Risk Coverage Ratio: A Leverage-Independent Method of Pricing based on Distribution of Return

ASTIN Topic: Reinsurance Pricing and Risk Load Considerations

David Ruhm, FCAS
The Hartford Insurance Group
690 Asylum Avenue, HO-GL-140
Hartford, CT 06115

Telephone: (860) 547-8815
FAX: (860) 547-4639
e-mail: David.Ruhm@TheHartford.com

Abstract

Various approaches have been taken to pricing insurance risk. Recently, a number of authors have written on relationships between insurance pricing and pricing in financial markets. A common central quantity in both insurance and finance is return on equity (ROE). The Risk Coverage Ratio (RCR) is introduced as a risk measure based on the ROE distribution that can be used to price insurance, even for risks that have unusually asymmetric or skewed distributions of return. The RCR calculation is independent of the leverage assumption, which results in pricing that is leverage-independent, a notable advantage. Several other advantages of RCR are discussed. RCR is conceptually related to the Default Rate on Surplus pricing method.

Keywords

ROE, risk criterion, risk measure, risk coverage ratio, leverage, downside risk, upside potential, layer additivity, default rate.

Introduction

The actuarial literature contains a variety of approaches to pricing insurance risk. Recently, a number of authors have written about risk pricing techniques that are related to techniques used in financial markets, notably WANG (2000) and MANGO (1999). Loss distributions, which are often the basis of insurance pricing techniques, do not always have an explicit analogy in other financial contexts. One quantity which is of central interest in both insurance and finance is return on equity (ROE). Pricing theory and methods based on the distribution of ROE, rather than on the distribution of losses, may offer the best potential for application across disciplines.

Asymmetric and highly skewed distributions present unusual technical challenges that make them more difficult to price. They arise in numerous real-world contexts, and can be skewed either towards the upside or towards the downside. Some contracts have aggregate limits that attenuate the downside severity, while leaving the upside unaffected. Other contracts that provide high insurance capacity can have the opposite profile: an upside potential that is nearly fixed, with tremendous potential for downside severity at low probabilities.

A single pricing method that produces satisfactory results across a wide spectrum of distributions makes it possible to price a great variety of risks in a consistent manner. At the center of such a pricing method, one can usually find a "risk criterion", a test that can be applied to the distribution of return to assess pricing adequacy. Under such a criterion-based method, pricing is defined to be adequate when the resulting distribution of return exactly meets the risk criterion. The risk load is then implicit in the pricing as the margin above expected breakeven.

An example of a risk criterion is: the expected return equals the risk-free rate plus twice the standard deviation of return:

$$E[R] = r + 2\sigma$$

Equivalently:

$$(E[R]-r)/\sigma = 2$$

This particular risk criterion uses the Sharpe ratio as a risk measure. The risk criterion is that the risk measure equals the specified value of two. This is an example of a risk criterion defined by a risk measure's being equal to a specified value, which is a common variety of risk criterion.

In this paper, several desirable characteristics for a risk measure are discussed. The Risk Coverage Ratio is introduced as a risk measure that has many of these desirable characteristics, and is discussed further.

Desirable Characteristics for a Risk Measure

The following is a list of desirable characteristics for a risk measure to have:

1) The risk measure applies to the ROE distribution (rather than the loss distribution).
2) The risk measure is independent of surplus allocation.
3) The risk measure and pricing method are explainable and acceptable to both management and underwriting staff.
4) Both downside risk and upside potential are reflected.

The reasons underlying the desirability of each of these characteristics are discussed below:

1) The risk measure applies to the ROE distribution (rather than the loss distribution): Risk measures focused on the ROE (or "Total Return") distribution are preferred since ROE includes the impacts of all sources of financial variability. While the largest influence on the ROE distribution is usually the loss distribution, differences between lines in terms of payout patterns and taxes need to be reflected, since they often have a significant impact on profitability and on associated risk.

2) The risk measure is independent of surplus allocation: The amount of risk and expected profit introduced to the company by underwriting an account or a line of business is a function of the pricing, and does not change with the assignment of surplus. Pricing should respond to risk and should be independent of leverage. Consequently, the risk measure that forms the basis for pricing should be independent of surplus allocation.

