FACING LTC RISKS

ANNAMARIA OLIVIERI°
Dipartimento di Economia
University of Parma

ERMANNO PITACCO*
Dipartimento di Matematica Applicata
University of Trieste

° Via JF Kennedy, 6 - I 43100 Parma (Italy)
phone +39 0521032387 fax +39 0521032385
email annamaria.olivieri@unipr.it or annamaria.olivieri@econ.univ.trieste.it

* Piazzale Europa, 1 - I 34127 Trieste (Italy)
phone +39 0406767070 fax +39 04054209
email ermanno.pitacco@econ.univ.trieste.it

ABSTRACT

This paper deals with Enhanced Pensions, i.e. Long Term Care (LTC) insurance covers for the elderly, providing a straight life annuity uplifted in case the insured becomes disabled (according to a given definition of LTC disability). The risks borne by the provider of the benefit, either an insurer, a pension scheme or a sickness fund, are focussed. With reference to demographical risks, some tools which can be used to face the risks themselves are discussed. In particular, a proper solvency reserve, tailored to the characteristics and magnitude of the risks, and a stop-loss reinsurance arrangement are analysed.

KEYWORDS

Long Term Care insurance, enhanced pension, projected mortality table, projected inception rates, longevity risk, pooling and non-pooling risks, solvency reserve, stop-loss reinsurance treaties

ACKNOWLEDGEMENT

This research work was supported by the Italian MURST (Project: *Models for the management of financial, insurance and operation risks*; Research Unit: *Models for the management of insurance risks*)

1. INTRODUCTION

Long Term Care (LTC) is care required in relation to chronic (or long-lasting) bad health conditions. LTC covers provide income support for the insured, who needs nursing and/or medical care, in the form either of a forfeiture annuity benefit or nursing and medical expense refunding. In the latter case benefits are usually provided by an insurer or a sickness fund, while in the former a pension scheme can also operate as a provider. A particular LTC cover consists in a straight life annuity uplifted in case the annuitant becomes disabled (according to a given definition of disability). Such a cover is commonly known as enhanced pension. A description of LTC covers and relevant actuarial issues can be found, for example, in Haberman and Pitacco (1999).

LTC covers, and in particular enhanced pensions, are affected by several types of risks. Disregarding the investment risk, which concerns most products in the area of the insurances of the person, we concentrate on risks originating from morbidity and life duration. Firstly, LTC covers are recent products so that experience data about LTC claims are still scanty. Moreover, given their lifetime duration, these covers are affected by demographical trends. In particular, the uncertainty in future trends originates the risk of systematic deviations from the demographical assumptions adopted in pricing and reserving.

This paper focusses on the risk coming from uncertainty in future demographical trends and the relevant impact on an LTC portfolio. Since trends affecting LTC covers involve mortality among healthy lives, mortality among disabled lives as well as senescent disability, the demographical risk concerned constitutes in some sense a generalization of the well known longevity risk, typically affecting straight life annuities.

The first step in analyzing the demographical risk consists in assessing the magnitude of the risk itself in terms of some monetary function, for example the random present value of benefits, the loss function or the behaviour of the portfolio random assets. Some analyses in this regard have been performed by Ferri and Olivieri (2000). The second step consists in investigating the possibility of facing the demographical risk. In the present paper, some tools are investigated. In particular, a proper solvency reserve, tailored to the characteristics and magnitude of the risk, and a stop-loss reinsurance arrangement are dealt with.

The paper is organized as follows. In Section 2 the enhanced pension is modelled within a multistate Markov framework. In Section 3 some demographic scenarios are discussed and the relevant projected models are introduced; a model allowing for the systematic risk originating from uncertainty in future trends is then presented. Section 4 deals with the solvency reserve facing demographical risks, including the risk of systematic deviations. The feasibility of reinsurance arrangements is discussed in Section 5. Finally, some conclusions are presented in Section 6.

2. ACTUARIAL MODEL

(a) Multistate model for the Enhanced Pension. LTC covers are usually modelled within a multistate framework. The evolution of a given insurance policy is then represented as a sample path of a time-discrete or, preferably, time-continuous, inhomogeneous Markov chain with finite state space $\mathcal{S}$, $\mathcal{S} = \{1, 2, \ldots, M\}$. The seminal contributions to multistate modelling in life insurance can be found in Hoem (1969, 1988). Overall perspectives on such approach are also given by Wolthuis (1994), Pitacco and Olivieri (1997) and Haberman and Pitacco (1999). We address to such references for a description of the main features of the multistate approach.

With reference to an LTC cover, the multistate model usually consists of the following states:

state 1 = "healthy" (or active), state 2 = "LTC disabled at level I", state 3 = "LTC disabled at level II", ..., state M = "dead". The commonly chronic character of LTC disability allows us to disregard the possibility of recovery (such assumption is suggested also by the lack of LTC insurance data); hence, hierarchical models are adopted. With reference to an enhanced pension, which usually considers one level of disability only, the multistate model is depicted in Figure 2.1. Note that such a model represents any LTC cover (for example, a stand alone) providing only one level of disability benefit.

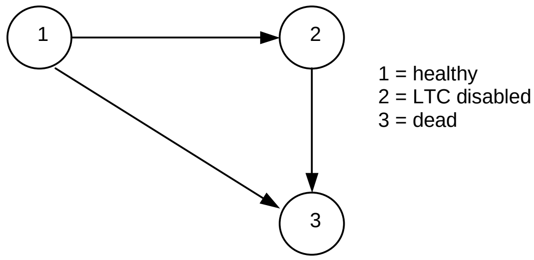

Figure 2.1 – A multistate model for LTC insurance with one level of disability

We adopt a time-continuous probabilistic model. We denote by $S(t)$ the unknown state of the policy at time t, $t \geq 0$, with 0 time of entry; we assume $S(0) = 1$. In order to perform actuarial calculations, transition probabilities

$$P_{ij}(t,u) = \Pr\{S(u) = j | S(t) = i\} \qquad 0 \leq t \leq u, \quad i, j \in \mathcal{S}$$

and transition intensities

$$\mu_{ij}(t) = \lim_{u \to t} \frac{P_{ij}(t,u)}{u-t} \qquad t \geq 0, \quad i, j \in \mathcal{S}, i \neq j$$

are used. We assume that such functions are well-defined for all the operations to be performed. In the three-state model of figure 2.1, the following relations hold

$$P_{11}(t,u) = e^{-\int_t^u (\mu_{12}(s)+\mu_{13}(s))\ ds} \tag{2.1}$$

$$P_{22}(t,u) = e^{-\int_t^u \mu_{23}(s)\ ds} \tag{2.2}$$

$$P_{12}(t,u) = \int_t^u P_{11}(t,s)\ \mu_{12}(s)\ P_{22}(s,u)\ ds \tag{2.3}$$

Relations (2.1) to (2.3) underpin the usual procedure, called TIA (Transition Intensities Approach), which consists in assigning the transition intensities and then deriving the transition probabilities.

