An Extension Model of Financially-balanced Bonus-Malus System

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1. Introduction

Bonus-malus systems have been studied by several authors under the framework of Markov chains. Optimal scales have been deduced by Norberg (1976), Borgan, Hoem and Norberg (1981) and Gilde and Sundt (1989). Centeno and Andrade (2002) deduced the optimal scales for bonus systems that were not first order Markovian processes. Andrade and Centeno (2005) proposed a premium given by geometric scale, i.e., the premiums between consecutive classes increase by a constant percentage. From a very different way of approximating an optimal BMS with an infinite number of classes, Coene and Doray (1996) gave a BMS with a finite number of classes, which stays in financial balance over the years, by minimizing a quadratic function of the difference between BMS and optimal BMS.

However, the above methods cannot be used directly in an open portfolio without the stationary distribution independent of the premium scales. Centeno and Andrade (2001) studied bonus-malus systems in an open portfolio, i.e. considering that a policyholder can transfer his policy to another insurance company at any time. Under the assumptions that (a) the market shares are exogenously given and (b) the probability that a policyholder leaves the company does not depend on the magnitude of the policyholder's premium, they proved that the long-run distribution of the policyholders among the classes of the BMS is independent of the market shares. But assuming that the leaving probabilities are independent of the premium scales is questionable. In fact, the leaving probabilities should be a function of the premium scales and the level of an insured's tolerance to the penalties for claims, as well as the cost of transferring his policy to a different insurance company. Particularly in China, due to lack of information disclosure among insurers, it may be feasible to "evade" a malus from one insurance company and create a premium savings greater than the cost of obtaining a new start with another company.

Generally speaking, given the level of the insured's tolerance in an open portfolio, the premium scales affect the stationary distribution and make the system difficult to model. For example, if the maluses are higher than the levels of the insured's tolerance, then the number of leaving policyholders may be higher than the number of new entrants, so the number of policyholders among the some of the classes of the BMS tends towards zero. Furthermore, if it is assumed that the insured will leave the company with 100% probability when the malus is beyond the policyholder's tolerance level, then the penalties for drivers with a claims history cannot be as high as an optimal BMS would dictate, since no drivers will remain with the insurance company to pay those penalties. The lower the policyholder tolerance level for penalties, the more difficult to establish a BMS that is financially-balanced, with the consequence that the model given by Coene and Doray (1996) may have no solution.

In this paper we extend the model of a financially-balanced BMS given by Coene and Doray (1996). We propose a BMS that is intended not only to produce a small approximation error to an optimal BMS, but also to reduce the financial imbalance. The extension model can be used to design a BMS in an open portfolio, and get a solution to the complex situation that the insured will leave the company with probability one when the malus is beyond the tolerance level. This solution is closer to the BMS scales that we see in practice, i.e., both bonus and malus scales are reduced to some milder levels than would be indicated in an optimal BMS system.

The remaining part of the paper is organized as follows. Section 2 introduces the open model proposed by Centeno and Andrade (2001) and the model of a financially-balanced BMS given by Coene and Doray (1996). Section 3 presents a case of leaving probabilities depending on the premium scales for an open portfolio. Section 4 presents the extension model and its simplified version for an open portfolio. Section 5 gives a simulation and some simple examples using the extension model. Finally, the paper presents some concluding remarks.

2. The open model and the financially-balanced BMS

In this section, we introduce the open model proposed by Centeno and Andrade (2001) and the model of a financially-balanced BMS given by Coene and Doray (1996).

Let K represent the number of classes of a bonus-malus system. The system is determined by three elements: The premium scale $C = (C_1, \ldots ,C_K)$, the initial class k and the transition rules that determine the transfer from one class to another when the number of claims is known. A new policyholder will start from class k. The transition rules are represented by a K times K matrix T, whose entries $T(i, j)$ is the set of numbers of claims that may lead a policyholder from class i in any period to class j in the next period. Table 1 in Section 5 of this paper illustrates a matrix T.

Each policyholder is characterized by a risk parameter θ, which is considered to be the observed value of a positive random variable with distribution function $U(.)$. We will use θ to denote the policyholder with risk parameter θ., which we assume to be stationary over time. Let M_n be the number of claims a policyholder made during the period $[n\text{-}1, n)$, $n = 1, 2, \cdots$. It is assumed that, given policyholder θ, M_n are i.i.d. random variables. Let $p_\theta(m)$ be the probability that policyholder θ makes m claims during the period $[n\text{ -}1, n]$.

For each i = 1, … , K, let $d_{m,\theta}(i)$, $0 \le d_{m,\theta}(i) \le 1$, be the conditional probability that policyholder θ, who was in class i in the period and made m claims in the period, leaves the company (for convenience of presentation, we assume that policyholders only leave the company at the end of a period). Let $d_\theta(i)$ be the conditional probability that a policyholder leaves the company at the end of a period when he was in class i during the period. Then we have

$$d_\theta(i) = \sum_{m=0}^{\infty} p_\theta(m) d_{m,\theta}(i),$$

where $p_\theta(m)$ is the probability that policyholder θ makes m claims during one insurance period.

This system can be treated by means of a Markov Chain with K+1 states, where the state labeled by K+1 refers to the outside world (i.e., the policyholder leaves the company). The first K states correspond to the BMS classes. Centeno and Andrade (2001) named this model as an "open model”. And in this model, the probabilities $d_{m,\theta}(i)$ do not necessarily depend on the premium scales $C = (C_1, \ldots , C_K)$.

