

BONUS-MALUS EXPERIENCE RATING AND RATING FACTORS*

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ABSTRACT

When a bonus-malus (BM) system is superimposed on an *a priori* rating system, the premium applicable to a risk of a particular tariff class is adjusted by multiplying a base premium by a coefficient which depends on the BM class of the risk. In this way, the premium of risks belonging to different tariff classes but to the same BM class, are updated by the same coefficient when reporting the same number of claims. However, in alternative models of experience rating, e.g. bayesian or credibility models, these adjustments depend also on the *a priori* premium level (based on the other rating factors and except for the BM class). Moreover, in bayesian systems the *a posteriori* premiums become closer and closer as time goes on. In this paper some different approaches of assessing the base premiums in a BM system are considered in order to obtain the above mentioned properties.

KEYWORDS

Bonus-malus systems. Experience rating.

1. INTRODUCTION

Bonus-malus (BM) systems are widely used in Europe and Asia to realise simple schemes of experience rating. However, some troubles connected with BM systems emerge when updating the *a priori* valuations owing to the experience: in particular, some reasonable properties that could be required are not in general satisfied.

In order to investigate which properties could be appreciated in an experience rating scheme, we can consider the bayesian claim experience rating (sometimes named, in the actuarial literature, BM system too) which evaluates premiums that reproduce exactly the expected claims *a posteriori*.

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For this purpose the simple Poisson-gamma model is recalled in § 2 and throughout this model we single out the following properties.

First, the *a posteriori* adjustments should depend on the *a priori* valuation. This aspect, very important when the BM system is applied to a portfolio of risks belonging to different tariff classes, has been dealt with for instance by Gisler (1996) and Taylor (1997). In particular, the bayesian model shows that, given the same claim experience, the premium adjustment coefficient should be higher for the low-risk policyholders.

Second, as time experience increases the *a priori* valuation should become less and less important and the *a posteriori* valuation should merely depend on the claim observations. As a consequence, risks having different *a priori* premiums, but showing the same claim experience, should get *a posteriori* premiums that become closer and closer as time goes on.

Third, the system should be financially balanced.

In § 3, we examine the usual multiplicative rule giving the BM premium. The premium for an insured risk is obtained by multiplying a base premium by a BM coefficient and by a tariff class relativity. By choosing as base premium the equilibrium premium, we satisfy the third property but not the first one.

In order to get the first property we can act assigning different base premiums to the various tariff classes. In § 4, we consider as base premiums the equilibrium premiums in the tariff classes. However, in this way the second property is not satisfied. Moreover, since the tariff classes are managed separately, the smoothing effect implied by the equilibrium premium calculation on the whole portfolio is reduced.

In § 5 we develop a method to determine the reference premiums in a BM system that makes all the above mentioned properties satisfied.

Finally, in § 6, following a suggestion by Taylor (1997) to build a BM scale adapted to the claim characteristics of the portfolio, we illustrate throughout a simple example how it is possible to build BM systems that produce an *a posteriori* premium valuation quite close to the bayesian one.

Another important aspect investigated in the paper is the solidarity implied by a BM system. Obviously, the solidarity level in a bayesian claim experience rating is zero, whereas BM systems show different types of solidarity: among BM classes in a given tariff class, among the different tariff classes and globally in the rating system.

2. BAYESIAN CLAIM EXPERIENCE RATING

Let us consider a portfolio of risks, e.g. motor insurance, subdivided into s tariff classes according to an *a priori* rating. Given a risk in the portfolio, let I be a random variable denoting his tariff class. For the claim number processes $N_1 | I = i, N_2 | I = i, \dots$ ($i = 1, \dots, s$) we make the following assumptions: their distributions depend on a random parameter Λ and

$$(1) \quad N_1 | I = i, \Lambda = \lambda, \quad N_2 | I = i, \Lambda = \lambda, \dots \text{ are i.i.d.,}$$

$$(2) \quad N_t | I = i, \Lambda = \lambda \quad \text{is Poisson distributed with parameter } \lambda \quad (t = 1, 2, \dots),$$

$$(3) \quad \Lambda | I = i \quad \text{is gamma distributed with density } f_i(\lambda) = \frac{\mu_i^{-r}}{\Gamma(r)} r^r \lambda^{r-1} \exp\left(-\frac{r}{\mu_i} \lambda\right).$$

