THE DIRECT AND "EXACT" ASSESSMENT OF ACTUARIAL RISK: APPLICATION WITH REGARD TO A PORTFOLIO OF LIFE POLICIES[1]

ANNA ATTIAS
Dipartimento di Matematica per le Decisioni Economiche, Finanziarie e Assicurative. Università di Roma "La Sapienza"

Via del Castro Laurenziano, 9 – 00168 Rome, Italy
Tel:+39.6.49766764 - Fax: +39.6.497667665 - e.mail: attias@scec.eco.uniroma1.it

AND

SANDRO TUMANI
Attuario INPS

ABSTRACT

One of the most frequently recurring subjects found in actuarial literature is the problem of how to assess the risk associated with a portfolio of life policies.
Almost all the studies are based on mathematical treatments which, by making various assumptions, provide solutions to the problem mentioned.
A number of contributions published recently[2] have suggested that the assessment of risk can be based on the construction of general algorithms; in these studies, estimates are made - in particular - for the value of ruin, the year of its occurrence and the probability distribution of the capital prior to it.
In other words, by contrast to the usual mathematical approach which, by making various assumptions, attempts to carry out valuations that are independent on the specific case, this paper proposes a different approach based on an algorithm that is founded on general hypotheses: an algorithm that requires just the **single** risk variable for the portfolio concerned to be defined.
The last part of this study develops a treatment of dependent risk variables that avoids calculating the covariances.

RÉSUMÉ

Il est de commune renommée que l'évaluation des risques eu égard pour un portefeuille de polices d'assurance est un des thèmes majeurs de toute la littérature actuarielle.
La quasi totalité des études y afférent se fonde sur un modèle mathématique qui, sous diverses hypothèses, propose des solutions.
Des publications récentes proposent d'aborder le thème de l'évaluation du risque par le biais d'algorithmes généraux; ces études prennent en compte le montant du dommage, son année et la distribution de probabilités eu égard le patrimoine avant l'évènement.
En d'autres termes, la présente note, au lieu du modèle mathématique traditionnel qui, sous diverses hypothèses, a comme objectif d'effectuer des évaluations indépendantes de l'application spécifique, propose une architecture différente basée sur un algorithme fondé sur des hypothèses générales, algorithme qui exige uniquement la définition d'une seule variable aléatoire du portefeuille de chaque application.
Enfin, au cours de l'étude, il se présente un intéressant développement de variables aléatoires dépendantes, ce qui permet d'éviter de calculer les covariances.

ZUSAMMENFASSUNG

Bekanntlich ist die Risikobewertung des Bestandes an Lebensversicherungsverträgen eines der in versicherungstechnischen Abhandlungen am häufigsten behandelten Themen.
Dabei verwendet das Gros der Studien einen mathematischen Ansatz, der unter Annahme verschiedener Hypothesen Lösungen zum erwähnten Problem erarbeitet.
In jüngster Vergangenheit sind einige Beiträge veröffentlicht worden, die eine Risikobewertung auf der Grundlage allgemeingültiger Algorithmen propagieren; in diesen Studien werden insbesondere die Höhe ungedeckter Verbindlichkeiten, das Jahr der Zahlungsunfähigkeit des Versicherers und die Wahrscheinlichkeitsverteilung des Vermögens vor Eintritt der Zahlungsunfähigkeit geschätzt.

---

[1] Even though this paper is the result of the joint research of the authors, it is specified that paragraphs I.1, I.1.1, I.1.2, I.1.3, I.3, I.3.1 are due to Anna Attias, paragraphs I.1.4, I.1.5, I.2, I.3.2, I.3.3 and the applications are due to Sandro Tumani

[2] Cfr.[5]

Mit anderen Worten wird im vorliegenden Dokument an Stelle des herkömmlichen mathematischen Ansatzes, der unter Annahme verschiedener Hypothesen zum Ziel hat, Bewertungen unabhängig von der spezifischen Anwendung vorzunehmen, ein unterschiedlicher Ansatz vorgeschlagen: Dieser Ansatz verwendet einen Algorithmus, der auf allgemeingültigen Hypothesen basiert und lediglich die Definition einer **einzigen** Zufallsvariablen des jeweiligen Bestandes erfordert.
Schließlich stützt sich die Studie auf die Verwendung abhängiger Zufallsvariablen, wodurch die Kovarianzberechnung überflüssig wird.

