M.A.R.C.: AN ACTUARIAL MODEL FOR CREDIT RISK

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ABSTRACT

In this paper an actuarial model to quantify and manage credit risk is presented. The model has been named M.A.R.C. (Actuarial Model for Credit Risk).
With the M.A.R.C. model we want to propose a general scheme useful for applying risk theory methods to credit portfolios.
To this purpose we have built up a model based on stochastic simulation (Monte Carlo method); this technique gives to the model the necessary flexibility.
Some numerical exercises are presented.
In the last part of the paper some statistical methods to evaluate and manage the output of the proposed stochastic procedure are described and applied.

KEY WORDS: actuarial models for credit risk; default risk; risk theory; stochastic simulation.

1. Introduction

The academic world, the financial markets and the control authorities have started to pay attention to the mathematical models for credit risk. Some important crises of the bank systems of several industrial countries have accelerated this process.
These models allow the user to measure and quantify credit risk both at portfolio and contributory level. It is easy to understand that these models may largely affect the lending business and accelerate the shift to "active" credit portfolio management.
This paper illustrates an actuarial approach in order to measure the so-called credit risk, to be intended only as "default" risk; we not deal with "migration" or market risk[1].
Our scheme is a statistical model of credit (default) risk. It doesn't make assumptions about the causes of default of the borrowers[2]. It considers default rates as random variables to model the uncertainty in the value of future rates of default.
The correlation between the various default rates (due to the state of the economy) is taken into account by using default rates distributions and classifying borrowers into sectors instead of using explicitly correlations into the model[3].

[1] Migration risk is the risk deriving from changes of solvency of the borrowers: when the credit rating of a borrower gets worse the value of its exposure diminishes; market risk is the risk generated by the variation in the structure of interest rates in the markets.
[2] This situation is very near to that taken in market risk monitoring to calculate the financial value at risk where there's no attempt to model the causes of the movements of prices.

Similar considerations are developed for the structure of recovery rates which are assumed to be correlated with default rates.
By applying the stochastic simulation as done in risk theory (see [4]) the scheme makes it possible to calculate a full loss distribution for a credit exposure portfolio.

1.1 A classification of the existing models for credit risk

The existing models for credit risk may be classified in some categories (see [3], [6]):

- VaR models (CreditMetrics, PortfolioManager, ExVaR, CreditVaR I, KMV);
- Econometric models (CreditPortfolioView);
- Actuarial models (Credit Risk Plus).

The approach proposed in this paper may be considered as a third class model (the actuarial one) and the model structure is intended to be immediately comparable with the Credit Risk Plus model[4].

1.2 Common elements of the various existing models

Even if the credit risk models may appear very different, it is possible to find out similarities; they are the followings (see [6]):

- *Joint default behaviour*. Default rates vary over the time because of the changing situation of the macroeconomic variables. So each borrower's default rate is determined by the "state of the world". The degree of "correlation" in the portfolio is reflected by conditional borrower's default rates which vary according to the different states;
- *Conditional distribution of portfolio default rate*. For each state, the conditional distribution of a homogeneous sub-portfolio's default rate may be calculated as if borrowers are independent.
- *Aggregation*. The unconditional distribution of portfolio defaults is obtained by combining homogeneous sub portfolio's conditional default rate distributions in each state.

2. The model variables

In order to measure and manage the credit (default) risk it is necessary to build up a mathematical model. The model is aimed at quantifying the credit loss for a bank deriving from the default of its debtors.
Therefore such a model needs some input data that are easily specified; they are the credit exposure of each debtor; the default rate of each position; the recovery rate for the single borrower. Indeed, it is necessary to detect the "correlation" between the joint defaults of the various debtors and the likely presence of a relationship between the default rates and the recovery rates.
The variables, that we have up to now mentioned, may be described as follows.

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[3] It is generally accepted that these correlations change rapidly in the time and that their estimates can become quickly inadequate.
[4] It uses a portfolio approach and mathematical techniques applied in the insurance industry and moves from the elimination of some hypotheses of the Credit Risk Plus to come to the same aim: the measurement and the management of credit risk.

2.1 The credit exposure

The term "credit exposure" means the value of a loan claimed by a bank from a single borrower.
This quantity is an amount of money and it depends on the characteristics of the credit; for example if we consider an instalment loan, the credit exposure may be calculated by discounting the future payments by means of deterministic interest rates[5].

