# STATISTICAL METHODS IN ESTIMATION OF AND INSURANCE AGAINST NATURAL RISKS

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Abstract

The paper describes fundamentals of statistical methods of estimation of natural risks relying on the theory of binary type systems with equivalence relation and statistical analysis of trains of insurance actions in insurance against natural risks. Algorithms and programs for quantitative analysis of aerial photographs and satellite images of the Earth's surface realizing these methods of assessment of natural risks were developed for digital systems. They can be used for monitoring of water areas, fire and seimically dangerous regions, etc.

**Keywords**

Statistical methods, binary type system with equivalence relation, natural risks, insurance actions.

## 1. Introduction

From the early 1990s, many researchers all over the world have been deeply involved in studies of natural risks using probabilistic methods. The major directions of this research are prediction of natural accidents (such as flood, wind, earthquake, etc.) in assessment of risks, synergic effects of natural accidents on the environment, estimation of vulnerability and damage caused by natural and man-induced accidents, theory and practice of assessment of natural risks, insurance and risk management.

The goal of this work was to develop fundamentals of statistical estimation of natural risks on the basis of the probability theory (sensitive to system attributes of objects**)** and also to elaborate the procedure for computer-aided measurements and visualization of these characteristics. The need for such characteristics is felt by the researchers engaged in identification and prediction of natural accidents and estimation of damage to industrial objects and population, by using, first of all, pattern recognition and quantitative analysis of aerial photographs and satellite images of the Earth’s surface.

Modern image analyzers either measure morphometric characteristics of the images or yield the integral characteristic of the image, without giving information on topological characteristics of the images. However, it is quite evident that it is just topology, and in particular a mutual arrangement of elements in the image, that determines to a large degree the properties of the objects studied. For this reason, from the theory of binary type systems with equivalence relation, we have introduced into our consideration informational characteristics [1] sensitive to the analysis and quantitative estimation of the system attributes of aerial photographs and satellite images of the Earth, which are among the most efficient tools used to obtain quantitative estimates of the probability of natural cataclysms. We have also developed methods for statistical analysis of the structure of insurance actions in insurance against natural risks.

## 2. Binary type systems with equivalence relation

The notion of the system has been known from ancient times, but only modern science employs the system approach to organization of the organic and inorganic nature [4]. However, there are still no strict unambiguous definitions of the system. The difficulty in elaboration of such definitions is that distinct formal signs or parameters by which one can distinguish a system from a nonsystem and one system from another are needed. For the system to correspond to its name, it must exhibit a constancy with respect to time and space. At present, there exists either a very generalized mathematical definition or a qualitative verbal definition of the system characterizing the system from the point of view of functionality, morphology, and informatics. This situation limits the analyzability of the system attributes of multicomponent objects, such as aerial photographs and satellite images, in estimation and prediction of natural risks.

A natural way to overcome these difficulties is to try to substantially reduce the considered set of the multicomponent objects related to systems to a subset, for the elements of which an axiomatic definition of the system can be given, and informational characteristics of the synthesis and functioning of such a system can be formed.

In this work, such a reduction was performed. From the set of material elements with relation (X,R), where X is the set of elements and R is the relation, only the sets A of finite power (referred to as Π-systems), for the elements of which the binary relation R satisfies the conditions of reflexivity, symmetry, and transitivity were considered. The fulfillment of these conditions leads to division of the set A by the equivalence relation R into the equivalence classes which do not intersect with each other. Since the power of these sets is high, we restrict ourselves to the case of metric spaces, when the relation R between elements is chosen to be their distance function in the Euclidean metric (image analysis) and in the metric based on determination of quantitative characteristics of a dependence (random processes).

When Π-systems satisfy the axioms of reflexivity, symmetry, and transitivity, each Π-system has a finite, strictly defined number of states. The state will be understood here as the structure of the graph (fully nonoriented, with multiplicity 1) that describes the Π-system. Since the Π-system is a binary type system with equivalence relation, its states are described by a complete graph or union of a finite number of complete subgraphs. The total number of states of the Π-system with a low power corresponds to the known theoretical numerical function for which Hardy and Ramanujan derived the asymptotic formula. We derived an exact formula for the Π-system with a power of not more than 8. Our Π-system is a complex system rather than a system whose characteristic is an additive composition of the characteristics of its components.

The material elements that form the Π-system can have both stochastic and nonstochastic nature. Fulfillment of the property of reflexivity and symmetry by the relation R depends exclusively on the nature of the elements, but the property of transitivity is satisfied only if the conditions expressed in terms of entropy are met. These conditions are formulated as limitations imposed on informational characteristics of the Π-systems.