KEYWORDS: time of ruin, capital immediately prior to ruin, capital at the time of ruin, individual random variables for each year, individual random variables for extendend periods.

I) METHODOLOGIES

I.1) The recent contribution by Gerber and Shiu[3]

The study presented by these two authors applies an interesting mathematical approach, with a wealth of elegant formal analysis.
Even so, considering the real circumstances of practical applications, their approach is founded on a somewhat restricted hypothesis, i.e. that the initial capital is not compounded[4].

I.1.1) General background to the study

As stated by the authors, the study addresses the problem of valuing American financial options[5] that can be approached using the traditional "probability of ruin", indicating both the value and time of its occurrence.
Under the classical approach to the valuation of these options:
- the share price progresses with continuous random excursions, following a Brownian geometric progression;
- the option right is exercised as soon as the value of the share exceeds the optimal exercise price, being the maximum price above the strike price.
- the price of the option is the discounted present value of the expected pay-off.

Gerber and Shiu attempt to generalize the progression of the random excursions, taking account of jumps in the value of the share.
Indeed, if it is assumed that the price of the share underlying the option can be represented by a Poisson curve, with positive or negative increments ("shifts"), then it is clear how a fall in the price of the underlying share price can be defined as "the amount of ruin" rather than "payment of the option price".

---

[3] Gerber and Shiu : "On the time value of ruin", NAAJ 1998, no. 2.

[4] In this regard, the following quotation is taken from the foot of the first column on page 50: "We should clarify that, while it can be very helpful to consider δ as a force of interest in this paper, we are dealing with the *classical model in which the surplus does not earn any interest*".

[5] The American option enables the buyer to acquire a given share within a certain period of time, at a price that is predetermined when entering into the option contract (the strike price).
On taking out the option, the buyer pays an amount (pay-off) equivalent to the discounted present value of the expected price for the share (underlying the option).
It is assumed that movements in the share price have a given distribution (Gauss, Poisson, lognormal); under these assumptions, the price fluctuations are smooth, i.e. without jumps or shifts.
Under these rules, the "optimal exercise boundary" is defined as being the maximum value of the share price during the period in which the given law of distribution is deemed to be valid.
It is assumed that the American option will be exercised as soon as the current price (i.e. the stockmarket price) of the share reaches the given maximum value (i.e. the "optimal exercise boundary" mentioned above).

I.1.2) The calculation

The authors make the following hypotheses for the calculation:
- the initial capital does NOT generate interest over time;
- premiums are paid on a continuous basis, at a rate c which remains constant over the entire period;
- the distribution of events is represented by a Poisson curve $\{S(t)\}$ with a parameter (mean and variance) of λ:

$$S(t) = \sum_{j=1}^{N(t)} X_j$$

Where $N(t)$ is a Poisson distribution with a mean, per time interval, of λ and X_j represents random variables settling the j-th event, such random variables being independent on each other;
- the distribution function $P(x)$ of the individual event X can be differentiated, with a first derivative of $p(x) = P'(x)$
- making $p_1 = E(X)$ gives rise to the relation:

$$c > \lambda \cdot p_1$$

This hypothesis means, in a given time interval, that the premium c is greater than the mean value of the number of events multiplied by the mean value of the settlement for each event;
- after a period $t \geq 0$, the value of the capital available at time t is discounted back in financial terms to the starting moment using the discounting factor $e^{-\delta t}$.

These hypotheses are essential to the elegant formal analysis conducted by the authors, but some of their assumptions (as already mentioned) - especially the lack of interest earned on the initial capital - do not take account of real-life conditions.

The authors examine the combined distribution of three random variables:
- The time of ruin
- The capital immediately prior to ruin
- The amount of ruin

These three random variables are summarized in a single random variable which is defined as the "discounted present value of the expected future loss"; this last variable is considered to be a function of the initial capital and satisfies, under the hypotheses made by the authors, a given equation for renewals.

The time of ruin is expressed using Laplace transformations which can be interpreted as a discount factor.

The authors define the three risk variables referred to above as follows.

