

THE "EXCESS VOLATILITY" REINSURANCE TREATY

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ABSTRACT

In this paper we propose a new reinsurance treaty called "Excess Volatility" (XV). This treaty aims at reducing the volatility of the underwriting result of a non-life insurance company. The volatility reduction is one of the key issues in today's highly competitive insurance environment where premium margins are eroding.

The XV treaty is based upon reciprocity between the direct insurer and the reinsurer: losses and profits are exchanged in order to stabilize the underwriting result over time. For this purpose a peculiar payment function is defined, providing a modular and flexible hedging instrument for the direct insurer and the reinsurer. Premiums are computed using the Proportional Hazard transform premium principle proposed by Wang (1995). The first results confirm the model is able to provide the needed protection against deviations in the loss experience, at a relatively low price.

KEYWORDS

Reinsurance, Volatility reduction, Proportional Hazard transform, Reciprocity.

1 Introduction

Any insurance portfolio is more or less structurally biased and unbalanced. This feature is striking for certain class of business (as, for example, nuclear power and aviation insurance) while it is less evident in other lines (as in motor insurance). The lack of homogeneity in the insured portfolio results in potential instability and volatility of the underwriting result that could lead to harmful effects both on the insurer's solvency and liquidity.

The pooling of risks is the first and most effective tool available to the direct (or primary) insurer to reduce the overall risk and the fluctuations of the underwriting result; unfortunately in certain business classes risk pooling is unable per se to smooth very high concentration of risks, so that reinsurance is the most common choice left.

Traditionally various choices have been available on the market among proportional and non-proportional treaties, featuring different advantages and disadvantages. Nowadays the market, due to the increasing level of sophistication and eroding margins, is demanding more flexible, efficient and tailor-made solutions. As a consequence, new and more sophisticated reinsurance covers are proposed.² The reinsurer's main task is therefore to choose, among these treaties, the scheme which is best suited to the client's needs, providing protection and optimizing profitability and business flexibility.

In today's highly competitive insurance market the profitability pattern is difficult to predict and the actuary is much more involved in controlling the variance of the underwriting result rather than setting a limit to the (overall) loss incurred. Put in other words, one of the main concerns of the actuary is to design an effective hedging strategy capable to reduce the risk of profit fluctuations.

Along this line it is the purpose of this paper to introduce a new flexible and customizable reinsurance scheme, aimed at reducing the volatility of the direct insurer's underwriting result. This model, called "Excess Volatility" reinsurance treaty, or in short XV, is conceived for primary insurers wishing to cut out the excess volatility of the underwriting result while retaining the physiological and, let us say, "normal" risk. The XV treaty is intended to be offered by leading reinsurers as a rather standardized product which can then be tailored to the client's needs.

The paper is structured as follows.

- ◇ In section two we present the main features of the XV treaty, discussing the shape of the payment function and establishing a comparison with option based hedging strategies.
- ◇ In section three, after a brief review of the premium principle adopted, we describe the premium computation procedure.
- ◇ In section four we provide the specific premium formulas for the XV treaty when the loss phenomena is modeled with a Weibull variate.
- ◇ In section five we give some numerical examples to provide a better understanding of the proposed treaty.

2 The XV treaty

2.1 The structure

The purpose of the XV reinsurance treaty is to provide the direct insurer with a comprehensive and customizable program to cut out the excess volatility of the underwriting result to a predefined level. This is accomplished through a two-way cover agreement between the primary insurer and the reinsurer where portions of the losses and profits are exchanged, rebalancing the underwriting result. Each of them has to pay the other depending on the sign and on the deviation from a predefined equilibrium point. In such a way large negative deviations of the underwriting result are offset by large positive deviations.

Let us now go deeper in the structure of the XV treaty. Instead of using an absolute measure of profitability, as it is the underwriting result, in the forthcoming of the paper we prefer a relative measure as the claims ratio $\rho = \mathbf{X}/\Pi$, where \mathbf{X} and Π denote, respectively, the aggregate claim amount and the gross premiums.³

In Figure 1 we represent a possible and realistic evolution, over time, of the claims ratio where low-volatility periods are followed by high volatility periods. In Figure 1 we can distinguish two zones: the "A" and the "B" zone.

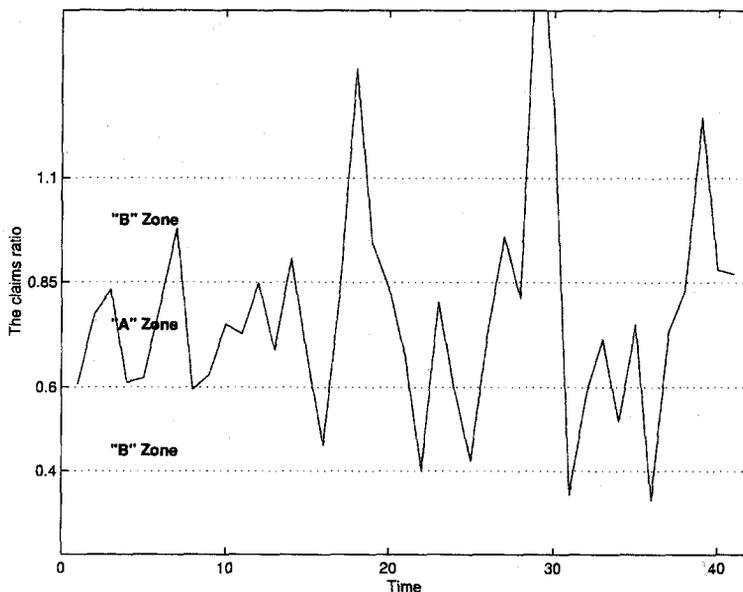


Figure 1: A possible evolution of the claims ratio.

