The Actuarial Profession
making financial sense of the future

A Simple Multi-State Reserving Model

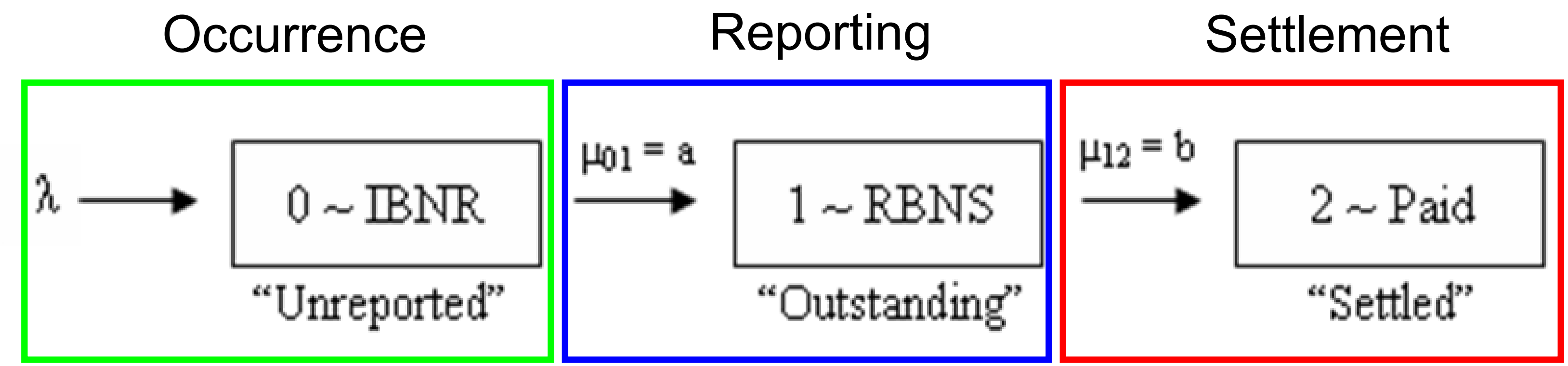
ASTIN Colloquium

Wednesday, 20th June 2007

What I'm Going to Talk to You About

- Treating claims development as a process
- Looking at simplest possible model
- Deriving closed form expressions
- Demonstrating practical extensions to model
- Discussing results of initial investigations
- Presenting a more general model form

The Simplest Model



- Inspired by Norberg, Hesselager and Hachtmeister
- Losses arriving as a Poisson process, rate λ
  - into Incurred But Not Reported (IBNR) State 0 ("Unreported")
- Transitioning at instantaneous rate *a*
  - into Reported But Not Settled (RBNS) State 1 ("Outstanding")
- Transitioning at instantaneous rate *b*
  - into Paid State 2 ("Settled")

Instantaneous Transition Rates

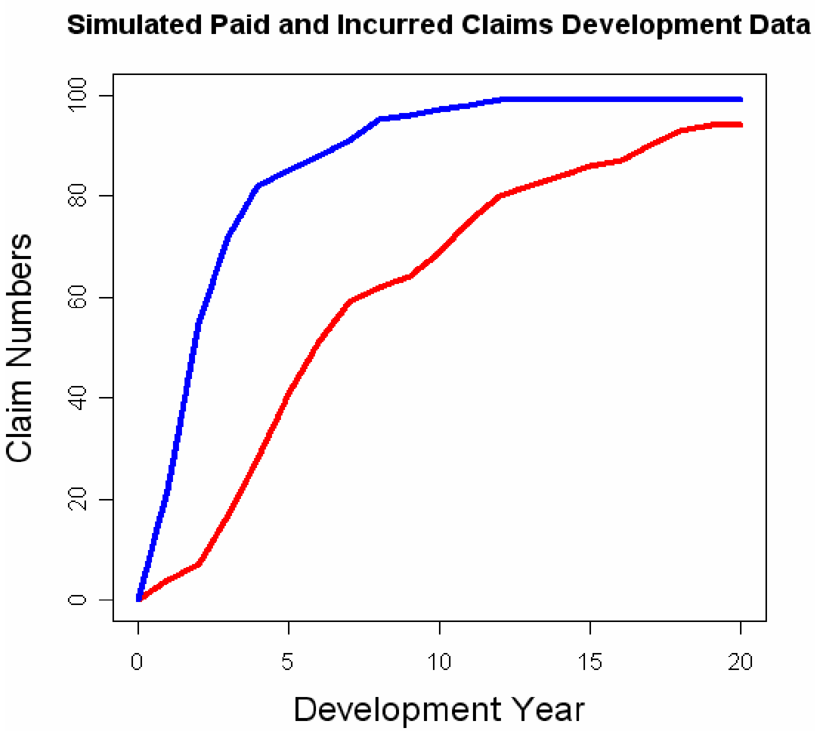
$$a = \lim_{h \to 0} \frac{\Pr\left(Claim\ in\ State\ 1\ at\ t+h \mid Loss\ in\ State\ 0\ at\ t\right)}{h}$$

- Limiting probability of moving from State 0 to State 1
  - analogous to “Force of Mortality” used by actuaries
- Waiting times in State 0 distributed as Exponential random variable
  - parameter value *a*
  - expected value of $1/a$ and variance of $(1/a)^2$
- Higher rate reduces waiting time in state
  - e.g. $a = 2$ implies average waiting time of 0.5 years