Given a risk in tariff class i , let EZ_i be the expected cost per claim and $X_t | I = i$ be the total loss in year t . Under the usual hypotheses leading to a compound distribution for $X_t | I = i$, the risk premium in year t , P_{it} , evaluated at the beginning of year 1 is:

$$P_{it} = P_{i1} = E(X_t | I = i) = EZ_i \mu_i.$$

The risk premium in year $t+1$, given the experience $H = (N_1 = n_1, \dots, N_t = n_t)$, is

$$(2.1) \quad P_{it+1}(H) = E(X_{t+1} | I = i, H) = EZ_i E(N_{t+1} | I = i, H) = P_{i1} \frac{r + \sum_{h=1}^t n_h}{r + t\mu_i}.$$

We want to stress some properties of the bayesian claim experience rating, under this particular hypothesis for the claim number process. First, it is interesting to note that the quantity

$$(2.2) \quad \frac{P_{it+1}(H)}{P_{i1}} = \frac{r + \sum_{h=1}^t n_h}{r + t\mu_i}$$

is an *adjustment coefficient* which depends, besides on the experience H , also on the *a priori* valuation of the expected claim number μ_i .

In addition, from (2.2), the following property is satisfied:

P1 Given two tariff classes i, k such that $\mu_i < \mu_k$ and given the same claim experience over t years, we have

$$\frac{P_{it+1}(H)}{P_{i1}} > \frac{P_{kt+1}(H)}{P_{k1}},$$

which means that the adjustment coefficient is higher for the risk having, a priori, the lower expected claim number.

In Table 1 are reported the adjustment coefficients calculated with respect to two types of risks in tariff classes described by the following parameters:

$$\begin{aligned} I = 1: & \quad r = 1,317230564, \mu_1 = 0,118248053, \\ I = 2: & \quad r = 1,317230564, \mu_2 = 0,213116286. \end{aligned}$$

We note that the adjustment coefficients for tariff class 1 are always higher than those for tariff class 2. Therefore if two risks, one in each tariff class, report no claims, the risk in tariff class 1 gets a lower premium reduction than the one recognised to the risk in tariff class 2. If both risks report the same number of claims, the risk in tariff class 1 gets a higher premium penalisation than the one applied to the risk in tariff class 2. For instance, after a five-year experience, if no claims are reported, the risk in tariff class 1 gets a 30% discount on the initial premium level, whereas the risk in tariff class 2 gets a 45% discount. If one claim is reported

by both risks, the premium of the risk in tariff class 1 is increased by 20% and the risk in tariff class 2 gets a 3% discount on the initial premium level.

TABLE 1

Adjustment coefficients for tariff class 1							
N. of years of observation	N. of claims						
	0	1	2	3	4	5	6
1	0.91762	1.61426	2.31089	3.00752	3.70415	4.40078	5.09742
5	0.69020	1.21418	1.73816	2.26214	2.78612	3.31010	3.83408
10	0.52695	0.92700	1.32705	1.72709	2.12714	2.52718	2.92723
15	0.42616	0.74968	1.07321	1.39673	1.72026	2.04378	2.36731

Adjustment coefficients for tariff class 2							
N. of years of observation	N. of claims						
	0	1	2	3	4	5	6
1	0.86074	1.51419	2.16763	2.82108	3.47453	4.12797	4.78142
5	0.55281	0.97248	1.39215	1.81182	2.23149	2.65117	3.07084
10	0.38198	0.67197	0.96196	1.25195	1.54194	1.83193	2.12192
15	0.29181	0.51335	0.73488	0.95641	1.17795	1.39948	1.62102

In addition to property P1, the bayesian experience premium rating satisfies also the following well-known properties (e.g. see Lemaire (1995)).

P2 *Given two risks belonging to different tariff classes (having therefore different a priori premium valuations) and reporting the same claim experience, as the time experience increases their final premium valuations tend to get near.*

In fact, if we consider two risks, belonging to the tariff classes i and k respectively, given the same claim experience over t years, the ratio between their *a posteriori* premiums in year $t+1$ is

$$(2.3) \quad \frac{P_{it+1}(H)}{P_{kt+1}(H)} = \frac{P_{i1}}{P_{k1}} \frac{r + t\mu_k}{r + t\mu_i} = \frac{\mu_i}{\mu_k} \frac{r + t\mu_k}{r + t\mu_i}.$$

This ratio tends to 1 as t tends to infinity, hence the difference $P_{it+1}(H) - P_{kt+1}(H)$ tends to 0. Therefore, property P2 is satisfied.

P3 *The merit rating system is financially balanced in any year.*

In fact, by calculating the premium adjustments following (2.1), in any year and for any risk, the expected total premium is equal to the expected total loss.