I.1.3) The "time of ruin" risk variable, in the case it is associated with the time at which the financial option is exercised

Occurrences of this random variable happen at various times T $(T = 1,2,\dots\dots)$ when the expected event (i.e. ruin) takes place, while the corresponding probabilities are given by:

$$p(u) = \Pr[T < \infty \mid U(0) = u] \tag{1}$$

Where T is the time of ruin defined by:

$$T = \inf\{t \mid U(t) < 0\} \qquad (2)$$

$U(0)$ is the initial capital at starting time 0, while t is a value for the finishing time (obviously, if $t = \infty$ then ruin does not occur).

I.1.4) The "capital immediately prior to ruin" random variable

Capital immediately prior to ruin is defined as $U(T-)$, being a random variable determined as follows:

$$U(t) = u + c \cdot t - S(t) \qquad (3)$$

where:

- u is the value of the initial capital $U(0) = u$;
- c is the continuous rate of premium presumed to be constant over time
- $S(t)$ is the Poisson curve describing the total value of the events

i.e.

$$S(t) = \sum_{j=1}^{N(t)} X_j \qquad (4)$$

Where $N(t)$ is a Poisson distribution with a mean, per time interval, of λ and X_j represents random variables settling the j-th event, such random variables being independent of each other;
The probability of the outcome $U(t)$ is given by:

$$\psi(u) = \Pr[t < \infty \mid U(0) = u] \qquad (5)$$

I.1.5) The "capital at the time of ruin" random variable

Capital at the time of ruin is defined as $U(T)$, being a random variable determined as follows:

$$U(T) = u + c \cdot T - S(T) < 0 \qquad (6)$$

using symbols defined earlier.
The probability of the outcome $U(T)$ is given by:

$$\psi(u) = \Pr[U(T) \mid u] \qquad (7)$$

It is useful to note that the authors, after much mathematical elaboration, arrive at a function that defines the expected value of ruin and the time of its occurrence. In other words, they do not calculate the probability distributions, but only the expected values.

I.2) The variability of interest rates: the technique of scenarios

A "scenario" is defined as a sequence of future events; in this case, a sequence of interest rates in various future years.
Under the process used below, the scenarios can be determined by applying an analysis of various macroeconomic forecasts, to which the actuary attributes a probability for their outcome.
It is assumed that the actuary has formulated m alternative scenarios and has attributed to each of these a probability of occurrence $p(j)$.
The following relation exists:

$$\sum_{j=1}^{m} p(j) = 1$$

The probabilities attributed to the various scenarios are determined by reference to forecasts made by the actuary regarding the yield from future investments.
This approach seems realistic and flexible, as well as easy to handle.

I.3) A different approach to calculating the probability of ruin, the time of ruin and the value of ruin

This approach assumes knowledge of the terms of each contract in the portfolio of the insurance company, as well as the initial capital available.
Given this, it is possible to define the individual random variables both year by year and for periods covering a number of years; these random variables comprise the probabilities for possible events, as well as the related outcomes (outcomes represented by services and the individual actuarial reserves accumulated).
It is important to recognize that the random variables define the insurer's risk: in other words, they define the events and the related expectancies for which the insurer is contractually committed to pay-out[6].
Considering the above, it is possible to imagine various scenarios regarding:
- the return on capital and
- the expectancies of life or death

In this way, it is possible to develop several types of random variable (one for each contract)

[6] An example will clarify the idea.
Assume a temporary death-cover policy covering a two-year period; the annual premium P is constant (paid at the start of each year), while the value of the service is C (paid at the end of the year).
In the first year, the random variable to be defined has just two occurrences and probabilities: the first occurrence represents the pay-out on death, less the annual premium paid, being an event measured by the probability of death; the second occurrence is zero and has the complement to 1 for the probability mentioned.
In the second year, the random variable comprises two events: the first concerns the probability that the insured survives the first year but dies during the second year, in which case the outcome will be (pay-out under contract - annual premium - actuarial reserve from the first year), all compounded for a year; the second event concerns the complement to 1 of that probability for an outcome equal to zero.

At this point, using reliable software[7], the random variables are summed (convolution) and all the required answers can be given.

It seems clear that:

- while the first (classic mathematical) approach seeks a general solution to the problem under numerous hypotheses, regardless of the specific case;
- the second approach uses a self-adjusting algorithm (with reference to the specific risk variables) to provide a specific solution for any real-world case.