Claims Number Development

t	IBNR N(0,t)	RBNS N(1,t)	Settled N(2,t)	Total N(.,t)
0	0	0	0	0
1	77	18	4	99
2	44	48	7	99
3	27	55	17	99
4	17	54	28	99
5	14	44	41	99
6	11	37	51	99
7	8	32	59	99
8	4	33	62	99
9	3	32	64	99
10	2	28	69	99
…	…	…	…	…
∞	0	0	99	99

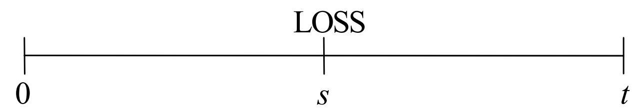
Observed Data (RBNS and Settled columns)



During Accident Year (t≤1)

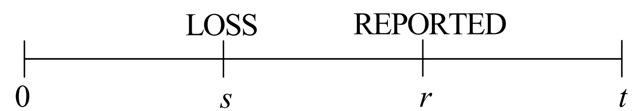
$$E[N(s,t)] = \lambda t * [\alpha(0,t), \alpha(1,t), \alpha(2,t)]$$

$$\lambda t * \alpha(0,t) = \int_0^t \lambda \exp[-a(t-s)]ds = \frac{\lambda}{a}[1-\exp(-at)]$$



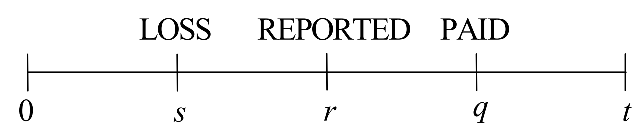
Loss arrives at *s* and stays in State 0 until *t*

$$\lambda t * \alpha(1,t) = \int_0^t \lambda \int_s^t a \exp[-a(r-s)]\exp[-b(t-r)]drds = \frac{\lambda a}{(a-b)b}[1-\exp(-bt)] - \frac{\lambda}{(a-b)}[1-\exp(-at)]$$



Loss arrives at *s*, leaves State 0 at *r* and stays in State 1 until *t*

$$\lambda t * \alpha(2,t) = \int_0^t \lambda \int_s^t a \exp[-a(r-s)] \int_r^t b \exp[-b(q-r)]dqdrds = \lambda a \left\{ \frac{t}{a} - \frac{[1-\exp(-at)]}{a^2} - \frac{[1-\exp(-bt)]}{(a-b)b} + \frac{[1-\exp(-at)]}{a(a-b)} \right\}$$



Loss arrives at *s*, leaves State 0 at *r*, then leaves State 1 at *q* and stays in State 2 forever

After Accident Year (t>1)

- Transition Intensity Matrix

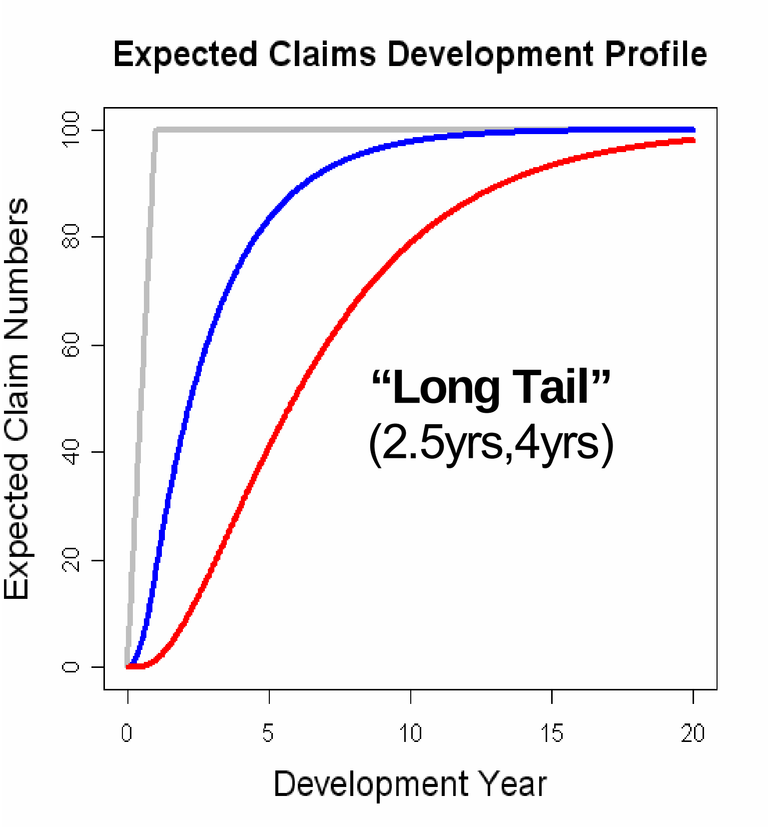
$$Q = \begin{pmatrix} -a & a & 0 \\ 0 & -b & b \\ 0 & 0 & 0 \end{pmatrix}$$

- Kolmogorov's Forward Equation

$$E[N(s,t)] = \lambda \pi(t)$$
$$= \lambda \pi(1) \exp[Q(t-1)]$$

where,

$$\pi(1) = [\ \alpha(0,1)\ ,\ \alpha(1,1)\ ,\ \alpha(2,1)\ ]$$



Conditional (Next Value) Expectations

- Beyond t=1, only transitions between states can occur
- Run-off process fully specified, by transition intensity matrix Q
- Transition probability matrix for reporting interval *d*

$$P(d) = \exp[Q.d]$$

- Expected next values can be determined from

$$E[N(s,y,t) \mid N(s,y,t-1)] = N(s,y,t-1)P(1)$$

  - need to assume total claims figure n_y
- Can be used to compare actual with expected values

$$\text{SoS} = \sum_{y=1}^{10}\sum_{t=1}^{11-y}\{N(s,y,t) - E[N(s,y,t) \mid N(s,y,t-1)]\}^2$$