- ◊ The "A" zone, which is like a corridor, defines a condition where the volatility of the claims ratio, and therefore the risk, assumes normal values. In this case the volatility is fully borne by the primary insurer and no external intervention is required. The upper and the lower limit of this zone (which in Figure 1, for illustrative purposes, have been put equal to 0.85 and 0.6) can be viewed as the treaty's trigger points, activating the reinsurer intervention.
- ◊ The "B" zone starts just beyond these trigger points and characterizes the non normal volatility condition. In this case the direct insurer's retention limits have been trespassed and an external intervention is required. Due to the reciprocity feature of the XV treaty, the "B" zone lays above and below the "A" zone: in the upper "B" zone non-normal losses are registered and they are partially or totally covered by the reinsurer; in the lower "B" zone non-normal profits are made and they have to be partially or totally transferred to the reinsurer.⁴ This *osmosis* takes place only to a predefined level which is the outer limit of the "B" zones: profits and losses beyond these levels (which in Figure 1 are equal to 1.1 and 0.4) remain to the primary insurer.

2.2 The payment function

In the XV treaty the splitting of the risk is defined through a "payment function" $h(\rho)$. In Figure 2 we represent the shape of the payment function which completely characterizes this reinsurance agreement.

Some features emerge from Figure 2.

- ◊ The payment function is zero within that range of values of ρ which corresponds to

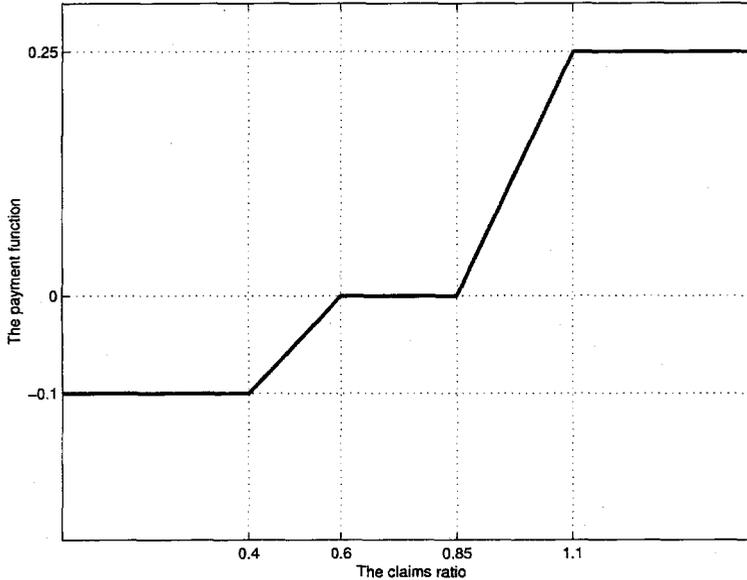


Figure 2: The payment function for the XV treaty

the "A" zone of Figure 1. In normal volatility conditions the risk belongs completely to the direct insurer, therefore the treaty is not activated. This is a sort of "free-zone" with no obligation between the two parties. Obviously, the expected value of the claims ratio, $E(\rho)$, should fall within this range.

- ◇ When the treaty is activated the payment function $h(\rho)$ could be either positive or negative depending on the value of ρ : if $h(\rho)$ is positive the reinsurer has to pay to the primary insurer, if it is negative the primary insurer has to pay to the reinsurer. Koller and Dettwyler (1997) proposed a reinsurance treaty, called APS (Adaptive Pivot Smoothing), featuring a similar payment function. Traditional treaties, on the other hand, bear always a non-negative payment function as long as they are one-way agreements where only the reinsurer has to pay.
- ◇ The payment function acts like an hedging instrument: if ρ is (sufficiently) higher than the expected value, the incurred losses are offset, in full or in part, by a positive payment; on the other hand, if ρ is (sufficiently) lower than the expected value, the unusual profits made are transferred, in full or in part, to the reinsurer through a negative payment.
- ◇ The full or partial exchange of losses and profits (i.e. the hedging) is represented, for illustrative purposes, in Figure 2. In the proposed configuration the payment function has different slopes: over-the-normal losses are fully refunded while unusual profits are only partially transferred to the reinsurer.
- ◇ When ρ remarkably deviates from the expected value, the payment function flattens: we have reached the limits of the treaty and beyond them no additional cover is provided. A floor and a ceiling value for $h(\rho)$ are therefore set corresponding to the limits of the treaty.

From Figure 2 it is clear how in the XV treaty the direct insurer and the reinsurer are intertwined. Therefore the XV treaty is more than a simple protection against high losses: it is a condition much more similar to a joint-venture where each party shares the fortunes/misfortunes of the other. Along this line the term direct (or primary) insurer is better suited to represent this two-way risk sharing agreement rather than the traditional term cedent would do.

2.2.1 The XV viewed as an insurance derivative

The XV treaty can provide a risk cover which is similar to what a *bull spread* or *range forward* written on ρ would offer. A range forward is basically a forward position with limited gain and losses. The typical payoff diagram is represented in Figure 3.