If we consider elements of a stochastic nature $A_i$, $A_k$, $A_j$, where $i \neq k \neq j$, it is quite natural to choose the relation R for these elements in the form of the stochastic interdependence. Evidently, it satisfies the conditions of reflexivity and symmetry. We have shown that, to satisfy transitivity, it is sufficient that the condition

$$H(A_i, A_j) \leq H(A_i \mid A_k) + H(A_j \mid A_k),$$

be fulfilled. Here, $H(A_i | A_k)$, $H(A_j \ | A_k)$ are the average conditional entropies of elements $A_i$ and $A_j$, and $H(A_i, A_j)$ is their joint entropy, i.e., the Π-system is formed if, for any three elements of this system, a decrease in the sum of conditional entropies of two elements caused by information on the third element does not make this sum less than the joint entropy.

Let $A_i$ be elements of a nonstochastic nature, however, they are such that the metric (pseudometric) $D_{ij}$ can be introduced for each pair of them, $A_i$ and $A_j$. Let us assume that the binary relation R for these elements is the condition $D_{ij} \leq C = \text{const}$. Apparently, this understanding of R satisfies the conditions of reflexivity and symmetry.

Consideration of the conditions under which the property of two elements "to be separated by a distance smaller than C" is the equivalence condition is beyond the scope of this treatment. Let us give here an example of the space satisfying the axioms of reflexivity, symmetry, and transitivity, which is important for understanding of further discussion. Note that fulfillment of transitivity of the initial relation can lead to equivalence relations caused by the metric. By using this approach and varying constant C, algorithms for pattern recognition, which is the basic theoretical tool in image analysis, can be developed. In particular, the following sufficient condition can be given: the distance between any two points is either smaller than C or larger than two C. This condition is compatible with nontriviality of division into classes, but, no doubt, it should be checked.

Let us assume that this space is a system of equidistant points. In particular, if $A_i$ are the elements in the plane, the relation R corresponds to the Euclidean distance between them not larger than constant C. Then the Π-system is formed if any three neighboring elements are at the vertices of an equilateral triangle with side C. In other words, the plane is covered with equilateral triangles in such a way that the distance between any vertices belonging to different triangles is larger than C by at least a small positive value. In the three-dimensional Euclidean space, this condition corresponds to tetrahedra with equal sides C, and in the n-dimensional space these will be simplexes with the edges equal to C. If the distance between two vertices of neighboring equilateral triangles exceeds C by the value much less than C, the space characterized by a uniform and isotropic distribution of elements results. The uniformity and isotropy mean here that the elements are distributed in space in such a way that any fixed unit region of this space contains the same number of elements.

Actual structures rarely exhibit strict uniformity and isotropy; however, they can approximate these conditions to one degree or another. Estimate of this degree is a quantitative measure of the informational characteristic of the synthesis of a static П-system. The algorithm for computer-aided measurements of the degree of uniformity and isotropy of distribution of elements in images is as follows:

1. The initial half-tone image is converted into a binary image with a large number of structural elements (for instance, the image of a polluted water surface of a river where smears are elements of the image).
2. Then the computer calculates the size $X_j$ of each element of the image as a diameter of the circle of an equal area.
3. The elements of each range of sizes are considered successively, positions of their centers of gravity are calculated, and then the distance between centers of gravity for each three neighboring elements of one size range is measured. After this the average distance for the distribution of these distances is found.
4. For each range of sizes, the field of the image is filled with regular triangles with the side equal to the calculated average distance between the centers of triads of the elements (a model of a uniform and isotropic structure with the maximum possible number of elements of a given range of sizes determined from the calculated average distance between elements is constructed).
5. By summarizing the data for all ranges of sizes, a histogram of an actual distribution of the number of elements and a histogram of the "predicted" distribution in the number of elements in the case of uniform and isotropic filling of the image is obtained.
6. The actual $F_n(x)$ and "predicted" $F(x)$ distributions of the number of elements in the case of uniform filling of the image are compared in the Kolmogorov metric. The value of e = $\sqrt{n}\left|F_n(x)-F(x)\right|$ is obtained. By using the obtained parameter e, the tabulated value of Kolmogorov function K(e) is found. A quantitative estimate of the degree of uniformity and isotropy of the image is the value of U = 1 - K( e).

For instance, monitoring of the Earth's surfaces covered by water or forests by using aerial photographs or satellite images gives a series of images. The developed methods and procedures of computer-aided analysis of a large series of such images (preliminarily scanned to convert the images into the digital form) allow quantitative estimation of the dynamics of variations in the condition of the Earth's surface by calculating the degree of uniformity and isotropy of each image as the integral characteristic of the estimate of the natural risk .

The methods and algorithms described here were realized as software for the "scanning electron microscope – computer" digital system operating at M.V. Lomonosov Moscow State University.

**3. Quantitative analysis of trains of actions in insurance against natural risks**

One of basic assumptions of the known dynamic models of the risk theory for a sequence of actions during a fixed time interval which is a realization of a random process of occurrence of actions from individual insurance contracts of high-risk projects is the independence and the same distribution of the time intervals between the moments the actions are brought. However, this assumption is not evident and requires checking based on the statistical data on natural risks. Earlier, we suggested a statistical method and algorithm for such checking and also described an example of their use [2,3,5].

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