2.2 The default rates

A default rate, representing the likelihood of a default event should be assigned to each borrower.
For listed corporations the default rates may be obtained by using the default statistics published by the rating agencies. These statistics classify the borrowers by rating category and assign a default rate to each category.
Obviously as not all the borrowers of a bank are "rated" it is necessary to use some other systems to calculate the default rates for these debtors. The literature and the professional practice have developed some useful techniques (see [1], [3] and [7] amongst the others).
It is important to note that the default rates vary over the time. Significant variations may occur from year to year because of the interrelation with the main macroeconomic variables of the national economies (see [2] and [11]). In periods of economic growth they will be lower; during periods of economic recession, the number of defaults may be much higher.
This imply that the default rate may be treated as a random variable.

Figure 1 A possible path of default rate

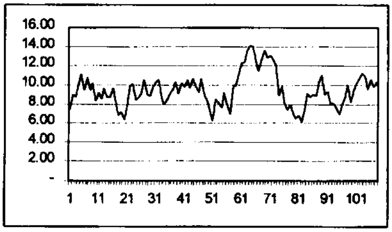
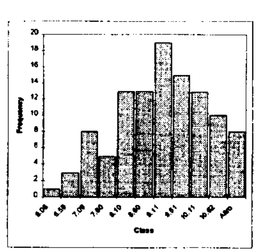


Figure 2 Frequency of default rate values



This variation over the time affects all the categories of debtors but not in the same way: for example, when there are troubles in the economy, the default rate of the construction sector usually grows faster than any other one[6].

[5] The using of a deterministic structure is coherent with our final aim that is the valuation of the credit risk only; to evaluate the market risk we can easily introduce a stochastic dynamic of the interest rates.
[6] There are several ways to estimate the expected rate of default for a borrower; the most important can be classified into three categories: subjective techniques, scoring approaches, market models. In the first class we can comprise all those analyses that are conducted when someone requests a loan from a bank: calculation of indexes about present and future economic and financial condition of the credit applicant, analysis of the management of the society and so on; the result is a qualitative judgement of the solvency of the borrower; the second class uses the weighting of a specified set of balance ratios to arrive at a numeric expression of the default probability of the firm; the third category uses techniques derived from the option theory or from the equilibrium models of stock/bond prices to obtain a numeric measure of the solvency. These arguments are dealt with [10].

2.3 Recovery rates

When a borrower defaults, a bank suffers a loss equal to the credit exposure of the debtor less a recovery amount. This latter amount is usually due to the existence of some real guarantees or foreclosure, liquidation, restructuring of the defaulted borrower or the sale of the claim. Recovery rates, like default rates, are subject to significant variations over the time and they differ according to the category of debtors.

2.4 The time horizon

The choice of time horizon is a key decision to build up a model for evaluating credit risk.
In general, the existing models use a constant time horizon (such as one year) or operate in a multi-period horizon.
By considering that actions to mitigate credit risk may be realised in one year and that the financial exercise and the accounting period are normally coincident with a single year, it is sensible to consider one year horizon.

3. The loss for the single borrower and for the whole portfolio

By considering the potential loss deriving from a single loan it is evident that the bank may suffer a loss equal to the credit exposure less a recovery amount if the borrower defaults and a loss equal to zero if the borrower doesn't default. The default rate may be considered as the probability of the default event.
In this way the loss deriving from the default of a single borrower is defined as a random variable (r.v.) assuming two values.
For the whole portfolio the loss is a random variable sum of the single r.v.'s describing the losses for each borrower. Its probability distribution is obtained as convolution.
The calculation of this probability distribution is the aim of this paper.

3.1 Expected loss for portfolio

The expected value of the portfolio loss is an important index for management's reserve policy: the higher the expected loss, the higher the reserves to be set aside.
More over the expected value of the loss is a basic element to detect if each credit position is conveniently priced or not.

3.2 The credit risk capital or economic capital

The economic capital (or credit risk capital) is normally defined as the additional maximum loss, above the expected value of the portfolio loss, where the additional maximum loss is determined by a specified probability level (e.g. 95%) over some time horizon.
As it is uneconomic for the bank to set aside an amount of capital coincident with the maximum loss, it is necessary to choose a level of capital sufficient to support the portfolio of transactions in not catastrophic losses (see [7]).