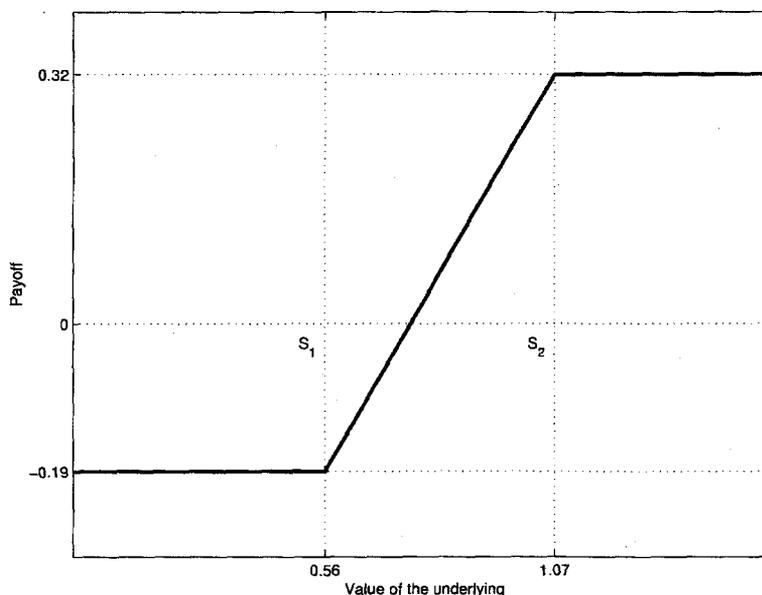


Figure 3: The range forward payoff diagram

These option strategies (or *packages*) can be viewed as a combination of simpler contracts. In this particular case we obtain a similar payoff combining a forward, written on the claims ratio, with a short position on a call and a long position on a put.⁵

In Figure 3 we denote with S_1 and S_2 , respectively, the strike prices of the put and of the call. These exercise prices are chosen such that the expected losses are offset by the expected profits or, better, the put premium equals the call premium. As a result the net premium payment is equal to zero by definition and no payment is required at the beginning of the contract.

2.2.2 Why a "free-zone"?

The main difference between Figure 2 and Figure 3 is the presence of the "free-zone", where $h = 0$ and no payments are made either by or to the primary insurer. Were this free-zone to collapse, the two payoffs diagrams would be almost identical.

In this subsection we are going to discuss the importance of this feature to obtain a more balanced and fair agreement.

- ◇ **No full hedge.** Most of the primary insurers want to achieve only a partial hedging of risk through reinsurance. They have in fact little interest in a full hedging which leaves no risk, a certain cost (the reinsurance premium) and little or no profit. On the contrary they are willing to retain the risk their financial and solvency standing allows, at the same time freeing themselves from the part of risk exceeding their underwriting capacity. It is exactly this risk selection that is accomplished through the free-zone, retaining profitability together with bearable and sustainable risks.
- ◇ **The "true" expected value of the claims ratio.** Let us now consider a no free-zone condition. The resulting payoff would be almost equal to that represented in Figure 3 which is in turn similar to the APS payment function. For the effectiveness and coherence of the hedge $h(E(\rho))$ must be zero, otherwise a positive or negative bias would be introduced in the hedge, creating a structural unbalance in the agreement. Unfortunately an accurate and reliable estimate of $E(\rho)$ can not always be achieved in all business classes.⁶ If we consider, on the other hand, a free-zone, sufficiently large to contain the "true" value of $E(\rho)$, these problems can be reduced and the treaty can be more balanced.
- ◇ **Moral Hazard.** The direct insurer could be willing to underestimate the true value of $E(\rho)$, introducing again a bias in the risk agreement. It is therefore essential for the reinsurer to perform a deep, accurate and detailed statistical analysis of the risk phenomena he is going to bear. Along this line the presence of a free-zone, which can include the "true" value of $E(\rho)$ and is sufficiently large to contain its possible fluctuation, can contribute to reduce the possible consequences of this phenomena.
- ◇ **Risk of commercial change.** The risk of deep commercial changes significantly altering the claims ratio is another possible risk for the reinsurer which can be (partially) avoided with the free-zone. Nowadays premium rates are often imposed by the market rather than being solely the result of an accurate statistical analysis. Therefore the claims ratio time series could display a rather sudden and structural shift to a different level.⁷ In these cases the terms of the reinsurance agreement must be redefined to reflect the new market conditions. Before this revision could take place, the free-zone can offer a first protection for the reinsurer against this kind of risk.

2.3 The payment function configuration

We can now write the payment function $h(\rho)$ in a formal way combining the lower and an upper layer in:

$$h(\rho) = \begin{cases} -rm & 0 \leq \rho < l - m, \\ r[\rho - l] & l - m \leq \rho < l, \\ 0 & l \leq \rho < L, \\ R[\rho - L] & L \leq \rho < L + M, \\ RM & L + M \leq \rho. \end{cases} \quad (1)$$

In (1) all the parameters are real and positive. Upper and lower case letters represent, respectively, the upper and lower layer settings. Figure 4 gives a visual representation of these configuration parameters.

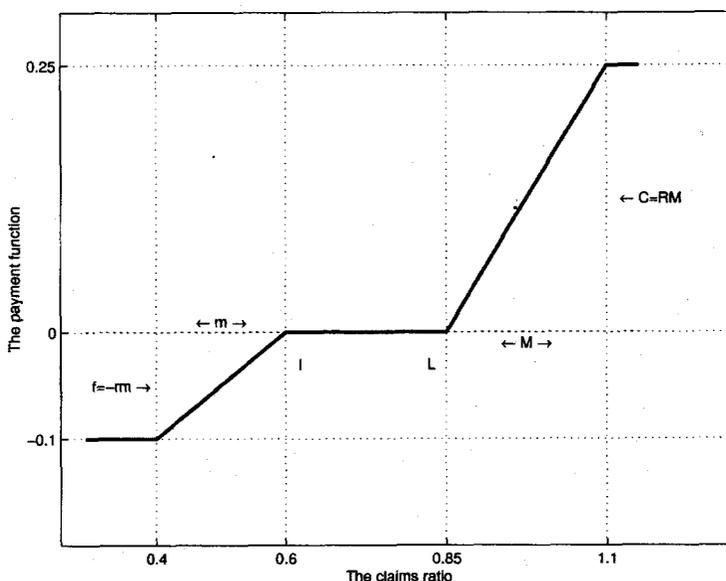


Figure 4: The payment function configuration parameters

With more detail:

- The interval $[l, L]$ delimits the free-zone.
- The intervals $[l - m, l]$ and $[L, L + M]$ define the non-normal condition, respectively below and above the free-zone.
- r and $R \in \mathbb{R}^+$ denote the slope of the payment function for the lower and upper layer; in a realistic view they essentially belong to the interval $(0, 1]$; depending on the value of r and R the XV treaty could feature a more or less proportional hedge.
- $C = RM$ and $f = -rm \in \mathbb{R}^+$ are the ceiling and the floor of the payment function.

