

TECHNICAL BASES FOR LTC COVERS INCLUDING MORTALITY AND DISABILITY PROJECTIONS

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ABSTRACT

Long Term Care (LTC) covers are insurance products for which it is difficult to choose proper pricing and valuation bases, since information coming from practical experiences is still scanty. Moreover, due to their lifetime duration, they are significantly affected by demographical trends.

This paper addresses the question of building technical bases for LTC covers. Uncertainty coming from future mortality trends and disability levels is stressed. The framework adopted is a multistate, time-continuous model; premiums and reserves are assessed relying on an inception-annuity scheme. A risk analysis with regard to both mortality and disability rates is performed.

KEYWORDS

Long Term Care covers, premium basis, valuation basis, loss function, projected mortality table, projected inception rates, longevity risk

1. INTRODUCTION

Long Term Care (LTC) is care required in relation to chronic (or long-lasting) bad health conditions. LTC insurance provides income support for the insured, who needs nursing and/or medical care, in the form either of a forfeiture annuity benefit or nursing and medical expense refunding. In some countries (e.g. Japan) LTC products are sold according to which the insured can choose, in case of a claim, between an annuity benefit or an appropriate care service provided by organizations offering nursing care services. Given the type of claim covered, LTC insurance has a lifetime duration.

The common approach to the representation of LTC covers is multistate modelling. Actuarial theory has developed models easy to handle from a formal viewpoint. The major task in implementations of such models is due to the fact that LTC covers are recent products and experience data are still scanty. Moreover, given their lifetime duration, these covers are affected by demographical trends. Because of the uncertainty in estimating the future levels of mortality and morbidity, considerable difficulties are therefore encountered in pricing and reserving.

This paper is devoted to the analysis of the risks coming from uncertainty of future

demographical trends. As it is well-known, such uncertainty suggests the adoption of projected tables. The use of mortality projections for pricing life covers sold to the elderly, such as annuities, is extensively widespread (and for reserving as well); on the contrary, such practice is not common for health insurances. Difficulties for these latter covers originate from the several causes of uncertainty by which they are affected, namely mortality levels, morbidity rates, amount of claims (when expense refunding covers are dealt with). In particular, the paucity of sickness data relating to very old ages increases the difficulty in estimating claim frequencies and amounts.

The main aim of this paper is the construction of projected mortality and disability tables for pricing LTC covers; the effects of uncertainty related to the future scenario, in terms of the risk borne by the insurer, are investigated.

For the sake of simplicity, the following hypotheses are adopted throughout the paper. Insurance covers with only one level of frailty and involving forfeiture benefits are considered. In particular, the paucity of data concerning annual claim amounts and frequencies suggests to disregard expense refunding covers (for which premiums are usually assessed with reference to the maximum amount insured). Randomness other than the demographic one is disregarded; in particular, a deterministic financial structure is adopted. Expense loading, profit assessment and reinsurance are not dealt with.

The paper is organized as follows. In Section 2 the main characteristics of the multistate approach are recalled and the basic instrument for performing the risk analysis, the loss function, is defined. In Section 3 demographical trends affecting LTC covers are discussed and some projected demographical functions are built. In Section 4 and 5 the riskiness of a Stand Alone cover and an Enhanced Pension are, respectively, analysed, considering in particular uncertainty of the future scenario in which the policies will evolve. Finally, Section 6 concludes with some final remarks.

2. MULTISTATE MODEL FOR LTC COVERS

(a) The Markov model. Consider a life insurance policy and model its random behaviour within a multistate framework. In particular, we represent its evolution as a sample path of a time-continuous, inhomogeneous Markov chain with a finite state space \mathcal{S} , $\mathcal{S} = \{1, 2, \dots, N\}$. The seminal contributions to multistate modelling in life insurance are given by Hoem (1969, 1988). An overall perspective on such approach is also given by Wolthuis (1994), Pitacco and Olivieri (1997) and Haberman and Pitacco (1999). We address to such references for a discussion of the main features of the multistate approach.

The following notation is adopted.

- $S(t)$ is the (unknown) state of the policy at time t in the period $[0, n]$ of the coverage; we assume $S(0) = 1$;
- $P_{ij}(t, u)$ ($0 \leq t \leq u$, $i, j \in \mathcal{S}$) are the transition probabilities

$$P_{ij}(t, u) = \Pr\{S(u) = j | S(t) = i\}$$

- $\mu_{ij}(t)$ ($t \geq 0$, $i, j \in \mathcal{S}$, $i \neq j$) are the transition intensities

$$\mu_{ij}(t) = \lim_{u \rightarrow t} \frac{P_{ij}(t, u)}{u - t}$$

It is assumed that such functions are well-defined for all the operations to be performed. With reference to an LTC cover, the multistate model usually consists of the following states: state 1 = “healthy” (or active), state 2 = “frailty (or disabled) at level I”, state 3 = “frailty at level II”, . . . , state N = “dead”. As far as transitions are concerned, usually hierarchic models are adopted, since the commonly chronic character of frailty allows to disregard the possibility of recovery. Note that such assumption is also suggested by the lack of LTC data. For an extensive examination of the characteristic of LTC products and the related multistate models, readers are referred to Haberman and Pitacco (1999) (also for a rich list of references). In numerical implementations, we consider one level of frailty only; hence the multistate model depicted in figure 2.1 is adopted.

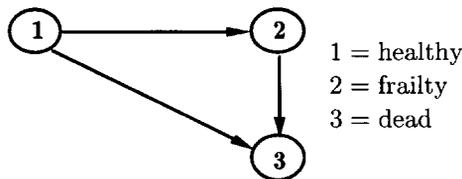


Figure 2.1 – A multistate model for LTC insurance with one level of frailty

We recall that in the three-state model of figure 2.1, solving the Kolmogorov differential equations leads to the following expressions for the transition probabilities used in premium and reserve calculation

$$P_{11}(t, u) = e^{-\int_t^u (\mu_{12}(s) + \mu_{13}(s)) ds} \quad (2.1)$$

$$P_{22}(t, u) = e^{-\int_t^u \mu_{23}(s) ds} \quad (2.2)$$

$$P_{12}(t, u) = \int_t^u P_{11}(t, s) \mu_{12}(s) P_{22}(s, u) ds \quad (2.3)$$

(b) Loss function. LTC products with forfeiture benefits involve annuities with a fixed amount. In the time-continuous model the following type of benefits is therefore considered:

- continuous annuities, paid at time t at a rate $b_j(t)$ if $S(t) = j$.

As regards to premiums, one of the following funding systems could be adopted

- continuous premiums, paid at time t at a rate $p_j(t)$ if $S(t) = j$;
- a single premium, π , paid at time 0.

Let I_E denote the indicator of event E . Assume a deterministic (and constant) force of interest δ and denote with $v = e^{-\delta}$ the annual discount factor. Further, let n be the maximum duration of the policy; given the lifetime duration of LTC covers, we have $n = \omega - x$ where x is the age at policy issue and ω the maximum age. The present value at time t of future benefits and premiums for a generic LTC policy are respectively given (for $t \geq 0$) by

$$Y(t) = \int_t^n v^{u-t} \sum_j b_j(u) I_{\{S(u)=j\}} du \quad (2.4)$$

$$X(t) = \int_t^n v^{u-t} \sum_j p_j(u) I_{\{S(u)=j\}} du \quad (2.5)$$

When the single premium is considered, $X(t)$ simply reduces to π for $t = 0$ and 0 for $t > 0$.

Since we can reasonably exclude that benefits and premiums are paid at the same time, only the functions $b_j(t)$ can be considered, meaning that they represent benefit payment when positive and premium payment when negative. Equations (2.4) and (2.5) can then be merged as follows

$$L(t) = \int_t^n v^{u-t} \sum_j b_j(u) I_{\{S(u)=j\}} du \quad (2.6)$$

The function $L(t)$ is commonly called loss function at time t , since it expresses the difference between future benefits and premiums (actually, for what said above we find $L(t) = Y(t) - X(t)$). In (2.6) the loss function is referred to a single policy. Expression of $L(t)$ in the case of a single premium is straightforward. We point out that an alternative (but equivalent) definition of loss function is given in Wolthuis (1994); we adopt (2.6) since it is easier to appraise.