Ending these considerations on the well-known Poisson-gamma model, we observe that here the gamma mixing distributions of the claim number processes have been expressed by using the parameterisation μ_i and r . The first parameter expresses the expected number of claims in tariff class i whereas the second one, which is concerned with the dispersion, is constant over the tariff classes. This choice is done for sake of simplicity, however, from

(2.1) and (2.2) we have that as t increases the effect of parameter r is reduced. For this reason, property P1 and P2 are asymptotically satisfied also in a more general situation. Note that in the following we are assuming $EZ_i = 1$, for any i .

3. EXPERIENCE RATING BY BONUS-MALUS SYSTEMS

Bonus-malus systems are widely used in motor insurance in order to update the *a priori* premium valuation according to the claim experience of the policyholder. In this way, the policyholders are subdivided into tariff classes, characterised by the values of some *a priori* tariff variables, and then these *a priori* valuations are adjusted *a posteriori* following the scheme provided by the BM system.

Given a BM system with J classes, let β_1, \dots, β_J be the *premium scale*. To calculate the premium a multiplicative model is generally used. If the *a priori* claim valuation is summarised by the relativities $\gamma_1, \dots, \gamma_s$, assumed constant over time, the *a posteriori* premium in year $t + 1$ for a risk in tariff class i and reporting the claim experience H , is given by

$$P_{t+1} \gamma_i \beta_{y(H)},$$

where P_{t+1} is a *reference* or *base premium* and $y(H)$ is the BM class to which the risk is assigned due to the claim experience H .

In order to have a financially balanced system (property P3), a suitable choice for the reference premium can be done.

In details, given a risk drawn from the portfolio let, as previously, I be his tariff class, $\{N_t, t \geq 1\}$ his claim number process and $\{Y_t, t \geq 1\}$ the process of the BM classes occupied by the risk.

We define *equilibrium premium* in year t the solution P_t^e of the following equation in P :

$$\sum_{i=1}^s \sum_{j=1}^J P \gamma_i \beta_j \Pr(I = i, Y_t = j) = E(N_t).$$

Therefore, if we choose P_t^e as reference premium in year t , the expected earned premium amount is equal to the expected claim amount all over the portfolio and the system is financially balanced (recall that $EZ_i = 1$ for $i = 1, \dots, s$).

In Figure 2 we compare the expected claim numbers $E(N_t | I = i, Y_t = j)$ with the BM premiums $P_t^e \gamma_i \beta_j$, in year $t = 15$. The calculations have been performed by the recursive procedure described in Gigante *et al.* (1999b), applied to a portfolio of risks in the two tariff classes of § 2 and with the following composition: $\Pr(I = 1) = 0.65$, $\Pr(I = 2) = 0.35$. We assume $\gamma_i = E(N_t | I = i)$ and the Italian BM system defined by the following transition rules

$$Y_{t+1} = \begin{cases} \max(Y_t - 1, 1) & N_t = 0 \\ \min(18, Y_t + 3N_t - 1) & N_t = 1, 2, 3, 4. \end{cases}$$

The premium scale is reported in the Table 2.

TABLE 2

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
β_j	0.50	0.53	0.56	0.59	0.62	0.66	0.70	0.74	0.78	0.82	0.88	0.94	1.00	1.15	1.30	1.50	1.75	2.00

Figure 1 shows, in year $t = 15$, the distribution of the policyholders among the BM classes in both the tariff classes. These distributions are relevant also when interpreting the numerical results reported in the sequel.

FIGURE 1

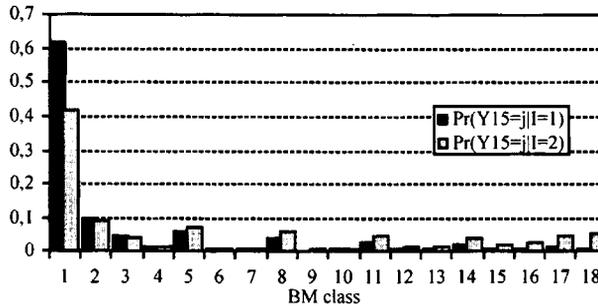
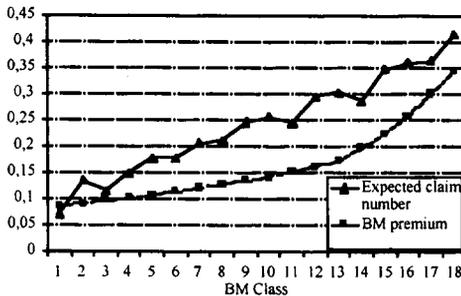
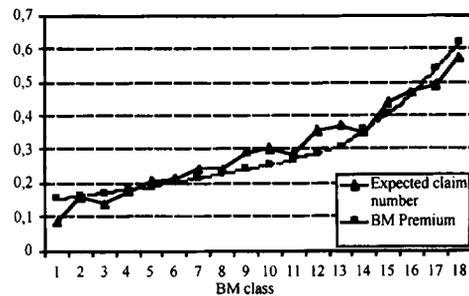


