)) representing Saudi pattern of mortality for both sex.

Figure(3)



Saudi Pattern of Mortality
0.15000
0.14000
0.13000
0.12000
0.11000
0.10000
0.09000
0.08000
0.07000
0.06000
0.05000
0.04000
0.03000
0.02000
0.01000
0.00000
mortality rate
1 13 25 37 49 61 73 85
Age
male
female

5. Monetary Saudi Tables

Commutation functions are derived by combining life table functions with compound interest functions, to construct the monetary tables based on the Saudi mortality rates obtained over the available range of ages, life tables can be built up by starting with any suitable radix, say 100000, and applying successfully the relations: [10]

$$d_x = l_x q_x \quad (7)$$

$$l_{x+1} = l_x - d_x \quad (8)$$

Also a computer program has been written in VISUAL BASIC.NET to compute the following functions for ages starting from 10 to 99 inclusively for both Saudi males and females, and interest rate 5%(as an example), the monetary tables have been represented in (Appendix II):

$$\left.\begin{aligned} D_x &= v^x \cdot l_x \\ N_x &= \sum_{t=0}^{\infty} D_{x+t} \\ S_x &= \sum_{t=0}^{\infty} N_{x+t} \end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned} C_x &= v^{x+1} \cdot d_x \\ M_x &= \sum_{t=0}^{\infty} C_{x+t} \\ R_x &= \sum_{t=0}^{\infty} M_{x+t} \end{aligned}\right\} \quad (10)$$

There are also other commutation functions used for the techniques of valuation of pension benefits, the researchers will deal with them in another article. To show some comparisons between Saudi life annuities for male and female using the calculated tables (Appendix II), and the following equations:-

$$\ddot{a}_x = \frac{N_x}{D_x} \qquad (11)$$

where, $\ddot{a}_x$: a life annuity-due which is a series of payments of one unit at the beginning of each year, payable as long as a life aged x is alive, and it can be expressed in terms of the elementary commutation functions as indicated. If the payments are made at the end of each year, the annuity is termed an immediate life annuity. It is denoted by the symbol a_x and has the following expression,

$$a_x = \frac{N_{x+1}}{D_x} \qquad (12)$$

Some comparisons are made to distinguish between the annuities of both sexes at different ages when the interest rate equal 5% in the following tables:-

Table(3)

The probable present value of a Saudi life annuity-due

Age(x)	$\ddot{a}_x$ Male(1)	$\ddot{a}_x$ Female(2)	Ratio(2/1)%
30	17.95	18.86	103.07
40	16.61	17.77	106.98
50	14.81	16.24	109.66
60	12.56	14.20	113.06

Table(4)

The probable present value of a Saudi immediate life annuity-due

Age(x)	a_x Male(1)	a_x Female(2)	Ratio(2/1)%
30	16.95	17.86	105.37
40	15.61	16.77	107.43
50	13.31	15.24	110.35
60	11.56	13.20	114.19

6. Conclusion

This article is adopting a law of mortality suggested by Heligman and Pollard(1980), to graduate the Saudi crude rates, it's mathematical formula contains three terms, each representing a distinct component of mortality. The curve is continuous and applicable over the entire age range, it has relatively few parameters all of which have demographic interpretation and together fully describe the Saudi age pattern of mortality for males and females. Also monetary tables have been constructed by using a computer program written in VISUAL BASIC.NET to assist calculating the actuarial present values for the Saudi life annuities (SPVA), some comparisons are made to distinguish between the annuities of both sex at different ages.