4. The mathematical definition of the main variables

Let's define with E_k the credit exposure referred to the k-th borrower ($k = 1,2,...,n$). As already said it represents the value of a single loan. In our context of valuation the credit exposure is a deterministic quantity calculated with the adoption of a deterministic structure of interest rates.

Let's define with X_k the default rate of the k-th borrower; X_k is a discrete random variable assuming values $x_k^{(s)} \geq 0$ ($s = 1,2,...,t$) with probability $p_k^{(s)}$.

Let W_k be the discrete r.v. "recovery rate" which takes values $w_k^{(h)}$ ($h = 1,2,...,m$) with probabilities $q_k^{(h)}$. Obviously it will be $0 \leq w_k^{(s)} \leq 1$.

The default event for a borrower can be described by the r.v. Y_k whose values are only two: 0 with probability $1 - x_k^{(s)}$ and 1 with probability $x_k^{(s)}$.

Now it is possible to introduce another r.v. describing the loss that the bank suffers when the k-th borrower defaults; it is equal to:

$$Z_k = E_k \cdot Y_k \cdot (1 - W_k)$$

Consequently, the loss for the entire portfolio may be obtained as sum of the various Z_k as follows:

$$\Pi = \sum_{k=1}^{n} Z_k$$

The calculation of the probability distribution of Π is the aim of our paper.

It is to be noted that the two r.v's X_k and W_k depend on a certain number of factors that affect the general condition of the economy; it is shown (see [2], [7] and [10]) that when default rates increase, recovery rates decrease. This normally happens in recession periods while the vice versa happens when the economy is growing.

In other words, this means that there is a correlation between X_k and W_k.

5. An Actuarial Model for Credit Risk (M.A.R.C.) based on the stochastic simulation approach (Monte Carlo Method)

In a previous paper (see [8]) an actuarial model called M.A.R.C. has been worked out by using the stochastic simulation[7].

In the M.A.R.C. model the borrowers are divided into independent categories[8] with assigned default and recovery rates. In the scheme each borrower is subjected to some "draws" to obtain the value of default rate, recovery rate and the default event. Obviously all the procedure is developed by taking into account the division by category and the correlation between the default rate and the recovery rate.

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[7] In that occasion (see [8]) MARC model has been compared with Credit Risk Plus, another actuarial model for credit risk developed by Credit Suisse that has the same aim of MARC but makes heavy assumptions to obtain similar results.

[8] See note 3.

By taking into account the results for the entire portfolio after a sufficient[9] number of replications, a certain number of values are obtained for Π; hence it is possible to derive the frequency distribution of Π and all the possible percentiles.
With the same scheme we can obtain also the risk contribution of each borrower; it is possible to measure, in respect of each borrower, the incremental effect on a chosen percentile level of the loss distribution when the borrower is removed from the existing portfolio; this quantity is called borrower risk contribution.
Risk contributions are important indicators because, as shown in the examples, they make it possible to measure the effect of a potential change in the portfolio (e.g. the removal of an exposure).
In this way a portfolio may be effectively managed by paying attention to a relatively few borrowers representing a significant proportion of the risk.

6. Some applications

In this paragraph we present some applications to illustrate the results of the procedure; the applications have been realised by considering an hypothetical credit portfolio formed by twenty five loans.

6.1 The credit portfolio and the borrowers' risk category

The following table shows the characteristics of the twenty five borrowers in terms of credit exposure and risk category.

[9] In order to establish the sufficient number of replications we divide the range between the minimum possible loss (that is equal to 0) and the maximum possible loss (the sum of the credit exposures) in v intervals with the generic interval indicated with I_u. We assume a binomial scheme by considering that the generic value of Π will belong to the interval I_u with probability p_u and, consequently, will not belong to the interval I_u with probability $1-p_u$ with $u=1,2,...,\nu$.