The latter approach can adopt two different criteria:

I) Definition of the individual random variables for each year
After defining random variables year by year, the convolution of these random variables is calculated to determine the risk for a non-specific year, independent on that for the other years.
This calculation identifies the risk for various periods, taking account of the interdependence of the risk among the various years; it is essential to calculate all the possible covariances for a year with respect to all the other years.

II) Definition of the individual random variables for extended periods
In this case, it is possible to define the random variables that define the risk for periods of several years; for example:

- one can define the random variable for a general policy for the first two years; then, summing these two-yearly random variables (which are independent for each policy), one can calculate the two-yearly risk without any covariance.

One proceeds in the same way for the first three years (defining the risk variables for the three-year period and convoluting) and, after that, for four years and so on.
The two processes are analyzed in detail below.

I.3.1) Definition of a general random variable year by year for each policy

The starting point for this approach is the definition of the random variables for each year of the insurance contracts held in the portfolio.
We consider the annual risk of the insurer as a random variable whose mean is the classical risk premium.
Assuming that the portfolio comprises S policies; for each year r $(r = 1,2,...,R.)$. We can define the random variable as $X_s^{(r)}$ (where $s = 1,2,..,S.$) which represents the occurrence affecting the s-th contract in its r-th year, if the rate of interest follows one of the predetermined scenarios.
The rate of interest for the year $(r-1,r)$ expected at time 0 in scenario c $(c = 1,2..,C)$ is $i_c(0;r-1,r)$.
The s-th contract envisages the pay-out for the service as a lump sum at the end of the year in which the event giving rise to the service takes place.
a) the payment of an amount $B_s^{(r)} > 0$ to the heirs of the insured at the end of the year in which the assured dies, if death occurs during the contract period (r = 1, 2, ..., n).

[7] Made available by prof. Alvaro Tomassetti

b) the payment of an amount $C_s^{(r)} \geq 0$ at the end of the year in which the insured becomes an invalid, if invalidity occurs during the contract period (r = 1, 2, ..., n).
c) the payment of an amount $E_s^{(r)} \geq 0$ if the insured survives to the end of the contract, i.e. at the end of year $(n-1, n)$
d) in the case of survival, the payment by the insured of an annual premium $P_s^{(r-1)}$ at the start of each year.
e) $V_{s,c}^{(r-1)}$ the actuarial reserve at time $r-1$, after the payment of the premium
In general year r for policy s the following random variable can be defined:

Table 1. Definition of random variable $X_s^{(r)}$ $(s = 1,2,..,S.)$ $(r = 1,2,...,n_s)$

Occurrences	Probabilities
$-B_s^{(r)} + V_{s,c}^{(r-1)} \cdot [1 + i_c(0; r-1, r)]$	${}_{r-1/}q_x^{(a,d)}$
$-C_s^{(r)} + V_{s,c}^{(r-1)} \cdot [1 + i_c(0; r-1, r)]$	${}_{r-1/}q_x^{(a,i)}$
0	$1 - {}_{r-1/}q_x^{(a,d)} - {}_{r-1/}q_x^{(a,i)}$

The variance of the random variable defined above represents a measure of the risk of an insurance policy.
Since the contracts are assumed to be mutually independent, the variance of the portfolio or, from now on, the portfolio risk, is the sum of the individual risks.
Similarly for policy s the random variable at the end of the contract will be:

Table 2. Definition of the individual random variable $X_s^{(n_s)}$ at contract maturity

Occurrences	Probabilities
$-B_s^{(n_s)} + V_{s,c}^{(r-1)} \cdot [1 + i_c(0; n_s - 1, n_s)]$	${}_{r-1/}q_x^{(a,d)}$
$-C_s^{(n_s)} + V_{s,c}^{(r-1)} \cdot [1 + i_c(0; n_s - 1, n_s)]$	${}_{r-1/}q_x^{(a,i)}$
$-E_s^{(n_s)} + V_{s,c}^{(r-1)} \cdot [1 + i_c(0; n_s - 1, n_s)]$	$1 - {}_{r-1/}q_x^{(a,d)} - {}_{r-1/}q_x^{(a,i)}$