(b) Random present value of future benefits. Enhanced pensions provide a straight life annuity, whose amount is uplifted in case the annuitant becomes LTC claimant. Benefits then consist in annuities paid in state 1 and 2. For the ease of numerical evaluations (which are discussed in Sections 4 and 5), discrete payments are considered. More precisely, we assume that an amount $b_j(h)$ is paid at time h ($h = 0, 1, \dots$) if $S(h) = j$ ($j = 1, 2$). Time may be measured in years, semesters, months, etc. In what follows, the time unit is assumed to be the year.
Usually, the amount of benefits is chosen so that $b_1(h) < b_2(h)$, the difference $b_2(h) - b_1(h)$ representing the disability benefit increase. Note that a model for the stand alone cover is found putting $b_1(h) = 0$ for all h. Trivially, if $b_1(h) = b_2(h)$ for all h, a traditional straight life annuity is represented.

Let I_E denote the indicator of event E. Assume an investment yield i_h in period (i.e. year) $(h-1,h)$ and denote by $v(h,u)=\prod_{s=h+1}^{u}(1+i_s)^{-1}$ the value at time h of a monetary unit at time u. The random present value at time h of future benefits for a given policy is (with $h=0,1,\ldots$)

$$Y(h)=\sum_{u=h}^{\infty} v(h,u) \sum_{j=1,2} b_j(u)\ I_{\{S(u)=j\}} \tag{2.4}$$

Consider now a given portfolio of enhanced pensions, with $N_j(h)$ contracts in state j at time h. Policies can differ one from the other because of the amount insured, the age at entry, the risk class, and so on. Denoting by $Y^{(k)}(h)$ the present value of future liabilities for policy k in the portfolio at time h, we define the random present value of future portfolio liabilities as follows

$$\mathcal{Y}(h)=\sum_{k=1}^{N_1(h)+N_2(h)} Y^{(k)}(h) \tag{2.5}$$

Denoting by $B(h)$ the amount globally paid at time h, the following relation also holds

$$\mathcal{Y}(h)=\sum_{u=h}^{\infty} B(u)\ v(h,u) \tag{2.5$'$}$$

In particular, in case the policies are homogeneous in terms of age at entry, annual amounts, risk class, etc., we can write

$$\mathcal{Y}(h)=\sum_{u=h}^{\infty}\sum_{j=1,2} N_j(u)\ b_j(u)\ v(h,u) \tag{2.5$''$}$$

(c) Premiums and reserves. Immediate enhanced pensions obviously require a single premium, π, paid at entry (dealing with LTC products sold to pensioners, it is anyhow reasonable to assume a single premium payment). Enhanced pensions are sold to healthy persons; hence the entry state is $S(0)=1$. We assume, as it is quite common for living benefits, that the equivalence principle is adopted. Let $H^{[\mathrm{p}]}$ denote the technical pricing basis, i.e. the set of hypotheses used for pricing; the single (pure) premium for a given policy is then

$$\pi=E[Y(0)|H^{[\mathrm{p}]},S(0)=1] \tag{2.6}$$

where we have pointed out the conditioning on the initial state, $S(0)=1$.
Specific reserves must be set up in the healthy and disabled state. Reserves are also commonly based on the equivalence principle. Hence, the individual (pure) reserve in state j at time h is

$$V_j(h)=E[Y(h)|H^{[\mathrm{r}]},S(h)=j] \qquad j=1,2 \tag{2.7}$$

where $H^{[\mathrm{r}]}$ denotes the set of hypotheses used for reserving. Usually, pricing and reserving are based on a deterministic financial structure; thus, in (2.6) and (2.7) a given (commonly constant) rate of interest is used.
The portfolio reserve at time h, $\mathcal{V}(h)$, is obtained by summing up the individual items. For example, if the portfolio is homogeneous

$$\mathcal{V}(h)=N_1(h)\ V_1(h)+N_2(h)\ V_2(h) \tag{2.8}$$

and in particular $\mathcal{V}(0)=N_1(0)\,V_1(0)=N_1(0)\ \pi$.

(d) Random value of assets. The liabilities of the insurer must be backed by proper investments. Let $A(h)$ denote the random value of assets at time h, pertaining to the portfolio for meeting pure liabilities. Adopting a time-discrete setting and excluding future equity flows, the behaviour of assets can be described as follows

$$A(h) = (A(h-1) - B(h-1))\ (1+i_h) \tag{2.9}$$

where the initial level of assets, $A(0)$, is given ($A(0) > 0$). We have also

$$A(h) = A(0)\ \frac{1}{v(0,h)} - \sum_{u=0}^{h-1} B(u) \frac{1}{v(0,u)} \tag{2.10}$$

In the case of an homogeneous portfolio, we have in particular

$$A(h) = (A(h-1) - \sum_{j=1,2} b_j(h-1)\ N_j(h-1))\ (1+i_h) \tag{2.9$'$}$$

$$A(h) = A(0)\ \frac{1}{v(0,h)} - \sum_{u=0}^{h-1} \sum_{j=1,2} b_j(u)\ N_j(u) \frac{1}{v(0,u)} \tag{2.10$'$}$$

It must be stressed that equations (2.9) and (2.10) (or (2.9′) and (2.10′)) do not supply us with an office model. Actually, several aspects of the management of the portfolio (such as, for example, expenses) are disregarded. This is due to the fact that pure liabilities only are dealt with in this paper.

3. DEMOGRAPHIC SCENARIOS AND RELATED RISKS

(a) Mortality and disability trends at adult ages. In order to choose appropriate technical bases (for pricing, reserving and ascertaining solvency), reasonable scenarios must be defined. Such a definition is particularly difficult when LTC covers are dealt with for two main reasons (see Ferri and Olivieri (2000)):

(i) LTC are recent products and experience data are rather scanty;

(ii) recent trends in mortality and morbidity witness significant changes that contribute in defining a moving scenario in which LTC products will evolve.