By indicating with $f_N^{(u)}$ the number of successes in N trials (the number of times that the generic value of Π belongs to I_u in N replications) then, from the Chebyschev inequality we obtain:

$$\Pr\left\{\left|\frac{f_N^{(u)}}{N}-p_u\right|\geq\varepsilon\right\}\leq\frac{1}{4N\varepsilon^2}$$

By prefixing a confidence level equal to α we have:

$$N\geq\frac{1}{4\varepsilon^2\alpha}$$

This number of replications assures that the relative frequency $\frac{f_N^{(u)}}{N}$ deriving from the repetition of the simulation procedure differs from the true probability p_u for a quantity not superior to ε with probability equal to $1-\alpha$.

Table 1
Composition of the credit portfolio

Name	Credit Exposure	Credit Category
1	476772	8
2	1449459	8
3	2393614	6
4	2571044	7
5	3082045	7
6	3206536	7
7	3527405	8
8	3933721	7
9	4173525	4
10	4261379	4
11	6287873	1
12	6424588	4
13	6533089	4
14	6555555	8
15	6706275	6
16	7076084	5
17	7229158	4
18	7338389	3
19	7666913	5
20	7777634	3
21	8600489	8
22	8618828	8
23	10277776	2
24	20496505	6
25	26917730	5

The category of the credits is a classification made on the basis of some factors as credit rating[10], geographical localisation, economic sector of activity, and so on. In our example we assumed eight debtor categories.

For each borrower a default rate is assigned; according to what said, the default rate is a random variable. The distributions of the eight r.v.'s is shown in the Appendix B together with the probability distributions of recovery rates and the correlation coefficients between default and recovery rates.

The procedure of simulation has been performed by fixing the parameter ε=0.001 and $\alpha = 0.05$ (see note 9).

The main features of the probability distribution are shown in the following table.

[10] The "rating" of a borrower is the measure of its solvency (see also par. 2.2).

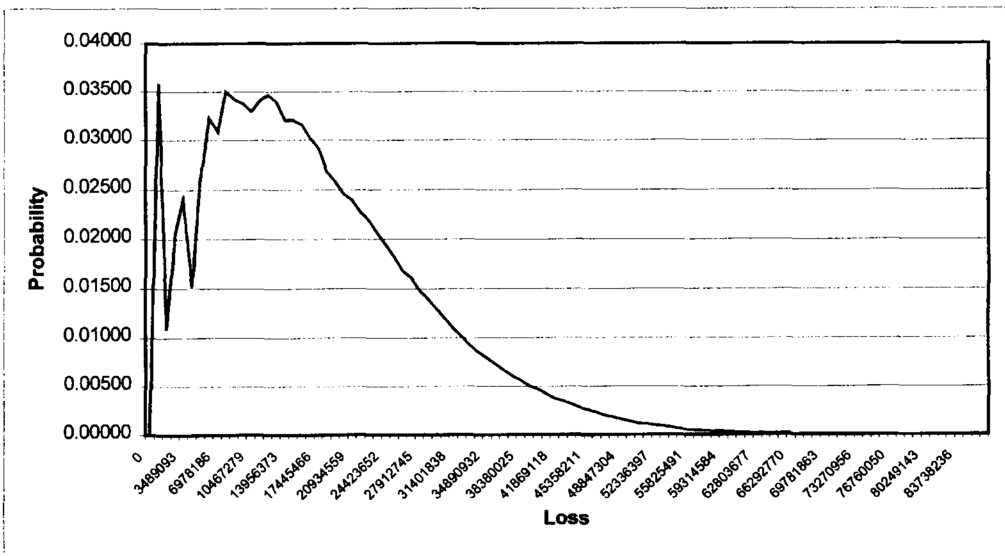
Table 2

Results of the procedure

Esposure	173582386
Expected loss	16800326
Expected loss / Esposure	9.679%
Loss standard deviation	11196575
Percentiles	Values[11]
50,00%	14884166
75,00%	23320700
90,00%	32237374
95,00%	38127126

The following graph shows the loss distribution for the considered portfolio.

Figure 3 The loss distribution for the entire portfolio



It's evident that the obtained distribution is not normal; a straightforward comparison may be made by considering the percentiles obtained with our procedure (already shown in table 2) and with a normal distribution having the same mean and standard deviation.

[11] The values corresponding to the generic percentile are the values of the portfolio loss whose cumulative probability is equal to the specified percentile.

Table 3
True percentiles and normality assumption

Percentiles	Normal distribution	"True" values
50,00%	16800326	14884166
75,00%	24352308	23320700
90,00%	31149306	32237374
95,00%	35217046	38127126

At the level of 95% the M.A.R.C. model give a more cautious estimate of the loss.