3) The risk measure and pricing method are explainable and acceptable to both management and underwriting staff: For the risk measure to be applied in practice, it must be accepted by management as an accurate measure of risk. It is also advantageous for underwriters to see that their lines of business are being evaluated fairly, using concepts of risk that they can understand. For these reasons, a risk measure and associated pricing method that are explainable and acceptable to non-technical staff provides an advantage in practical applications.

4) Both downside risk and upside potential are reflected: Clearly, the risk measure has to reflect the degree of downside risk so that pricing can compensate for the risk. However, if only downside risk is reflected and upside potential is ignored, two different lines could be priced to have identical downside risks but different upside potentials, which would be inequitable. Therefore, a risk measure should reflect both the downside risk and the upside potential, rather than just the downside risk alone.

The Risk Coverage Ratio

Researching risk measures and evaluating them using the above criteria led to development of the Risk Coverage Ratio, or "RCR":

$$RCR = (R - r) / X$$

, where: $R = E[ROE]$
r = risk-free rate
$X = E[\max(0, r\text{-}ROE)]$

The numerator of RCR is the expected excess return above the risk-free rate, which is the expected marginal gain from writing insurance, in comparison to the alternative of investing in risk-free securities. This value represents the amount of compensation for assuming the risk. The denominator is a measure of the amount of risk. The ratio's intuitive meaning is how many times the risk is “covered” by the expected return, hence the name “Risk Coverage Ratio”. (All rates and yields in this paper are on an after-tax basis.)

The denominator, X, is the expected amount by which the return falls below the risk-free rate. The formula for X is very similar to an expected excess formula, but in the downward direction, since the distribution variable is return rather than loss. Since any return on surplus that is below the risk-free rate is equivalent to a loss of surplus on a net present value basis, X measures the exposure to surplus loss that is created by assuming the risk. If there is no possibility of surplus loss, X equals zero.

Exhibit 1 shows a graph of a normal return distribution in which X is decomposed into frequency and severity factors[1]. The average of the results below the risk-free rate “r” is denoted by “T”, which stands for average tail value. Then, the gap between r and T, r-T, is the average severity of surplus loss. The variable P represents the probability of a surplus loss, corresponding to the area in the tail. Combining the frequency and severity yields the risk to surplus, (r-T)P. This is equal to E[max(0, r-ROE)], the expression for X given above.

Example of a RCR Calculation

Consider a hypothetical normal ROE distribution with mean equal to 16.00% and standard deviation equal to 10.00%. For this example, assume the risk-free rate is 4.00%. Then the expected excess return is:

$$R\text{-}r = 16.00\% - 4.00\% = 12.00\%.$$

The value for X can be calculated using an elementary integral (the calculation is given in Appendix 1):

$$X = 0.56\%$$

Dividing these two results gives the Risk Coverage Ratio:

$$RCR = 12.00\% / 0.56\% = 21.4$$

While this ratio might appear very large at first, it corresponds to a risk load of 1.20 standard deviations ((16%-4%)/10%), and an 11.5% probability of surplus loss.

In an actual pricing application, this figure would now be compared to the risk level that has been selected for pricing. If the risk criterion for pricing is RCR=20, the premium underlying the ROE distribution would have to be reduced for the ROE distribution to exactly meet the risk criterion. If the risk criterion is instead RCR=25, the premium would have to be increased.

Evaluation of RCR vs. Desirable Characteristics

How well does the Risk Coverage Ratio perform in terms of the desirable characteristics listed above?

1) The risk measure applies to the ROE distribution (rather than the loss distribution): The RCR calculation is applied to the ROE distribution, as demonstrated in the example above.

2) The risk measure is independent of surplus allocation: Since both the numerator and denominator of RCR are directly proportional to leverage (as shown in Appendix 2), RCR is unaffected by leverage, and consequently is independent of surplus allocation.

3) The risk measure and pricing method are explainable and acceptable to both management and underwriting staff: The name "Risk Coverage Ratio" facilitates the explanation of this risk measure, since the concept of measuring how many times risk is covered by expected return is easily grasped. The numerator's meaning, the marginal expected return from underwriting, becomes apparent once it is understood that the risk-free rate is available without assuming any underwriting risk. The denominator is not as straightforward to explain as the numerator is, but the frequency-severity diagram discussed above (Exhibit 1) has proven very useful in elucidating the concept. Once the Risk Coverage Ratio is understood, the author has found that it is generally accepted very positively by management and staff, because it clearly reflects frequency and severity of risk, as well as upside potential.