Aspect (i) is usually overcome by resorting to population medical data, properly transformed to reflect selection of insured people with respect to the general population. Moreover, medical data are usually in the form of prevalence rates, whereas incidence rates are required for insurance pricing and reserving; hence, a transformation is necessary also in this regard. Such problems are not dealt with in this paper (see, for example, Gatenby (1991), Olivieri (1996) and Haberman and Pitacco (1999)).

Aspect (ii) has emerged in rather recent years. Mortality trends at adult and old ages reveal decreasing annual probabilities of death (see for example Benjamin and Soliman (1993), Macdonald (1997), Macdonald et al. (1998)). In particular, the following phenomena are observed in populations including both healthy and disabled lives:

(1) an increasing concentration of deaths around the mode of the density function of the future lifetime distribution (the so-called "curve of deaths");

(2) a forward shift of the mode of the curve of deaths.

These changes clearly affect any cover involving lifetime benefits. In particular, the former aspect reduces uncertainty since with higher probability the actual duration of life might coincide with its mode; on the contrary, the latter increases the risk inherent in the management of a policy, the magnitude of the above mentioned shift being unknown.

In the case of health covers, such as LTC, risk emerges further from uncertainty concerning the time spent in the disability state. Actually, when living benefits are paid in case of disability, it is not merely important how long one lives, but also how long he/she lives in a condition of disability.

Although it is reasonable to assume a relationship between mortality and morbidity, the relevant definition is difficult due to the complexity of such a link and to the impossibility of defining and measuring disability objectively. Three main theories have been formulated about the evolution of senescent morbidity (as pointed out, for example, in Swiss Re (1999)).

(i) "Compression theory" (see Fries (1980)): chronic degenerative diseases will be postponed until the latest years of life because of medical advances. Assuming there is a maximum age, these improvements will result in a compression of the period of morbidity.

(ii) "Pandemic theory" (see Gruenberg (1977) and Kramer (1980)): the reduction in mortality rates is not accompanied by a decrease of morbidity rates; hence, the number of disabled people will increase steadily.

(iii) "Equilibrium theory" (see Manton (1982)): most of the changes in mortality are related to specific pathologies. The onset of chronic degenerative diseases and disability will be postponed and the time of death as well.

The scenarios depicted by the above mentioned theories produce rather different consequences for the insurer; in particular, Compression theory suggests optimistic views, whilst Pandemic theory pessimistic ones. The deep differences among such theories imply a high level of uncertainty about the evolution of senescent disability. The adoption of projected tables for the evaluation of insured benefits seems then appropriate. However, since the three theories imply rather different scenarios, the mentioned uncertainty should be included in the actuarial model used for evaluating benefits.

It should be stressed that uncertainty in future mortality and senescent disability trends implies the risk of systematic deviations from the scenario used to calculate expected values (and in particular premiums and reserves), hence a model risk. So, the demographic risk inherent in LTC covers consists of a random fluctuation component and a systematic deviation component as well.

(b) Projected scenarios. In order to appraise the risk inherent in LTC covers, uncertainty of the future evolution of mortality and disability must be explicitly taken into account (see Ferri and Olivieri (2000)). To this aim, several scenarios must be considered, each one including a specific projection of mortality and disability trends.

It must be pointed out that the traditional actuarial approach to demographic projections consists in extrapolations of (recent) trends as far as these can be perceived from observed data. On the contrary, the approach adopted in this paper (as well as by Ferri and Olivieri (2000)) leads to models expressing the basic characteristics of the evolving scenario in which demographical changes take place. In this latter approach, the use of analytical laws for mortality and disability rates is required, whose parameters are functions of the calendar year. The adequacy of the projection model can be checked comparing the behaviour of some quantities with the scenario characteristics suggested by the three theories mentioned in point (a). So, possible scenarios have been defined (and their adequacy tested) in terms of the evolution of the expected time spent in the healthy state and in the disability state.

Let us consider a person in state i at time t, where t denotes the time elapsed since policy issue. In a non-projected multistate context, denote by $\bar{e}_{ij}(t)$ the time expected to be spent in state j from time t, i.e. from age $x+t$ (with x age at entry). We have:

$$\bar{e}_{ij}(t) = \int_t^{\infty} P_{ij}(t,u)\, du \tag{3.1}$$

In particular, referring to the three-state model of figure 2.1, the following quantities can be defined

- $\bar{e}_{11}(t)$ expected time spent in the healthy state for a healthy person, i.e. healthy life expectancy for a healthy person;
- $\bar{e}_{12}(t)$ expected time spent in the disability state for a healthy person, i.e. disability life expectancy for a healthy person;
- $\bar{e}_{22}(t)$ life expectancy for a disabled person.

Note that the total life expectancy for a healthy life is

$$\bar{e}_{1.}(t) = \bar{e}_{11}(t) + \bar{e}_{12}(t) \tag{3.2}$$

In a projected context, functions depend on the calendar year. Let y denote the calendar year in which the person enters insurance. So, transition probabilities $P_{ij}(t, u; y)$ (instead of $P_{ij}(t, u)$) and expected values $\bar{e}_{ij}(t; y)$ (instead of $\bar{e}_{ij}(t)$) shall be considered. In this framework, the theories mentioned in point (a) can be expressed in terms of the evolution of life expectancy, for any t, as follows:

(i) $\bar{e}_{1.}(t; y)$ increases as y increases, with a major contribution (in relative terms) from $\bar{e}_{11}(t; y)$;

(ii) $\bar{e}_{1.}(t; y)$ increases as y increases, with a major contribution (in relative terms) from $\bar{e}_{12}(t; y)$;

(iii) $\bar{e}_{11}(t; y)$ and $\bar{e}_{12}(t; y)$ increase as y increases, at similar rates.

In what follows, we focus on one generation of insureds only, entering the LTC cover (at age x) in the same year. Hence, the calendar year y can be omitted in the notation.

In numerical evaluations, we have represented mortality in terms of the Weibull law, since it reflects more easily than other laws some specific future trends of mortality (see Olivieri and Pitacco (1999)). More precisely, we have assumed (for a generation entering insurance in a given year y, for example the current year):

$$\mu_{13}(t) = \frac{\beta}{\alpha}\left(\frac{x+t}{\alpha}\right)^{\beta-1} \qquad \alpha, \beta > 0 \tag{3.3}$$

$$\mu_{23}(t) = (1+\gamma)\,\mu_{13}(t) \qquad \gamma \geq 0 \tag{3.4}$$

Note that according to (3.4) mortality of disabled lives is supposed to keep higher than that of healthy people. Although specific data at this regard are lacking, such hypothesis seems quite reasonable.

