USE OF MARKOV PROCESS THEORY AND THIELE'S DIFFERENTIAL EQUATION IN PRACTICAL CLAIMS RESERVING

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Abstract

A straightforward derivation of Thiele's differential equations for the moments of present values of a payment stream governed by a discrete time Markov process is presented. Two different applications of the theory are then discussed. The first deals with the problem of rehabilitation on disability annuity claims and the second bases predictions on case estimates. Emphasis is put on straightforwardness in deriving the computational scheme and exemplifying the features relevant in an implementation.

Keywords

Claims reserving, disability annuities, rehabilitation, Markov process theory and Thiele's differential equations.

1. Introduction

Traditionally methods for claims reserving works on a macro level regarding aggregated payments or claim numbers. With the development of computers it has however become tractable to model the development on a micro level that is regarding and modeling the stochastic behavior of a single claim.

There have in the latest years been some theoretical works on micro models (e.g. Arjas 1989, Hesselager 1993, Norberg 1995 and Haastrup 1997) though it is the impression that the practical experiences with micro models are still limited.

In this paper the foundations for and implementation of a micro model based on the theory of Markov processes will be described. This type of model is well known in a life insurance context e.g. in the Danish technical basis G-82, whereas the use of a Markov model for calculation of claims reserves has been suggested by Hesselager (1993). The model frame is very flexible and its computational scheme based on the Thiele differential equations is easy to implement using ones favorite software package.

The calculation of moments of present values is thoroughly examined by Møller and Norberg in several articles but where these works emphasize theoretical deliberations we here emphasize the practical aspects. We therefore present a, hopefully, more intuitive approach, the aim being to encourage others to use these models in practice.

We start by deriving the Thiele equation from a quite straightforward method inspired by Møller (1994).

2. The theoretical basis

2.1 Thiele's differential equations

The development of a claim is regarded as a stochastic process $(X_t)_{t\in\{0,1\ldots T\}}$ observable at the time epochs $t_0 = 0, \ldots, t_n = T$ with T some appropriately chosen time horizon. Time can be measured from the occurrence time of the claim or from the reporting time depending on the characteristics of the problem. At each point t_i in time the process is in one of a finite number of states, which in this context each represent some classification of the claim. The state space of the process is denoted $\mathfrak{I}=\{1\ldots I\}$ and all claims are assumed to develop independently.

Furthermore we assume that the X-process is a Markov process why the transition probabilities fulfill:

$$p_{ij}(s,t) = P[X_t = j \mid X_s = i, X_0 = i_0, \ldots X_{s-1} = i_{s-1}] = P[X_t = j \mid X_s = i].$$

Subsequently the probabilities fulfill the Chapman-Kolmogorov equation:

$$p_{ij}(s,t) = \sum_k p_{ik}(s,u)\,p_{kj}(u,t) \qquad s \le u \le t$$

The transition probabilities are ordered in the transition matrix $\underline{\underline{P}}(s,t)$ with $p_{ij}(s,t)$ as element (i,j). Using Matrix notation the Chapman-Kolmogorov equation becomes:

$$\underline{\underline{P}}(s,t) = \underline{\underline{P}}(s,u)\,\underline{\underline{P}}(u,t) \qquad s \le u \le t$$

Throughout the "life" of a claim payments of different kinds can occur. The word "payment" is used in the broad sense, which means that the money is not necessarily physically paid to the insured but can be allocated either as a not yet paid amount or as a case estimate as we shall see in the second implementation.

We distinguish between two types of payments: Pension type payments and lump sum payments, denoted by the functions $(c_i)_{i\in\mathcal{J}}$ and $(c_{ij})_{i,j\in\mathcal{J}}$ respectively.

The pension payment c_i is an amount paid if the X-process is in state i at time t (that is $X_t = i$), and c_{ij} is an amount paid if the process makes a transition from state i to state j at time t (that is $X_{t-1} = i$, and $X_t = j$). The payments are in the sequel assumed to be constants but making them time dependant (yet still deterministic) is of course only a trivial extension of the model.

By introducing the counting processes $(N_{ij}(t))_{t\in\{0,\ 1,\ \ldots\ ,T\}}$ which counts the number of transitions from state i to state j up to and including time t, and the indicator functions $I(X_t = i)$ we can identify the aggregated amount paid on the claim at time t as:

$$B_t = \sum_{i=1}^{t} \sum_{k,j} c_{kj} \Delta N_{kj}(i) + \sum_{l} c_l I(X_{t-1} = l)$$

With $B_0 = 0$, $c_{kk} = 0$ for all k and $\Delta N_{kj}(t) = N_{kj}(t) - N_{kj}(t-1)$.

In practice this means that if the X-process at the time of the calculation is in state j and at the time of the last calculation was in state i we have a payment of c_{ij}. Furthermore the amount c_i is paid regardless of whether a transition has taken place or not. We notice that there is no payment at time 0 since the payment is due at the end of the interval.