- Can search for parameter values n_y, *a* and *b*

Conditional Distribution

- For a multi-state Markov chain
  - conditional upon total number of claims = n_y
  - number of claims in each state at time t ($\geq$1)
    - distributed as Multinomial(n_y,π(t))

$$\Pr(n_{IBNR}, n_{RBNS}, n_{Paid}) = \frac{n_{IBNR}!\,n_{RBNS}!\,n_{Paid}!}{(n_{IBNR}+n_{RBNS}+n_{Paid})!}(p_{IBNR})^{n_{IBNR}}(p_{RBNS})^{n_{RBNS}}(p_{Paid})^{n_{Paid}}$$

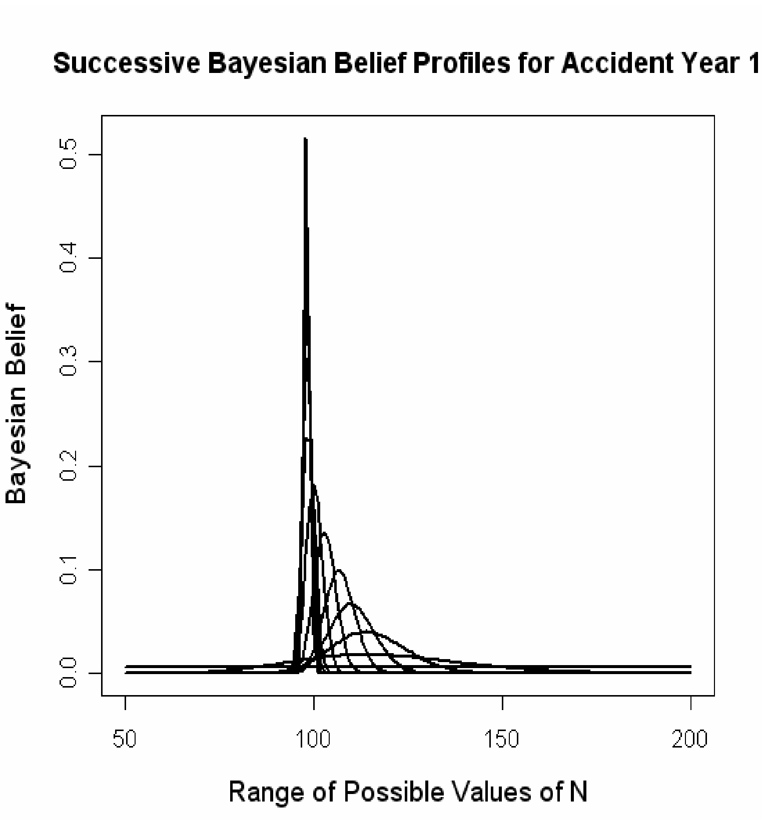
- where
  - n_{IBNR}, n_{RBNS}, n_{Paid} are claims counts at time *t*
  - p_{IBNR}, p_{RBNS}, p_{Paid} are π(0,t), π(1,t) and π(2,t) respectively

Bayesian Formulation

$$\Pr\left(N_y \mid N\left(s \in \{1,2\}, y, t\right)\right) = \frac{\Pr\left(N\left(s \in \{1,2\}, y, t\right) \mid N_y\right)\Pr(N_y)}{\sum_{\forall N_y} \Pr\left(N\left(s \in \{1,2\}, y, t\right) \mid N_y\right)\Pr\left(N_y\right)}$$

- Given O/S (State 1) and Paid (State 2) counts at time *t*
- Can calculate probabilities of different value of N_y
- Assume that $n_{IBNR} = N_y - n_{RBNS} - n_{Paid}$

Bayesian Results



Expected Values of N_y :

	Development Year										
Accident Year	0	1	2	3	4	5	6	7	8	9	10
1	125.0	118.5	115.0	110.4	107.0	103.0	100.2	98.6	98.4	98.2	98.2
2	125.0	118.5	118.3	119.1	117.1	118.1	118.1	119.7	120.7	121.5	
3	125.0	123.7	119.3	118.5	116.9	114.1	112.1	110.8	109.6		
4	125.0	113.4	102.4	103.8	103.2	103.6	103.2	103.3			
5	125.0	103.1	100.2	100.1	104.4	107.1	109.1				
6	125.0	83.1	82.7	85.2	84.4	87.3					
7	125.0	97.9	99.2	98.8	98.8						
8	125.0	87.8	93.8	95.9							
9	125.0	113.4	92.6								
10	125.0	70.8									

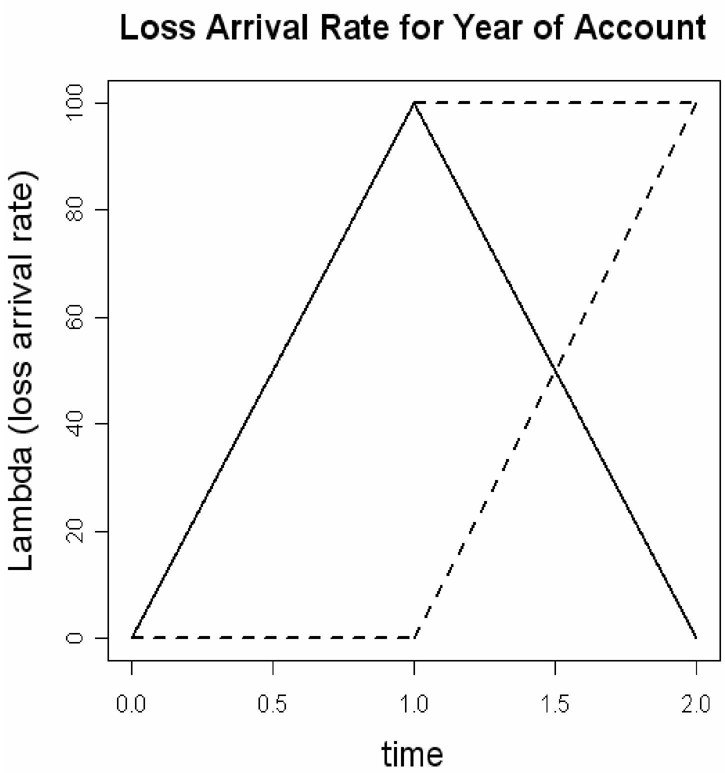
Standard Deviation (i.e square-root of Variance) for N_y :