The payment function configuration it is of crucial importance for the effectiveness of the treaty. Its definition depends on greater extent on the experience and technical knowledge of the reinsurer.

3 The XV premium

3.1 The price for a real valued risk

As we said the primary insurer and the reinsurer agree to exchange portions of losses and profits in a "reciprocity" condition. Coherently each of the parties has a role in the premium definition, setting the price for the specific risk borne.

The XV premium, $\Pi[h(\rho)]$, is the combination of two price components: the first component is set by the primary insurer and takes into account the negative portion of the payment function; the second component is defined by the reinsurer and covers the positive portion of the payment function. This latter component, as usual, has a positive sign; the first component, on the other hand, has a negative sign. Alternatively the lower layer premium can be viewed as a sort of discount on the upper layer premium for the potential transferring of extraprofits.

3.2 The Wang's premium principle

The usual moment-based premium principles are not perfectly suited for the XV treaty pricing needs: they appear rather cumbersome and not flexible enough to follow the complex shapes of the payment function. A more flexible and, let us say, "modular" premium principle has to be found for this cover agreement.

Along this line we adopt the premium principle proposed by Wang (1995) and further discussed by Young (1999). In this subsection we recall the main features of the Wang's premium principle.

Let us consider an insurance risk denoted by a non negative loss random \mathbf{X} , where $\mathbf{X} \in [0, +\infty]^8$. Any premium principle is a rule H that assigns a non negative number to every risk: therefore $H(\mathbf{X})$ can be viewed as an alteration of the expected value of the risk $E(\mathbf{X})$.

Let us now denote by $F_{\mathbf{X}}(t)$ the cumulative distribution function (cdf) of \mathbf{X} , i.e. $\mathbb{P}[X \leq t]$, and by $S_{\mathbf{X}}(t) = 1 - F_{\mathbf{X}}(t)$ the decumulative distribution function (ddf) or $\mathbb{P}[X > t]$.⁹ It is well known that

$$E(\mathbf{X}) = \int_0^{+\infty} S_{\mathbf{X}}(t) dt \quad . \quad (2)$$

Wang (1995) proposed a premium principle based on a Proportional Hazard transform (PH-t), defined as

$$\pi_{\theta}(\mathbf{X}) = \int_0^{+\infty} [S_{\mathbf{X}}(t)]^{1/\theta} dt \quad , \quad (3)$$

where $\theta \geq 1$ is a risk aversion index. If $\theta = 1$, obviously, (3) reduces to (2).

Besides Wang 1996a proposed a more general class of principles, called "risk adjusted premium principles", applying a *distortion* function g to $S_{\mathbf{X}}(t)$, i.e.

$$H_g(\mathbf{X}) = \int_0^{+\infty} g[S_{\mathbf{X}}(t)] dt \quad , \quad (4)$$

where g is an increasing, concave function such that $g : [0, 1] \rightarrow [0, 1]$. In other words, H_g is simply the expected value of a distorted distribution of the original risk. The PH-t premium principle of formula (3) is the simplest member of this premiums class H_g and has close connections with the dual theory of choice under risk of Yaari (1987).

In Figure 5 we report the $S_X(t)$ and the $g[S_X(t)]$ in the PH-t case for a Weibull variate. The area below $S_X(t)$ represents the expected value $E(X)$ while the area below $g[S_X(t)]$ represents the premium $H_g(X)$. Obviously, the area between the two lines represent the risk loading.

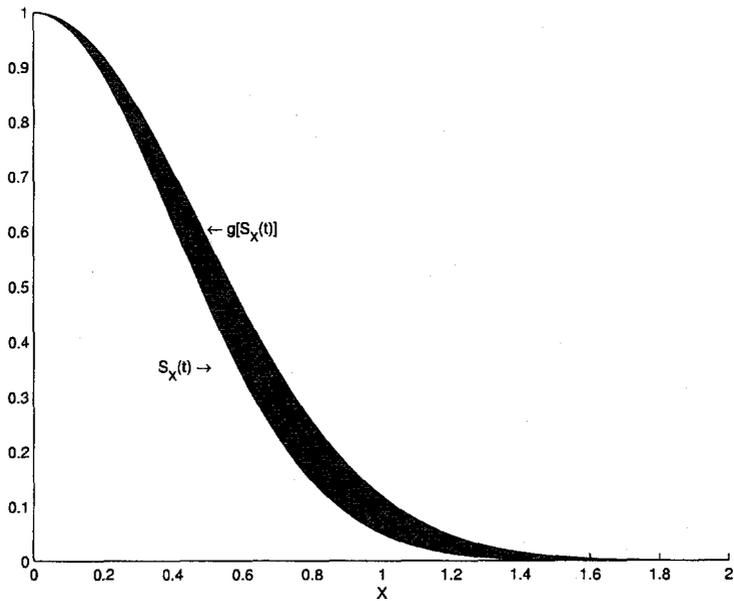


Figure 5: The ddf function $S_X(t)$ and $g[S_X(t)]$ for a Weibull variate X

The PH-t premium principle (3) satisfies the following main properties:

- $E(X) \leq H_g(X) \leq \max(X)$.
- $\theta \in [1, +\infty]$ defines the shape of g and is a measure of the risk aversion: the higher the value of θ , the higher the risk loading. To obtain feasible and consistent prices in the real world a positive risk loading is required. It follows that is $\theta > 1$: if and only if this condition is fulfilled, the function $g(y) = y^{1/\theta}$ is strictly concave and $E(X) < H_g(X)$.
- If $\theta > 1$, due to strict concavity, the relative loading for an infinitesimal layer, $(t, t + dt]$, is an increasing function of t : smaller risks determine a relatively light loading while higher risks are relatively more heavily loaded, allowing for a more efficient differentiation of risks.
- The premium principle H_g is both scale-invariant and translation-invariant so that $H_g(c_1 X + c_2) = c_1 H_g(X) + c_2$ with $c_1 \geq 0, c_2 \geq 0$.