As far as disability is concerned, medical data suggest an exponential behaviour of inception rates; hence we have assumed the Gompertz law

$$\mu_{12}(t) = \eta\, e^{\lambda\,(x+t)} \qquad \eta, \lambda > 0 \tag{3.5}$$

With reference to male people aged 65 in the current year, following Ferri and Olivieri (2000) we have adopted the five scenarios listed in table 3.1. Scenario H_C has been taken as a starting point for the construction of projected scenarios $H_1, H_2, \ldots, H_5$, since it comes from cross-sectional observations of mortality and disability of elderly people. In particular, $\mu_{12}^{(H_C)}(t)$ (with obvious meaning of the symbol) comes from appropriate transformations of OPCS survey data on the prevalence of disability among adults (see Gatenby (1991) and Olivieri (1996)).

The five projected scenarios have been built with reference to their impact on the expected time spent in the healthy and in the disability state. Table 3.2 shows the evaluation of life expectancies $\bar{e}_{11}(0)$, $\bar{e}_{12}(0)$, $\bar{e}_{1.}(0)$ and $\bar{e}_{22}(0)$ under each set of hypotheses, together with the changes of such quantities with respect to scenario H_C.

	α	β	γ	η	λ
H_C	82	7	0.1	8.27e-06	0.095599
H_1	83.5	8	0.1	1.08e-05	0.090437
H_2	85.2	9.15	0.1	1.08e-05	0.090437
H_3	85.2	9.15	0.1	8.27e-06	0.095599
H_4	85.2	9.15	0.1	5.75e-06	0.102944
H_5	87	10.45	0.1	5.75e-06	0.102944

Table 3.1 – Parameters of the transition intensities

Firstly, note that it has been assumed that total life expectancy will increase anyhow. This is suggested by mortality trends in populations that include both healthy and disabled people. In absolute terms, the changes in $\bar{e}_{1.}(0)$ mainly depend on the changes in $\bar{e}_{11}(0)$. Therefore, consequences suggested by Compression, Equilibrium and Pandemic theory must be checked by looking at the relative contributions of $\bar{e}_{11}(0)$ and $\bar{e}_{12}(0)$ (as shown, for example, by the quantities $\bar{e}_{1j}^{(H)}(0)/\bar{e}_{1j}^{(H_C)}(0) - 1$, $j = 1, 2$, where $\bar{e}_{1j}^{(H)}(0)$ is the life expectancy calculated according to the generic scenario H).

For the insurer, H_1 represents a scenario that should involve lower costs than the others for two reasons. Firstly, there is a slight increase in total expected life. Secondly, the change in the time expected to be spent in the healthy state is bigger in percentage than the one related to disability, which actually falls down to a negative value. This evolutionary hypothesis has therefore been chosen because it seems one of the best suited to represent consequences depicted by Compression theory. On the other side, H_5 represents the scenario with presumably the highest costs involved since it assumes a fall in mortality rates accompanied by a substantial rise in disability rates, which imply a considerable increase in disability life expectancy as well as total life expectancy. Therefore, this scenario can reasonably express the Pandemic theory evolutionary hypothesis, although the latter does not lead to any particular increase in disability rates. However, taking into account the scope of this paper, this more pessimistic scenario can also reasonably include the risk of an underestimation of transition intensities $\mu_{12}(t)$, due to the fact that they are obtained from medical data and not from insurance experiences. Scenario H_3 assumes a "medium" decrease in mortality rates in respect of H_C. Moreover, no change in respect of H_C has been considered for the disability law. The result is a projection which is somehow intermediate between the above depicted scenarios, with a change in total life expectancies which receives almost equal (relative) contributions from changes in healthy and disability life expectancies. For this reason, scenario H_3 can be considered as reflecting the evolutionary projection of Equilibrium theory. Finally, scenarios H_2 and H_4 depict projections that are intermediate between scenario H_3 and the other two "extreme" scenarios H_1 and H_5.

It must be stressed that an unchanged link between $\mu_{13}(t)$ and $\mu_{23}(t)$ has been assumed in projections. This choice has been suggested by the lack of specific observations on mortality of active and disabled lives; usually, $\mu_{13}(t)$ is set equal to the general population mortality, where both healthy and disabled people are included, and $\mu_{23}(t)$ is slightly increased in its respect. We have adopted this simplification also for the projected functions.

	$\bar{e}_{11}^{(H)}(0)$	$\bar{e}_{12}^{(H)}(0)$	$\bar{e}_{1.}^{(H)}(0)$	$\bar{e}_{22}^{(H)}(0)$	$\frac{\bar{e}_{11}^{(H)}(0)}{\bar{e}_{11}^{(H_C)}(0)}-1$	$\frac{\bar{e}_{12}^{(H)}(0)}{\bar{e}_{12}^{(H_C)}(0)}-1$	$\frac{\bar{e}_{1.}^{(H)}(0)}{\bar{e}_{1.}^{(H_C)}(0)}-1$
H_C	14.428	1.566	15.995	15.307	0.00%	0.00%	0.00%
H_1	15.156	1.435	16.591	15.931	5.05%	-8.41%	3.73%
H_2	16.042	1.563	17.605	16.983	11.19%	-0.25%	10.07%
H_3	15.844	1.749	17.593	16.983	9.82%	11.65%	10.00%
H_4	15.501	2.073	17.574	16.983	7.43%	32.36%	9.88%
H_5	16.577	2.366	18.943	18.397	14.89%	51.05%	18.44%

Table 3.2 – Life expectancies

(c) Assessing the risk of systematic deviations: deterministic versus stochastic approach. Let us consider a monetary function, for example the random present value, at time 0, of future benefits at a portfolio level, $\mathcal{Y}(0)$. Typically, we are interested in events like $\{\mathcal{Y}(0) \leq y\}$, where y denotes a real number. The probability of such an event obviously depends on the hypotheses adopted to represent future mortality and disability, i.e. the scenario assumed.
Denote by H a particular scenario. For any given H, a probabilistic structure conditional on H can be built up, and hence a conditional distribution function