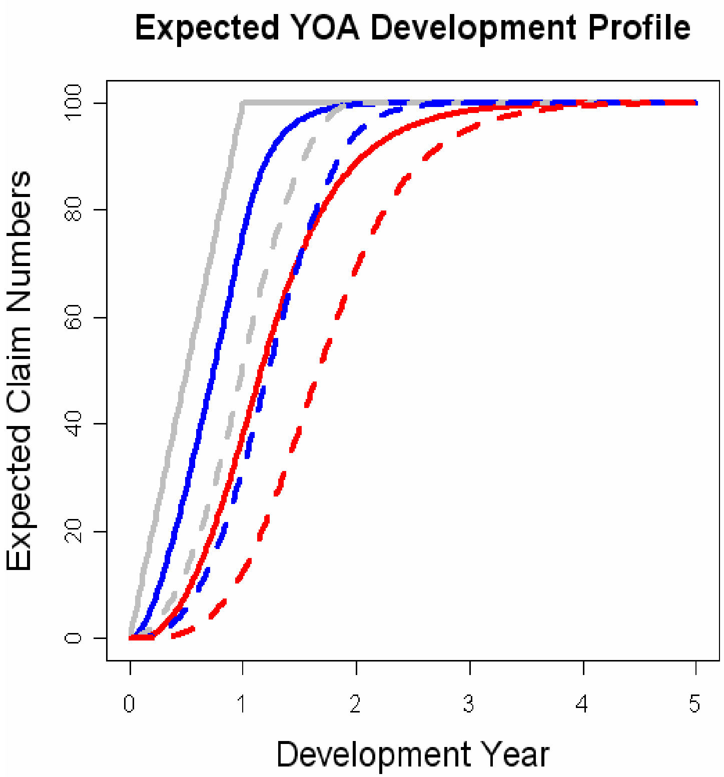
	Development Year										
Accident Year	0	1	2	3	4	5	6	7	8	9	10
1	43.6	22.2	10.0	6.0	4.0	2.9	2.2	1.7	1.3	1.0	0.8
2	43.6	22.2	10.1	6.1	4.2	3.0	2.2	1.6	1.2	0.9	
3	43.6	22.6	10.2	6.1	4.2	3.0	2.3	1.8	1.4		
4	43.6	21.8	9.4	5.7	3.9	2.8	2.1	1.6			
5	43.6	20.8	9.3	5.6	3.8	2.7	2.0				
6	43.6	18.0	8.4	5.2	3.5	2.5					
7	43.6	20.3	9.2	5.6	3.8						
8	43.6	18.9	8.9	5.5							
9	43.6	21.8	9.1								
10	43.6	14.7									

75% Percentile (actually, the first integer greater than the 75% percentile) Values for N_y :

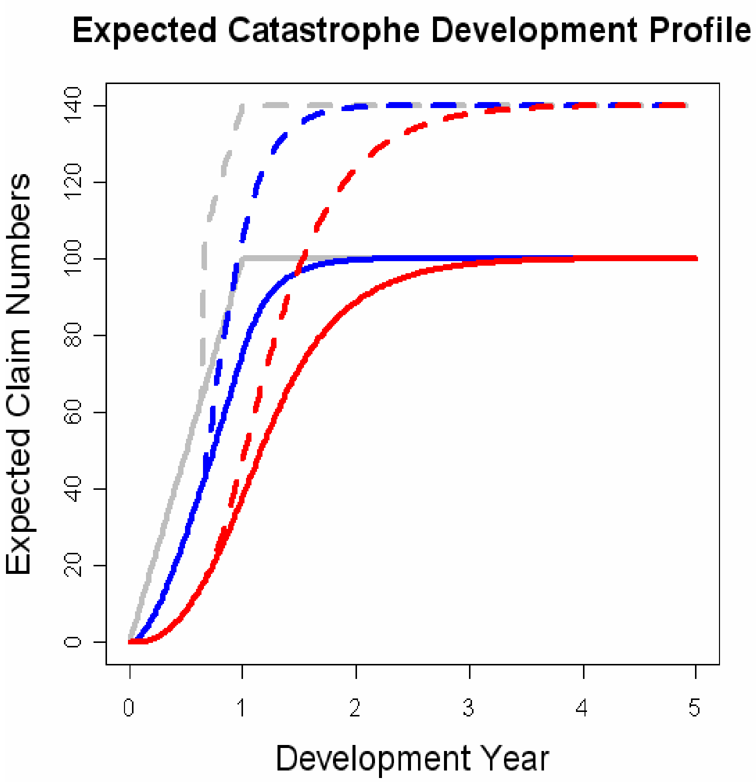
	Development Year										
Accident Year	0	1	2	3	4	5	6	7	8	9	10
1	163	133	121	114	110	105	102	100	99	99	99
2	163	133	125	123	120	120	120	121	121	122	
3	163	138	126	123	120	116	114	112	110		
4	163	127	108	108	106	105	105	104			
5	163	116	106	104	107	109	110				
6	163	94	88	89	87	89					
7	163	111	105	102	101						
8	163	100	100	99							
9	163	127	98								
10	163	79									