We now seek expressions for the moments of the stochastic present value at time t of the future payments using the constant interest rate $(1+r) = v^{-1}$.

Let R_t denote this present value. We can then write R_t^q as:

$$R_t^q = \left[\sum_{i=t+1}^{T} \sum_{k,l} v^{i-t} (c_k + c_{kl}) \Delta N_{kl}(i) \right]^q$$

with :

$$\Delta N_{kk}(i) = 1 - \sum_{j \neq k} \Delta N_{kj}(i)\text{, and } c_{kk} = 0.$$

Further calculations including the use of the binomial formula produce:

$$R_t^q = \left[\sum_{k,l} v(c_k + c_{kl})\Delta N_{kl}(t+1) + \sum_{i=t+2}^{T} v^{i-t}\sum_{k,l}(c_k + c_{kl})\Delta N_{kl}(i)\right]^q$$
$$= \left[\sum_{k,l} v(c_k + c_{kl})\Delta N_{kl}(t+1) + vR_{t+1}\right]^q$$
$$= v^q R_{t+1}^q + v^q \sum_{p=1}^{q}\binom{q}{p}\left(\sum_{k,l}(c_k + c_{kl})\Delta N_{kl}(t+1)\right)^p R_{t+1}^{q-p}$$
$$= v^q R_{t+1}^q + v^q \sum_{p=1}^{q}\binom{q}{p} R_{t+1}^{q-p}\sum_{k,l}(c_k + c_{kl})^p \Delta N_{kl}(t+1)$$

We now have an expression for the entity R_t. It is then possible by use of the tower property of conditional expectations and the Markov property of the X-process to calculate expressions for the state wise moments of R_t.

$$V_{j,t}^{(q)} = E\left[R_t^q \middle| X_t = j\right]$$
$$= v^q E\left[R_{t+1}^q \middle| X_t = j\right] + v^q E\left[\sum_{p=1}^{q}\binom{q}{p}\sum_{k,l}(c_k + c_{kl})^p \Delta N_{kl}(t+1) R_{t+1}^{q-p} \middle| X_t = j\right]$$
$$= v^q E\left[\sum_m I(X_{t+1} = m) E\left(R_{t+1}^q \middle| X_{t+1} = m\right) \middle| X_t = j\right]$$
$$+ v^q \sum_{p=1}^{q}\binom{q}{p} E\left[\sum_{k,l}(c_k + c_{kl})^p \Delta N_{kl}(t+1)\sum_m I(X_{t+1} = m) E\left(R_{t+1}^{q-p} \middle| X_{t+1} = m\right) \middle| X_t = j\right]$$
$$= v^q \sum_m V_{m,t+1}^{(q)} p_{jm}(t,t+1) + v^q \sum_{p=1}^{q}\binom{q}{p}\sum_m V_{m,t+1}^{(q-p)}(c_j + c_{jm})^p p_{jm}(t,t+1)$$

Now let $\underline{V}_t^q$ denote the vector of the state wise moments, let # denote elementwise multiplication and let $\underline{\underline{C}}$ be the matrix containing the payments (that is with *$(c_{ij} + c_i)$* as element *(i,j)*). We can then put up the difference equation on the matrix form:

$$\underline{V}_t^q = v^q \underline{\underline{P}}(t,t+1) * \underline{V}_{t+1}^q + v^q \sum_{p=1}^{q}\binom{q}{p}\left(\underline{\underline{C}}^{\#p} \# \underline{\underline{P}}(t,t+1)\right) * \underline{V}_{t+1}^{q-p}.$$

The equation is solved recursively starting with the lowest moments and the side condition $\underline{V}_T^q = \underline{0}$ for $q>0$ and $\underline{V}_T^0 = \underline{1}$.

The difference equation is of course the discrete time version of the differential equation of Norberg (1995) and Møller (1994) and enables us to besides calculating the expected value of the future payments also to calculate the moments of the distribution. Knowledge of the distribution is relevant in order to calculate a safety loading.

The Matrix notation makes the difference equation immediately applicable for numerical calculations especially if one is using a matrix based programming language as APL or the IML facility of the SAS system.

2.2 Flexibility of the state space

Equipped with the difference equation we have, for a convenient definition of the state space, appropriate payment functions and estimates of the transition probabilities, a tool for fast calculation of the desired moments.

The Markov property is of course a restrictive assumption which could seem to, nice computational schemes or not, make a model very unlikely to reflect the reality it is supposed to model. This is to a certain extent due to the straightforwardness of the assumption. Assumptions just as crude or even cruder could easy be made, hidden in assumptions of special families of distributions etc.

As illustrated in the first example below the Markov assumption can in practice be relaxed by adding more states to the state space. If for instance claims are categorized due to an evaluation of severity made by the claims handler, a claim that is categorized as severe after at an earlier stage having been categorized as a refusal might well have different properties than a claim categorized as severe at initiation. This can be taken care of by introducing an additional state for claims that have had their categorization altered. In general it is obvious that if the conditions assumed to influence the distributions can be expressed in simple terms (i.e. in terms of indicators), introducing them in the model can be made by adding additional states.