$$F_{\mathcal{Y}(0)}(y|H) = \Pr\{\mathcal{Y}(0) \leq y|H\} \tag{3.6}$$

can be evaluated.
Moreover, for a given (finite) set of scenarios

$$\mathcal{H} = \{H_1, H_2, \ldots, H_s\} \tag{3.7}$$

we can compare the conditional distribution functions $F_{\mathcal{Y}(0)}(y|H_h)$, $h = 1, 2, \ldots, s$. According to this approach, different reasonable scenarios are considered and the relevant calculations are performed. Such an approach is usually called scenario testing. It should be stressed that scenario testing allows for the random fluctuations risk, whereas it provides just a rough information about the systematic deviations risk, typically through ranges for some results (for example: expected values, variances, percentiles, etc). As it does not take explicitly into account the systematic deviations risk, it represents a deterministic approach to the analysis of the demographical risk.
Conversely, a stochastic approach consists in considering each scenario as a possible outcome to which a probability is assigned. Referring to the set $\mathcal{H}$, denote by ρ_h, $h = 1, 2, \ldots, s$, the probabilities respectively attributed to the s scenarios. Then, an unconditional probabilistic structure can be built up, and hence an unconditional distribution function

$$F_{\mathcal{Y}(0)}(y) = \Pr\{\mathcal{Y}(0) \leq y\} = \sum_{h=1}^{s} \rho_h \; F_{\mathcal{Y}(0)}(y|H_h) \tag{3.8}$$

can be evaluated.
It should be stressed that the stochastic approach allows us to incorporate the systematic deviations risk into the actuarial model. In particular, as we will see in the following Sections, in this framework it is possible to point out the importance of the systematic deviations component of demographic risk with respect to the random fluctuations component.

4. SOLVENCY RESERVE

(a) Solvency and solvency reserve. Among the tools available to the insurer to face LTC risks, a primarily role should be played by the allocation of a suitable capital, i.e. tailored to the severity and features of the risks affecting the portfolio. For this reason we refer to such capital as to a solvency reserve; in this sense, its amount must be determined so that a given solvency requirement is satisfied.
Solvency is a key concept in insurance theory and practice; however it is not univocally defined. What is commonly adopted is a stochastic approach to risk analysis. In this framework, solvency is meant as the ability to meet, with an assigned (high) probability, the random future liabilities as described by a realistic (experience based) probabilistic structure.
Future liabilities can be meant in different ways. The traditional approach consists in measuring liabilities through the portfolio reserve, $\mathcal{V}(h)$. However, the portfolio reserve is made up

by the individual reserves (see (2.8)), which are expected values based on a given conservative (and not realistic) scenario. Such scenario may be rather different from the one in which the portfolio will evolve.
An alternative approach involves the random present value of future liabilities, $\mathcal{Y}(h)$, which is compared to the value of assets and investigated within the same scenario (for a deeper discussion on the two approaches, see Olivieri and Pitacco (2000)).
In this paper we adopt the latter approach, given that it allows us to analyse liabilities consistently with assets backing them. Hence the following solvency requirement is considered at time h $(h = 0, 1, \dots)$

$$\Pr\left\{ \bigwedge_{u=h}^{T} A(u) - \mathcal{Y}(u) \geq 0 | A(h), \mathcal{I}(h) \right\} = 1 - \varepsilon \tag{4.1}$$

where T is the time horizon within which the insurer solvency is investigated, ε the accepted probability of ruin, $A(h)$ the initial level of assets (the term "initial" being here referred to the time of investigation) and $\mathcal{I}(h)$ collects all information available at time h about portfolio composition, future interest rates, mortality and disability scenario, etc.
Let n be the maximum residual duration of policies at time h (whence $\mathcal{Y}(n) = 0$). It can be easily shown that

$$\begin{aligned} &\Pr\left\{ \bigwedge_{u=h}^{T} A(u) - \mathcal{Y}(u) \geq 0 | A(h), \mathcal{I}(h) \right\} \\ &= \Pr\left\{ A(T) - \mathcal{Y}(T) \geq 0 | A(h), \mathcal{I}(h) \right\} \\ &= \Pr\left\{ A(n) \geq 0 | A(h), \mathcal{I}(h) \right\} \end{aligned} \tag{4.2}$$

Thus, a long term view is implicitly adopted in requirement (4.1).
Probability (4.2) can be calculated adopting a run-off or a going concern approach in regards to the concept of "portfolio". Actually, the portfolio can be thought of as a "closed" collection of policies already in force, as well as an "open" collection of policies to which also future business contributes.
For a given choice (i.e. accepted level) of the ruin probability ε, under a run-off or going concern approach, condition (4.1) (that according to (4.2) is independent of the time span T) leads to a required initial (total) investment $A^*(h)$; we call $A^*(h)$ the required (or minimum) solvency reserve. Given the portfolio reserve at time h, we define required (or minimum) solvency margin (to be assigned to the portfolio at time h) the quantity

$$M^*(h) = A^*(h) - \mathcal{V}(h) \tag{4.3}$$

(b) Valuation of the required solvency reserve. The minimum solvency reserve has been assessed with reference to a cohort of policies entering insurance at the same time (time 0) and age, with the same amount of benefits, in the same risk class, etc. The insured positions are assumed to be independent under any given demographical hypothesis H.
The entry age (assumed to be the retirement age) is $x = 65$. Scenario H_3 (see Section 3) has been chosen for pricing and reserving; the technical rate of interest is $i = 0.03$ (constant).
The amount of benefits has been determined supposing that the insured, eligible at retirement for a traditional straight life annuity (basic pension) of annual (constant) amount b, switches to an enhanced pension with annual (constant) amounts b_1, $b_1 < b$, when healthy and b_2, $b_2 > b$, when LTC disabled (according to a given definition of LTC disability). The single premium is given by the actuarial value of the basic pension, calculated as follows

$$b \sum_{h=0}^{\infty} v(0,h) \; (P_{11}(0,h) + P_{12}(0,h)) = \pi \tag{4.4}$$

where $v(0,h)$, $P_{11}(0,h)$ and $P_{12}(0,h)$ are determined under the hypotheses mentioned above. For a given b_1, the amount b_2 is found by solving

$$\pi = b_1 \sum_{h=0}^{\infty} v(0,h)\; P_{11}(0,h) + b_2 \sum_{h=0}^{\infty} v(0,h)\; P_{12}(0,h) \tag{4.5}$$