More generally, we will consider also the loss function for a given portfolio. Let S° be the subset of S in which insurance covers are no longer in force (conversely, $S - S^\circ$ is the set of states in which some policies are still in force). Furthermore, let $N(t)$ be the number of contracts in force at time t and $N_i(t)$ the number of contracts in state i at that time (viz $N(t) = \sum_{i \notin S^\circ} N_i(t)$). Denoting with $L^{(h)}(t)$ the loss function for the h -th policy in the portfolio at time t , we define the portfolio loss function as follows

$$\mathcal{L}(t) = \sum_{h=1}^{N(t)} L^{(h)}(t) = \sum_{i \notin S^\circ} \sum_{h=1}^{N_i(t)} L^{(h)}(t) \quad (2.7)$$

Valuation of a generic policy or a portfolio involves some moments of the loss function. Moments for the present value of benefits and premiums for a generic policy are discussed, for example, in Norberg (1991, 1995). For the sake of brevity, in the sequel we derive moments of the loss function just for the types of cover examined in numerical evaluations.

(c) Premiums and reserve. We assume, as it is quite common for lifetime covers, that premiums and reserves are calculated according to the equivalence principle. For this purpose, a set of assumptions describing the future scenario, i.e. the future expected levels of interest, mortality and morbidity, must be assigned (usually on a conservative basis). Such set of hypotheses is usually called technical basis.

Let $\mathcal{H}^{[p]}$ and $\mathcal{H}^{[r]}$ be the technical basis used, respectively, for pricing and reserving; usually $\mathcal{H}^{[r]}$ coincides with $\mathcal{H}^{[p]}$, unless further information is gained on the future scenario during the insurance period. We assume that the equivalence principle must be fulfilled at policy level; hence for each policy premiums must be calculated so that

$$E[L(0)|S(0) = 1; \mathcal{H}^{[p]}] = 0 \quad (2.8)$$

i.e.

$$E[X(0)|S(0) = 1; \mathcal{H}^{[p]}] = E[Y(0)|S(0) = 1; \mathcal{H}^{[p]}] \quad (2.8')$$

The reserve for a policy in state i at time t is then defined, for an assigned level of premiums, as follows

$$V_i(t) = E[L(t)|S(t) = i; \mathcal{H}^{[r]}] \quad (2.9)$$

3. CHOICE OF THE TECHNICAL BASIS

(a) Mortality and disability trends at adult ages. The choice of a proper valuation basis is particularly difficult when LTC covers are dealt with for two main reasons

- (i) LTC are recent products and experience data are rather scanty;
- (ii) recent trends in mortality and morbidity witness significant changes that contribute in defining a moving scenario in which LTC products will evolve.

Aspect (i) is usually overcome by resorting to medical data, properly transformed to reflect selection of insured people with respect to the general population. Moreover, medical data are usually in the form of prevalence rates, whereas incidence rates are required for insurance pricing; hence, a transformation is necessary also in this regard. Such problems are not dealt with in this paper (see, for example, Gatenby (1991), Olivieri (1996) and Haberman and Pitacco (1999)).

Aspect (ii) has emerged in recent years. Mortality trends at adult ages reveal decreasing annual probabilities of death (see for example Benjamin and Soliman (1993), Macdonald (1997), Macdonald et al. (1998)). In particular, the following phenomena are observed in populations including both healthy and disabled lives

- (1) an increasing concentration of deaths around the mode of the density function of the future lifetime distribution (the so-called “curve of deaths”)
- (2) a forward shift of the mode of the curve of deaths.

These changes clearly affect any cover involving lifetime benefits. In particular, whilst the former aspect reduces uncertainty since with higher probability the actual duration of life might coincide with its mode, the latter increases the risk inherent in the management of a policy, the magnitude of the above mentioned shift being unknown.

In the case of health covers, such as LTC, risk emerges further from uncertainty concerning the time spent in the disability state. Actually, when living benefits are paid in case of disability it is not merely important how long one lives, but also how long he/she lives in a condition of disability.

Although it is reasonable to assume a relationship between mortality and morbidity, the relevant definition is difficult due to the complexity of such a link and to the impossibility of defining and measuring disability objectively. Three main theories have been formulated about the evolution of senescent morbidity (as pointed out, for example, in Swiss Re (1999)).

- (i) “Compression theory” (see Fries (1980)): chronic degenerative diseases will be postponed until the latest years of life because of medical advances. Assuming there is a maximum age, these improvements will result in a compression of the period of morbidity.
- (ii) “Pandemic theory” (see Gruenberg (1977) and Kramer (1980)): the reduction in mortality rates is not accompanied by a decrease of morbidity rates; hence, the number of frailty people will increase steadily.

- (iii) “Equilibrium theory” (see Manton (1982)): most of the changes in mortality are related to specific pathologies. The onset of chronic degenerative diseases and disability will be postponed and the time of death as well.

The scenarios depicted by the above mentioned theories produce rather different consequences for the insurer; in particular, Compression theory suggests optimistic views, whilst Pandemic theory pessimistic ones. The deep differences among such theories imply a high level of uncertainty about the evolution of senescent morbidity. The adoption of projected tables for the evaluation of insured benefits seems then appropriate. However, since the three theories imply rather different scenarios, the mentioned uncertainty should be included in the projection model.

(b) Projected scenarios. In order to appraise the risk inherent in a tariff structure for LTC covers, we have explicitly considered uncertainty of the future actual evolution of mortality and morbidity. To this aim, we have built several scenarios, each including a given projection of the future mortality and disability trends.

It must be pointed out that the traditional actuarial approach to demographic projections consists in extrapolations of (recent) trends as far as these can be perceived from observed data. On the contrary, the approach adopted in this paper (more common in the field of demography) leads to models expressing the basic characteristics of the evolving scenario in which demographical changes take place. In this latter approach, the use of analytical laws for mortality and disability rates is required, whose parameters are functions of the calendar year. The adequacy of the projection model can be checked comparing the behaviour of some quantities with the scenario characteristics suggested by the three theories mentioned in point (a). With reference to the multistate model of figure 2.1, we have defined possible scenarios (and tested their adequacy) in terms of the evolution of the expected time spent in the healthy state and in the frailty state.

Let us consider a person in state i at time t . We define the time expected to be spent in state j in the period (t, n) as follows

$$\bar{e}_{ij}(t) = \int_t^n P_{ij}(t, u) du \quad (3.1)$$

In the three-state model of figure 2.1, given the meaning of n ($n = \omega - x$), the following quantities can be defined

- $\bar{e}_{11}(t)$ expected time spent in the healthy state for a healthy person, i.e. healthy life expectancy for a healthy person;
- $\bar{e}_{12}(t)$ expected time spent in the frailty state for a healthy person, i.e. frailty life expectancy for a healthy person;
- $\bar{e}_{22}(t)$ life expectancy for a disabled person.

Note that the total life expectancy for a healthy life is

$$\bar{e}_1(t) = \bar{e}_{11}(t) + \bar{e}_{12}(t) \quad (3.2)$$

With reference to a person entering insurance in the healthy state (at a given age), the theories mentioned in point (a) can be expressed in terms of the evolution of life expectancy as follows:

- (i) when compared to observed data, $\bar{e}_1(t)$ increases with a major contribution (in relative terms) from $\bar{e}_{11}(t)$;
- (ii) $\bar{e}_1(t)$ increases with a major contribution (in relative terms) from $\bar{e}_{12}(t)$;
- (iii) $\bar{e}_{11}(t)$ and $\bar{e}_{12}(t)$ increase at similar rates.

In numerical evaluations, we have represented mortality in terms of the Weibull law, since it reflects more easily than other laws some specific future trends of mortality (see Olivieri and Pitacco (1999)). More precisely, we have assumed

$$\mu_{13}(t) = \frac{\beta}{\alpha} \left(\frac{x+t}{\alpha} \right)^{\beta-1} \quad \alpha, \beta > 0 \tag{3.3}$$

$$\mu_{23}(t) = (1 + \gamma) \mu_{13}(t) \quad \gamma \geq 0 \tag{3.4}$$

Note that according to (3.4) mortality of frailty lives is supposed to keep higher than that of healthy people. Although specific data at this regard are lacking, such hypothesis seems quite reasonable.