6.2 The meaning of the results

The scheme previously described is enough flexible to verify the effect of eliminating some borrowers from the portfolio.
In the following example we analyse the consequences on the loss distribution coming from the removal of two debtors: the number 21 and the number 22.
They are characterised by high values of both credit exposure and default rate.
The effect on the results are shown in the following table and graph.

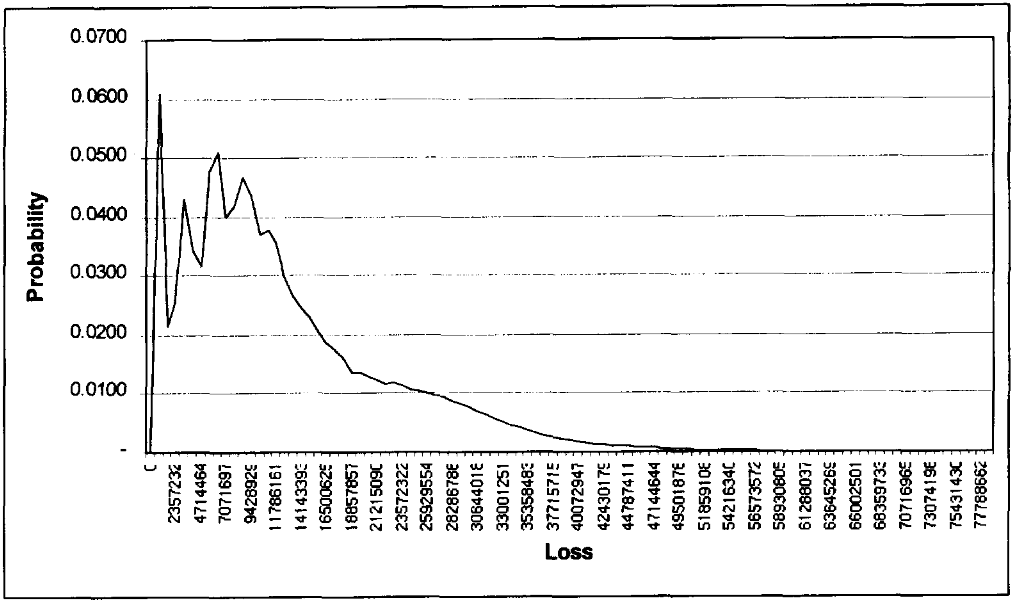
Table 4
Results of the procedure after
the removal of the borrower 21 and 22

Esposure	156363069
Expected loss	12212368
Expected loss / Esposure	7.810%
Loss standard deviation	9688497
Percentiles	Values[12]
50,00%	9696105
75,00%	16919033
90,00%	23557744
95,00%	31572167

A shift of the distribution to the left may be immediately observed by comparing the following graph with figure 3.

[12] The values corresponding to the generic percentile are the values of the portfolio loss whose cumulative probability is equal to the specified percentile.

Figure 4 The loss distribution for the entire portfolio after the removal of the borrower 21 and 22



The following table may be obtained by considering the relative differences between the two exercises.

Table 5
Differences in percentage between the results
of Table 3 (entire portfolio) and 4 (removal of the borrower 21 and 22)

Esposure	-9.920%
Expected loss	-27.309%
Expected loss / Esposure	-10.193%
Loss standard deviation	-13.469%
Percentiles	Values[13]
50,00%	-34.856%
75,00%	-27.451%
90,00%	-17.618%
95,00%	-17.192%

6.3 The borrowers' risk contributions

It is important to calculate the risk contribution of each borrower.
As already said the risk contribution of an exposure is a measure of the incremental effect on a chosen percentile when the exposure is removed from the original loss distribution.

[13] The values corresponding to the generic percentile are the values of the portfolio loss whose cumulative probability is equal to the specified percentile.