- The premium for a risk layer can be computed in a simple and effective way. This is an important feature since it is a common practice in the reinsurance market, particularly in non-proportional agreements, to divide the risk in different layers. Let us consider a generic layer $(\omega, \omega + \delta]$ of the risk \mathbf{X} which is defined as the loss from an excess-of-loss cover, or

$$\mathbf{X}_{(\omega, \omega + \delta]} = \begin{cases} 0, & 0 \leq \rho < \omega, \\ \mathbf{X} - \omega, & \omega \leq \mathbf{X} < \omega + \delta, \\ \delta, & \omega + \delta \leq \mathbf{X} \end{cases}, \quad (5)$$

where ω is the attachment point or retention and δ is the limit. The PH-transform premium for this layer is simply

$$H_g(\mathbf{X}_{(\omega, \omega + \delta]}) = \int_{\omega}^{\omega + \delta} g[S_{\mathbf{X}}(t)] dt \quad (6)$$

- Another important feature of the PH-t principle is layer additivity: when a risk is divided into layers the sum of the premiums of each layer gives the premium for the overall risk.
- So far we assumed $\mathbf{X} \in [0, +\infty]$. If we consider real valued risk $\mathbf{X} \in [-\infty, +\infty]$, the premium $H_g(\mathbf{X})$ can be written ¹⁰ as

$$H_g(\mathbf{X}) = \int_{-\infty}^0 \{g[S_{\mathbf{X}}(t)] - 1\} dt + \int_0^{+\infty} g[S_{\mathbf{X}}(t)] dt \quad (7)$$

All the above properties can be resumed in just one feature: "modularity". In other words, no matter what the nature of the risk is, we can add layers, split them, transform them or make a shift, and the overall premium still continues to be the sum of the premiums for each individual "piece" of risk. This modularity (i.e. the absence of any grouping effect) makes the PH-t premium principle perfectly suited for the peculiar payment function $h(\rho)$ and the best choice for the pricing of the XV treaty.

The main disadvantage of the PH-t principle, on the other hand, is that either $F_{\mathbf{X}}(t)$ or $S_{\mathbf{X}}(t)$ must be fully specified, requiring more statistical and computational effort than usual moment based methods. To some extent, this is the price to pay for a more precise and better tuned premium rate.

3.3 How to compute the XV premium

We are now able to define the computation procedure for the XV premium $\Pi[h(\rho)]$ which, we recall, is split in the lower and upper layer premium. For this purpose two different values of the risk index, θ_1 and θ_2 , are used by the direct insurer and the reinsurer. If we denote with h^- and h^+ , respectively, the negative and positive portion of the payment function, we have that

$$\Pi[h(\rho)] = -H_{g_1}(h^-) + H_{g_2}(h^+) \quad (8)$$

where the subscripts g_1 and g_2 reflect the different risk index adopted. At this stage of the model we assume that there are no costs for the establishment and management of the cover agreement.

The relation between θ_1 and θ_2 must be discussed with detail. Usually the direct insurer is conceived as more risk averse than the reinsurer or, in other words, the direct insurer's risk index is higher than that of the reinsurer. In this specific case the values of the risk indexes are a part of a binding agreement, explicitly stated in the contract and not uniquely defined by the risk attitude of the parties. It is therefore reasonable to presume that the reinsurer, who has a greater control on the contract conditions, will require, in place of reciprocity, a relation $\theta_1 < \theta_2$. In other words, the reinsurer agrees to pay a premium for the potential transferring of extraprofits (which will have a discount effect on $\Pi[h(\rho)]$) but, at the same time, requires a risk loading θ_1 lower than his risk attitude θ_2 .

3.3.1 The upper layer premium

Following (6) the upper layer premium is

$$H_{g_2}(h^+) = R \int_L^{L+M} [S_\rho(t)]^{1/\theta_2} dt \quad (9)$$

3.3.2 The lower layer premium

A little more reasoning is needed for the price of the negative portion h^- . In this case, in the primary insurer's view, the risk is measured by the difference $l - \rho$: the more the distance of the claims ratio from the limit l the greater the payment to be arranged. Let us now denote this risk as ψ , where

$$\psi = \begin{cases} l - \rho & 0 \leq \rho < l, \\ 0 & l \leq \rho. \end{cases} \quad (10)$$

The expected value of the risk ψ can be written through the ddf function S_ψ as

$$E[\psi] = \int_0^l S_\psi(\tau) d\tau \quad (11)$$

Since we have

$$S_\psi(\tau) = \mathbb{P}[\psi > \tau] = \mathbb{P}[\rho < l - \tau] = F_\rho(l - \tau), \quad (12)$$

we can rewrite (11) as

$$E[\psi] = \int_0^l F_\rho(t) dt \quad (13)$$

Along this line our proposal for the lower layer premium, instead of (7), is

$$H_{g_1}(h^-) = r \int_{l-m}^l [F_\rho(t)]^{1/\theta_1} dt \quad (14)$$

4 The claims ratio distribution function

A natural candidate to model the claims ratio is a Weibull variate. This variate has been extensively used for this purpose by several authors, together with the Gamma and the Burr variate. The cdf of a Weibull variate with parameter a and b , $W_{a,b}(t)$, is

$$W_{a,b}(t) = \mathbb{P}[\rho \leq t] = \int_0^t ab\rho^{b-1} e^{-a\rho^b} d\rho = 1 - e^{-at^b}, \quad (15)$$

where both a and $b \in \mathbb{R}^+$ and $t \geq 0$.