As far as disability is concerned, medical data suggest an exponential behaviour of inception rates; hence we have assumed the Gompertz law

$$\mu_{12}(t) = \eta e^{\lambda(x+t)} \quad \eta, \lambda > 0 \tag{3.5}$$

With reference to male people aged 65 in the current year (time 0), we have adopted the five scenarios listed in table 3.1. Scenario \mathcal{H}_C has been taken as a starting point for the projection of scenarios $\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_5$, since it comes from cross-sectional observations of mortality and disability of elderly people. In particular, $\mu_{12}^{(\mathcal{H}_C)}(t)$ (with obvious meaning of the symbol) comes from appropriate transformations of OPCS survey data on the prevalence of disability among adults (see Gatenby (1991) and Olivieri (1996)). The five projected scenarios have been built with reference to their impact on the expected time spent in the healthy and in the frailty state. Table 3.2 shows the evaluation of life expectancies $\bar{e}_{11}(0)$, $\bar{e}_{12}(0)$, $\bar{e}_1(0)$ and $\bar{e}_{22}(0)$ under each set of hypotheses, together with the changes of such quantities with respect to scenario \mathcal{H}_C .

| | α | β | γ | η | λ |
|-----------------|----------|---------|----------|----------|-----------|
| \mathcal{H}_C | 82 | 7 | 0.1 | 8.27e-06 | 0.095599 |
| \mathcal{H}_1 | 83.5 | 8 | 0.1 | 1.08e-05 | 0.090437 |
| \mathcal{H}_2 | 85.2 | 9.15 | 0.1 | 1.08e-05 | 0.090437 |
| \mathcal{H}_3 | 85.2 | 9.15 | 0.1 | 8.27e-06 | 0.095599 |
| \mathcal{H}_4 | 85.2 | 9.15 | 0.1 | 5.75e-06 | 0.102944 |
| \mathcal{H}_5 | 87 | 10.45 | 0.1 | 5.75e-06 | 0.102944 |

Table 3.1 – Parameters of the transition intensities

Firstly, note that we have assumed that total life expectancy will increase anyhow. This is suggested by mortality trends in populations that include both healthy and frailty people. In absolute terms, the changes in $\bar{e}_1(0)$ mainly depend on the changes in $\bar{e}_{11}(0)$. Therefore, consequences suggested by Compression, Equilibrium and Pandemic theory must be checked by looking at the relative contributions of $\bar{e}_{11}(0)$ and $\bar{e}_{12}(0)$ (as shown, for example, by the quantities $\bar{e}_{1i}^{(\mathcal{H}_i)}(0)/\bar{e}_{1i}^{(\mathcal{H}_C)}(0) - 1$, $i = 1, 2$, where $\bar{e}_{1i}^{(\mathcal{H}_i)}(0)$ is the life expectancy calculated according to scenario \mathcal{H}_i).

A close look at the relative increases reveals that for the insurer \mathcal{H}_1 represents a scenario that should involve lower costs than the others for two reasons. Firstly, there is a slight

increase in total expected life. Secondly, the change in the time expected to be spent in the healthy state is bigger in percentage than the one related to disability, which actually falls down to a negative value. We have therefore chosen this evolutionary hypothesis because it seems one of the best suited to represent consequences depicted by Compression theory.

On the other side, \mathcal{H}_5 represents the scenario with the highest costs involved since it assumes a fall in mortality rates accompanied by a substantial rise in disability rates, which imply a considerable increase in frailty as well as total life expectancies. Therefore, this scenario can reasonably express the Pandemic theory evolutionary hypothesis, although the latter does not lead to any particular increase in disability rates. However, taking into account the scope of this paper, this more pessimistic scenario can also reasonably include the risk of an underestimation of transition intensities $\mu_{12}(t)$, due to the fact that they are obtained from medical data and not from insurance experiences. Scenario \mathcal{H}_3 assumes a “medium” decrease in mortality rates in respect of \mathcal{H}_C . Moreover, no change in respect of \mathcal{H}_C has been considered for the disability law. The result is a projection which is somehow intermediate between the above depicted scenarios, with a change in total life expectancies which receives almost equal (relative) contributions from changes in healthy and frailty life expectancies. For this reason, this scenario can be considered as reflecting the evolutionary projection of Equilibrium theory.

Finally, scenarios \mathcal{H}_2 and \mathcal{H}_4 depict projections that are intermediate between scenario \mathcal{H}_3 and the other two “extreme” scenarios \mathcal{H}_1 and \mathcal{H}_5 .

It must be stressed that an unchanged link between $\mu_{13}(t)$ and $\mu_{23}(t)$ has been assumed in projections. This choice has been suggested by the lack of specific observations on mortality of active and disabled lives; usually, $\mu_{13}(t)$ is set equal to the general population mortality, where both healthy and frailty people are included, and $\mu_{23}(t)$ is slightly increased in its respect. We have adopted this simplification also for the projected functions.

| | $\bar{e}_{11}^{(\mathcal{H})}(0)$ | $\bar{e}_{12}^{(\mathcal{H})}(0)$ | $\bar{e}_1^{(\mathcal{H})}(0)$ | $\bar{e}_{22}^{(\mathcal{H})}(0)$ | $\frac{\bar{e}_{11}^{(\mathcal{H})}(0)}{\bar{e}_{11}^{(\mathcal{H}_C)}(0)} - 1$ | $\frac{\bar{e}_{12}^{(\mathcal{H})}(0)}{\bar{e}_{12}^{(\mathcal{H}_C)}(0)} - 1$ | $\frac{\bar{e}_1^{(\mathcal{H})}(0)}{\bar{e}_1^{(\mathcal{H}_C)}(0)} - 1$ |
|-----------------|-----------------------------------|-----------------------------------|--------------------------------|-----------------------------------|---|---|---|
| \mathcal{H}_C | 14.428 | 1.566 | 15.995 | 15.307 | 0.00% | 0.00% | 0.00% |
| \mathcal{H}_1 | 15.156 | 1.435 | 16.591 | 15.931 | 5.05% | -8.41% | 3.73% |
| \mathcal{H}_2 | 16.042 | 1.563 | 17.605 | 16.983 | 11.19% | -0.25% | 10.07% |
| \mathcal{H}_3 | 15.844 | 1.749 | 17.593 | 16.983 | 9.82% | 11.65% | 10.00% |
| \mathcal{H}_4 | 15.501 | 2.073 | 17.574 | 16.983 | 7.43% | 32.36% | 9.88% |
| \mathcal{H}_5 | 16.577 | 2.366 | 18.943 | 18.397 | 14.89% | 51.05% | 18.44% |

Table 3.2 – Life expectancies

As far as numerical evaluations are concerned, calculations have been performed using the matrix language module of SAS software (SAS/IML); in particular, the QUAD procedure (based on adaptive Romberg-type integration techniques), has been adopted for numerical integration.

4. RISKINESS OF THE STAND ALONE COVER

(a) **Loss function.** A stand alone LTC insurance provides a fixed amount in the case of LTC need. The amount of the annuity can be defined as a function of the frailty

level. Since only one level of frailty has been considered in this paper, it is assumed that only one level of benefit is provided. For brevity, we adopt a constant amount, b_2 . Consider a policy issued to an elderly person; reasonably, such cover is financed with a single premium. The loss function at individual level is for $t = 0$

$$L(0) = b_2 \int_0^n v^u I_{\{S(u)=2\}} du - \pi \quad (4.1)$$

and for $t > 0$

$$L(t) = b_2 \int_t^n v^{u-t} I_{\{S(u)=2\}} du \quad (4.2)$$

For the sake of simplicity, portfolio analyses will be developed considering only a cohort of policies, i.e. a group of policies entering at the same time and age, insured for the same amount, similar in terms of risk class, etc. Such policies, in particular, pay the same premium. The portfolio loss function at time t is

$$\mathcal{L}(t) = \sum_{i=1,2}^{N_i(t)} \sum_{h=1} L^{(h)}(t) \quad (4.3)$$

Since we aim at valuing the risk inherent in a given pricing basis, in the following the loss function will be analysed mainly at time 0.

