ASTIN Colloquium
Understanding Split Credibility

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Ground Rules

- Follow US Anti-trust Laws, s'il vous plait!
  - Violators will be sent to the guillotine.
- Ask questions of understanding anytime
  - Wait till later to debate.
- There may be a glaringly obvious error in this presentation
  - Catch me later in the bar to tell me what it is.

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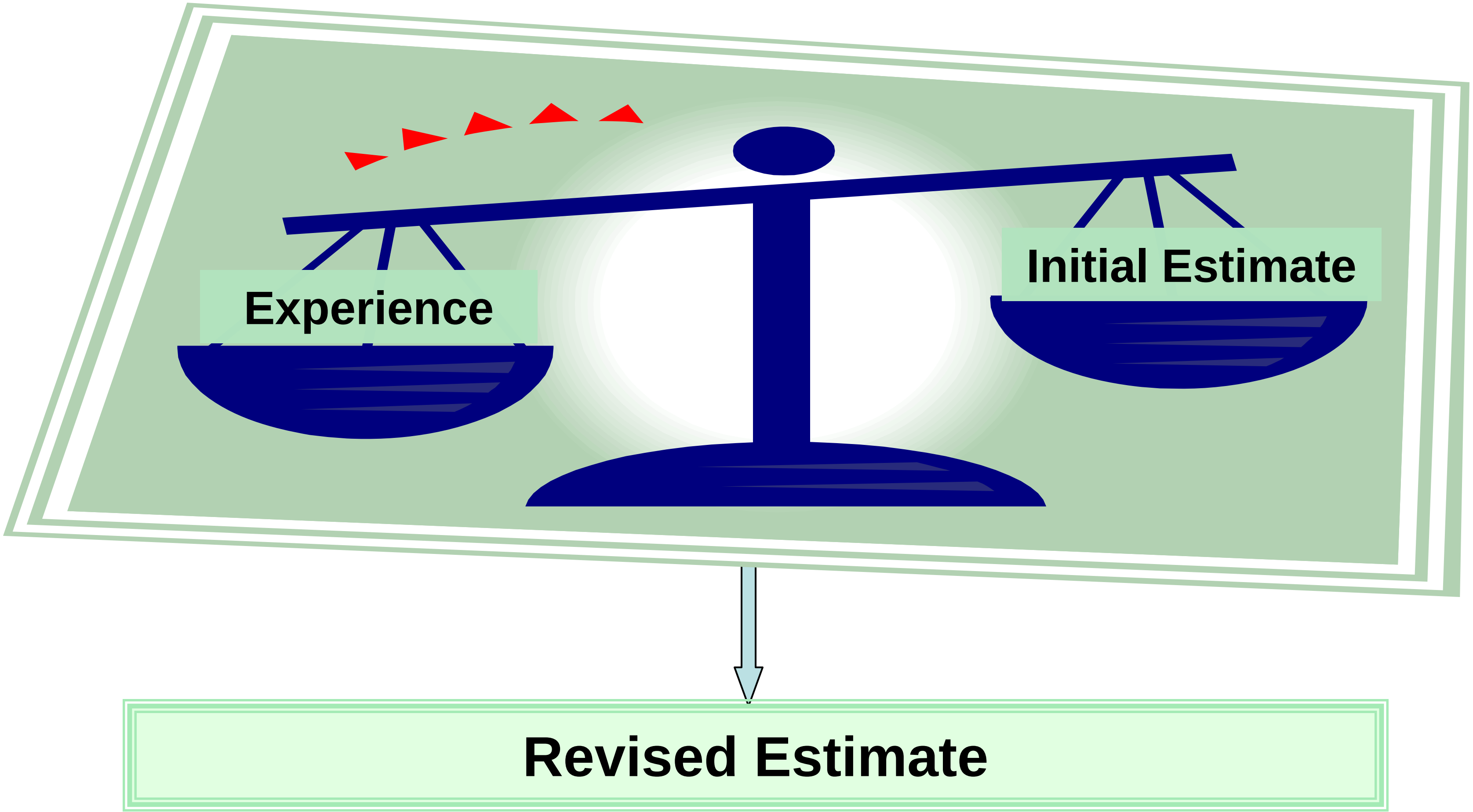
- The methods may or may not meet with regulatory approval
- Examples are for illustration only.
- If something I say gets me into trouble, you are all witnesses that I never said it.