In this specific case the adoption of a Weibull variate gives an additional benefit since its ddf has a very simple form, $S_\rho(t) = e^{-at^b}$, yielding straightforward premium computations.

Combining (9) and (14) we can rewrite (8) as

$$\Pi[h(\rho)] = -r \int_{l-m}^l [1 - e^{-at^b}]^{1/\theta_1} dt + R \int_L^{L+M} [e^{-at^b}]^{1/\theta_2} dt. \quad (16)$$

5 First Results

In this section we report some first results to give a quantitative measure of the potential effects of the XV treaty. We considered two scenarios with different degrees of protection and volatility reduction. Before going deeper in the analysis we wish to remark that these scenarios are just two possible examples of the capabilities of the XV treaty and that, due to the high flexibility and modularity of the agreement, more complex and even better tuned configurations can be proposed.

Let us now review the steps we followed in these analyses.

- To model the claims ratio we adopted a Weibull variate $W_{3,2}$ with a positive shift $\xi = +0.3$. Although these parameters are not the result of a rigorous statistical fitting, they appear to give a risk profile which is close to and consistent with the real experience of certain business classes. In Figure 6 and 7 we report the shape of the probability density function and the ddf in this specific case.

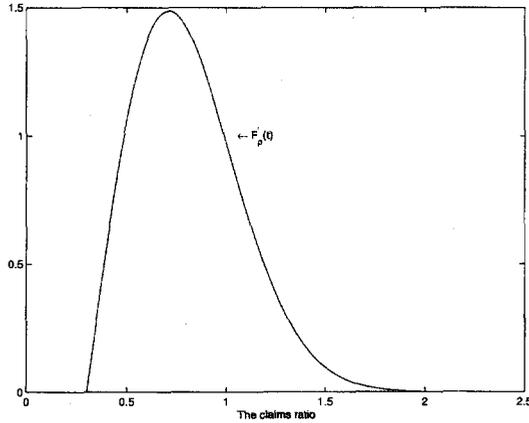


Figure 6: The pdf for a $W_{3,2}$ shifted by $\xi = +0.3$

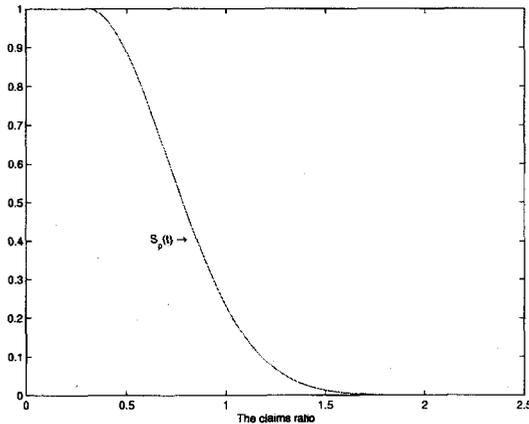


Figure 7: The ddf for a $W_{3,2}$ shifted by $\xi = +0.3$

- We simulated first a sample of 300 observations from this distribution, obtaining the claims ratio time series before the intervention of the XV treaty. The mean, the variance and the skewness of the claims ratio for this specific sample are, respectively, 0.81423, 0.06615 and 0.61531; for a comparison the theoretical values are $E(\rho) = 0.81166$, $\sigma^2(\rho) = 0.071533$ and $\gamma(\rho) = 0.61950$. Then we computed the underwriting result under the hypothesis that even the primary insurer adopts the Wang's premium principle. With a risk index $\theta = 1.2$ the net premium (before the expenses loading) charged to the insured is 0.8605: this is equivalent to a mark-up of 5.6% on the mean value $E(\rho)$. Obviously, to make rigorous comparisons these time series have been kept constant in the different scenarios.
- The risk indexes adopted for the XV treaty are respectively $\theta_1 = 1.1$ and $\theta_2 = 1.185$.
- To keep things simple, we put r and R equal to 1 obtaining a more symmetrical payment function.

- Since we assume that the primary insurer’s expenses matches the expense loading, these two components are offset in the computation of the underwriting result.

5.1 Scenario A

This scenario is conceived to give a significant protection against losses, at the same time preserving the profitability of the primary insurer. The lower and upper layer configuration parameters are respectively $[0.6, 0.7]$ and $[0.86, 1.1]$, ensuring protection as soon as the claims ratio exceeds the net premiums and leaving a rather wide free-zone.

The lower, upper and overall premium, expressed in terms of the gross premium Π , are $H_g(h^-) = 0.037006$, $H_g(h^+) = 0.075636$, $\Pi[h(\rho)] = 0.038630$.

In Figure 8 and in Figure 9 we report, respectively, the claims ratio and the underwriting result of the primary insurer before and after the intervention of the XV treaty. The thinner line depicts the time series before the XV treaty, while the thicker mark is the time series after the XV intervention. For a better visualization only the first 100 observations are reported.

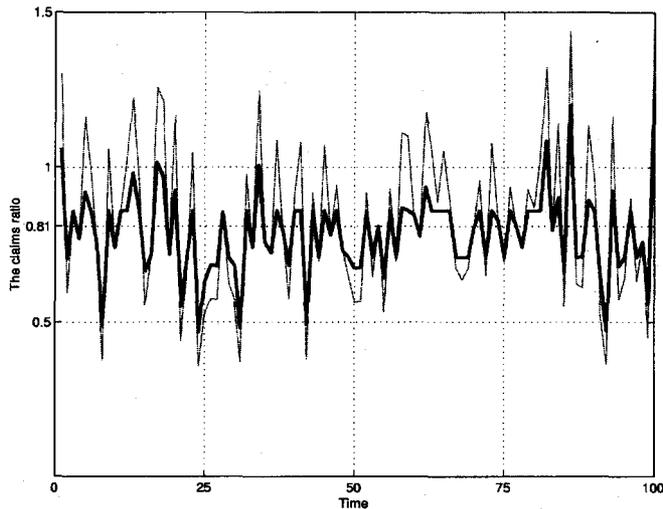


Figure 8: Scenario A: the claims ratio before and after the XV treaty.