FIGURE 2

Tariff class 1



Tariff class 2



We see from Figure 2 that for both the tariff classes 1 and 2, the risks in BM class 1 pay for a premium higher than their expected claims. This means that this premium calculation method introduces a solidarity effect among the policyholders. However, since the same happens also for the risks in tariff class 2 and BM classes 17 or 18, this is not the usual solidarity which makes the low-risk policyholders to finance the high-risk ones. In fact, if we calculate for each tariff class the difference between the expected earned premium and the expected claim number:

$$\sum_{j=1}^J \Pr(Y_t = j | I = i) P_t^c \gamma_i \beta_j - E(N_t | I = i),$$

we have that it amounts to -0.01498 for tariff class 1 and to 0.02783 for tariff class 2: the high-risk policyholders finance the low-risk ones.

A measure of the global solidarity level introduced by this premium calculation method is given by

$$\sum_{i=1}^s \sum_{j=1}^J \Pr(I = i, Y_t = j) [P_t^e \gamma_i \beta_j - E(N_t | I = i, Y_t = j)]_+ = 0.01825,$$

where $[P_t^e \gamma_i \beta_j - E(N_t | I = i, Y_t = j)]_+ = \max (P_t^e \gamma_i \beta_j - E(N_t | I = i, Y_t = j), 0)$.

As far as the adjustment coefficients are concerned, given a reference premium P_{t+1} and the starting BM class h , they have the following expression

$$\frac{P_{t+1} \gamma_i \beta_{y(H)}}{P_t \gamma_i \beta_h} = \frac{P_{t+1} \beta_{y(H)}}{P_t \beta_h} \quad (i = 1, \dots, s).$$

As a difference to the bayesian experience rating, they do not depend on the *a priori* valuation summarised by the tariff class, which means that in this case property P1 is not in force. It is clear that, if we want a BM system to satisfy property P1, we need to affect the reference premiums that cannot be the same for all types of risks.

4. REFERENCE PREMIUM DEFINED BY THE EQUILIBRIUM PREMIUM IN EACH TARIFF CLASS

Given a portfolio of risks subdivided into s tariff classes, as in Gigante *et al.* (1999a) we can determine for each tariff class the equilibrium premium P_{it}^e ($i = 1, \dots, s$) such that

$$P_{it}^e \sum_{j=1}^J \beta_j \Pr(Y_t = j | I = i) = E(N_{it} | I = i).$$

The quantity

$$b_{it} = \sum_{j=1}^J \beta_j \Pr(Y_t = j | I = i)$$

is the *average premium level* in tariff class i and in year t .

Denoting by H the claim experience over t years of a risk in tariff class i and starting from BM class h , the premium adjustment coefficient is given by

$$\frac{P_{it+1}(H)}{P_{it}^e} = \frac{P_{it+1}^e \beta_{y(H)}}{P_{it}^e \beta_h}.$$

Let us consider two risks belonging to tariff classes i and k respectively, starting from the same BM class and having the same claim experience over t years. If we want to compare their premium adjustment coefficients, we just need to compare $\frac{P_{it+1}^e}{P_{it}^e}$ and $\frac{P_{kt+1}^e}{P_{kt}^e}$. Since $E(N_{t+1} | I = i) = P_{it+1}^e b_{it+1}$, $E(N_t | I = i) = P_{it}^e b_{it}$ and, in our hypothesis, $E(N_{t+1} | I = i) = E(N_t | I = i)$, the comparison is reduced to look at $\frac{b_{it}}{b_{it+1}}$ and $\frac{b_{kt}}{b_{kt+1}}$.