Credibility



No Split-Credibility

- Credibility Weighted Estimate

$$\mu^* = zA + (1 - z)E$$

- E = initial (prior) mean
- A = mean of actual data
- z = credibility

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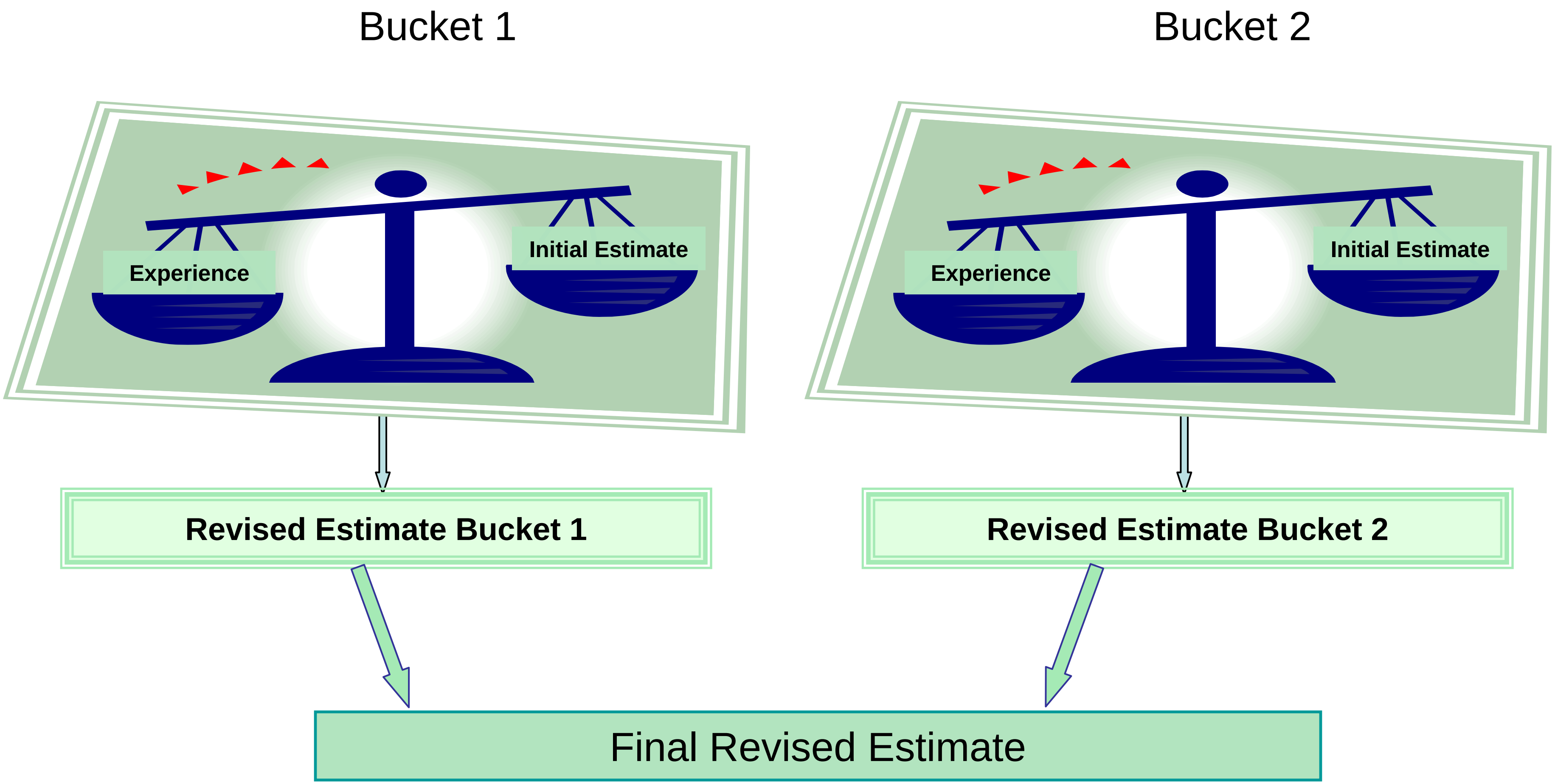
Split Credibility - Basic Idea

- Divide loss $ into buckets
- Derive separate credibility weighted estimates for each bucket
- Add together to get the final estimate

Split Credibility



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Split Credibility – Basic Formula