An example could be that the one-step transition probabilities beside the age depended upon the duration in the states. This could also be handled by adding additional states in this case a state for each value of the duration that is supposed to influence the transition probability, thereby defining a subgroup of the state space. The transition probabilities for the states in the subgroup must then reflect that once the process has entered a state in the subgroup the only possible transitions are transition out of the subgroup, or transition to the state in the subgroup that represents one more period of sojourn.

3. A model for handling rehabilitation on a portfolio of disabillity annuity claims

3.1 The model

In Denmark the pricing of disability annuities is usually calculated from the Danish technical basis G-82 which is a traditional three state Markov model in continuous time. Likewise the reserves on claims are calculated with use of the parameters inherent in G-82. The G-82 basis does however not operate with the possibility of rehabilitation why a substantial positive run-off on the claims reserve is observed. A way to handle this problem could be to introduce a rehabilitation intensity as sketched by the dotted arrow on figure 1 and recalculate the reserves using either numerical integration or solving Thiele's differential equations.

This procedure does however raise questions about which form the rehabilitation intensity should have, and secondly whether it is at all reasonable to have it depend on the age of the insured? Of course a constant intensity could be used but it seems apparent that if one becomes disabled it is the period from the time of disabling to the time where ones condition is stable that is critical in terms of rehabilitation. This is the time where one would expect a higher death intensity and also a higher rehabilitation intensity.

The case with intensities depending both on time since entry into the state and age is dealt with in continuous time by Møller (1994) but the resulting two dimensional differential equations do not seem inviting from a practical point of view.

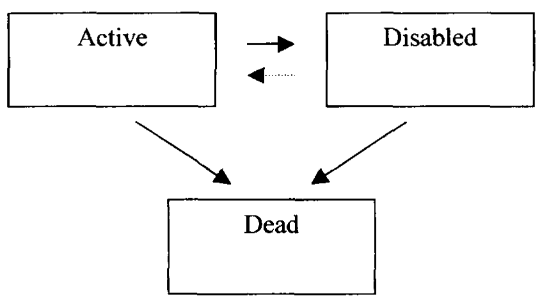

Figure 1: *The G-82 model*

An alternative procedure was chosen in the present case. The model is made up of two submodels the first depending on time since reporting and the second being the G-82 basis. The first submodel then predicts payments for claims being younger than a certain threshold age for the time until they reach the threshold age. The second submodel then deals with payments due when the claim is older than the threshold age. The model was implementeted on a monthly basis since the company produces monthly accounts.

When constructing the model it is of course necessary to work from the available information. The present data material of the portfolio of disability annuity claims consists of a number of records comprising information on annual payment, claims date, a case estimate, a registration date and information on whether the insured has deceased. The case estimate is the claim handlers expectation of the claim and has three possible values 0, 50 and 100 corresponding to the percentage of the annual payment the insured is expected to be entitled to. The case estimate is of course liable to change during the handling of the claim, and changes from 50% or 100% to 0% are regarded as a rehabilitation. If the insured is rehabilitated he will still be entitled to payments corresponding to the period he was disabled.

In the context of a state space model this means that the claims can be in one of four states one for each value of the case estimate and one state representing death. The payment scheme includes no lump sum payments but a pension type payment of 1 in the state of 100% disability and a pension type payment of 0.5 in the state of 50% disability.

Empirical analysis of the data material indicated that claims which had a case estimate and had previously been rehabilitated (a reopened claim) had different properties than claims which had not been reopened. The state space was therefore enhanced by adding three additional states, one for each possible value of the case estimate. The two states corresponding to 0% thus separates claims that are rehabilitated and claims which are expected to be rejected. The state space is shown in figure 2. Furthermore the data suggested a threshold age for the model of 36 months partly due to the data material only comprising four years of claims. It is however easy to change this value at a later stage if further data suggests this.

Now let x denote the age of the insured, t the time since reporting, T the threshold age and $\mathfrak{I}$ the state space. The total reserve is then given by:

$$V_{i,t,x}^{RBNS} = V_{i,t} + \sum_{j \in \mathfrak{I}} p_{ij}(t,T) V_{j,x+(T-t)_+}^{G-82},$$

with $V_{i,t,x} = 0$ and $p_{ij}(t,T) = \delta_{ij}$ for t≥T and δ_{ij} the Kronecker delta.

Interpreting the two terms in the expression for the reserve one finds that the first term covers expenses within the first T months from reporting and the second term is the expected reserve calculated on G-82 depending on the age of the insured when his claim is T months old.