The risk for the insurer is measured through the variance of its loss function. Valuation of the moments of the loss function can be performed either disregarding or considering uncertainty concerning the future evolution of mortality and disability. In the former approach (“deterministic” approach), a (projected) scenario is assigned; different scenarios can be compared in terms of the different value they imply for the moments of the loss function. In the latter approach (“stochastic” approach) a set of scenarios is considered, to which a probability distribution is assigned.

(b) Deterministic approach. Let us consider a given scenario \mathcal{H} , i.e. a given set of hypotheses concerning mortality and morbidity. The first and second moment of the loss function at individual level are for $t > 0$

$$E[L(t)|S(t) = i; \mathcal{H}] = b_2 \int_t^n v^{u-t} P_{i2}^{(\mathcal{H})}(t, u) du \quad i = 1, 2 \quad (4.4)$$

$$\begin{aligned} E[(L(t))^2|S(t) = i; \mathcal{H}] &= 2(b_2)^2 \int_t^n v^{2(u-t)} P_{i2}^{(\mathcal{H})}(t, u) \\ &\times \left[\int_u^n v^{r-u} P_{22}^{(\mathcal{H})}(u, r) dr \right] du \quad i = 1, 2 \end{aligned} \quad (4.5)$$

where notation $P_{ij}^{(\mathcal{H})}(t, u)$ indicates the transition probabilities calculated according to scenario \mathcal{H} (a similar notation is below adopted for transition intensities). After some manipulations (and in particular using relation (2.3)), we find the expressions (easier to handle)

$$E[L(t)|S(t) = 2; \mathcal{H}] = b_2 \int_t^n v^{u-t} P_{22}^{(\mathcal{H})}(t, u) du \quad (4.4')$$

$$E[L(t)|S(t) = 1; \mathcal{H}] = \int_t^n v^{u-t} P_{11}^{(\mathcal{H})}(t, u) \mu_{12}^{(\mathcal{H})}(u) E[L(u)|S(u) = 2; \mathcal{H}] du \quad (4.4'')$$

$$E[(L(t))^2|S(t) = 2; \mathcal{H}] = 2b_2 \int_t^n v^{2(u-t)} P_{22}^{(\mathcal{H})}(t, u) E[L(u)|S(u) = 2; \mathcal{H}] du \quad (4.5')$$

$$E[(L(t))^2|S(t) = 1; \mathcal{H}] = \int_t^n v^{2(u-t)} P_{11}^{(\mathcal{H})}(t, u) \mu_{12}^{(\mathcal{H})}(u) E[(L(u))^2|S(u) = 2; \mathcal{H}] du \quad (4.5'')$$

Note, in particular, that (4.4'') reflects an inception-annuity scheme, being based on the probability of entering into state 2 ($\mu_{12}^{(\mathcal{H})}(u) du$) and on the annuity thereon paid ($E[L(u)|S(u) = 2; \mathcal{H}]$).

The variance of the individual loss function is then

$$Var[L(t)|S(t) = i; \mathcal{H}] = E[(L(t))^2|S(t) = i; \mathcal{H}] - (E[L(t)|S(t) = i; \mathcal{H}])^2 \quad (4.6)$$

Expressions of the expected value and variance of the loss function at time 0 are straightforward.

As far as valuations at portfolio level are concerned, let N be the initial size of the cohort. Assuming that the insured lives are independent risks conditional on scenario \mathcal{H} , expected value and variance of the loss function at time t are

$$E[\mathcal{L}(t)|\mathcal{H}] = \sum_{i=1,2} E[N_i(t)|\mathcal{H}] E[L(t)|S(t) = i; \mathcal{H}] \quad (4.7)$$

$$\begin{aligned} Var[\mathcal{L}(t)|\mathcal{H}] &= \sum_{i=1,2} E[N_i(t)|\mathcal{H}] Var[L(t)|S(t) = i; \mathcal{H}] \\ &+ \sum_{i=1,2} Var[N_i(t)|\mathcal{H}] (E[L(t)|S(t) = i; \mathcal{H}])^2 \\ &+ 2 Cov[N_1(t), N_2(t)|\mathcal{H}] \prod_{i=1,2} E[L(t)|S(t) = i; \mathcal{H}] \end{aligned} \quad (4.8)$$

where, having assumed $S(0) = 1$

$$E[N_i(t)|\mathcal{H}] = N P_{1i}^{(\mathcal{H})}(0, t) \quad (4.9)$$

$$Var[N_i(t)|\mathcal{H}] = N P_{1i}^{(\mathcal{H})}(0, t) (1 - P_{1i}^{(\mathcal{H})}(0, t)) \quad (4.10)$$

$$Cov[N_1(t), N_2(t)|\mathcal{H}] = -N P_{11}^{(\mathcal{H})}(0, t) P_{12}^{(\mathcal{H})}(0, t) \quad (4.11)$$

In particular, at time 0 all policies are in state 1 (i.e. $N_1(0) = N(0) = N$); (4.7) and (4.8) thus reduce as follows

$$E[\mathcal{L}(0)|\mathcal{H}] = N E[L(0)|S(0) = 1; \mathcal{H}] \quad (4.7')$$

$$Var[\mathcal{L}(0)|\mathcal{H}] = N Var[L(0)|S(0) = 1; \mathcal{H}] \quad (4.8')$$

It must be stressed that at any time t the variance of the portfolio loss function is proportional to the initial size of the portfolio. In order to have an index of relative riskiness, which shows how risk evolves with N , the standard deviation of the portfolio

loss function at time 0 (which measures risk throughout the whole coverage period) can be compared to premiums received at time 0 (which finance the whole benefits of the portfolio); we find

$$r^{(\mathcal{H})} = \frac{\sqrt{\text{Var}[\mathcal{L}(0)|\mathcal{H}]}}{N \pi} = \frac{1}{\sqrt{N}} \frac{\sqrt{\text{Var}[L(0)|S(0) = 1; \mathcal{H}]}}{\pi} \quad (4.12)$$

Let $\mathcal{Y}(t)$ be the random present value of benefits at portfolio level. Having adopted the equivalence principle and given the single premium funding system, $\text{Var}[\mathcal{Y}(t)|\mathcal{H}] = \text{Var}[\mathcal{L}(t)|\mathcal{H}]$ for any t , $N \pi = E[\mathcal{Y}(0)|\mathcal{H}]$ whence the index $r^{(\mathcal{H})}$ is the coefficient of variation of $\mathcal{Y}(0)$. Relation (4.12) shows that risk per unit of premium vanishes with the size of the portfolio (actually, $r^{(\mathcal{H})} \rightarrow 0$ when $N \rightarrow +\infty$). This is due to the fact that under our hypotheses only the risk of random fluctuations (which is a pooling risk) is taken into account.