- Final Estimate =Sum of Credibility Weighted Estimates

$$\mu^* = \mu_1^* + \mu_2^*$$

$$= \left\{ z_1 A_1 + (1 - z_1) E_1 \right\}$$

$$+ \left\{ z_2 A_2 + (1 - z_2) E_2 \right\}$$

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Split Credibility – US Workers Comp

- US Workers Comp Experience Rating
- Split-z procedure
- Primary and Excess Split
- Has been successfully used for many years

Credibility Example-Loss Data

Loss Experience

Claim Number	Total Loss	Primary Loss	Excess Loss
1	1,000	1,000	-
2	1,500	1,500	-
3	2,500	2,500	-
4	4,000	4,000	-
5	15,000	5,000	10,000
6	80,000	5,000	75,000
Total	**104,000**	**19,000**	**85,000**

Based on Split point = 5,000

Credibility Example – Non-Split vs Split

No Split Plan	Actual Loss	Credibility	Expected Loss	Cred wtd estimate
Total	**104,000**	**50%**	**100,000**	**102,000**

Split Plan	Actual Loss	Credibility	Expected Loss	Cred wtd estimate
Primary	19,000	**70%**	30,000	22,300
Excess	85,000	**20%**	70,000	73,000
Combined Split Estimate				**95,300**

Why Split?

- One idea: Splitting Reduces Volatility
  - Primary Layer less volatile than Total? Yes.
  - Excess Layer less volatile than Total? No!
- Volatility Analysis Incomplete
  - Volatility = Process risk
  - Credibility depends on both process and parameter risk
  - High credibility is not the same as low volatility

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Our Goals

- Improve understanding of split credibility
- Present alternative interpretation of credibility as parameter risk reduction factor
- Present key equation explaining when split credibility will be effective
- Examine some disconcerting results when split credibility is applied to the Gamma-Poisson, Gamma-Exponential loss model

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No-Split Credibility Notation

- Define
  - $\mu(\theta) = E[A(\theta)]$ the conditional mean
  - $\sigma^2(\theta) = Var(A(\theta))$ the conditional process variance
- Take expectations wrt to θ to define:
  - $E = E[\mu(\theta)]$ the unconditional prior mean
  - $\sigma^2 = E[\sigma^2(\theta)]$ the process risk
  - $\tau^2 = Var(\mu(\theta))$ the parameter risk
- Set $\lambda^2 = \sigma^2 + \tau^2$ = the total variance of A.

Mean Square Error of Credibility Estimate

- Given arbitrary credibility, z, the mean square error (MSE) is given as:

$$\varepsilon^2 = E[\left(zA + (1-z)E - \mu(\theta)\right)^2]$$

$$= z^2 \cdot E[\left(A - \mu(\theta)\right)^2] + (1-z)^2 \cdot E[\left(E - \mu(\theta)\right)^2]$$

$$= z^2\sigma^2 + (1-z)^2\tau^2$$

- Note when z=0 that MSE = τ^2

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Optimal Credibility

- The z which minimizes mean square error is given as z* where:

$$z^* = \frac{\tau^2}{\tau^2 + \sigma^2} = \frac{\tau^2}{\lambda^2}$$

- What increases optimal credibility?
  - Reducing process risk
  - Increasing parameter risk

Mean Square Error at Optimal Credibility

- At the optimal credibility, the mean square error is given as:

$$\varepsilon_0^2(NS) = \frac{\tau^2\sigma^2}{\tau^2+\sigma^2} = \tau^2\left(1-\frac{\tau^2}{\lambda^2}\right)$$

Alternate Interpretation of Optimal Credibility

- Plug in z* formula to rewrite as

$$\varepsilon_0^2(NS) = \tau^2\left(1-\frac{\tau^2}{\lambda^2}\right) = \tau^2(1-z^*)$$

- Initial mean square error is τ^2
- Therefore z*= the factor by which parameter risk is reduced using optimal weighting

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Example of Error Reduction Interpretation

- Let τ^2 =100 and σ^2= 300.
- It follows that:
- λ^2 = 400 and z* = 100/400 = 25%

$$MSE = E[(\mu - \mu^*)^2]$$

$$= (.25)^2 \cdot 300 + (.75)^2 \cdot 100$$

$$= 100 \cdot (1 - .25) = 75$$

Split Credibility Notation-Basic Set-up

- Split $A = A_1 + A_2$
- Conditional means and process variances
  - $\mu_1(\theta) = E[A_1(\theta)]$ and $\mu_2(\theta) = E[A_2(\theta)]$
  - $\sigma_1^2(\theta) = Var(A_1(\theta))$ and $\sigma_2^2(\theta) = Var(A_2(\theta))$
- Unconditional means and process variances
  - $E_1 = E[\mu_1(\theta)]$ and $E_2 = E[\mu_2(\theta)]$
  - $\sigma_1^2 = E[\sigma_1^2(\theta)]$ and $\sigma_2^2 = E[\sigma_2^2(\theta)]$