The obvious way to implement the model was to tabulate the state wise reserve functions. Since the present type of policy has an expiration date this would present a small problem for claims which expire before the claim reaches the threshold age. This problem can be dealt with in several manners. Firstly one could choose to disregard the problem since the impact on the total reserves will usually be small. There are usually only a small number of such claims and the reserves are small since the time to expiration is short. Secondly one could tabulate $V_{i,t,x}$ for each of the 34 ages for which the problem is relevant, with all transition probabilities set to zero from the appropriate age and thirdly the reserve could be corrected by a factor e.g. the quotient between the remaining number of months and the threshold age. In the present case we have chosen to disregard the problem since the expiration age of all the policies is 60 years and only 1 % of the claims in the data material have been reported after the insured has reached the age of 57.

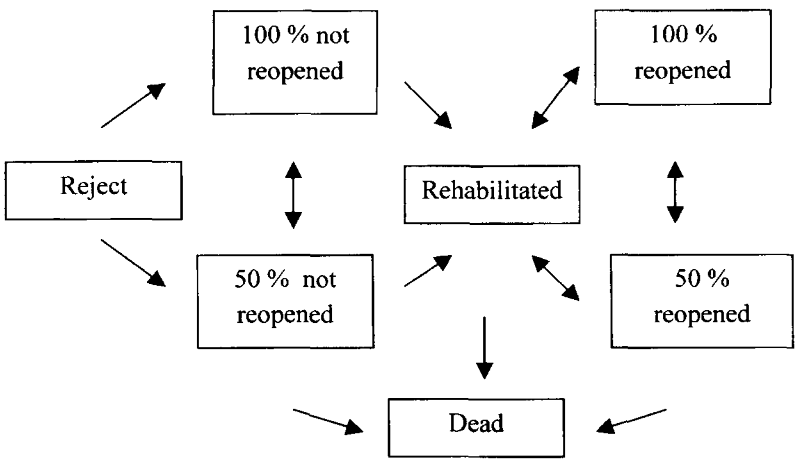

Figure 2: *The statespace for submodel 1. Notice that the state 'dead' can be reached from all other states and that a claim starts in the left part of the scheme and only switches to the right part if the insured is rehabilitated.*

3.2 Estimation techniques

Using the Markov model approach and the difference equation of course demands the estimation of the one step probabilities (intensities) for the various transitions.

Techniques for this is sought in the vast arsenal of estimation procedures developed for use in survival analysis including use of parametric functions, piecewise constant intensities, least squares methods, and kernel smoothing techniques.

Normally when working with developments of claims it will be difficult to base assumptions of parametric intensity functions on intuitive grounds similar to the intuitive interpretation of the Gompertz-Makeham intensity function in the life insurance model. This of course speaks in favor of using some kind of smoothing of raw rates like for instance Kernel smoothing with the classical moving average as a special case. These methods adjust the estimates to the special characteristics of the problem in question, but of course to some extent, depending on the degree of smoothing (the choice of bandwidth), limits the estimates to the areas for which events have indeed occurred. This of course puts some requirements on the amount of data available for the estimation. In the implementation of the present model a moving average technique with a bandwidth of 3 months was used.

3.3 Discussion of the estimated intensities

The data material for the implementation was limited why it has not been possible to examine the long term run-off of the model. We therefore limit ourselves to a brief discussion of the intensities estimated and compare the resulting state wise reserve functions with the G-82 basis.

In figures 3 to 6 estimated intensities for different transitions within submodel 1 are shown.

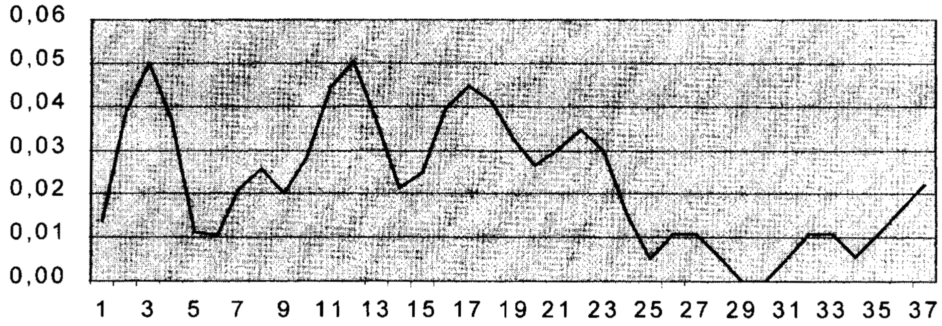

Figure 3: *Estimated one-step transition probability for transition between state 50% not reopened and state rehabilitated.*