4) Both downside risk and upside potential are reflected: Downside risk is clearly reflected in the denominator. It is also reflected in the numerator in that it influences expected return (denoted by the variable R). The numerator reflects upside potential in the same way, since a line of business that has higher upside potential will have higher expected return, all else being equal.

While RCR reflects both downside risk and upside potential, there is a bias toward the measurement of downside risk. For example, if two symmetric ROE distributions have the same mean and one of them is heavier-tailed (on both the left and the right tails), the heavier-tailed distribution will have both higher downside risk and higher upside potential. The heavier-tailed distribution will also have a lower RCR value, meaning that the risk is considered higher relative to the expected return. Therefore, the bias towards measurement of downside risk means that RCR contains an implicit penalty for volatility, which is a desirable feature.

Layer-Additivity and Decreasing Rate-on-Line Criteria

The principle of layer additivity is that the price for underwriting several layers combined should equal the sum of the individual prices. For example, the price for the $200,000 xs $200,000 layer should equal the price of the $100,000 xs $200,000 layer plus the price of the $100,000 xs $300,000 layer. This principle has been discussed in several articles, and has been referred to by VENTER (1991) as a "no-arbitrage" condition.

In general, the Risk Coverage Ratio pricing method does not produce perfect layer additivity. A simple counterexample using a discrete loss distribution is provided in Appendix 3. In this counterexample and in other examples developed by the author, the discrepancies are relatively insignificant in magnitude, and do not appear to detract from the method's usability.

The principle of decreasing rate-on-line is that the rate-on-line (cost per unit of coverage) for a loss layer should be less than the rate-on-line for any lower layer, because losses in the layer are conditional on losses occurring in all lower layers[2]. This property is also referred to by VENTER (1991) as a no-arbitrage condition. Pricing by Risk Coverage Ratio appears to generally satisfy this condition.

Comparison to Default Rate on Surplus

Risk Coverage Ratio is similar in concept to a simplified version of Default Rate on Surplus, a pricing method developed by MANGO (1999). Like RCR, the Default Rate method measures the risk to surplus and balances this risk against the expected operating gain.

As MANGO (1999) explains, there is a connection between insurance pricing based on risk to surplus and the pricing of bonds in financial markets. A bond's price is strongly influenced by its default risk, the risk that interest or principal will not be paid as scheduled. In general, the higher a bond's default risk, the higher the necessary expected return to compensate. If surplus is viewed as bond principal on which there is an expected return (derived from the combination of underwriting and investing), then investment in bonds and insurance underwriting are analogous and the pricing concepts from one discipline can be applied to the other.

Recent developments have increased the significance of this relationship, as insurance-based securitizations have become more common. At some point, one would expect the actuarial pricing of an insurance risk and the financial pricing of the same risk within a bond to be mathematically consistent with each other.

While conceptually similar, there are a number of significant distinctions between the RCR method and the Default Rate method. First, the Default Rate calculation is focused on the loss distribution, while RCR is based on total return. As discussed earlier, a method that is based on the total return distribution is more likely to include all components of profitability and risk.

Second, the Default Rate method requires an explicit allocation of surplus, and Default Rate pricing varies with the amount of surplus that is allocated (although not by much in various examples). In contrast, RCR pricing is independent of the selected allocation of surplus. Leverage independence is desirable for a pricing method, for the reasons discussed above.

A third difference is Default Rate's division of surplus into layers. Whereas the denominator of RCR uses a simple expected value calculation to measure the risk to surplus, the Default Rate method involves dividing the surplus into layers and assigning weights to each of the layers before taking the expectation. The denominator of RCR is analogous to a Default Rate calculation in which there is only one surplus layer (the entire surplus), with an assigned weight of unity. While the use of layers certainly makes the Default Rate method more flexible, the simplicity of the denominator might facilitate explanation of the method to non-technical management and staff.

Conclusion

Risk Coverage Ratio is relatively easy to calculate and implement as a component of a ROE model, and it has the four advantages listed above. RCR is based on the ROE distribution, rather than the loss distribution. At the same time, RCR is independent of leverage, so that surplus allocation has no influence on pricing. This is considered by the author to be an important advantage since there are many methods for allocating surplus to individual lines but actual surplus effects are only realized at the company level, not at the line level. Also, the risk and expected operating gain from underwriting is a function of the pricing, and does not change with the selected assignment of surplus.