Extension: YOA, not Accident Year

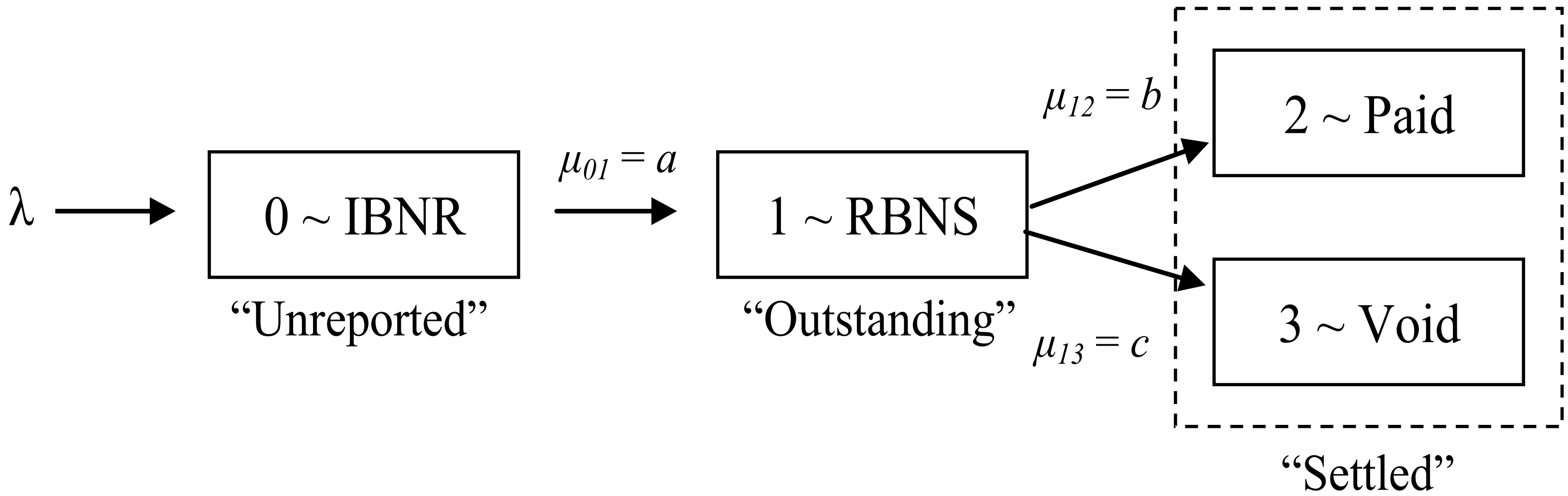




Extension: Catastrophes

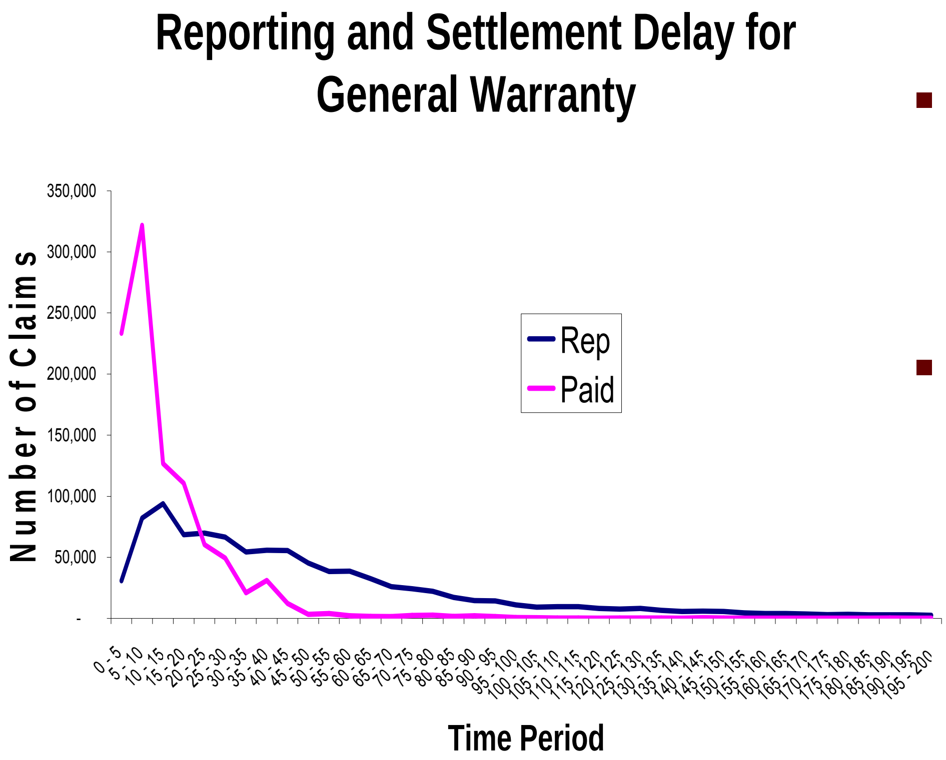


Negative Development - "Void" Claims



Negative Development - “Void” Claims

Initial Investigations



- Initial analysis of claims level data
  - indicates Exponential assumption not valid
- Three likely reasons
  - mixing of business
  - changes in processing rates over time
  - multiple stages in settlement process