Given a BM system, there are tariff classes characterised by a claim frequency lower than the one for which the system has been designed. These risks are expected to concentrate in the bonus classes and their average premium levels are decreasing. In addition, the rate of decrement is higher for the less risky class. As a result, if i and k are two tariff classes of this type and

$$(4.1) \quad E(N_t | I = i) < E(N_t | I = k)$$

we have

$$(4.2) \quad \frac{b_{i1}}{b_{i,t+1}} > \frac{b_{k1}}{b_{k,t+1}}.$$

To illustrate this aspect, in Table 3 are reported the average premium levels and the ratios $\frac{P_{it+1}^e}{P_{i1}^e}$ calculated for the BM system and the portfolio composition described in § 3.

TABLE 3

Year t	Tariff class 1		Tariff class 2	
	$b_{1,t+1}$	$P_{1,t+1}^e/P_{11}^e$	$b_{2,t+1}$	$P_{2,t+1}^e/P_{21}^e$
	(1)	(2)	(3)	(4)
1	0.789206	1.000000	0.944137	1.000000
5	0.686688	1.149278	0.862780	1.094115
10	0.633773	1.245216	0.814548	1.158691
15	0.607745	1.298528	0.785823	1.200854
20	0.593069	1.330646	0.767237	1.229764
25	0.585582	1.347643	0.757233	1.245843

Note that, in $t = 10$, $\frac{P_{1,t+1}^e}{P_{11}^e}$ is about 1.24, whereas $\frac{P_{2,t+1}^e}{P_{21}^e}$ is about 1.16. Therefore if two risks, one in tariff class 1 and the other in tariff class 2, enter the system in BM class 11 and do not report claims, then after ten years they are both in BM class 1, but the percentage discount on the premium, given by $1 - \frac{P_{111}^e \beta_1}{P_{11}^e \beta_{11}}$, is 30% for the risk in tariff class 1 and 34% for the risk in tariff class 2. In case both risks are in BM class 4 (having reported one claim in ten years) the premium discounts are 17% and 22%, respectively.

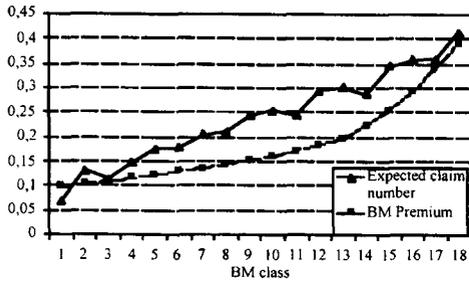
Notice that if we consider two tariff classes satisfying (4.1), one at least characterised by a claim frequency higher than the one for which the BM system has been designed, relation (4.2) still holds.

Therefore, by assuming as reference premium in each tariff class the equilibrium premium, the experience adjustment of the premium is substantially similar to the bayesian adjustment, in the sense that it assigns the highest adjustment coefficient to the less risky class. This fact implies that, in this way, property P1 holds for a BM system too.

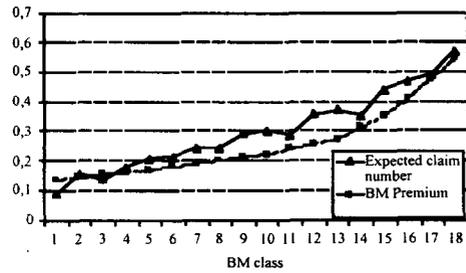
In Figure 3 we compare the expected claim numbers $E(N_t | I = i, Y_t = j)$ with the BM premiums $P_{it}^e \beta_j$, in year $t = 15$.

FIGURE 3

Tariff class 1



Tariff class 2



Note that, since we apply as reference premium in each tariff class its equilibrium premium, the tariff classes are financially balanced and property P3 holds. However, since the two classes have been managed separately, there is no more solidarity effect between them. It remains a solidarity effect among the BM classes in each tariff class. This solidarity effect can be measured by

$$\sum_{j=1}^J \Pr(Y_t = j | I = i) [P_{it}^e \beta_j - E(N_t | I = i, Y_t = j)]_+.$$

It amounts to 0.01718 for tariff class 1 and to 0.02100 for tariff class 2. The higher solidarity level in tariff class 2 does not clearly emerge from Figure 3. This is due, in part, on the different scales on the y-axes in the two graphs. Anyway, to really appreciate the solidarity effect in each tariff class, we have to take also into account the probability distributions in Figure 1, which show a concentration of the risks in BM class 1 much more remarkable in tariff class 1 than in tariff class 2.