Table 4.1 shows expected value and variance of the present value of benefits at time 0 at individual level, under the five projected scenarios; we have assumed $b_2 = 1$ and $\delta = \ln 1.03$. For the equivalence principle, the single premium is equal to $E[Y(0)|S(0) = 1; \mathcal{H}]$, whilst the variance of the individual loss function is equal to $\text{Var}[Y(0)|S(0) = 1; \mathcal{H}]$. Values at portfolio level can be found simply by multiplying both quantities by the initial size of the cohort. It must be noted that the five scenarios imply an increasing expected cost of benefits and an increasing risk for the insurer as well. However, in relative terms risk declines with the severity of the projection, as shown by the index $r^{(\mathcal{H})}$, since a higher premium results. The Stand Alone cover is anyway a risky product, since the variance of the loss function has a large magnitude when compared to premiums.

| | $E[Y(0) S(0)=1;\mathcal{H}]$ $=\pi^{(\mathcal{H})}$ | $\text{Var}[Y(0) S(0)=1;\mathcal{H}]$ $=\text{Var}[L(0) S(0)=1;\mathcal{H}]$ | $r^{(\mathcal{H})}$ |
|-----------------|--|---|---------------------|
| \mathcal{H}_1 | 0.85299 | 6.37087 | 2.73293 |
| \mathcal{H}_2 | 0.92916 | 6.92783 | 2.73057 |
| \mathcal{H}_3 | 1.03702 | 7.54546 | 2.69743 |
| \mathcal{H}_4 | 1.22605 | 8.59303 | 2.64739 |
| \mathcal{H}_5 | 1.38711 | 9.65429 | 2.63818 |

Table 4.1 - Stand Alone

Figures 4.1 and 4.2 show the risk profile of a policy in the healthy and in the frailty state, respectively, under the different scenarios. Clearly, $\text{Var}[L(t)|S(t) = 2; \mathcal{H}]$ is affected by mortality only; hence in figure 4.2 only scenarios involving different mortality levels are considered. Risk decreases with time. Moreover, the risk profile varies with the projection, the lowest being implied by \mathcal{H}_5 . This witnesses that in our projections the mortality assumptions have been chosen so that they induce an increasing concentration of the curve of deaths and a forward shift of its mode (see point (a) in Section 3). Risk decreases with time also when the healthy state is considered. However, the different projections induce profiles that cross each other. This is due to the interactions between mortality and disability levels. Figure 4.1 shows that in the first years of the coverage, the variance of the loss function is mostly affected by disability levels (actually, $\text{Var}[L(t)|S(t) = 1; \mathcal{H}]$ is higher under scenarios \mathcal{H}_4 and \mathcal{H}_5 , which imply the highest inception rates); as time passes, the effect of mortality concentration becomes

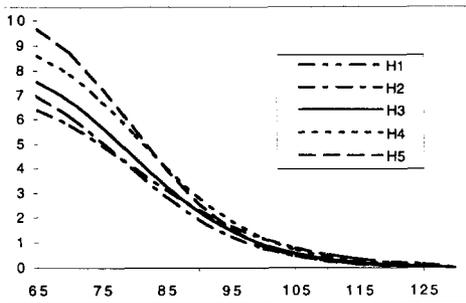


Figure 4.1 – Stand Alone
 $Var[L(t)|S(t)=1; \mathcal{H}], t \geq 0$

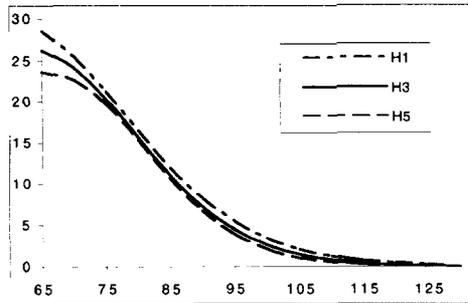


Figure 4.2 – Stand Alone
 $Var[L(t)|S(t)=2; \mathcal{H}], t \geq 0$

predominant (in the latest years \mathcal{H}_5 , which is based on the highest mortality levels as well, produces the lowest variance).

As far as the risk profile at portfolio level is concerned, (4.8) shows that $Var[\mathcal{L}(t)|\mathcal{H}]$ is affected both by variability of payments at individual level and by variability of the portfolio composition. It is therefore difficult to interpret the behaviour of $Var[\mathcal{L}(t)|\mathcal{H}]$ in terms of the consequences produced by the different scenarios. We have preferred to calculate the variance of the portfolio loss function only in relation to three assigned patterns of the number of active and disabled people, determined as follows

$$N_i^{[med]}(t) = E[N_i(t)|\mathcal{H}] \quad i = 1, 2 \quad (4.13a)$$

$$N_i^{[min]}(t) = E[N_i(t)|\mathcal{H}] - 1.96 \sqrt{Var[N_i(t)|\mathcal{H}]} \quad i = 1, 2 \quad (4.13b)$$

$$N_i^{[max]}(t) = E[N_i(t)|\mathcal{H}] + 1.96 \sqrt{Var[N_i(t)|\mathcal{H}]} \quad i = 1, 2 \quad (4.13c)$$

where the constant 1.96 comes from a normal approximation of the distribution of $N_i(t)$. In figure 4.3 the profile of the variance of the portfolio loss function is depicted, for a portfolio of initial $N = 1,000$ insureds. Note that for a given pattern of $N_1(t), N_2(t)$ the variance of the portfolio loss function reduces to the first term in (4.8).

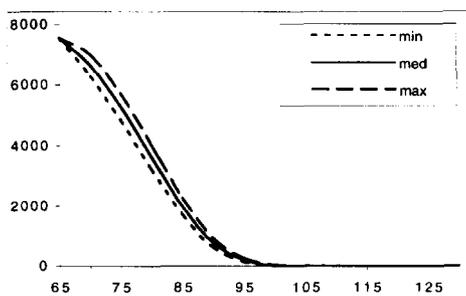


Figure 4.3 – Stand Alone
 $Var[\mathcal{L}(t)|\mathcal{H}; N_1^1(t), N_2^1(t)], t \geq 0$

In the sequel, scenario \mathcal{H}_3 is adopted as premium and reserving basis ($\mathcal{H}^{[p]} = \mathcal{H}^{[r]} = \mathcal{H}_3$). Hence $\pi = 1.03702$. Note that a safety loading could be explicitly included in this premium considering either the risk of the cover (in terms of the variance of the

loss function) or an appropriate function of the different premium levels that would be required under the other scenarios. In figure 4.4 and 4.5 the reserves for a healthy and a frailty policy are respectively depicted.

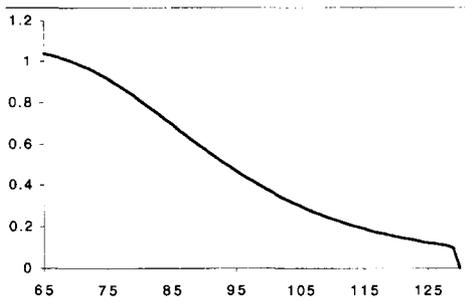


Figure 4.4 – Stand Alone

$$V_1(t) = E[L(t)|S(t)=1; \mathcal{H}_3], t \geq 0$$

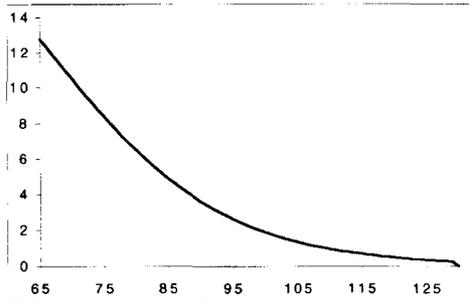


Figure 4.5 – Stand Alone

$$V_2(t) = E[L(t)|S(t)=2; \mathcal{H}_3], t \geq 0$$

(c) **Stochastic approach.** The risk analysis is now performed considering explicitly uncertainty in demographic projections. To this aim we consider the five projected scenarios \mathcal{H}_k ($k = 1, 2, \dots, 5$) of table 3.1, interpreting each of them as a possible outcome of the actual future mortality and disability rates. The weights ρ_k ($k = 1, 2, \dots, 5$) represent the “degrees of belief” of such sets of hypotheses. For the sake of brevity, risk analysis is performed at time 0 only.

Adopting the same hypotheses of point (b) (in particular, assuming that the policyholders are independent risks under each scenario), expected value and variance of the portfolio loss function at time 0 are now given by the following expressions

$$E[\mathcal{L}(0)] = E[E[\mathcal{L}(0)|\mathcal{H}]] = N E[E[L(0)|S(0) = 1; \mathcal{H}]] \quad (4.14)$$

$$\begin{aligned} \text{Var}[\mathcal{L}(0)] &= E[\text{Var}[\mathcal{L}(0)|\mathcal{H}]] + \text{Var}[E[\mathcal{L}(0)|\mathcal{H}]] \\ &= N E[\text{Var}[L(0)|S(0) = 1; \mathcal{H}]] + N^2 \text{Var}[E[L(0)|S(0) = 1; \mathcal{H}]] \end{aligned} \quad (4.15)$$

It must be pointed out that the variance of $\mathcal{L}(0)$ is now a second degree function of the size of the portfolio. The first term of the variance gives account of random fluctuations around expected values, whilst the second term expresses systematic deviations of observed values from expected ones. In terms of the index considered in point (b) we have

$$r = \frac{\sqrt{\text{Var}[\mathcal{L}(0)]}}{N \pi} = \left(\frac{1}{N} \frac{E[\text{Var}[L(0)|S(0) = 1; \mathcal{H}]]}{\pi^2} + \frac{\text{Var}[E[L(0)|S(0) = 1; \mathcal{H}]]}{\pi^2} \right)^{1/2} \quad (4.16)$$

The index r consists now of two terms. The first one is similar to expression (4.12); actually it witnesses the (pooling) risk of random fluctuations and tends to zero as N increases. The second one is constant with respect to the size of the portfolio; it shows the risk of systematic deviations, which is a non-pooling risk. Note, finally, that r has now a positive limiting value, given by the square root of the second term in (4.16).