NOTES

1. The author is grateful to Russell Bingham for this diagram and line of explanation.
2. Technically, the condition is actually “non-increasing”, rather than “decreasing”.

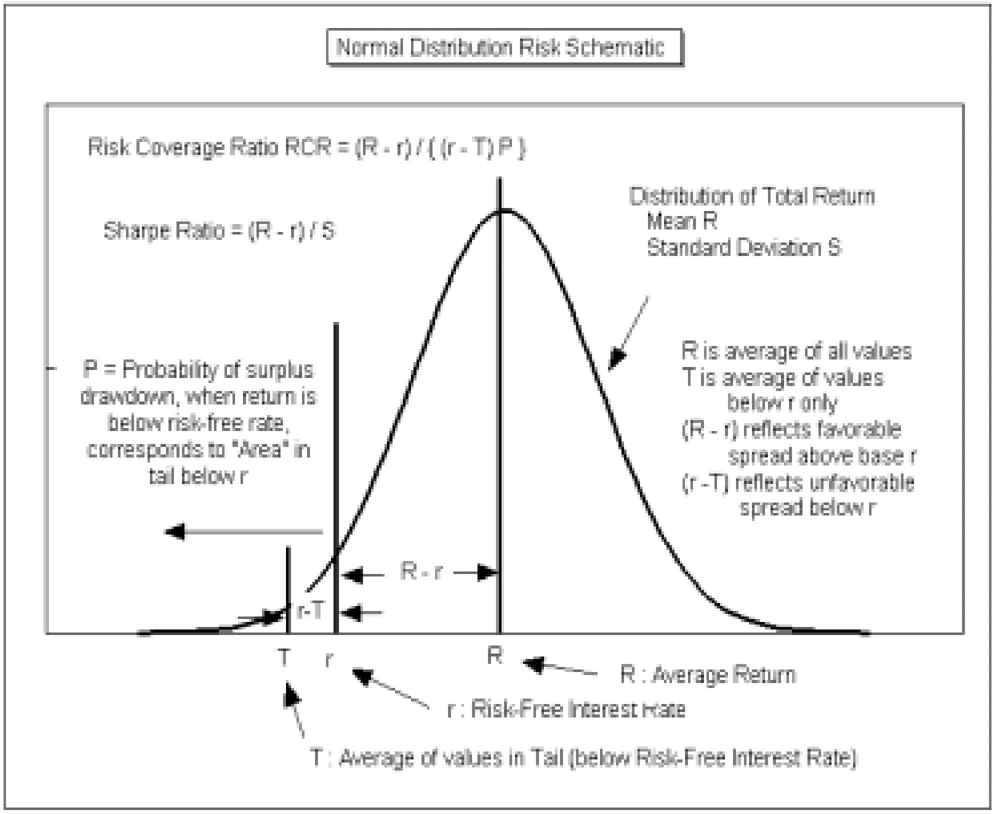
REFERENCES

MANGO, D. F. (1999) Risk load and the default rate of surplus. *Securitization of Risk Discussion Paper Program (Casualty Actuarial Society)*, 175-222.

VENTER, G. G. (1991) Premium calculation implications of reinsurance without arbitrage. A*STIN Bulletin* **21**, 223-230.

WANG, S. S. (2000) A class of distortion operators for pricing financial and insurance risks. *Journal of Risk and Insurance* **67**, 15-36.

Exhibit 1: Frequency and Severity of Surplus Drawdown



Appendix 1: Calculation of RCR Denominator

The calculation is as follows:

$$X = E[\max(0, r-ROE)] = \int (r-x)\, f(x)\, dx$$

, where the limits of integration are from $-\infty$ to r. Noting that:

$$f(R) = \exp(-.5[(x-R)/\sigma]^2)/(\sigma\sqrt{2\pi})$$

and making the substitution:

$$y = (x-R)/\sigma$$

the expression now becomes:

$$X = (1/\sqrt{2\pi}) \int (r-R-\sigma y) \exp(-.5y^2)\, dy$$

where the limits of integration are now from $-\infty$ to $(r-R)/\sigma$. Integrating and evaluating yields:

$$X = (\sigma/\sqrt{2\pi}) \exp(-.5[(R-r)/\sigma]^2) - (R-r)\, \Phi[(r-R)/\sigma]$$