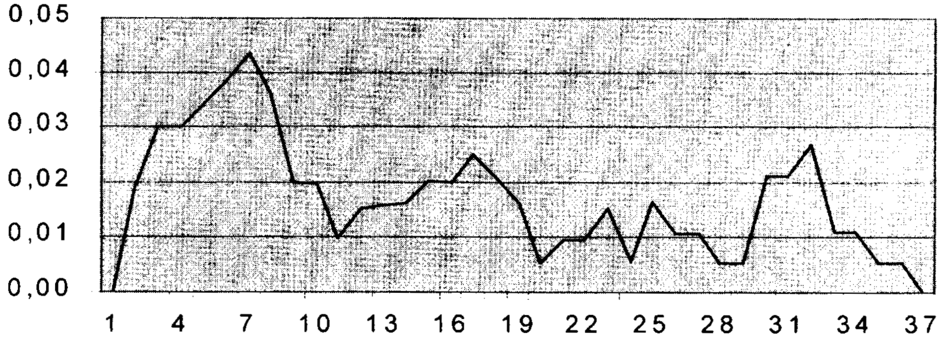

Figure 4: *Estimated one-step transition probability for transition between state 100% not reopened and state rehabilitated.*

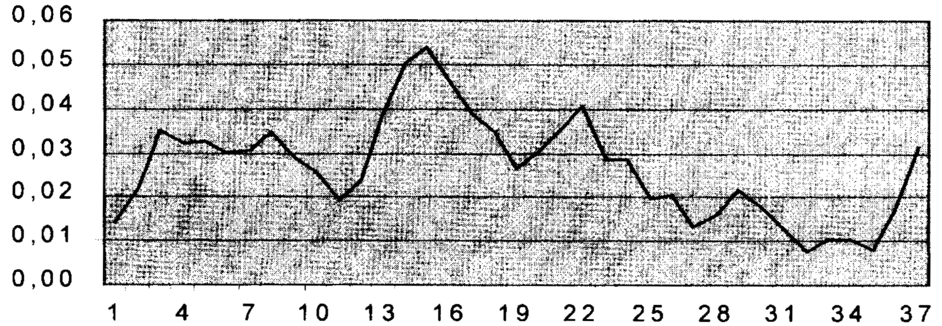

Figure 5: *Estimated one-step transition probability for transition between state 100% not reopened and state 50% not reopened.*

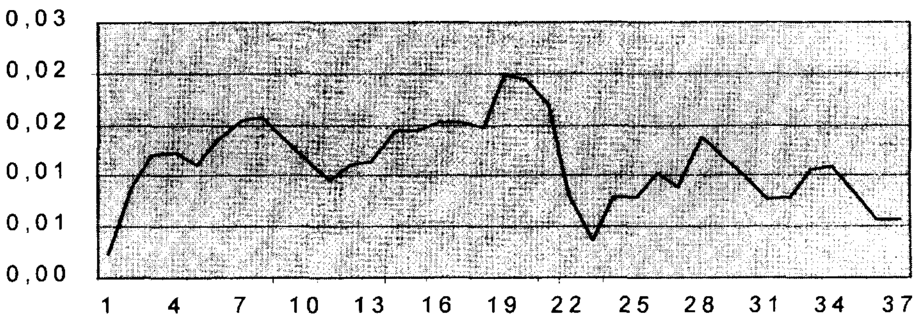

Figure 6: *Estimated one-step transition probability for transition between state 50% not reopened and state 100% not reopened.*

We see the rehabilitation probabilities (fig. 3 and 4) show a peak in the young ages where after the level drops. After around one year from the time of reporting the level again increases and then fades out. The events in the young ages are cases where the insured recovers quickly but where the claim handler at the time of reporting either made a conservative estimate of the disability degree or where the information brought to the knowledge of the claim handler was insufficient.

The level change in rehabilitation probability after around one year corresponds well with the expected duration of medical examinations and recreation periods.

The intensities for transition between 50% and 100% degree of disability also indicate that transitions start to occur after the initial medical examinations have been made during the first year after reporting.

The intensity for reopening of the claim with a 100% degree of disability is shown in figure 7. Compared to the other intensities this is of a much smaller magnitude but indicates that a reopening of a rehabilitated claim is a feature relevant mainly on young claims. If the insured has been subject to a medical examination leading to rehabilitation he has very little chance (or risk) of getting his case reopened.

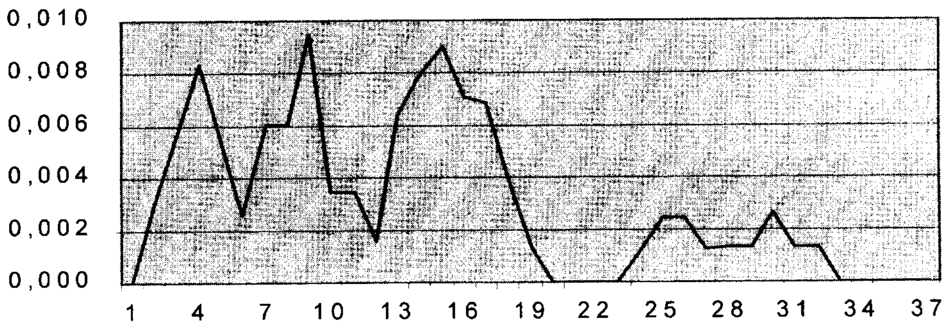

Figure 7: *Estimated one-step transition probability for transition between state rehabilitated and state 100% reopened.*

Concluding on the intensities shown in the figures it must be said that they support the choice of including the time since reporting in the reserving model. Of course other definitions of the state space could possibly be relevant, just experiments with parametric intensity functions etc. could be done. If one has access to a very large data material for a similar portfolio introducing two dimensional intensities would be an interesting extension.