(see (2.6)), where the financial and demographical parameters are the same as in (4.4). We have chosen $b = 100$ monetary units (m.u.), $b_1 = 90$ m.u. and found $b_2 = 221.22$ m.u.
Table 4.1 and 4.2 quote the required solvency reserve and the required solvency margin assessed at time 0 respectively within a deterministic and stochastic approach to the future demographical scenario (results have been obtained through stochastic simulation). In any case, the analysis has been performed disregarding profit; hence no specific safety loading has been included in the single premium or in reserves. In particular, a (constant) investment yield equal to the technical rate of interest ($i = 0.03$) has been assumed.
In Table 4.1 scenario H_3 has been adopted as a representation of the future scenario; since a deterministic approach is followed with regard to future mortality and morbidity, the available information in (4.1) is

$$\mathcal{I}(0) = \{N_1(0), i_h = i = 0.03, H_3\} \tag{4.6}$$

The required solvency margin faces the risk of random fluctuations and given that this is a pooling risk the minimum solvency margin itself decreases, in relative terms, as the size of the portfolio increases. Trivially, the lower is the accepted ruin probability ε, the higher is the required solvency reserve.
For the sake of brevity we do not quote similar results obtained under the other scenarios. We just point out that, as it is quite intuitive, the more severe is the projection, the higher is the minimum solvency margin.
In Table 4.2 a stochastic approach has been adopted. The set $\mathcal{H} = \{H_1, H_2, H_3, H_4, H_5\}$ (see Section 3) has been considered; the following weights have been assigned to the five scenarios: $\rho_1 = \rho_5 = 0.05$, $\rho_2 = \rho_4 = 0.15$, $\rho_3 = 0.6$. The available information in (4.1) is then

$$\mathcal{I}(0) = \{N_1(0), i_h = i = 0.03, (\mathcal{H} = \{H_1, H_2, \ldots, H_5\}, \{\rho_1, \rho_2, \ldots, \rho_5\})\} \tag{4.7}$$

The magnitude of the required solvency margin is higher than in Table 4.1, witnessing the severity of the risk of systematic deviations. The relative value of the required solvency margin slightly decreases with the portfolio size, due to the presence of the risk of random fluctuations; however, contrarily to the deterministic case there seems to be a minimum positive value of the required solvency margin independent of the portfolio size, due to the systematic (i.e. non pooling) risk component.

		ε=0.01		ε=0.025		ε=0.05	
$N_1(0)$	$\mathcal{V}(0)$	$A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$	$A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$	$A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$
100	136,035	151,443	11.326%	149,401	9.825%	147,179	8.192%
200	272,070	295,439	8.589%	291,386	7.100%	288,233	5.941%
300	408,105	435,232	6.647%	431,269	5.676%	426,540	4.517%
400	544,140	575,250	5.717%	571,422	5.014%	566,979	4.197%
500	680,175	715,539	5.199%	709,534	4.316%	705,451	3.716%
600	816,210	853,711	4.594%	849,447	4.072%	844,283	3.439%
700	952,245	993,444	4.326%	987,919	3.746%	981,051	3.025%
800	1,088,280	1,129,484	3.786%	1,125,641	3.433%	1,120,146	2.928%
900	1,224,315	1,272,069	3.900%	1,263,782	3.224%	1,258,378	2.782%
1,000	1,360,350	1,402,735	3.116%	1,396,460	2.654%	1,390,785	2.237%
2,000	2,720,700	2,787,999	2.474%	2,780,099	2.183%	2,769,741	1.802%
3,000	4,081,050	4,174,577	2.292%	4,157,763	1.880%	4,141,550	1.482%
4,000	5,441,401	5,525,220	1.540%	5,512,469	1.306%	5,503,452	1.140%
5,000	6,801,751	6,906,187	1.535%	6,892,375	1.332%	6,876,162	1.094%

Table 4.1 – Required solvency reserve and margin; scenario H_3

$N_1(0)$	$\mathcal{V}(0)$	ε=0.01 $A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$	ε=0.025 $A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$	ε=0.05 $A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$
100	136,035	154,963	13.914%	151,612	11.451%	148,973	9.511%
200	272,070	304,625	11.966%	298,779	9.817%	293,319	7.810%
300	408,105	455,318	11.569%	445,527	9.170%	435,577	6.732%
400	544,140	604,445	11.083%	593,161	9.009%	579,180	6.439%
500	680,175	754,323	10.901%	741,187	8.970%	721,460	6.070%
600	816,210	904,332	10.796%	889,640	8.997%	865,212	6.004%
700	952,245	1,052,567	10.535%	1,037,489	8.952%	1,009,613	6.025%
800	1,088,280	1,203,524	10.590%	1,185,584	8.941%	1,153,909	6.030%
900	1,224,315	1,350,793	10.330%	1,333,604	8.927%	1,294,647	5.745%
1,000	1,360,350	1,501,214	10.355%	1,483,238	9.034%	1,437,263	5.654%
2,000	2,720,700	2,991,104	9.939%	2,965,069	8.982%	2,886,963	6.111%
3,000	4,081,050	4,480,350	9.784%	4,447,603	8.982%	4,296,875	5.288%
4,000	5,441,401	5,965,233	9.627%	5,929,172	8.964%	5,761,719	5.887%
5,000	6,801,751	7,452,583	9.569%	7,412,720	8.983%	7,226,563	6.246%

Table 4.2 – Required solvency reserve and margin; set of scenarios $\mathcal{H}$=$\{H_1,H_2,\ldots,H_5\}$

5. REINSURANCE

(a) The reinsurance arrangement. In the previous Section the solvency reserve has been defined and assessed; however, ways of financing it have not been discussed. From (4.3) it emerges that at least the required solvency margin must be funded with shareholders' capital. Moreover, equity capital higher than the required solvency margin is needed in case a portfolio reserve higher than what funded by premiums must be set up.
The insurer can, partially, face LTC risks through reinsurance; a proper reinsurance arrangement could, intuitively, reduce the required solvency reserve and hence indirectly help in financing the solvency reserve itself. In the following we investigate this aspect.
We consider a stop-loss-like reinsurance arrangement, where the loss is defined according to the comparison of the solvency reserve (i.e. assets, inclusive of the solvency margin) and the portfolio reserve. In order to specify the characteristics of the reinsurance arrangement, first of all it must be stressed that the solvency requirement (4.1) leads to a long term capital allocation strategy, as it emerges from (4.2). The required solvency reserve determined at time h, $A^*(h)$, represents therefore a capital facing long term needs; on the contrary, short term needs such as, for example, liquidity problems or the funding of the portfolio reserve are disregarded in this setting. The design of the reinsurance arrangement relies on the same long term view. Hence, reference is made to the assets strictly facing obligations to the insured. Moreover, when defining the intervention of the reinsurer it must be considered that a reinsurance treaty should refer to some "official" quantities, i.e. quantities which are or can be certified, as explained below.
We assume that a time interval is fixed, say z; every z years the reinsurer intervenes if assets are lower than a given portion of the portfolio reserve. Let $R(s)$ be the payment made by the reinsurer at time s ($s = z, 2z, \ldots$). The asset process is now described as follows

$$A(h) = A(0)\ \frac{1}{v(0,h)} - \sum_{u=0}^{h-1} B(u)\ \frac{1}{v(u,h)} + \sum_{u=1}^{[h/z]} R(uz)\ \frac{1}{v(uz,h)} \tag{5.1}$$