With respect to property P2, this premium calculation method does not grant that it holds, in general. In fact, if we consider two risks in tariff classes i and k respectively, who are both in the same BM class in year t , the ratio of their premiums is

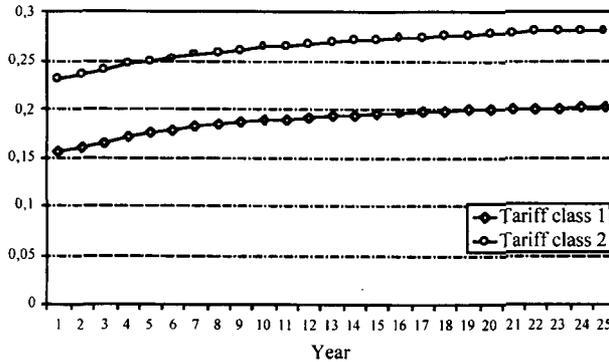
$$\frac{P_{it}^e}{P_{kt}^e} = \frac{E(N_t | I = i)}{E(N_t | I = k)} \frac{\sum_{j=1}^J \beta_j \Pr(Y_t = j | I = k)}{\sum_{j=1}^J \beta_j \Pr(Y_t = j | I = i)}.$$

Since the second hand side of the preceding expression depends on the premium scale it cannot be granted in general that the ratio $\frac{P_{it}^e}{P_{kt}^e}$ tends to 1.

In Figure 4 are reported the results of some numerical valuations for our portfolio. Note that the differences between the two premiums remain substantially constant.

In the next paragraph a different approach is proposed to overcome the various inconveniences implied by this approach.

FIGURE 4 – Equilibrium premiums



5. AN ALTERNATIVE PROPOSE FOR THE REFERENCE PREMIUM

We have seen in the previous paragraph how it is possible to manage the reference premium in order to make the BM system satisfy properties P1 and P3. In this paragraph we want to pursue, beside these results, also property P2, but giving up the equilibrium condition in each tariff class.

The equilibrium condition (property P3) on the whole portfolio, in presence of different reference premiums for the various tariff classes, is

$$(5.1) \quad \sum_{i=1}^s \sum_{j=1}^J P_{it} \beta_j \Pr(I = i, Y_t = j) = E(N_t).$$

It is possible to write the reference premiums as $P_{it} = P_{1t} (1 + f_i(t))$, where $f_i(t) = 0$.

In order to achieve property P2, the functions $f_i(t)$, $i \neq 1$, should tend to 0 as t tends to infinity and a suggestion on how to define them comes from the bayesian adjustment scheme of § 2. As previously, let us assume the portfolio of risks be subdivided into s tariff classes and let $\mu_1 < \dots < \mu_s$.

Recalling (2.3), the ratio between the premiums in year $t+1$ for a risk in class i and a risk in class 1, if they have reported the same claim experience, is

$$\frac{P_{it+1}}{P_{1t+1}} = \frac{\mu_i}{\mu_1} \frac{r + t\mu_1}{r + t\mu_i}.$$

Hence, in this model we assume $P_{it+1} = P_{1t+1} (1 + f_i(t+1))$, where

$$(5.2) \quad f_i(t+1) = \frac{\mu_i}{\mu_1} \frac{r + t\mu_1}{r + t\mu_i} - 1.$$

Now, turning back to the BM system, in order to get property P2 we could define the functions $f_i(t)$ just as in (5.2). Then, by solving (5.1) in P_{1t} , we obtain a BM system that satisfies properties P2 and P3: given the same claim experience, the premiums applied to risks belonging to different tariff classes tend to become near and near as the time experience

increases; moreover, the system is financially balanced. In addition, it is interesting to note that by defining the reference premium in this way also property P1 is satisfied. In fact, given two risks in tariff classes i and k respectively, such that $\mu_i < \mu_k$, and having the same initial and final BM classes, their premium adjustment coefficients in year t are

$$\frac{P_{it}}{P_{i1}} = \frac{P_{it}(1 + f_i(t))}{P_{i1}(1 + f_i(1))}, \quad \frac{P_{kt}}{P_{k1}} = \frac{P_{kt}(1 + f_k(t))}{P_{k1}(1 + f_k(1))}.$$