In table 1 some selected reserve function values are listed together with the reserve if a procedure of using the technical reserve from G-82 was implemented. The figures are computed for a male aged *45* years, with expiration at 60 years and a constant annual interest rate of 3%.

t	State	V^{G-82}	V^{model}
0	*0% not reopened*	*0*	*4,876643*
	100% not reopened	*11,954488*	*5,888517*
	50% reopened	-	-
12	*0% not reopened*	*0*	*5,115589*
	100% not reopened	*11,954488*	*6,882652*
	50%t reopened	*11,954488*	*3,219735*
24	*0% not reopened*	*0*	*4,625987*
	100% not reopened	*11,954488*	*8,843348*
	50% reopened	*11,954488*	*5,559008*

Table 1: *Values of the reserve from the model for x=45 in different states and different values of time since reporting.*

It is seen that for a new claim in state 100% the reserve is only about half that of the G-82 reserve. After 24 months the reserve constitutes about two thirds of the G-82 reserve. It is interesting to note that the reserve in state 100% reopened after 24 months is only 5,56 indicating that the model expects a high probability for a rehabilitation or transfer to the 50% reopened state. Because of the sparse data material one should be careful to draw to rigid conclusions from this figure but claims that have been reopened are usually claims where there is an ongoing dispute between the company and the policy holder (and his lawyers). It would therefore not be surprising if these claims also in the future turned out to have a higher rehabilitation frequency since disputes of whether a claim is to be rejected or not usually turn out to the benefit of the company.

4. Calculation of the RBNS reserve using case estimates

In the second example of an implementation we will try to combine the Markov model with the estimates made by the case handlers on the individual claims. We regard a portfolio of disability sums which are policies that entitles the policy holder to a lump sum if he becomes disabled. The sum is calculated as the sum insured (which varies from one policy to another) times the disability degree which is a percentage determined after one or more medical examinations.

The implementation uses case estimates as the starting point.

4.1 Case estimates

In many lines of business the claim handler estimates the expected total expenses on the claim using the information available. This estimate is called a case estimate. In most cases the case estimate, minus the sum already paid, is regarded as 'the reserve' and alone decides the company's RBNS reserve on the claim.

Being a product of a human consideration based on perhaps very complex information (i.e. medical or juridical), reserves consisting of case estimates are of course sensitive in a different way than reserves calculated by the use of a pure statistical model. A natural thought would therefore be to combine the two procedures using the information doubtlessly included in the case estimate for making better statistical predictions.

As a matter of fact whether to use case estimates or not is a current topic for discussion among practising actuaries and heavy arguments are found both pro and con. It is for certain that the case estimates comprise a large amount of information since the evaluation made by an experienced case handler uses many more explanatory variables than a statistical model is able to comprise. At the same time the human factor is also a major drawback since the case estimates are sensitive to changing conditions in the case handling department, new unexperienced staff and so on.

In real life applications the approach of regarding the case estimate is often used. As mentioned the case estimate minus the sum already paid, is often regarded by the company as 'the reserve' on the claim. For the practising actuary it is therefore natural to let the RBNS reserve comprise this adjusted with a correction based on a statistical analysis of the case estimates. This is of course an assumption of the claim handlers are using the same conditions for making the case estimates all the time, a point often used as an argument against using the case estimates.

In the present work we choose to base the predictions on the case estimate using actual payments only as a covariate. This idea can be formalized by the following.

At time t after initiation let c_t denote the case estimate, that is the expected total liability, and let p_t denote the aggregated payments made on the claim. The actual amount the company has set aside to cover the future liabilities on the claim, the 'case-out', is $c_{out}(t) = c_t - p_t$ i.e. an estimate of the future expenses affiliated with the claim.

Since the handling time of a claim is finite, at some point in time the payments made on the claim equal the case estimate, that is the case-out equals zero. If we let T denote some appropriately chosen time horizon we have that : $c_T \approx p_T$

Usually actuarial models (normally disregarding the case estimates) try to predict the sum of the future payments. In our notation this corresponds to predicting the difference $p_T - p_t$. However, since the case estimate shows what the company actually reckons the final cost to be, the object of interest might be to predict how far the current estimate is from the 'final' estimate c_T.

We therefore choose to make the prediction concerning the 'case estimate-error' R_t defined by the relation $R_t = c_T - c_t$. R_t is of course a stochastic variable at time t depending on the future behavior of the case estimate. This could lead to the company adjusting the case-out by the expected value of R_t conditioned on some appropriate information.