(with $[c]$ integer part of number c). The payment from the reinsurer is then defined as follows

$$R(s) = \max\{(1-r)\ \mathcal{V}(s) - A(s), 0\} \tag{5.2}$$

where r determines the reinsurance excess limit.
Some further remarks on the arrangement described above. First of all, note that in (5.2) the insurer liabilities are defined in terms of the portfolio reserve, $\mathcal{V}(s)$, and not of the present value

of future obligations, $\mathcal{Y}(s)$; this is due to the need, mentioned above, that the intervention of the reinsurer relies on official quantities (to this regard, it must be pointed out that the value of assets, although affected by the insurer capital allocation strategy, can also be certified). Moreover, the time interval which defines the interventions of the reinsurer also allows for some revisions of the reinsurance premium (which is likely to be calculated according to the percentile principle). Given that we aim at investigating the solvency reserve (under the hypotheses discussed above), we disregard the pricing of the reinsurance contract.

(b) Solvency reserve. According to the obligation of the reinsurer, the future liabilities of the insurer can be defined as follows

$$\mathcal{Y}(h) = \sum_{u=h}^{\infty} B(u)\ v(h,u) - \sum_{u=[h/z]+1}^{\infty} R(uz)\ v(h,uz) \tag{5.3}$$

LTC risks are not fully met by the reinsurance treaty; hence a proper capital (intuitively lower than what assessed in Section 4) should be allocated to the portfolio. We still adopt the solvency requirement (4.1) (equations (4.2) hold as well), with assets and liabilities defined as in (5.1) and (5.3), respectively.
Numerical evaluations, through stochastic simulation, led to the results quoted in Tables 5.1–5.2 and in Figures 5.1–5.2. A comparison between Table 5.1 and Table 5.2 involves considerations similar to those in Section 4. It is important, on the other hand, to compare Table 5.1 with Table 4.1 and Table 5.2 with Table 4.2. The relief deriving from the reinsurance arrangement is evident, in particular as far as the systematic risk is concerned (actually, the reduction of the required solvency margin is higher in the stochastic setting than in the deterministic one).

$N_1(0)$	$\mathcal{V}(0)$	ε=0.01 $A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$	ε=0.025 $A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$	ε=0.05 $A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$
100	136,035	148,242	8.973%	143,893	5.777%	141,547	4.052%
200	272,070	289,389	6.366%	285,040	4.767%	280,310	3.029%
300	408,105	428,857	5.085%	421,838	3.365%	417,642	2.337%
400	544,140	569,660	4.690%	563,061	3.477%	557,720	2.496%
500	680,175	709,472	4.307%	702,796	3.326%	696,960	2.468%
600	816,210	847,491	3.832%	840,014	2.916%	832,842	2.038%
700	952,245	987,645	3.718%	979,177	2.828%	971,166	1.987%
800	1,088,280	1,126,122	3.477%	1,119,179	2.839%	1,110,253	2.019%
900	1,224,315	1,264,675	3.296%	1,255,596	2.555%	1,247,508	1.894%
1,000	1,360,350	1,400,633	2.961%	1,393,919	2.468%	1,382,323	1.615%
2,000	2,720,700	2,779,294	2.154%	2,764,646	1.615%	2,753,354	1.200%
3,000	4,081,050	4,154,293	1.795%	4,137,050	1.372%	4,123,622	1.043%
4,000	5,441,401	5,520,746	1.458%	5,504,267	1.155%	5,491,144	0.914%
5,000	6,801,751	6,892,693	1.337%	6,877,434	1.113%	6,863,396	0.906%

Table 5.1 – Required solvency reserve and margin with reinsurance (r=0.1); scenario H_3

When interpreting the results, it should be remembered that short term capital allocation problems are not dealt with in this paper; for this reason the problems of reinsurance pricing and funding a portion or the whole reserve at time h, $h = 1, 2, \ldots$, have been disregarded (we recall that the problem of funding the reserve has been disregarded also in Section 4, where the solvency requirement adopted leads to a long term approach to capital allocation). Future work should concern also these short term capital needs.

$N_1(0)$	$\mathcal{V}(0)$	ε=0.01		ε=0.025		ε=0.05	
		$A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$	$A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$	$A^*(0)$	$\frac{M^*(0)}{\mathcal{V}(0)}$
100	136,035	150,683	10.768%	145,869	7.229%	142,155	4.499%
200	272,070	293,890	8.020%	287,946	5.835%	282,360	3.782%
300	408,105	438,604	7.473%	427,584	4.773%	420,794	3.109%
400	544,140	586,254	7.740%	570,009	4.754%	561,018	3.102%
500	680,175	731,686	7.573%	711,208	4.563%	700,925	3.051%
600	816,210	877,270	7.481%	851,064	4.270%	838,240	2.699%
700	952,245	1,024,419	7.579%	991,127	4.083%	978,339	2.740%
800	1,088,280	1,170,678	7.571%	1,128,936	3.736%	1,116,925	2.632%
900	1,224,315	1,318,309	7.677%	1,268,858	3.638%	1,255,010	2.507%
1,000	1,360,350	1,463,500	7.583%	1,406,826	3.416%	1,393,279	2.421%
2,000	2,720,700	2,939,359	8.037%	2,798,248	2.850%	2,773,864	1.954%
3,000	4,081,050	4,413,692	8.151%	4,189,228	2.651%	4,157,014	1.861%
4,000	5,441,401	5,897,944	8.390%	5,577,051	2.493%	5,539,061	1.795%
5,000	6,801,751	7,364,190	8.269%	6,962,374	2.362%	6,916,883	1.693%

Table 5.2 – Required solvency reserve and margin with reinsurance (r=0.1); set of scenarios $\mathcal{H}$=$\{H_1,H_2,\ldots,H_5\}$