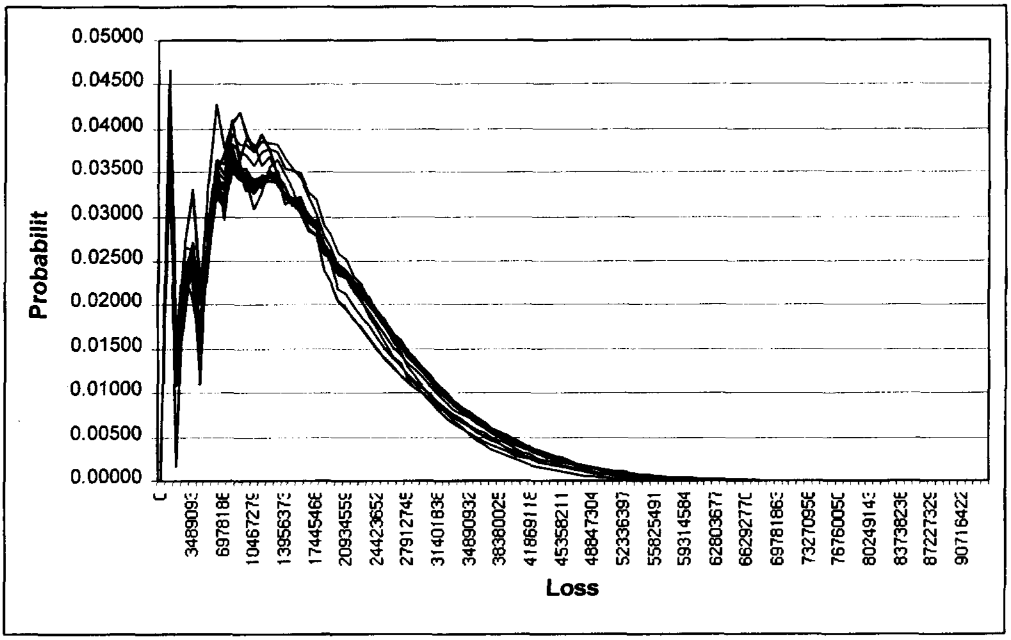
If the chosen percentile level is the same percentile used for calculating economic capital (see par. 3.2), the risk contribution is equal to the incremental effect on the amount of economic capital required for the portfolio.
Therefore risk contributions may be used in portfolio management by identifying the borrowers requiring the largest economic capital.
In our exercise the risk contributions for the 25 borrowers of our portfolio have been calculated at the percentile level of 95%.
The results are as follows.

Table 6
Risk contributions of the borrowers

Name	Credit Exposure	Credit Category	Risk Contribution
1	476772	8	147131
2	1449459	8	495075
3	2393614	6	274124
4	2571044	7	452233
5	3082045	7	447263
6	3206536	7	474503
7	3527405	8	1282960
8	3933721	7	669100
9	4173525	4	245747
10	4261379	4	228990
11	6287873	1	104073
12	6424588	4	378279
13	6533089	4	342723
14	6555555	8	2432679
15	6706275	6	829379
16	7076084	5	685207
17	7229158	4	413092
18	7338389	3	235113
19	7666913	5	763305
20	7777634	3	364160
21	8600489	8	3310347
22	8618828	8	3331871
23	10277776	2	233828
24	20496505	6	4147109
25	26917730	5	5547801

The significance of the risk contributions i.e. their relationships with the risk of the portfolio is shown in the following graph, where each distribution is obtained by removing one debtor each time.

Figure 5 Loss distribution for the portfolios after the removal of the borrowers one at time



7. Stochastic simulation and some statistical techniques to study the probability distribution of the loss of the portfolio

In this part of the paper some statistical techniques are illustrated. In some cases they may be useful to manage the simulation output data.
In fact, the stochastic simulation approach needs a lot of trials (as already seen in note 7).
For instance for each of our exercises we have made over 3 millions of replications in order to achieve the necessary accuracy in calculating relative frequencies of the loss distributions.
However, should this procedure be considered too heavy or too costly, consolidated statistical methods might help in simplifying the calculations.
A possible way is described here.
First of all by using M.A.R.C. we have obtained a sample of 10000 results relevant to the entire portfolio loss. Afterwards some parametric families of continuous type distribution functions (such as Beta, Gamma, Lognormal and Weibull) have been chosen and their parameters have been calculated by the maximum likelihood method.
The results coming from the simulation (over 3 millions replications) and those obtained by the latter procedure have been compared. In the table that follows the results coming from the simulation have been named "true" results.