Once the sum of the random variables has been defined – for a given year - convolution takes place to obtain the mean, variance and asymmetry for assessment of the risk.
Formally, this is:

$$X^{(r)} = \sum_{s=1}^{S} X_s^{(r)} \tag{8}$$

and in addition, given the mutual independence of each policy, in a given year, the following will be true:

$$Var[X^{(r)}] = \sum_{s=1}^{S} Var[X_s^{(r)}] \tag{9}$$

Should one wish to calculate the risk for a period of years, given that the risk variable for the year n is dependent on the analogous variable in all the preceding years, it should be necessary to calculate all the various possible covariances.
This means, wanting to determine the risk in period (1,m), having to write:

$$Var[X^{(1,m)}] = \sum_{r=1}^{m}\sum_{s=1}^{S} Var[X_s^{(r)}] + 2\sum_{r_1=r_0}^{m-1}\sum_{r_2=r_1+1}^{m}\sum_{s=1}^{S} Cov[X_s^{(r_1)}, X_s^{(r_2)}] \quad (10)$$

where:

$$Cov[X_s^{(r_1)}, X_s^{(r_2)}] = E[(X_s^{(r_1)} - E(X_s^{(r_1)}))\cdot(X_s^{(r_2)} - E(X_s^{(r_2)}))] \quad (11)$$

In [5] an interesting relation about the covariances is shown.
In order to determine the above, it will be necessary (for each s and for $r_1 > r_2$):

- to calculate all the pairs of possible random variables produced;
- to determine the resulting means

This calculation becomes particularly onerous once the time horizon exceeds three-five years. *Accordingly, an alternative approach that eliminates the calculation of the covariances becomes highly attractive, given that the same end-results are achieved.*

I.3.2) Definition of the random variables for extended periods
With reference to the general policy s and to a period of several years ranging from 0 to m ($0 \le m < n_s$), the following random variable can be defined:

Table 3. Definition of the individual random variable $X_s^{(1,2,\dots,m)}$ for the period (1,m)

Occurrences	Probabilities
$-B_s^{(1)} + V_{s,c}^{(0)}\cdot[1+i_c(0;0,1)]$	${}_{0/}q_x^{(a,d)}$
$-C_s^{(1)} + V_{s,c}^{(0)}\cdot[1+i_c(0;0,1)]$	${}_{0/}q_x^{(a,i)}$
$-B_s^{(2)} + V_{s,c}^{(1)}\cdot[1+i_c(0;1,2)]$	${}_{1/}q_x^{(a,d)}$
$-C_s^{(2)} + V_{s,c}^{(1)}\cdot[1+i_c(0;1,2)]$	${}_{1/}q_x^{(a,i)}$
⋮	⋮
$-B_s^{(m)} + V_{s,c}^{(m-1)}\cdot[1+i_c(0;m-1,m)]$	${}_{m-1/}q_x^{(a,d)}$
$-C_s^{(m)} + V_{s,c}^{(m-1)}\cdot[1+i_c(0;m-1,m)]$	${}_{m-1/}q_x^{(a,i)}$
$-E_s^{(m)} + V_{s,c}^{(m-1)}\cdot[1+i_c(0;m-1,m)]$	$1-\sum_{j=0}^{m-1} {}_{j/}q_x^{(a,d)} - \sum_{j=0}^{m-1} {}_{j/}q_x^{(a,i)}$

At this point, it is possible to write that the variance of the sum of the random variables is equal to the sum of the variances of each random variable, since the random variables $X_s^{(1,2...,m)}$ $(s = 1,2,..,S)$ are mutually independent, being:

$$Var\left[\sum_{s=1}^{S} X_s^{(1,2,...,m)}\right] = \sum_{s=1}^{n} Var\left[X_s^{(1,2,...,m)}\right] \qquad (12)$$

I. 3.3) A useful relation between the sum of the variances of random variables, both as random variables each relating to a period 1,2,...,m and as random variables relating to a specific year ($1 \le m \le n$)

Given the above analysis, the following equality can be written:

$$\sum_{s=1}^{S} Var[X_s^{(1,2,...m)}] = \sum_{r=1}^{m}\sum_{s=1}^{S} Var[X_s^{(r)}] + 2\sum_{r_1=r_0}^{m-1}\sum_{r_2=r_1+1}^{m}\sum_{s=1}^{S} Cov\left[X_s^{(r_1)}, X_s^{(r_2)}\right] \qquad (13)$$

which demonstrates the hypothesis that the risk associated with a group of policies can be calculated with the same result either

a) using (12) summing JUST the variances of each policy (after defining the random variables for the period);

b) using (13) summing the variances for each policy and all the possible covariances (if the random variables are only defined on a year-by-year basis).