A Not So Simple General Model

- Structure
- Simulation
- Testing Projection Methods
- Link to Bornhuetter-Ferguson Method

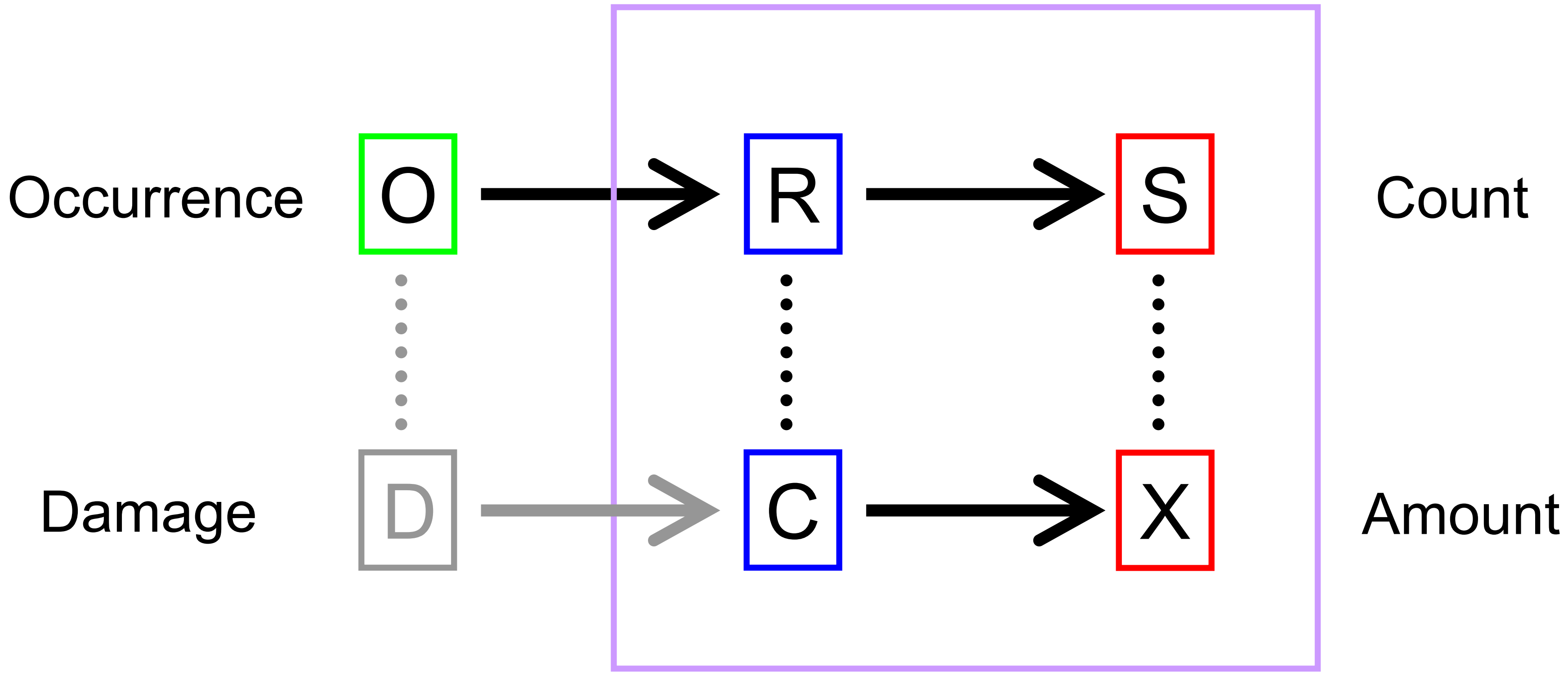
Modelling the Claims Process

O	=	Occurrence Process
R	=	Reporting Process
S	=	Settlement Process
C	=	Case Estimation Process
X	=	Claims Amount Process

Case Estimates, Outstanding and Incurred Claims

- On notification of claim
  - individual case estimate struck
  - held as “Outstanding” claim until paid
  - “Paid” + “Outstanding” = “Incurred” (actually “Reported”)
- Many causes of settlement delays
  - loss-adjusting
  - quotations for work
  - restoration, repair or replacement
  - disputes – investigation, arbitration, trial

Modelling the Claims Process



Simulation is Easy…

- O_i = occurrence time for i'th claim
- R_i = reporting time for i'th claim
- S_i = settlement time for i'th claim
- CE_i = case estimate for i'th claim
- X_i = paid amount for i'th claim
- So, claim i is reported by time t if $t \geq R_i$
- and, claim i is paid by time t if $t \geq S_i$