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Split Credibility Notation – Var and Cov

- Process Covariances
  - $C(\theta) = Cov(A_1(\theta), A_2(\theta))$ and $\rho = E[C(\theta)]$
- Parameter Variances and Covariance
  - $\tau_1^2 = Var(\mu_1(\theta))$ and $\tau_2^2 = Var(\mu_2(\theta))$
  - $\pi = Cov(\mu_1(\theta), \mu_2(\theta))$

Split Credibility Notation- Total Variances

Total Component Variances

- $\lambda_1^2 = \sigma_1^2 + \tau_1^2$ and $\lambda_2^2 = \sigma_2^2 + \tau_2^2$

Total Covariance

- $\kappa = \rho + \pi$

Total Variances

- Total: $\lambda^2 = \lambda_1^2 + \lambda_2^2 + 2\kappa$
- Process: $\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho$
- Parameter: $\tau^2 = \tau_1^2 + \tau_2^2 + 2\pi$

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Mean Square Error with Arbitrary Credibilities

$$\varepsilon^2 = E[\left(z_1A_1 + (1-z_1)E_1 - \mu_1(\theta) + z_2A_2 + (1-z_2)E_2 - \mu_2(\theta)\right)^2]$$

$$= z_1^{\ 2} \cdot E[\left(A_1 - \mu_1(\theta)\right)^2] + (1-z_1)^2 \cdot E[\left(E_1 - \mu_1(\theta)\right)^2]$$

$$+z_1^{\ 2} \cdot E[\left(A_2 - \mu_2(\theta)\right)^2] + (1-z_2)^2 \cdot E[\left(E_2 - \mu_2(\theta)\right)^2]$$

$$+2z_1z_2E[C(\theta)] + 2(1-z_1)(1-z_2)Cov(\mu_1(\theta), \mu_2(\theta))$$

$$\varepsilon^2 = \tau^2 + z_1^2\lambda_1^2 - 2z_1(\tau_1^2 + \pi) + z_2^2\lambda_2^2 - 2z_2(\tau_2^2 + \pi) + 2z_1z_2\kappa$$

Optimal Split Credibilty Formulas

$$z_1 = \frac{\lambda_2^2(\tau_1^2 + \pi) - \kappa(\tau_2^2 + \pi)}{D}$$

$$z_2 = \frac{\lambda_1^2(\tau_2^2 + \pi) - \kappa(\tau_1^2 + \pi)}{D}$$

where $D = \lambda_1^2 \lambda_2^2 - \kappa^2$

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Mean Square Error with Optimal Credibilities

$$\varepsilon_0^2(SP) = (\tau_1^2 + \pi)(1 - z_1^*) + (\tau_2^2 + \pi)(1 - z_2^*)$$

- Error reduction interpretation
  - Each component starts with its own parameter risk plus the parameter covariance
  - The separate "z" are the factors by which the parameter error is reduced for each component

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The Key Formula: Difference in MSE

$$\Delta(\varepsilon_0^2) = \varepsilon_0^2(NS) - \varepsilon_0^2(SP) =$$

$$\frac{1}{D\lambda^2}\Big((\tau_1^2+\pi)(\sigma_2^2+\rho)-(\sigma_1^2+\rho)(\tau_2^2+\pi)\Big)^2$$

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When Is a Split Plan Effective?

- Split plan is effective if it reduces MSE versus the No-split plan
- This happens when Δ is maximized.
- Largest possible Δ is achieved if:
  - One component gets all the parameter risk
  - The other gets all the process risk
  - Covariances are zero
- Intuition: A Split works to the degree that it separates noise from signal!

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Effectiveness Due to Differential Allocation of Variance

- Key Result : Split plan improves on No-split Plan when the Split Induces a Differential Allocation of Process and Parameter Risk!
- A Split does not necessarily improve on a No-split plan.
  - An arbitrary split may or may not induce such a differential allocation
  - Example: Toss a fair coin to classify a loss a type 1 or type 2.