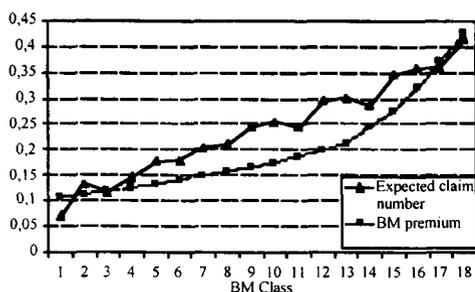
It is easy to prove that from $\mu_i < \mu_k$ it follows

$$\frac{1 + f_i(t)}{1 + f_i(1)} > \frac{1 + f_k(t)}{1 + f_k(1)}.$$

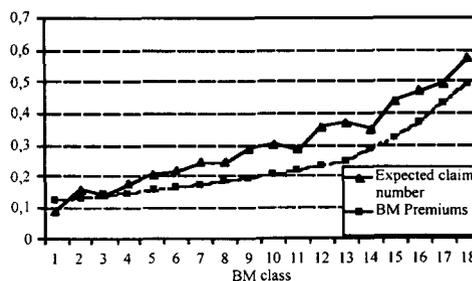
In the next Figure 5, we compare the expected claim numbers $E(N_t | I = i, Y_t = j)$ with the BM premiums $P_{it}(1 + f_i(t))\beta_j$, in year $t = 15$.

FIGURE 5

Tariff class 1



Tariff class 2



Now, a certain solidarity is introduced between the two tariff classes. To appreciate this effect we can calculate for each tariff class, the following quantity:

$$\sum_{j=1}^J \Pr(Y_t = j | I = i) P_{it} \beta_j - E(N_t | I = i).$$

For tariff class 1 it amounts to 0.01035 and for tariff class 2 it amounts to -0.01923 . Therefore, we have a compensation between the two classes: the 65% of low-risk policyholders in the portfolio partially finance the 35% of high-risk policyholders. A measure of the global solidarity level here introduced is given by

$$(5.3) \quad \sum_{i=1}^s \sum_{j=1}^J \Pr(I = i, Y_t = j) [P_{it} \beta_j - E(N_t | I = i, Y_t = j)]_+ = 0.02029.$$

The solidarity level is therefore essentially of the same amount of the one obtained in § 3. Here we have chosen the functions $f_i(t)$ on the basis of the simple Poisson-gamma model. Other model assumptions on the claim number process or considerations on the

compensation among BM classes and tariff classes could suggest the use of different functional types.

All the considered approaches to determine the references premiums do not produce good results in the matching between the risk and the BM premiums. This is partly due to the fact that the evaluations have been performed with reference to an assigned premium scale, namely the Italian BM scale, which does not take into account the risk characteristics of our portfolio. When projecting a BM system it is possible to define a BM scale adapted to the portfolio characteristics summarised by the *a priori* claim valuation. This problem is dealt with in the next paragraph.

6. A BONUS-MALUS SCALE ADJUSTED BY A PRIORI RATING

Taylor (1997) considers the problem of defining a BM scale that takes into account the *a priori* characteristics of the risks in the portfolio, with the aim of recognising “the differentiation of underlying claim frequency by experience, but only to the extent that this differentiation is not already recognised within base premiums”. For this purpose, assuming that the risks in the portfolio are subdivided into s tariff classes, he defines the ratios $r_j^{(t)}$ that, in our notation, can be written as follows:

$$r_j^{(t)} = \frac{\sum_{i=1}^s E(N_t | Y_t = j, I = i) \Pr(I = i | Y_t = j)}{\sum_{i=1}^s E(N_t | I = i) \Pr(I = i | Y_t = j)}.$$

Let

$$\lambda_j^{(t)} = E(N_t | Y_t = j) = \sum_{i=1}^s E(N_t | Y_t = j, I = i) \Pr(I = i | Y_t = j)$$

$$\mu_j^{(t)} = \sum_{i=1}^s E(N_t | I = i) \Pr(I = i | Y_t = j).$$

Therefore one gets the following factorisation of the risk premium $\lambda_j^{(t)}$ for a risk in BM class j at time t :

$$\lambda_j^{(t)} = \mu_j^{(t)} r_j^{(t)}.$$

Since $\mu_j^{(t)}$ are the expected earned premiums paid by the risks in BM class j on the basis of their *a priori* valuations, Taylor explains $r_j^{(t)}$ as the factor “by which experience revises the average prior risk premium in BM level j ”. By choosing the BM class h as the reference class, a BM scale justified by the experience in each BM class j should be defined as follows:

$$(6.1) \quad \beta_j^{(t)} = \frac{r_j^{(t)}}{r_h^{(t)}} \quad (j = 1, \dots, J).$$

It is easy to see that if the premium of a risk in BM class j in year t is given by his *a priori* valuation $E(N_t | I = i)$ multiplied by $r_j^{(t)}$, then each BM class is financially balanced and so is the whole portfolio. In fact,

$$\sum_{j=1}^J \Pr(Y_t = j) \sum_{i=1}^S r_j^{(t)} E(N_t | I = i) \Pr(I = i | Y_t = j) = \sum_{j=1}^J \Pr(Y_t = j) E(N_t | Y_t = j) = E(N_t).$$