Simulation is Easy…

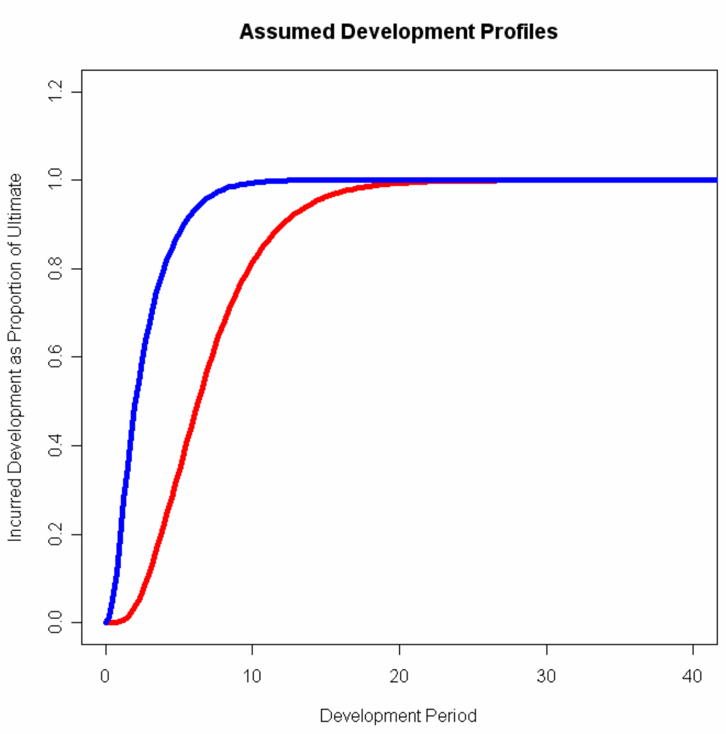
- Red: Paid Claims
- Blue: Reported Claims
- Green: (Estimated) Ultimate
  - applying Incurred Bornhuetter-Ferguson (IBF) method
  - based on benchmark incurred development patterns

Underlying Model

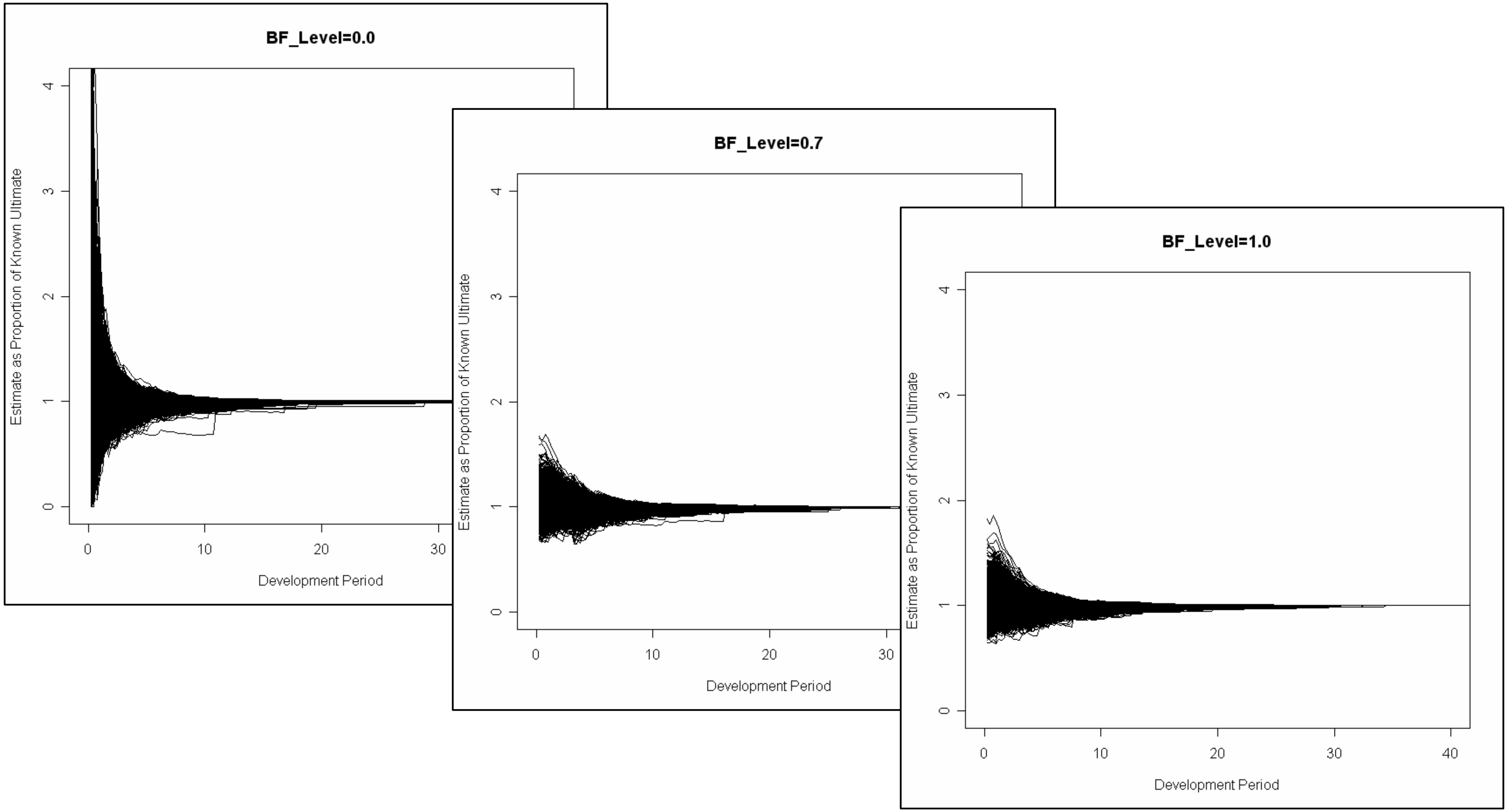
- Inception: all annual covers, incepting at 1/1
- O Poisson loss process, Lambda = 100
- R-O Exponential reporting delays, 2.5 years ave
- S-R Gamma settlement delays
  - equivalent to sum of two Exponential random variables
    - average 2 years
  - giving average delay of 4 years
- CE Estimation Errors ~Normal(+10%,5%)
  - note positive bias, or "redundancy"
- X Paid Amounts ~logNormal(8.8,0.8)