7.1 The results of the estimates and the comparison with the "true" distribution

The parameters obtained by the 10000 sample are listed in the table below. It contains also some useful statistics.

Table 7
MLE's estimation parameters

Distribution	Weibull	Beta	Gamma	Lognormal
Parameter 1	1.448745	1.598456	1.654285	2.047673e+7
Parameter 2	1.852446e+7	6.564999	1.022125e+7	2.783286e+7
Chi square	8.717617e-8	1.165705e-7	1.309851e-7	5.746548e-7
KS statistic	0.026668	0.028524	0.054338	0.114641

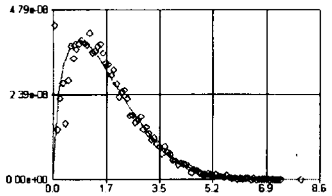
In the following table the values of some percentiles for the estimated distribution functions and the "true" distribution[14] are compared.

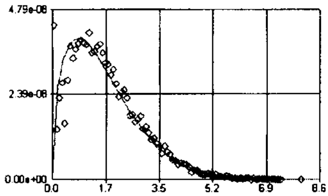
Table 8
"True" distribution (3 millions replications) and estimation from analytical functions

Percentiles	*"True" distribution*	*Weibull*	*Beta*	*Gamma*	*Lognormal*
10%	3989019	3918738	4131365	3696422	3271050
20%	7092546	6578154	6815230	6116968	5130038
30%	9638686	9092859	9358217	8467362	7096612
40%	12212850	11651450	11951442	10932290	9364220
50%	14884166	14383846	14720993	13650522	12134614
60%	17803971	17439702	17807972	16796267	15724586
70%	21308393	21056752	21430982	20662355	20749125
80%	25663204	25727822	26030319	25877773	28703192
90%	32237374	32943214	32882360	34407516	45015654
95%	38127126	39505092	38772872	42638995	65279766

As shown Weibull and Beta well fit the right tail of the distribution while Gamma and Lognormal distributions do not give good results.
The following graph shows the estimated Weibull and Beta probability distributions compared with the "true" values.

Figure 6 Comparison between Weibull (left, line), Beta (right, line) distributions and the "true" data (scatter)





8. Some risk - management indications

Beside the indicators already mentioned, the construction of the loss distribution and its percentiles suggest other important observations:

- The expected loss may be interpreted as an amount of money to set aside in a financial year to "cover" the credit exposures expected to be in default;

[14] We recall here that the results for the so-called "true" distribution are those coming from 3 millions of replications of the simulation procedure; this number of replications assures (see note 9) that $\varepsilon = 0.001$ and $\alpha = 0.05$.

- The provision of a further amount of money may be seen as a solvency margin for the bank. As said this quantity is usually equal to the difference between the value corresponding to a prefixed percentile (for example the 95th) and the expected loss;
- The losses exceeding the prefixed percentile may be considered catastrophic events whose probability is very low. This "catastrophic" loss might be continuously controlled by means of the proposed procedure.

9. Conclusions and future lines of research

The model presented in this paper is based on an actuarial approach: the single credit has been considered as an insurance contract whose value depends on the occurrence of the default event.
By taking into account the evolution of the default rates over the time we have described a method to calculate the loss distribution of the entire portfolio.
From the loss distribution some statistical indexes may be obtained and they are useful to decide upon adequate strategies of funding and to quantify a solvency margin for the bank.
In conclusion we think that the M.A.R.C. model seems a good tool for applying the typical techniques of the risk theory to credit portfolios. We deem that these techniques may pay an important role in the active default risk management of banks and may create an important link between the actuarial science and the credit industrial sector.

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Appendix A

1. The Weibull distribution

Probability density function

$$f(x)=\alpha\beta^{-\alpha}x^{\alpha-1}e^{-\left[\frac{x}{\beta}\right]^{\alpha}}$$

Distribution

$$F(x)=1-e^{-\left[\frac{x}{\beta}\right]^{\alpha}}$$

Parameters
$\alpha>0,\beta>0$

2. The Lognormal distribution

Probability density function

$$f(x)=\frac{1}{x\sqrt{2\pi\sigma_1^2}}e^{\frac{-[\ln x-\mu_1]^2}{2\sigma_1^2}}$$

where

$$\mu_1=\ln\left[\frac{\mu^2}{\sqrt{\sigma^2+\mu^2}}\right],\sigma_1=\sqrt{\ln\left[\frac{\sigma^2+\mu^2}{\mu^2}\right]}$$