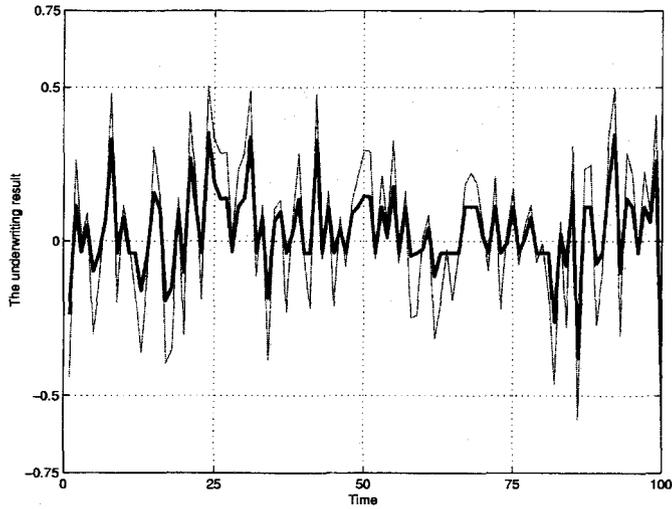


Figure 9: Scenario A: the underwriting result before and after the XV treaty.

From these figures it is clear the asymmetric effect on the claims ratio which can be achieved through this configuration of the XV treaty. The dangerous peaks are consistently smoothed and reduced while favourable conditions experience only a very little worsening. This asymmetric effect is obviously less evident in the underwriting result time series for the presence of the XV premium.

To provide a measure of this reduction effect, we computed the fundamental moments of the claims ratio and of the underwriting result on a single year basis. In Table 1 we report the mean, the variance and the skewness of the claims ratio together with the 5% and the 95% percentile of:

- the claims ratio before the XV treaty, ρ , denoted by ρ_b ;
- the claims ratio after the XV intervention, $\rho - h(\rho)$, denoted by ρ_a ;
- the primary insurer's underwriting result before the XV treaty, $0.8605 - \rho$, denoted by udr_b ;
- the primary insurer's underwriting result after the XV treaty, $0.8605 - \rho - \Pi[h(\rho)] + h(\rho)$, denoted by udr_a ;
- the reinsurer's underwriting result, $\Pi[h(\rho)] - h(\rho)$, denoted by udr_{Re} .

	ρ_b	ρ_a	udr_b	udr_a	udr_{Re}
E	0.81423	0.78578	0.04627	0.03609	0.01018
σ^2	0.06615	0.02282	0.06615	0.02282	0.01405
γ	0.61531	0.97690	-0.61531	-0.97690	-0.59777
5%	0.41808	0.62808	-0.39085	-0.01553	-0.37532
95%	1.25135	0.86000	0.44242	0.21639	0.22603

Table 1

As we can see the treaty can significantly reduce the volatility of the primary insurer's underwriting result which, after the XV intervention, is only 34% of the original value. Obviously there is a price to pay for this volatility reduction which can be recognized in the decrease of the mean value of udr after the XV treaty activation: this decrease from 0.04627 to 0.03609 is perfectly reflected in the udr_{Re} mean value.

If we consider the skewness of the underwriting result, before and after the XV treaty, it is interesting to note that the distribution of udr_a is now much more skewed to the left than udr_b . The skewness increases by more than 58% from -0.61531 to -0.97690 , witnessing a lower probability of negative values.

To understand the behaviour of the treaty over a longer period, we simulated 60 times the application of a XV treaty over an horizon of 5 years. At the end of each simulation we computed the sum of the claims ratio and of the underwriting result. In Table 2 we report the fundamental moments of these sums. As we can see the volatility reduction effect is very close to that shown in Table 1.

	udr_b	udr_a	udr_{Re}
E	0.23134	0.18044	0.05089
σ^2	0.38573	0.13300	0.08151
γ	-0.08142	-0.20485	-0.00417

Table 2

5.2 Scenario B

This is a stronger protection scenario where a more complete hedge is arranged: the lower and upper layer limits are now $[0.6, 0.81]$ and $[0.86, 1.3]$. In this case a much wider range of protection is required reaching 130% of the gross premiums (in scenario A the upper limit was just 110%). In place of this extended cover as soon as ρ falls below 0.81, which is the mean value of the claims ratio, the extraprofit made is immediately transferred to the reinsurer. Note how, in this case, the free-zone, $[0.81, 0.86]$, is very narrow and corresponds to the security loading charged by the primary insurer.

The premiums for this scenario are, respectively, $H_g(h^-) = 0.086009$, $H_g(h^+) = 0.102038$ and $\Pi[h(\rho)] = 0.016028$.

As we did for the previous scenario, in Figure 10 and in Figure 11 we report the claims ratio and the underwriting result before and after the XV treaty. As we can see the volatility reduction effect is now much stronger: the values are more concentrated near the mean value and the peaks are almost completely removed.