More Model Details

- "Benchmark" incurred development
  - running simulation model & averaging
- In "Bornhuetter-Ferguson" framework
  - Initial estimate on known parameters
    - average (logNormal) claim size ~£9,140
    - Poisson loss arrival rate = 100
    - so, BF_Estimate ~£914k
    - "cheating"
      - just stochastic variation to worry about
  - "BF_Level" varied (0%,70%,100%)
    - percentage developed at which "Incurred Development Factor" method applied
    - using benchmark development profile



Testing Projection Methods



Unreported & Unpaid Claims Distribution

- N = total number of claims in Accident Year
  - assume N ~ Poisson(Lambda=100)
- ${}_{R}N_{t}$ = total Reported claims | Reported claims at t
- ${}_{P}N_{t}$ = total Paid claims | Paid claims at t
- Then
  - ${}_{R}N_{t=0} \equiv {}_{P}N_{t=0} \equiv N$ a Poisson r.v.
  - ${}_{R}N_{t=\infty} \equiv {}_{P}N_{t=\infty} \equiv n$ realisation of N

Unreported & Unpaid Claims Distribution

- ${}_{R}N_t \sim$ Poisson(Lambda x Pr[R>t])
  \+ Number of Claims Reported by time t
- ${}_{P}N_t \sim$ Poisson(Lambda x Pr[S>t])
  \+ Number of Claims Paid by time t
- Now
  - 1-Pr[R>t] $\equiv$ Incurred Development Profile
  - 1-Pr[S>t] $\equiv$ Paid Development Profile

Unreported & Unpaid Claims Distribution

- Take expected values for Incurred claims

$E[{}_RN_t] = E[\text{Poisson}(\text{Lambda} \times \Pr[R>t])$
$\quad + \text{ Number of Claims Reported by time } t]$

$= E[\text{Poisson}(\text{Lambda})] \times \Pr[R>t]$
$\quad + \text{ Number of Claims Reported by time } t$

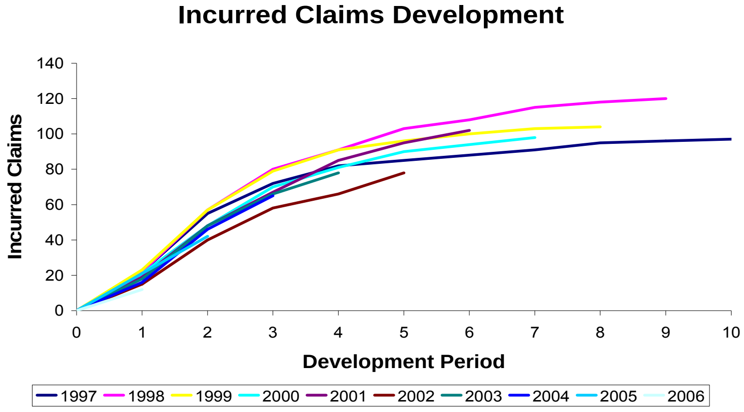
$= \text{Initial Estimate} \times (1-\text{IncDevProf})$
$\quad + \text{ Number of Claims Reported by time } t$

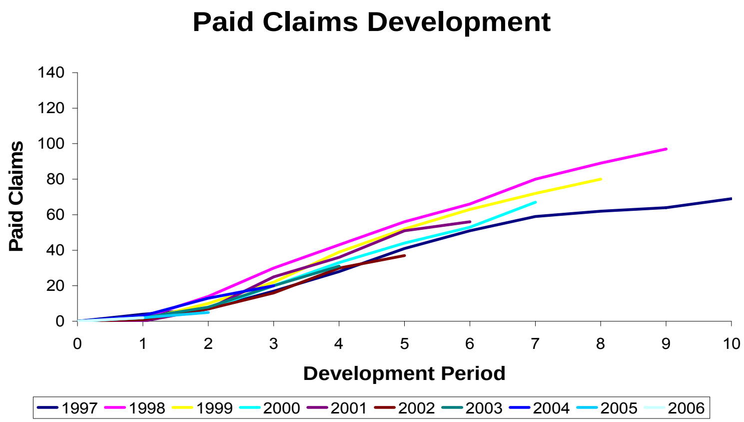
i.e. Bornhuetter-Ferguson Method

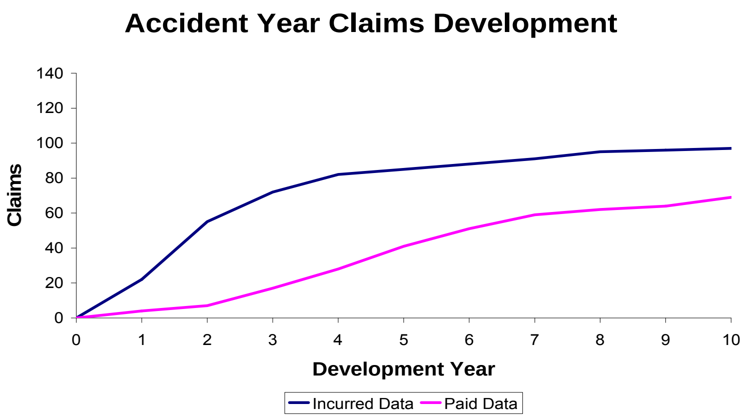
- Similar result holds for Paid claims

Development Curves

- Claims v. Development Period
- Check consistency of development
- Point towards higher/lower ultimates
- Incurred and Paid should converge







Why Look at Claims Level Approach?

- Linking business processes to reserving process
- Greater computing power
- Ready capture of claims processing data
- Alternative foundation for stochastic reserving and ICAs
- Can allow for known/anticipated changes in processing

What I've Talked About

- Treated claims development as a process
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Any Questions?