Example-Split Plan A

Unsplit Plan	
Process Var	300.0
Parameter Var	100.0
Total Var	400.0
Credibility	25%
MSE of z-wtd estimate	75.00

Split Plan A	Combined	Component 1	Component 2	CoVar
Process Var	300.0	200.0	60.0	20.0
Parameter Var	100.0	66.7	20.0	6.7
Total Var	400.0	266.7	80.0	26.7
D	20,622			
Credibility		25%	25%	
MSE of z-wtd estimate	75.00			

Example-Split Plan B

Unsplit Plan	
Process Var	300.0
Parameter Var	100.0
Total Var	400.0
Credibility	25%
MSE of z-wtd estimate	75.00

Split Plan B	Combined	Component 1	Component 2	CoVar
Process Var	300.0	150.0	130.0	10.0
Parameter Var	100.0	80.0	10.0	5.0
Total Var	400.0	230.0	140.0	15.0
D	31,975			
Credibility		37%	7%	
MSE of z-wtd estimate	67.94			

Quiz

- Which was a more effective split?
  - A
  - B
  - Both equally effective
- Split Plan B does not work as well as a no-split plan because component 2 has a credibility of only 7% versus 25% for the non-split plan.
  - True
  - False

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Loss Models and Per Occ Split Credibility

- Much of the overall process risk is due to severity.
- A Split tends to allocate a disproportionate part of severity-driven process risk to the excess layer.
- The proportion of parameter risk allocated to the excess layer can be less than, equal to, or greater than the proportion of process risk.
- Conclusion: In general, a per occ split may or may not be effective! It all depends on the loss model.

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Collective Risk Model (CRM)

- Number of Counts: conditionally Poisson with mean ($n\chi$) where χ is Gamma with $E[\chi] = 1$ and $Var(\chi) = c =$ contagion.
- Claim Severity: conditionally exponential with mean (s/β) where $h=1/\beta$ is Gamma with $E[h] = 1$ and $Var(h) = 1+b$. "b" is the mixing parameter.

Optimal Non-split Z for CRM

$$z^* = \frac{n^2 s^2 \cdot ((1+c)(1+b)-1)}{n^2 s^2 \cdot ((1+c)(1+b)-1) + 2ns^2(1+b)}$$

$$= \frac{n^2 \cdot ((1+c)(1+b)-1)}{n^2 \cdot ((1+c)(1+b)-1) + 2n(1+b)}$$

Collective Risk Model Example: Assumptions

Inputs		
Variable	**Notation**	**Value**
Mean Claim Count	n	40.000
Mean Severity	s	10.000
Severity Mixing Parameter	b	0.040
Claim Count Contagion	c	0.010
Split Point	k	10.000
Split Point to Mean Severity F	k/s	1.000

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Collective Risk Model Example: Variance Allocation

Results				
	Total	**Primary**	**Excess**	**Split Plan**
Loss				
Mean	400.000	250.063	149.937	
Process Variance	8,320	2,083	3,239	
Parameter Variance	8,064	1,046	3,874	
Parameter CV	0.224	0.129	0.415	
Process Covariance				1,499
Parameter Covariance				1,572
Total Covariance				3,072
Total Variance	16,384	3,129	7,112	

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Collective Risk Model Example: Credibility and Error Reduction

Results				
	Total	**Primary**	**Excess**	**Split Plan**
Credibility				
Numerator	8,064	1,894,863	8,995,885	
Denominator	16,384	12,817,500	12,817,500	
Optimal z	**49.2%**	**14.8%**	**70.2%**	
Error				
Initial MSE	8,064			8,064
Initial Parameter CV	0.224			0.224
MSE with Optimal z	4,095			3,855
Final Parameter CV	0.160			0.155
CV Improvement %				
From Initial to No-split	28.7%			
From No-split to Split				2.98%

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Inversion of Primary and Excess Z's

- CRM Example exhibits an inversion of the primary and excess credibilities
  - The excess is more credible than the primary!
    - Many believe this can't be true.
  - Inversion occurs with the CRM model when severity parameter uncertainty is large and claim count parameter uncertainty is small.
- The US Workers Compensation Split plan (NCCI) is constructed so inversions cannot occur.

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Conclusions and Questions

- Split Credibility struggles in a Single Severity model with Scale Parameter Uncertainty.
  - Structure mitigates against differential allocation of process and parameter risk
- A Multiple Claim Type model may be a more fruitful approach for establishing the effectiveness of split credibility in US Workers Compensation.
- Questions??