Now, if we calculate the premium of a risk in tariff class i and BM class j at time t as follows

$$(6.2) \quad \beta_j^{(t)} r_j^{(t)} E(N_t | I = i),$$

it results that the adjustment on the prior valuation takes properly into account the claim experience reported by the policyholder. In addition the system is financially balanced. However, in (6.2) the premium scale is recalculated every year whereas in traditional BM system it is constant over time. If we want to set a fixed BM scale, but with the property of being adapted to the *a priori* claim valuation of the portfolio we can, for instance, substitute in (6.1) the asymptotic values of $r_j^{(t)}$ and $r_h^{(t)}$, as suggested by Taylor, or we can calculate a suitable weighted average of the ratios $r_j^{(t)}$ relative to a finite time horizon of interest. Obviously, the problem of assigning the reference premiums granting properties P1, P2, P3 emerges again and it can be solved as in § 5.

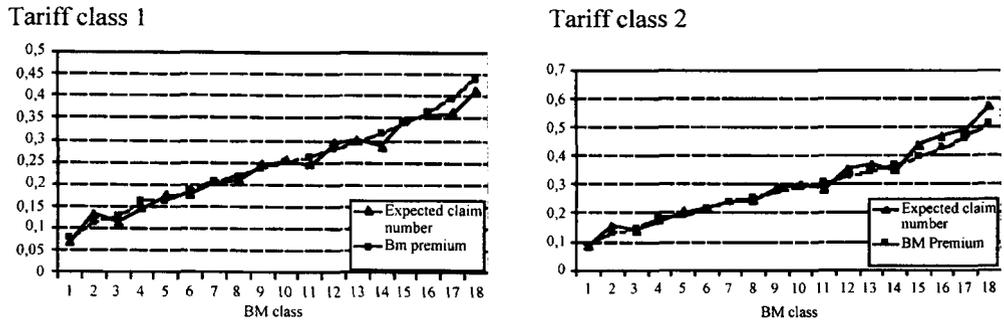
TABLE 4

BM class j	$r_j^{(15)}$	$r_j^{(20)}$	$r_j^{(25)}$	Mean	β_j
1	0.376344	0.500922	0.559298	0.478855	0.256580
2	0.650175	0.689890	0.790243	0.710103	0.380488
3	0.650173	0.922253	0.814639	0.795688	0.426346
4	0.925482	0.965062	1.165658	1.018734	0.545859
5	0.913438	1.051360	1.127448	1.030749	0.552297
6	1.062215	1.181341	1.249696	1.164417	0.623919
7	1.200484	1.352587	1.365901	1.306324	0.699956
8	1.167136	1.425855	1.521407	1.371466	0.734860
9	1.471782	1.390938	1.605841	1.489520	0.798116
10	1.487774	1.673229	1.556973	1.572659	0.842663
11	1.420214	1.710834	1.834730	1.655259	0.886923
12	1.779041	1.620652	1.890319	1.763337	0.944833
13	1.817222	1.979378	1.802286	1.866295	1.000000
14	1.721195	2.043060	2.157736	1.973997	1.057709
15	2.164752	1.971908	2.248610	2.128423	1.140454
16	2.263619	2.393694	2.251643	2.302985	1.233988
17	2.307368	2.535319	2.619946	2.487544	1.332878
18	2.758803	2.728788	2.801555	2.763049	1.480499

In Table 4 are reported the ratios $r_j^{(t)}$ calculated in years 15, 20, 25 for a BM system with the same transition rules of the Italian one. The average values are reported in the fourth column. By choosing as reference class the 13th (as in the Italian BM system) we get the premium scale reported in the fifth column. In Figure 6 the expected claim numbers are compared with the BM premiums calculated for a BM system with the same transition rules of the Italian one and premium scale the one in Table 4. The reference premiums are those defined in (5.1) and the functions f_i are given by (5.2). It is interesting to observe that this BM system

produces a remarkably matching between expected claims and BM premiums whereas on the other side the solidarity effect is reduced. Now the solidarity, measured by (5.3), amounts to 0.00576.

FIGURE 6



As a closing remark, we have to stress that the very simple portfolio composition, here considered, obviously do not reproduce a real portfolio, but it has just the aim of illustrating the various methods used to set the reference premiums in an assigned BM system. In particular, the numerical examples help to appreciate some effects connected to properties P1, P2 and P3.

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