Substituting the values R=16%, r=4%, and σ=10% gives the result for the example:

$$X = (.1/\sqrt{2\pi}) \exp(-.72) - (.12)\, \Phi[-1.2]$$

$$X = 0.56\%$$

Appendix 2: RCR is independent of leverage

Both the numerator and the denominator of RCR are proportional to leverage. This is shown as follows:

By definition,

$$ROE = G/s + r$$

, where: G = Operating Gain from underwriting
s = surplus
r = risk-free rate

The only random variable is G. Evaluating the numerator of RCR:

$$\text{Numerator of RCR} = R\text{-}r = E[ROE]\text{-}r = E[G]/s + r - r = E[G]/s$$

The right-hand-side of this equation is inversely proportional to surplus, since operating gain is independent of surplus. Therefore, the numerator of RCR is inversely proportional to surplus, and proportional to leverage.

Evaluating the denominator of RCR:

$$\text{Denominator of RCR} = X = E[\max(0, r\text{-}ROE)]$$

$$X = E[\max(0, \text{-}G/s)] = E[\max(0, \text{-}G)]/s$$

Again, since operating gain is independent of surplus, it follows that X is inversely proportional to surplus, and proportional to leverage.

As both the numerator and denominator of RCR are proportional to leverage, RCR is independent of leverage. In fact, by combining the above calculations it is possible to formulate RCR in terms of the operating gain variable rather than the ROE variable, eliminating all references to leverage or surplus:

$$RCR = E[G] / E[\max(0, \text{-}G)]$$

Appendix 3: A counterexample to layer additivity for RCR pricing

Consider the following discrete loss distribution:

$$\text{Loss} = \begin{cases} 0 \text{ with } 20\% \text{ probability} \\ 10 \text{ with } 50\% \text{ probability} \\ 20 \text{ with } 30\% \text{ probability} \end{cases}$$

Loss is payable exactly one year in the future, with certainty. The risk-free rate of interest is 5%. Other than premium and surplus, these are the only quantities (no expenses or taxes).

Using a risk criterion of RCR=20 and surplus of 10 (any amount will suffice, since RCR is independent of surplus), the indicated premium for the ground-up layer 0-10 is 9.412. This is verified as follows:

$$R = [P(1+r) – E[\text{Capped Loss}]] / s + r$$
$$R\text{-}r = [(9.412)(1.05) – 8] / 10$$
$$R\text{-}r = 18.8\%$$
$$X = E[\max(0, r\text{-}R)]$$
$$X = (20\%)(0) + (80\%)(5.0\% - \{[(9.412)(1.05) – 10]/10 + 5.0\%\})$$
$$X = 0.94\%$$
$$RCR = (R\text{-}r)/X = 20.0$$

Using the same risk criterion and surplus, the indicated premium for the 10 xs 10 layer is 8.571:

$$R = [P(1+r) – E[\text{Capped Loss}]] / s + r$$
$$R\text{-}r = [(8.571)(1.05) – 3] / 10$$
$$R\text{-}r = 60.0\%$$
$$X = E[\max(0, r\text{-}R)]$$
$$X = (70\%)(0) + (30\%)(5.0\% - \{[(8.571)(1.05) – 10]/10 + 5.0\%\})$$
$$X = 3.00\%$$
$$RCR = (R\text{-}r)/X = 20.0$$

The sum of these two premiums is 9.412 + 8.571 = 17.983. This is more than the indicated premium for the combination of the two layers, which is the entire risk, of 17.823 (using surplus of 20 to keep all quantities in scale with the previous calculations):

$$R = [P(1+r) – E[\text{Loss}]] / s + r$$
$$R\text{-}r = [(17.823)(1.05) – 11] / 20$$
$$R\text{-}r = 38.6\%$$
$$X = E[\max(0, r\text{-}R)]$$
$$X = (70\%)(0) + (30\%)(5.0\% - \{[(17.823)(1.05) – 20]/20 + 5.0\%\})$$
$$X = 1.93\%$$
$$RCR = (R\text{-}r)/X = 20.0$$

The discrepancy in this example is 0.160, or about 0.9% of premium. The author believes that this is the typical order of magnitude for discrepancies in layer additivity when RCR is used for pricing. While not completely negligible, discrepancies of this magnitude do not invalidate the use of the Risk Coverage Ratio as a good working model that is approximately additive.