In table 4.2 evaluation of the expected value and variance of the loss function is quoted. As an example, we have chosen $\rho_1 = \rho_5 = 0.05$, $\rho_2 = \rho_4 = 0.15$, $\rho_3 = 0.6$, expressing the fact that the most realistic scenario has been adopted for premium calculation, whilst the most extreme scenarios occur with a low probability. What emerges from table 4.2 is the heavy riskiness linked to systematic deviations. The so-called longevity risk, related to changes in mortality and disability levels, can be identified in such component of the variance of the loss function. However, to a large extent the risk of random fluctuations keeps higher than that of systematic ones. Actually, the two components of $Var[\mathcal{L}(0)]$ are equal when the initial size of the portfolio is

$$\bar{N} = \frac{E[Var[L(0)|S(0) = 1; \mathcal{H}]]}{Var[E[L(0)|S(0) = 1; \mathcal{H}]} \quad (4.17)$$

When $N > \bar{N}$ the non-pooling risk is predominant, whilst when $N < \bar{N}$ the pooling risk is larger than the other. In the example of table 4.2, we find $\bar{N} = 528$.

It must be stressed that the stochastic approach leads to a rather different risk assessment than the deterministic one. However, the risk of systematic deviations although neglected in the deterministic approach is obviously present any time a future behaviour is concerned. The advantage of the stochastic approach is that it allows to explicit such risk, whilst in a deterministic setting only random fluctuations can be formally represented.

With reference to table 4.2, note finally that having adopted \mathcal{H}_3 for premium calculation, the expected value of the loss function is not zero. At this regard, as mentioned in point (b), a loading could be explicitly included into the premium so that $E[\mathcal{L}(0)] = 0$. Obviously, a (further) loading could also be determined in relation to the variance of the loss function.

| N | $E[\mathcal{L}(0)]$ | $E[Var[\mathcal{L}(0) \mathcal{H}]]$ | $Var[E[\mathcal{L}(0) \mathcal{H}]]$ | $Var[\mathcal{L}(0)]$ |
|---------|---------------------|--------------------------------------|--------------------------------------|-----------------------|
| 1 | 0.020 | 7.657 | 0.015 | 7.671 |
| 10 | 0.205 | 76.567 | 1.451 | 78.017 |
| 100 | 2.048 | 765.667 | 145.076 | 910.743 |
| 1,000 | 20.480 | 7,656.667 | 14,507.60 | 22,164.27 |
| 10,000 | 204.802 | 76,566.666 | 1,450,760.0 | 1,527,323.7 |
| 100,000 | 2,048.02 | 765,666.658 | 145,076,000 | 145,841,667 |

Table 4.2 – Stand Alone

In table 4.3 index r is quoted for different initial sizes of the portfolio, both in the deterministic and in the stochastic approach. Note that whilst for $N = 1$ the two approaches lead to a similar risk evaluation (the magnitude of the relative risk is the same), for $N > 1$ the latter approach leads to a relative riskiness which is higher and has a positive limiting value. As already explained, this is due to the systematic component of the variance of the loss function.

We point out that arguments similar to the above ones hold when a different set of weights ρ_k is chosen (unless they lead back to the deterministic case).

Finally, in figure 4.6 three possible profiles of $Var[\mathcal{L}(t)]$ are depicted, related to three patterns of $N_i(t)$. The three patterns have been chosen as follows

$$N_i^{[\text{med}]}(t) = E[N_i(t)] = E[E[N_i(t)|\mathcal{H}]] \quad i = 1, 2 \quad (4.18a)$$

| N | $r^{(\mathcal{H}_3)}$ | r |
|-----------|-----------------------|-------------------|
| | (det. approach) | (stoch. approach) |
| 1 | 2.69743 | 2.67082 |
| 10 | 0.26974 | 0.85175 |
| 100 | 0.02697 | 0.29101 |
| 1,000 | 0.00270 | 0.14356 |
| 10,000 | 0.00027 | 0.11917 |
| 100,000 | 0.00003 | 0.11645 |
| $+\infty$ | 0 | 0.11615 |

Table 4.3 – Stand Alone

$$\begin{aligned}
 N_i^{[\min]}(t) &= E[N_i(t)] - 1.96 \sqrt{\text{Var}[N_i(t)]} \\
 &= E[E[N_i(t)|\mathcal{H}]] - 1.96 \sqrt{E[\text{Var}[N_i(t)|\mathcal{H}]] + \text{Var}[E[N_i(t)|\mathcal{H}]]} \quad i = 1, 2
 \end{aligned}
 \tag{4.18b}$$

$$\begin{aligned}
 N_i^{[\max]}(t) &= E[N_i(t)] + 1.96 \sqrt{\text{Var}[N_i(t)]} \\
 &= E[E[N_i(t)|\mathcal{H}]] + 1.96 \sqrt{E[\text{Var}[N_i(t)|\mathcal{H}]] + \text{Var}[E[N_i(t)|\mathcal{H}]]} \quad i = 1, 2
 \end{aligned}
 \tag{4.18c}$$

The expression of the variance of the portfolio loss function at time t under these three patterns is

$$\begin{aligned}
 \text{Var}[\mathcal{L}(t)|N_i^{[1]}(t)] &= \sum_{i=1,2} N_i^{[1]}(t) E[\text{Var}[L(t)|S(t) = i; \mathcal{H}]] \\
 &+ \sum_{i=1,2} (N_i^{[1]}(t))^2 \text{Var}[E[L(t)|S(t) = i; \mathcal{H}]]
 \end{aligned}
 \tag{4.19}$$

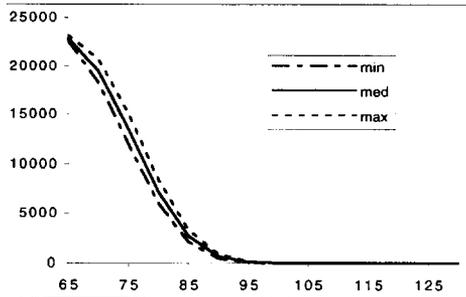


Figure 4.6 – Stand Alone

$$\text{Var}[\mathcal{L}(t)|N_1^{[1]}(t), N_2^{[1]}(t)], t \geq 0$$

5. RISKINESS OF THE ENHANCED PENSION COVER

(a) Loss function. The enhanced pension is sold to an annuitant and is a combination of a standard pension annuity paid while the policyholder is healthy and an uplifted income paid while he/she is claiming LTC benefits. For a given amount of single premium, the cost of the uplift is a reduction in the initial pension income.

With reference to the multistate model of figure 2.1, assume that a person at retirement is eligible for a straight life annuity (basic pension) at a rate b . He/she can switch this

benefit to a combination of an annuity while he/she is in state 1 at a rate b_1 , $b_1 < b$, and an LTC annuity (enhanced pension) at a rate b_2 , $b_2 > b$.