The applications will use (12), since it is vastly easier to calculate with respect to (13) once the reference period exceeds just a few years.

II) APPLICATIONS

The calculation of the above refers to a portfolio of life policies comprising 10,000 policies providing temporary death cover and the doubling of capital (the details of these contracts are set out in the appendix).

It is important to note that the valuation has been made using a real compound return on capital of 2% and assuming an initial capital in the first case of 10,000 and, in the second case, of 20,000 (conventional units).

Table 4. Initial capital of 10,000 monetary units. Net real compounding rate 2%. Parameters representative of the sum of the random variables $X_s^{(1,2,...,m)}$

Year	Mean	Standard deviation	Skewness
1	9891,779	63.104	0.209
2	9393,049	151.951	0.164
3	8446,415	263.847	0.135
4	6964,235	403.505	0.117
5	4772,783	569.203	0.101
6	1907,107	762.742	0.091
7	-1925,671	990.235	0.082

Table 5. Initial capital of 20,000 monetary units. Net real compounding rate 2%. Parameters representative of the sum of the random variables $X_s^{(1,2,\ldots,m)}$

Year	Mean	Standard deviation	Skewness
1	20,091.779	63.104	0.209
2	19,797.049	151.951	0.164
3	19,058.495	263.847	0.135
4	17,788.556	403.505	0.117
5	15,813.591	569.203	0.101
6	13,168.731	762.742	0.091
7	9,561.185	990.235	0.082
8	4,882.936	1,248.861	0.079
9	-957.452	1,537.530	0.067

The appendix presents the probability distributions for the first three years.

III) APPENDIX

The appendix presents three tables:

1) Table A1: Composition of the portfolio of policies
2) Table A2: Probability of invalidity of the insured population
3) Table A3: Probability distribution of the random variables $X_s^{(1)}$, $X_s^{(1,2)}$, $X_s^{(1,2,3)}$

Table A1: Composition of the portfolio of policies

Group	Sex	Age	$B_s^{(r)}$	$C_s^{(r)}$	$E_s^{(r)}$	n_s	Policies
1	Female	30	100	100	200	30	1500
2	Male	35	150	150	300	30	2500
3	Female	40	400	400	0	20	2000
4	Male	50	500	500	0	15	4000

Table A2: Probabilities of invalidity of the insured population

age	Male	Female	age	Male	Female	age	Male	Female
30	0.000070	0.000065	42	0.000265	0.000405	54	0.001345	0.001020
31	0.000075	0.000075	43	0.000320	0.000480	55	0.001530	0.001020
32	0.000080	0.000085	44	0.000380	0.000575	56	0.001710	0.001020
33	0.000090	0.000105	45	0.000430	0.000665	57	0.001835	0.001020
34	0.000100	0.000130	46	0.000475	0.000755	58	0.001690	0.001020
35	0.000115	0.000155	47	0.000520	0.000805	59	0.001260	0.001020
36	0.000135	0.000180	48	0.000585	0.000875	60	0.001260	0.001020
37	0.000150	0.000200	49	0.000690	0.000980	61	0.001260	0.000000
38	0.000160	0.000225	50	0.000810	0.001145	62	0.001260	0.000000
39	0.000170	0.000240	51	0.000935	0.001305	63	0.001260	0.000000
40	0.000185	0.000280	52	0.001050	0.001435	64	0.001260	0.000000
41	0.000215	0.000330	53	0.001180	0.001325	65	0.001260	0.000000

The probabilities of mortality are the ones of the 1991 Italian census.