Distribution
No closed form

Parameters
$\mu>0,\sigma>0$

3. Beta distribution

Probability density function

$$f(x)=\frac{x^{\alpha_1-1}(1-x)^{\alpha_2-1}}{B(\alpha_1,\alpha_2)}$$

where

$$B(\alpha_1,\alpha_2)=\int_0^1 t^{\alpha_1-1}(1-t)^{\alpha_2-1}\,dt$$

Distribution
No closed form

Parameters

$\alpha_1 > 0, \alpha_2 > 0$

4. Gamma distribution

Probability density function

$$f(x) = \frac{\beta^{-\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)}$$

where

$\Gamma(\alpha)$ is the Gamma function

Distribution

No closed form

Parameters

$\alpha > 0, \beta > 0$

Appendix B

In this Appendix we briefly summarise the input data of the applications presented in the paper.

Three tables are shown. They contain the hypotheses about default rates, recovery rates and correlation between default and recovery rate for each category of borrowers.

The data don't need any explanation; the only clarification is about the structure of the recovery rates that, as shown in table B.2, are equal for each category.

This equality derives only from the need of using input data such that the results may be easily understood: in fact if two categories of borrowers have similar expected default rates but their expected recovery rates are heavily different the consequence will be that the two categories will affect the total risk of the credit portfolio in two different manners even if, at a first glance, they could seem very similar because both the categories have similar default rates. Substantially the choice of the data shown in the following tables is consistent with the aim of highlighting in a simple way the basic characteristics of the proposed model.

Table B.1

Default rates distributions for the different categories of borrowers

Categories								
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Probabilities**
0.0054	0.0057	0.0108	0.0180	0.0269	0.0359	0.0539	0.1078	0.1000
0.0087	0.0093	0.0174	0.0290	0.0434	0.0579	0.0869	0.1738	0.2000
0.0111	0.0118	0.0221	0.0369	0.0553	0.0738	0.1107	0.2213	0.3000
0.0131	0.0140	0.0262	0.0437	0.0655	0.0873	0.1310	0.2620	0.4000
0.0150	0.0160	0.0300	0.0500	0.0750	0.1000	0.1500	0.3000	0.5000
0.0169	0.0180	0.0338	0.0563	0.0845	0.1127	0.1690	0.3380	0.6000
0.0189	0.0202	0.0379	0.0631	0.0947	0.1262	0.1893	0.3787	0.7000
0.0213	0.0227	0.0426	0.0710	0.1066	0.1421	0.2131	0.4262	0.8000
0.0246	0.0263	0.0492	0.0820	0.1231	0.1641	0.2461	0.4922	0.9000
0.0382	0.0407	0.0764	0.1273	0.1909	0.2545	0.3818	0.7635	1.0000

Table B.2

Recovery rates distributions for the different categories of borrowers

Categories								
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Probabilities**
0.07184	0.07184	0.07184	0.07184	0.07184	0.07184	0.07184	0.07184	0.1000
0.11584	0.11584	0.11584	0.11584	0.11584	0.11584	0.11584	0.11584	0.2000
0.14756	0.14756	0.14756	0.14756	0.14756	0.14756	0.14756	0.14756	0.3000
0.17467	0.17467	0.17467	0.17467	0.17467	0.17467	0.17467	0.17467	0.4000
0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.5000
0.22533	0.22533	0.22533	0.22533	0.22533	0.22533	0.22533	0.22533	0.6000
0.25244	0.25244	0.25244	0.25244	0.25244	0.25244	0.25244	0.25244	0.7000
0.28416	0.28416	0.28416	0.28416	0.28416	0.28416	0.28416	0.28416	0.8000
0.32816	0.32816	0.32816	0.32816	0.32816	0.32816	0.32816	0.32816	0.9000
0.50902	0.50902	0.50902	0.50902	0.50902	0.50902	0.50902	0.50902	1.0000

Table B.3

Recovery rates and default rates correlation
for the different categories of borrowers

Categories	Correlation
1	-0.46
2	-0.56
3	-0.53
4	-0.37
5	-0.66
6	-0.49
7	-0.61
8	-0.37