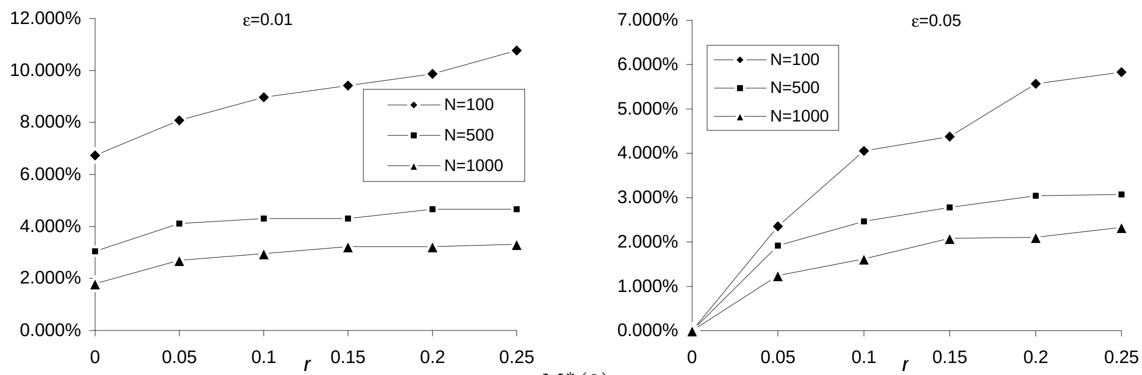

Figure 5.1 – $\frac{M^*(0)}{\mathcal{V}(0)}$; scenario H_3

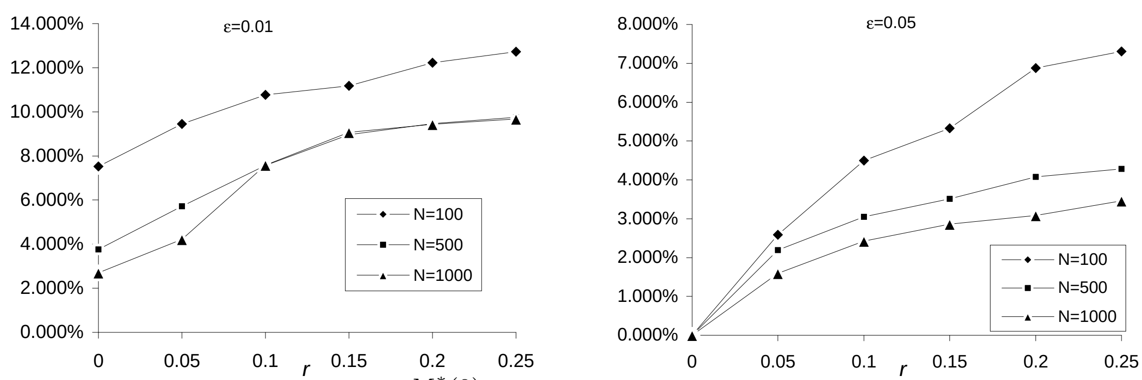

Figure 5.2 – $\frac{M^*(0)}{\mathcal{V}(0)}$; set of scenarios $\mathcal{H}$=$\{H_1,H_2,\ldots,H_5\}$

6. FINAL REMARKS

The risk coming from uncertainty in future demographical trends and the relevant impact on LTC covers have been analysed, referring to a portfolio of enhanced pensions. To face this risk, proper tools must be used. In this paper, the allocation of solvency margin and a particular reinsurance arrangement have been dealt with. The numerical results obtained suggest some conclusions.

(i) The LTC enhanced pension is affected by a high riskiness, mainly due to possible systematic deviations from the projected scenarios assumed as pricing and reserving bases. Consequently, the required solvency margin implies the allocation of important resources.

(ii) A stop-loss-like reinsurance arrangement can help in keeping lower the required solvency margin and hence the need of proper capital. Of course, the choice of an appropriate reinsur-

ance strategy must be based, in particular, on the level of the reinsurance premium. Further research is needed to build up a model leading to an "optimal" choice, taking into account the cost of reinsurance as well as the cost of capital allocation.
(iii) Partially ceding risk inherent in an enhanced pension portfolio (or other LTC covers) can constitute a good compromise even for pension schemes (or sickness funds) providing this type of benefits. In this sense, the role of insurers (acting as "reinsurers") on healthcare markets can be enhanced.

REFERENCES

Benjamin J and AS Soliman (1993), *Mortality on the move*, Actuarial Education Service, Oxford

Ferri S and A Olivieri (2000), Technical bases for LTC covers including mortality and disability projections, *Proceedings of the XXXI International ASTIN Colloquium*, Tokyo: 135–155

Fries JF (1980), Aging, natural death and the compression of morbidity, *N. Engl. Journal of Medecine*: 303

Gatenby P (1991), *Long Term Care*, Paper presented to the Staple Inn Actuarial Society, London

Gruenberg EM (1977), The failure of success, *Milbank Memorial foundation Q./Health Society*: 55

Haberman S and E Pitacco (1999), *Actuarial models for disability insurance*, Chapman & Hall/CRC, Boca Raton

Hoem JM (1969), Markov chain models in life insurance, *Blätter der deutscen Gesellschaft für Versicherungsmathematic*, **9**: 91–107

Hoem JM (1988), The versatility of the Markov chain as a tool in the mathematics of life insurance, *Transactions of the 23rd International Congress of Actuaries*, Helsinki, **R**: 171–202

Kramer M (1980), The rising pandemic of mental disorders and associated chronic diseases and disabilities, *Acta Psychiatrica Scandinavica*: 62

Macdonald AS (ed.) (1997), *The Second Actuarial Study of Mortality in Europe*, Groupe Consultatif des Associations d'Actuaires des Pays des Communautes Europeennes, Oxford

Macdonald AS et al (1998), An international comparison of recent trends in population mortality, *British Actuarial Journal*, **4**: 3–141

Manton KG (1982), Changing concepts of morbidity and mortality in the elderly population, *Milbank Memorial foundation Q./Health Society*

Olivieri A (1996), Sulle basi tecniche per coperture "Long Term Care", *Giornale dell'Istituto Italiano degli Attuari*, **49**: 87–116

Olivieri A and E Pitacco (1999), Funding sickness benefits for the elderly, *Proceedings of the XXX International ASTIN Colloquium*, Tokyo, 135–155

Olivieri A and E Pitacco (2000), Solvency requirements for life annuities, *Proceedings of the International AFIR 2000 Colloquium*, Tromsø547–571

Pitacco E and A Olivieri (1997), *Introduzione alla teoria attuariale delle assicurazioni di persone*, Quaderni dell'Unione Matematica Italiana n. 42, Pitagora Editrice, Bologna

Swiss Re (1999), *Long Term Care Data Pack*, London

Wolthuis H (1994), *Life insurance mathematics (The Markovian model)*, CAIRE Education Series, no. 2, Bruxelles