4.2 Stochastic payment functions

We have now defined the entity R_t as the future change in case estimate. This is easily put into the Markov frame if we define a suitable state space and appropriate payment functions. It would be natural to let the definition of the state space depend on the level of the case estimate since a separation of severe claims from small seems reasonable (of course the same would be the case if we regarded payments in stead of case estimates). Since many values of case estimates are possible the number of states would be so high that even if one imposes parametric forms on the intensities calculations would be impossible. Therefore some sort of grouping of the states is called for. This however makes the transition amounts stochastic since claims with different values of the case estimate will be grouped together. Furthermore it will be possible to change case estimate without leaving the group.

If we choose to disregard the transitions within the group we are faced with stochastic transition amounts. Hesselager (1993) treats this situation by assuming that the payments arise independently of each other and independently of the process showing the classification of the claim. With this assumption Thiele's differential equations can again be derived. However the assumptions of the transition amounts both being mutually independent as well as independent of the Markov process are crucial, but present some difficulties of interpretation.

If the states are defined by the use of the level of the case estimate or the accumulated payments the independency is compromised. Knowledge of the history of the claim could change the expected value of possible future transition amounts. In that case just knowledge of the size of the payment resulting in the present classification might have a substantial influence on the expected value of a future transition amount. If we for instance regard two claims that reside in the interval between 500 thousand Danish kroner (TDK) and 1.000 TDK. The distribution of the amount payable if the claim makes a transition to the state with payments in the interval 1.000 and 1.200 TDK will now be different for the claim that has entered with a payment of 200 TDK than for the one having entered with a payment of 900 TDK.

This problem can be dealt with by assuming stationary distributions within the groups that is assuming the distribution of the claims in a group on the different levels of case estimate (or payments) comprised by the group is time invariant. With this perhaps rather crude assumption the Thiele equations still holds.

4.3 Implementation of the Markov model

The implementation on the disability portfolio comprised regarding different definitions of the state space, using different time horizons and different techniques for the estimation of the intensities. All calculations were made using months as time units and time was like in the first implementation measured from the time of reporting.

The performance of the model was examined by an as-if analysis simulating an implementation over the period January 1997 to November 1999 and regarding the monthly run-offs. The monthly run-off is defined by:

$$RO_j = (C_{j>} - C_{j\geq}) + (P_{j>} - P_{j\geq}),$$

with $C_{j\geq}$ the total case estimate at the beginning of month j, $C_{j>}$ the total case estimate on the same claims at the end of month j and P denoting the corresponding predicted future changes.

In figure 8 the monthly run-off on the uncorrected case estimates is shown. It is apparent that in the first part of the period the run-off has been positive thus indicating a tendency for the

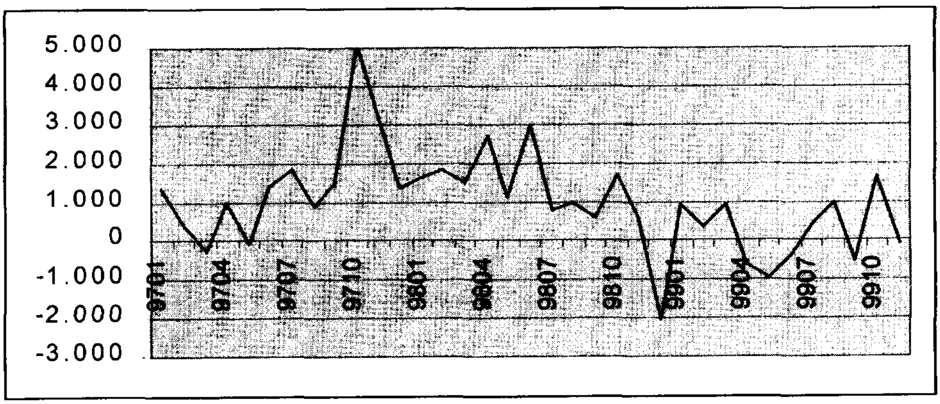

Figure 8: Run-off on case estimates in the period January 1997 to November 1999.

claim handlers to overestimate the claims. This behavior has gradually changed why in 1999 the run-offs have fluctuated around zero.

A closer inquiry revealed that some time around 1996 the claims handlers had been told that they had a tendency to underestimate the claims. This led to a new regime where the claims were often overestimated. This tendency is apparently to some extent worn out since by 1999 the run-offs do not show the same regularity as in 1997 and 1998.

4.5 Results of the implementation

The model was implemented using a state space comprising six states, a 36 months time horizon an estimation period of 48 months and estimates of the intensities calculated by a moving average technique. The state space was made up in a two dimensional manner as shown on figure 9 with each state comprising an interval of both the case estimate and the amount paid. The payment functions consisted of lump sums only. The values were calculated as the averages of the observed changes in disability degree upon the different types of transitions.