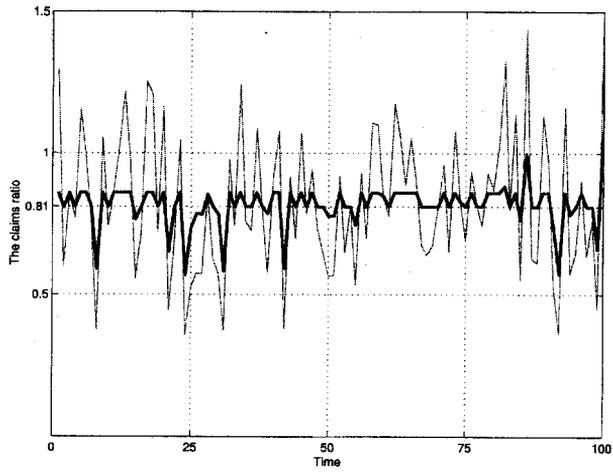


Figure 10: Scenario B: the claims ratio before and after the XV treaty.

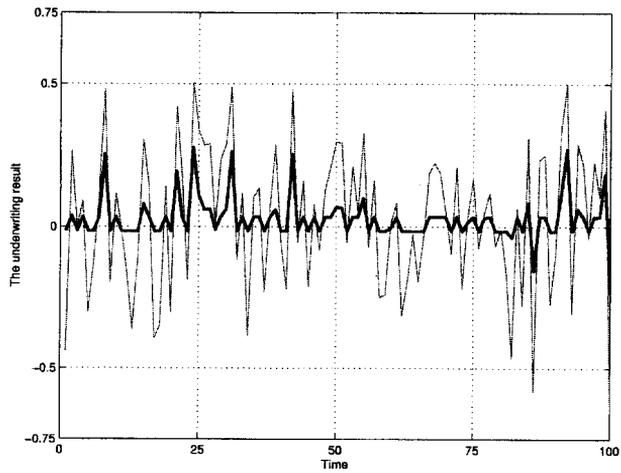


Figure 11: Scenario B: the underwriting result before and after the XV treaty.

In Table 3 and Table 4 the usual moments and percentiles of the key variables, before and after the intervention of the XV treaty on a single year and over 5 years, are reported.

	ρ_b	ρ_a	udr_b	udr_a	udr_{Re}
E	0.81423	0.81613	0.04627	0.02834	0.01793
σ^2	0.06615	0.00773	0.06615	0.00773	0.03510
γ	0.61531	0.58159	-0.61531	-0.58159	-0.75449
5%	0.41808	0.52808	-0.39085	-0.18948	-0.20137
95%	1.25135	1.01135	0.44242	0.29379	0.14863

Table 3

	udr_b	udr_a	udr_{Re}
E	0.23134	0.14170	0.08964
σ^2	0.38573	0.04384	0.19956
γ	-0.08142	0.27007	-0.15092

Table 4

From these values it clearly emerges that the potential risks have been "sterilized". The effect on the volatility (still measured on the variance of udr) is strong, leaving to the primary insurer less than 12% of the original variance. This hedging effect can even be appreciated measuring the gap between the percentiles: after the XV intervention this difference is much narrower belonging to the interval $[-0.18948, 0.29379]$.

The price for this smoothness and sterilization is, of course, higher than that paid in scenario A: as a consequence the mean value of the underwriting result now drops from 0.04627 to 0.02834. In other words, the primary insurer retains nearly 61% of the original underwriting result.¹¹

So far we considered the point of view of the primary insurer. On the reinsurer's side, even in this complete hedge scenario, the price/risk profile appear to be fair and in line with his greater attitude toward risk.

6 First Conclusions

In this paper we introduced a new reinsurance treaty aimed at reducing the volatility of the underwriting result of a non-life insurance company. This smoothing effect can effectively help the direct insurer in optimizing his profitability and, ultimately, his underwriting capability and strength. This treaty follows a rather different approach, concentrating more on the harmful effects caused by the volatility, rather than focusing on the absolute amount of losses.

We recall now the main features and properties of the XV treaty.

- ◊ It can provide an effective hedge against unfavourable events and, at the same time, the retention of a certain degree of profitability.

- ◊ This kind of risk cover can not be replicated by separate and traditional reinsurance strategies: its peculiar payment function is designed in order to enhance the cooperation between the two parties that are intertwined in a profit and loss exchange.
- ◊ The treaty features a modular structure which can be fully adapted to respond to the client's needs.
- ◊ It is easy to administer and bears very little managing costs.

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Notes

¹The present work is due to the close collaboration of the two authors. Along this line sections 1, 2.1, 2.3 and 6 belong to both the authors. Sections 2.2, 3, 4 and 5 belong to A. Iannizzotto. All the computations were performed by A. Iannizzotto.

²It is now common to distinguish between the so-called "traditional reinsurance" agreements and the "non-traditional" ones. Proportional and non-proportional treaties belong to the first group while ART (Alternative Risk Transfers) and more complex, option-like, structures belong to the latter group.

³Bold faces denote throughout the paper random variables except for the greek letters. We consider premiums as given. The underwriting result can simply be written as $1 - \rho$.

⁴These unusual profits are used to stabilize the underwriting result. As we shall see this feature will have a critical impact in premium quantification, allowing to obtain a low overall price for the cover.

⁵These products are common in Foreign Exchange Markets and are also known as "flexible forward", "cylinder option", "option fence" and "forward band".

⁶This could be a consequence for example of a lacking sound statistical information, of structural risk change due to exogenous phenomena and very high volatility of the claims ratio over time.

⁷A striking example could be the hail market in Italy where, due to a strong competition, premiums dropped and claims ratios soared to unprecedented levels.

⁸In the forthcoming of the paper we shall relax this assumption.

⁹We assume that X is a continuous variate with continuous cdf and ddf.

¹⁰See Wang and Young (1998) and Young (1999).

¹¹In comparing the tradeoff between the XV price and the risk reduction effect, it is important to highlight that these analysis do not consider the tangible benefits that such a cover would give both on the solvency and on the underwriting capacity of the primary insurer.

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