Let ${}_t p_x$ be the survival probability for a person aged x (either healthy or disabled). Having assumed that mortality of active people coincides with that of the general population, ${}_t p_x = P_{11}(0, t) + P_{12}(0, t)$. The equivalence principle at policy issue requires

$$b \int_0^n v^t (P_{11}(0, t) + P_{12}(0, t)) dt = b_1 \int_0^n v^t P_{11}(0, t) dt + b_2 \int_0^n v^t P_{12}(0, t) dt \quad (5.1)$$

Once $b - b_1$ has been chosen and a pricing basis $\mathcal{H}^{[p]}$ has been assigned, (5.1) allows to calculate benefit b_2 . We denote with π the actuarial value of benefits calculated according to $\mathcal{H}^{[p]}$.

The loss function at individual level at time t ($t > 0$) is

$$L(t) = b_1 \int_t^n v^{u-t} I_{\{S(u)=1\}} du + b_2 \int_t^n v^{u-t} I_{\{S(u)=2\}} du \quad (5.2)$$

Expression for $L(0)$ is straightforward. With reference to a cohort of policies, issued at the same time, age, with the same amount assured and similar in terms of risk class, the portfolio loss function is still given by (4.3).

(b) Deterministic approach. Let us consider a given scenario \mathcal{H} . The first and second moment of the loss function at individual level are for $t > 0$

$$E[L(t)|S(t) = 1; \mathcal{H}] = \sum_{i=1,2} b_i \int_t^n v^{u-t} P_{1i}^{(\mathcal{H})}(t, u) du \quad (5.3)$$

$$E[L(t)|S(t) = 2; \mathcal{H}] = b_2 \int_t^n v^{u-t} P_{22}^{(\mathcal{H})}(t, u) du \quad (5.4)$$

$$\begin{aligned} E[(L(t))^2|S(t) = 1; \mathcal{H}] &= 2(b_1)^2 \int_t^n v^{2(u-t)} P_{11}^{(\mathcal{H})}(t, u) \left[\int_u^n v^{r-u} P_{11}^{(\mathcal{H})}(u, r) dr \right] du \\ &+ 2(b_2)^2 \int_t^n v^{2(u-t)} P_{12}^{(\mathcal{H})}(t, u) \left[\int_u^n v^{r-u} P_{22}^{(\mathcal{H})}(u, r) dr \right] du \\ &+ b_1 b_2 \int_t^n v^{2(u-t)} P_{11}^{(\mathcal{H})}(t, u) \left[\int_u^n v^{r-u} P_{12}^{(\mathcal{H})}(u, r) dr \right] du \end{aligned} \quad (5.5)$$

$$\begin{aligned} E[(L(t))^2|S(t) = 2; \mathcal{H}] &= 2(b_2)^2 \int_t^n v^{2(u-t)} P_{22}^{(\mathcal{H})}(t, u) \left[\int_u^n v^{r-u} P_{22}^{(\mathcal{H})}(u, r) dr \right] du \end{aligned} \quad (5.6)$$

After some manipulations we find

$$\begin{aligned} E[L(t)|S(t) = 1; \mathcal{H}] &= b_1 \int_t^n v^{u-t} P_{11}^{(\mathcal{H})}(t, u) du \\ &+ \int_t^n v^{u-t} P_{11}^{(\mathcal{H})}(t, u) \mu_{12}^{(\mathcal{H})}(u) E[L(u)|S(u) = 2; \mathcal{H}] du \end{aligned} \quad (5.3')$$

$$E[(L(t))^2|S(t) = 2; \mathcal{H}] = 2b_2 \int_t^n v^{2(u-t)} P_{22}^{(\mathcal{H})}(t, u) E[L(u)|S(u) = 2; \mathcal{H}] du \quad (5.6')$$

$$E[(L(t))^2|S(t) = 1; \mathcal{H}] = \int_t^n v^{2(u-t)} P_{11}^{(\mathcal{H})}(t, u) \mu_{12}^{(\mathcal{H})}(u) E[(L(u))^2|S(u) = 2; \mathcal{H}] du + 2b_1 \int_t^n v^{2(u-t)} P_{11}^{(\mathcal{H})}(t, u) E[L(u)|S(u) = 1; \mathcal{H}] du \quad (5.5')$$

The variance of the individual loss function is still given by (4.6); expected value and variance for $t = 0$ are straightforward. As far as the portfolio loss function is concerned, expected value and variance are as in Section 4 (see (4.7), (4.8)).

Table 5.1 shows expected value and variance of the loss function at individual level (portfolio values can be obtained simply by multiplying such quantities by the initial size of the cohort). As far as benefits are concerned, we have chosen $b = 1$ and $b_1 = 0.9$. The conversion factor has been based on scenario \mathcal{H}_3 ; the single premium is thus $\pi = \pi^{(\mathcal{H}_3)} = 13.14962$. We have obtained $b_2 = 2.21105$; a great increase in the LTC benefit can therefore be gained with a modest decrease in the current benefit. In order to facilitate the comparison among the results in different scenarios, the amount of such benefits has been kept fixed in any numerical evaluations (in particular, in table 5.1 we have quoted the quantity $r^{(\mathcal{H})} = \frac{\sqrt{\text{Var}[L(0)|S(0)=1; \mathcal{H}]}}{\pi^{(\mathcal{H}_3)}}$).

Considerations similar to those relating to the Stand Alone cover can be expressed. Note in particular that, due to the different levels of benefits, \mathcal{H}_4 is the most risky scenario. Anyhow, the relative riskiness of the Enhanced Pension is significantly lower than that of the Stand Alone. This is due to the fact that in the Enhanced Pension benefits are paid starting from policy issue, whence a high initial funding is necessary. In the Stand Alone case, on the contrary, the occurrence of benefit payment is random, whence a lower premium is required.

Examples in figures 5.1 to 5.5 are similar to those considered in point (b) of Section 4. Similar arguments hold.

| | $E[L(0) S(0)=1; \mathcal{H}]$ | $\text{Var}[L(0) S(0)=1; \mathcal{H}]$ | $r^{(\mathcal{H})}$ |
|-----------------|-------------------------------|--|---------------------|
| \mathcal{H}_1 | 12.31263 | 43.23329 | 0.50003 |
| \mathcal{H}_2 | 13.01303 | 41.62918 | 0.49067 |
| \mathcal{H}_3 | 13.14962 | 43.71386 | 0.50280 |
| \mathcal{H}_4 | 13.38909 | 47.28529 | 0.52294 |
| \mathcal{H}_5 | 14.37080 | 46.34328 | 0.51770 |

Table 5.1 – Enhanced pension

(c) Stochastic approach. The analysis is now performed considering explicitly uncertainty of the future scenario. Expected value and variance of the portfolio loss function are still given by equations (4.14) and (4.15). Numerical evaluations, performed under the same hypotheses of Section 4, are quoted in tables 5.2 and 5.3 and in figure 5.6. Similar considerations hold. In particular, the size of the portfolio that let random fluctuations and systematic risk have the same contribution to the variance of the loss function is $\bar{N} = 368$ (see (4.17)). Such result is not fully comparable to the Stand Alone case, due to the different level of benefits.

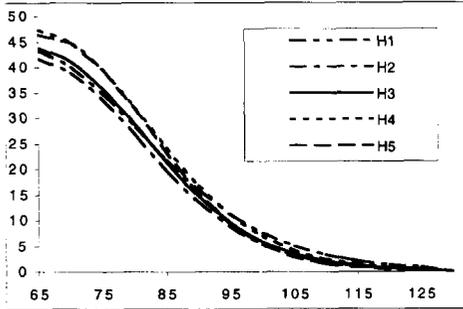


Figure 5.1 – Enhanced Pension
 $Var[L(t)|S(t)=1; \mathcal{H}], t \geq 0$

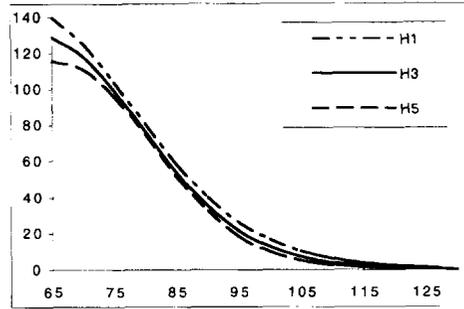


Figure 5.2 – Enhanced Pension
 $Var[L(t)|S(t)=2; \mathcal{H}], t \geq 0$

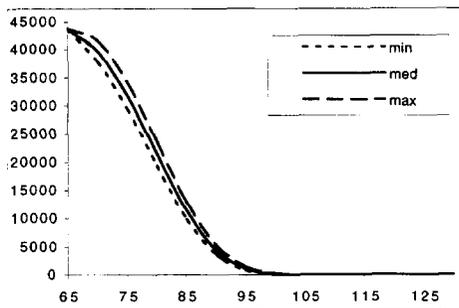


Figure 5.3 – Enhanced Pension
 $Var[\mathcal{L}(t)|\mathcal{H}; \bar{N}_1(t), \bar{N}_2(t)], t \geq 0$