The performance of the model is measured in relation to not only the changes made by claims known for less than the time horizon of the model but on all claims. Furthermore run-offs from transitions within the subgroups are also included. From a theoretical point

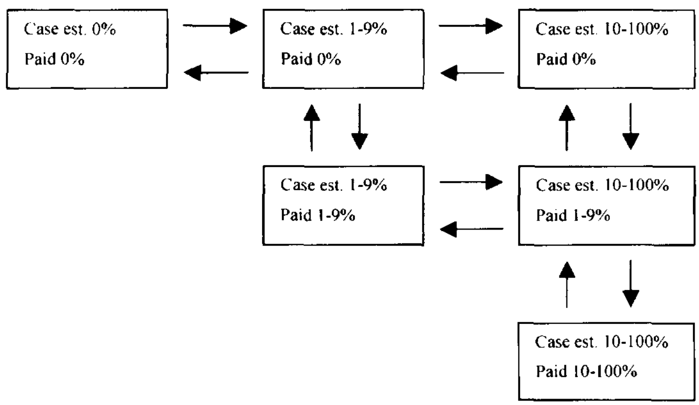

Figure 9: *State space for the case estimate model with each state comprising a combination of case estimate and amount paid both in degrees of disability. Notice that not all possible transitions are shown and that transitions upwards in the state space indicating a repayment, though not assumed impossible, has not been observed.*

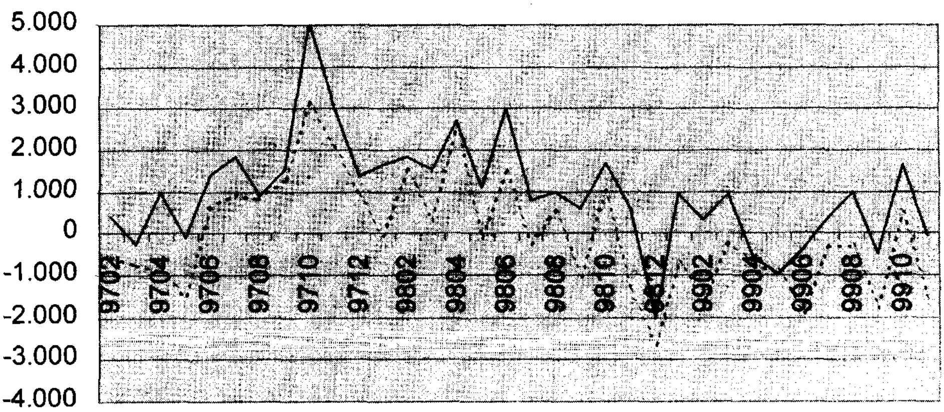

Figure 10 : *Run-off of both case estimates alone and corrected by the Markov model (dotted line)*

of view this is perhaps not adequate but in a practical implementation the test of the model will be the "manager test" and the managers probably won't make a distinction between jumps within and between subgroups. The problem with the old claims where the data material is usually sparse can be solved by introducing another model for these or perhaps by assigning a constant amount for claims older than the time limit.

In figure 10 the run-offs of both the original and the corrected case estimates are shown. If we regard the period as a whole it is apparent from the figure that introducing the correction has reduced the fluctuations. In the period where the case estimates showed large positive run-offs the reduction is larger than in the later part of the period. In this period the model has had difficulties in catching the fluctuations, though it is seen that the two run-off curves both show smaller fluctuations.

The figure demonstrates the demand for the claims handlers to behave in a stable manner if the case estimates are to be used as a starting point in a claims reserving model. If the claim handlers start making actuarial calculations by themselves the model cannot keep up.

5. Conclusion

The two examples of implementation of the Markov model frame demonstrate the major parts of the deliberations needed for using the model frame for practical work. The choice of state space is important but the choice of estimation technique for the intensities often plays an even more important role. In the second implementation the largest contributions to the total reserve were due to either transitions from the state with a case estimate of 0% to the states with a case estimate between 1% and 9% or vice versa. Modeling the intensities for these transitions right will therefore be more important than whether there are one or two intervals for very large claims. In this way the model frame can be used as an analytical tool identifying some relevant features of the problem.

Claims reserving is only one area where this model frame can be used. Another could be in calculating the present value of the net worth of a client defined as the expected value of the payment stream induced by a particular client. This would involve estimating cancellation intensities, claim intensities, and all kind of expenses affiliated with a claim, herein the claims handlers salary and other administrative expenses. In such a model it would be natural to let the cancellation intensity depend on the time since the last claim. For this purpose the idea of defining a subgroup as mentioned at page 5 could be applied.

As noted in the introduction it is the impression that few implementations of both claims reserving and other kinds of models have been made using the Markov frame and Thiele's differential equations. It is therefore hoped that the present presentation has encouraged some to engage in new implementations and hopefully to share them with the rest of the actuarial world.

Litterature:

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[4] C.M. Møller (1994) : Lecture notes for the course FM1 (handwritten). Published 1997 Laboratory of Actuarial Mathematics University of Copenhagen

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