Table A3: Probability distribution of the random variables $X_s^{(1)}$, $X_s^{(1,2)}$, $X_s^{(1,2,3)}$

Occurrences	$X_s^{(1)}$ p.d.	$X_s^{(1,2)}$ p.d.	$X_s^{(1,2,3)}$ p.d.
$\mu - 3.00 \cdot \sigma$	0.000345	0.000518	0.000648
$\mu - 2.50 \cdot \sigma$	0.003108	0.003764	0.004190
$\mu - 2.00 \cdot \sigma$	0.016981	0.017964	0.018836
$\mu - 1.50 \cdot \sigma$	0.060470	0.062129	0.062701
$\mu - \sigma$	0.160491	0.157975	0.158333
$\mu - 0.75 \cdot \sigma$	0.229659	0.230090	0.229390
$\mu - 0.50 \cdot \sigma$	0.317001	0.316215	0.314342
$\mu - 0.25 \cdot \sigma$	0.414864	0.412206	0.409294
μ	0.515350	0.509709	0.508747
$\mu + 0.25 \cdot \sigma$	0.607926	0.607886	0.606576
$\mu + 0.50 \cdot \sigma$	0.698915	0.698473	0.697136
$\mu + 0.75 \cdot \sigma$	0.777316	0.777296	0.776182
$\mu + \sigma$	0.842065	0.842114	0.841355
$\mu + 1.50 \cdot \sigma$	0.927273	0.929055	0.929795
$\mu + 2.00 \cdot \sigma$	0.972035	0.973206	0.973780
$\mu + 2.50 \cdot \sigma$	0.990418	0.991247	0.991720
$\mu + 3.00 \cdot \sigma$	0.997250	0.997589	0.997782
$\mu + 4.00 \cdot \sigma$	0.999843	0.999881	0.999901
$\mu + 5.00 \cdot \sigma$	0.999995	0.999997	0.999998
$\mu + 8.00 \cdot \sigma$	1.000000	1.000000	1.000000
Moments			
μ	308.221	1010.951	2165.665
σ	63.105	151.062	263.847
Skewness	0.209	0.164	0.135
Kurtosys	3.044	3.028	3.019

Bibliography

[1] M.H. Amsler (1992): "Risque de deces et risque de ruine: Reflexions sur la mesure du risque de ruine", ASTIN 22, no. 1, 107-119.
[2] N.L. Bowers, H.U. Gerber, J.C. Hickman, D.A. Jones, C.J. Nesbitt (1986, 1997): "Actuarial Mathematics", The Society of Actuaries, Schaumburg, Illinois.
[3] M..G. Bruno (1999) "An alternative method for calculating the probability of ruin in risk theory: theoretical approach and applications". Sent to the Italian Institute of Actuaries for the "young actuaries" competition, Italy.
[4] M.G. Bruno (1999) "Calculation methods for evaluating Asian options" submitted for publication in the new Journal of AMASES "Decision in Economics and Finance: A Journal of Applied Mathematics", Italy.

[5] M.G. Bruno, E. Camerini, A. Tomassetti (1998): "Financial and Demographic Risks of a Portfolio of Life Insurance Policies with Stochastic Interest Rates: Evaluation Methods and Applications", currently being printed in the North-American Actuarial Journal, USA.
[6] J.D. Cummins, B.D. Smith, R.N.Vance, J.L. VanDerhei(1983): Risk classification in life insurance", Huebner International Series on Risk, Insurance and Economic Security, Kluwer-Nijhoff Publishing, Boston USA.
[7] C.D. Daykin, T. Pentikainen, M. Pesonen (1994): "Practical risk theory for actuaries", Monographs on Statistics and Applied Probability 53, Chapman & Hall, London.
[8] F.E. De Vylder (1996): "Advanced risk theory- a self-contained introduction", Collection Actuariat, Editions de l'Université de Bruxelles, Belgique.
[9] H.U. Gerber, E.S.W. Shiu (1998): "On the Time Value of Ruin", North-American Actuarial Journal 2, no. 1, 48-78, USA.
[10] H.U. Gerber, E.S.W. Shiu (1997): "From ruin theory to option pricing", Joint day proceedings, volume of the XXVIII th International Astin Colloquium / 7-th International AFIR Colloquium, Cairns, Australia, pages 157-176
[11] A. Tomassetti, A. Manna, S. Pucci (1995): "Procedimenti esatti in orizzonti temporali finiti per la valutazione delle distribuzioni dell'ammontare delle varie "rovine" nonché delle corrispondenti epoche, nella teoria individuale del rischio", Giornale dell'Istituto Italiano degli Attuari, LVIII, Italy.