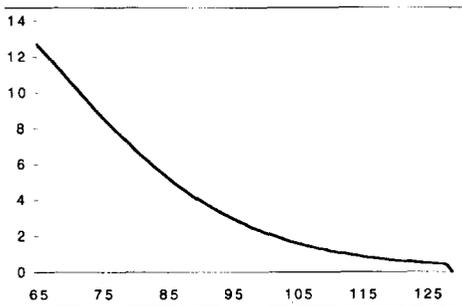


Figure 5.4 – Enhanced Pension
 $E[L(t)|S(t)=1; \mathcal{H}_3], t \geq 0$

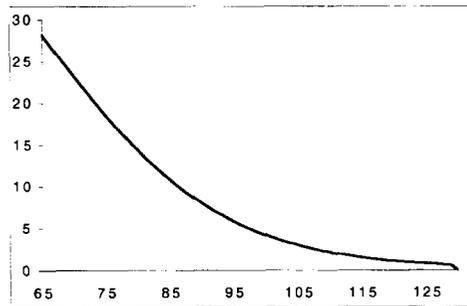


Figure 5.5 – Enhanced Pension
 $E[L(t)|S(t)=2; \mathcal{H}_3], t \geq 0$

| N | $E[\mathcal{L}(0)]$ | $E[Var[\mathcal{L}(0) \mathcal{H}]]$ | $Var[E[\mathcal{L}(0) \mathcal{H}]]$ | $Var[\mathcal{L}(0)]$ |
|---------|---------------------|--------------------------------------|--------------------------------------|-----------------------|
| 1 | 0.035 | 44.044 | 0.120 | 44.164 |
| 10 | 0.346 | 440.443 | 11.979 | 452.422 |
| 100 | 3.464 | 4,404.431 | 1,197.919 | 5,602.350 |
| 1,000 | 34.641 | 44,044.31 | 119,791.91 | 163,836.22 |
| 10,000 | 346.408 | 440,443.1 | 11,979,191.4 | 12,419,634.5 |
| 100,000 | 3,464.082 | 4,404,431 | 1,197,919,142 | 1,202,323,573 |

Table 5.2 – Enhanced Pension

| N | $r^{(\mathcal{H}_3)}$ | r |
|-----------|-----------------------|-------------------|
| | (det. approach) | (stoch. approach) |
| 1 | 0.50280 | 0.50538 |
| 10 | 0.05028 | 0.16176 |
| 100 | 0.00503 | 0.05692 |
| 1,000 | 0.00050 | 0.03078 |
| 10,000 | 0.00005 | 0.02680 |
| 100,000 | 0 | 0.02637 |
| $+\infty$ | 0 | 0.02632 |

Table 5.3 – Enhanced Pension

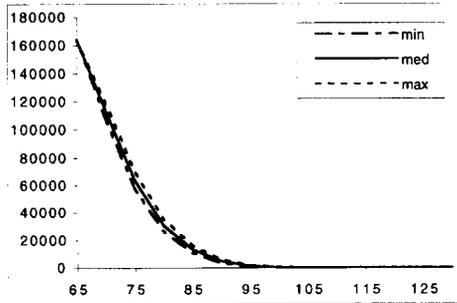


Figure 5.6 – Enhanced Pension

$$\text{Var}[\mathcal{L}(t) | \bar{N}_1(t), \bar{N}_2(t)], t \geq 0$$

6. CONCLUDING REMARKS

In this paper we have investigated the risk inherent in a given pricing basis for LTC products. In particular, the risk coming from the uncertainty of the future mortality and morbidity levels (i.e. the longevity risk) has been highlighted. In order to conclude, we wish to express some remarks on the hypotheses under which both analytical and numerical results have been obtained.

Firstly, only a cohort of homogeneous and (conditional on a given scenario) independent policies has been considered. When a more general portfolio is dealt with, the risk of random fluctuations and systematic deviations could be more serious than what assessed in this paper, due to differences among the insured positions and to the possible interrelations among them.

Secondly, only male people have been considered. When female policyholders are dealt with, risk could result higher because female populations show both lower mortality levels and longer stays in the frailty state than males.

Other causes of randomness should be included in the analysis when actual portfolios are studied (such as, for example, interest rates). In particular, we recall that only forfeiture benefits have been analysed in this paper. When expense refunding covers are examined, careful attention should be given to the annual claim frequencies as well as claim sizes, whose uncertainty is in particular caused by the paucity of data.

Finally, it must be stressed that the risk profiles examined in this paper are strictly linked to our choice of future scenarios. Obviously, any risk analysis depends heavily on the hypotheses adopted; hence, different choices would have led to a different assessment of risk. However, the seriousness of systematic risk is to a large extent independent of our choices; in particular, it is obviously present even if the model does not allow for it and becomes heavier with the size of the portfolio. In order to contain the systematic

risk, the insurer should resort to suitable reinsurance programmes and/or set up a proper fund financed possibly with the safety loading included into premiums, but mainly with shareholders' funds.

REFERENCES

- Benjamin J and AS Soliman (1993), *Mortality on the move*, Actuarial Education Service, Oxford
- Fries JF (1980), Aging, natural death and the compression of morbidity, *N. Engl. Journal of Medecine*: 303
- Gatenby P (1991), *Long Term Care*, Paper presented to the Staple Inn Actuarial Society, London
- Gruenberg EM (1977), The failure of success, *Milbank Memorial foundation Q./Health Society*: 55
- Haberman S and E Pitacco (1999), *Actuarial models for disability insurance*, Chapman & Hall/CRC, Boca Raton
- Hoem JM (1969), Markov chain models in life insurance, *Blätter der deutschen Gesellschaft für Versicherungsmathematik*, **9**: 91–107
- Hoem JM (1988), The versatility of the Markov chain as a tool in the mathematics of life insurance, *Transactions of the 23rd International Congress of Actuaries*, Helsinki, **R**: 171–202
- Kramer M (1980), The rising pandemic of mental disorders and associated chronic diseases and disabilities, *Acta Psychiatrica Scandinavica*: 62
- Macdonald AS (ed.) (1997), *The Second Actuarial Study of Mortality in Europe*, Groupe Consultatif des Associations d'Actuaires des Pays des Communautés Europeennes, Oxford
- Macdonald AS et al (1998), An international comparison of recent trends in population mortality, *British Actuarial Journal*, **4**: 3–141
- Manton KG (1982), Changing concepts of morbidity and mortality in the elderly population, *Milbank Memorial foundation Q./Health Society*
- Norberg R (1991), Reserves in life and pension insurance, *Scandinavian Actuarial Journal*, **1991**: 3–24
- Norberg R (1995), Differential equations for moments of present values in life insurance, *Insurance: Mathematics & Economics*, **17**: 245–256
- Olivieri A (1996), Sulle basi tecniche per coperture "Long Term Care", *Giornale dell'Istituto Italiano degli Attuari*, **49**: 87–116
- Olivieri A and E Pitacco (1999), Funding sickness benefits for the elderly, *Proceedings of the XXX International ASTIN Colloquium*, Tokyo, 135–155
- Pitacco E and A Olivieri (1997), *Introduzione alla teoria attuariale delle assicurazioni di persone*, Quaderni dell'Unione Matematica Italiana n. 42, Pitagora Editrice, Bologna
- Swiss Re (1999), *Long Term Care Data Pack*, London
- Wolhuis H (1994), *Life insurance mathematics (The Markovian model)*, CAIRE Education Series, no. 2, Bruxelles