In a closed portfolio, it is usually assumed that Θ is Gamma-distributed with parameters α and β, and $p_\theta(m)$ is Poisson-distributed. Then the distribution of the number of claims in one year has a Negative Binomial distribution, see Lemaire (1995). Under these assumptions, Lemaire (1995) has shown that for the optimal

BMS, (i.e., the policyholder's premium is comparable to his expected claim frequency), the premium for a driver who made N claims in t years is given by

$$C_{(N,t)} = \frac{\beta}{\alpha}\frac{\alpha + N}{\beta + t}$$

where we assume that the premium for a new policyholder (N = 0, t = 0) is 1. It is obvious that the optimal BMS has an infinite number of classes (though many of them will be very sparsely populated). It is easy to show that this optimal BMS is financially-balanced, i.e. the average premium received each year by the insurer is equal to the average claims.

Coene and Doray (1996) proposed a financially-balanced BMS with a finite number of classes. Their method is to minimize a quadratic function of the difference between the premium of the optimal BMS and the premium of a BMS with a finite number of classes, weighted by the stationary probability of being in a certain class. It is equivalent to solving the following quadratic programming problem,

$$\min \sum_{l=1}^{K} \sum_{(N,t)} f_l \left[C_l - C_{(N,t)} \right]^2 \tag{1}$$

with the constraints

$$\begin{cases} C_{l+1} - C_l \geq 0, \quad l = 1,\ldots,K-1 \\ C_k = 1 \\ \sum_{l=1}^{K} f_l C_l \geq 1 \end{cases}$$

where f_l is the stationary probability of being in class l. The constraint $C_{l+1} - C_l \geq 0$, $l = 1,\ldots,K-1$ ensures that the premium increases (or at least does not decrease) as we move to higher (more malus) BMS classes.. The inequality constraint $\sum_{l=1}^{K} f_l C_l \geq 1$ is imposed to ensure that the portfolio is financially-balanced in the long term, i.e., that the average premium collected is at least equal to the average number of claims (we are ignoring expenses here). More constraints on the premiums may be added in this model, which was pointed out by Coene and Doray (1996), such as the maximum given to a driver and the maximum penalty a bad driver could be limited with the constraints $C_1 \geq A$ and $C_K \leq B$.

3. A case of leaving probabilities depending on the premium scales for an open portfolio

We assume that the new policies of the portfolio enter the system in the same class *k*, and a policyholder leaves the company at the end of the period, if his premium for the upcoming period is higher than his tolerance level denoted by *G*, no matter what class he was in and how many claims he made in the expiring period. That means all policyholders have the same "evasion" manner. This assumption is simplistic, but we will show some heuristic results when the policyholder's leaving probabilities depend only on the premium scales for an open portfolio. According to the above assumption, if claims number $m \in T(i, j)$, then

$$d_{m,\theta}(i) = \begin{cases} 1, & \text{if } C_j > G \\ 0, & \text{if } C_j \le G \end{cases} \tag{2}$$

So the leaving probability $d_{m,\theta}(i)$ is a function of the premium scale *C* and the tolerance level *G*. Since the stationary probabilities of the Markov chain is determined by its transition probability matrix, then f_l, which is the stationary probability of being in a certain class *l*, is also a function of *C* and *G*. Therefore the model (1) could be changed to

$$\min\left\{\sum_{l=1}^{K}\sum_{(N,t)} f_l(C,G)\left[C_l - C_{(N,t)}\right]^2\right\} \tag{3}$$

$$\begin{cases} C_{l+1} - C_l \ge 0, \quad l = 1,\dots,K-1 \\ C_k = 1 \\ \sum_{l=1}^{K} f_l(C,G)C_l \ge 1 \end{cases}$$

We can see that this model is more difficult to solve. First, the long-run distribution has no explicit form. Second, the restriction of financial balance may not be satisfied in some common cases, which will be illuminated in section 5.

4. The extension model for an open portfolio with policyholder evasion

We propose an extension model based on the model (1), that is

$$\min\left\{\sum_{l=1}^{K}\sum_{(N,t)} f_l^{(n)}(C,G)\left[C_l - C_{(N,t)}\right]^2 + M \times \left[\sum_{l=1}^{K} f_l^{(n)}(C,G)C_l - 1\right]^2\right\} \tag{4}$$

$$\begin{cases} C_{l+1} - C_l \geq 0, \quad l = 1,\dots,K-1 \\ C_k = 1 \end{cases} \tag{5}$$

The first part of model (4) represents the difference between proposed BMS scales and optimal BMS scales, so it measures the approximation error when proposed BMS scales are used to replace optimal BMS. The second part of model (4) represents the financial imbalance of the proposed BMS. So the target of model (4) is to minimize the approximation error and the financial imbalance at the same time. M is a financial-balancing weight in model (4). By adjusting the value of M between zero and infinity, we can put different emphasis on the financial balance. The larger the value of M is, the more financially-balanced the proposed BMS will be. When M = 1, the BMS approximation error and financial imbalance will be equivalently weighted $f_l^{(n)}$ is the proportion of policies being in a class l during period n. Here we use the distribution during period n instead of the stationary distribution that may not exist. To the extent that the distribution changes over time, the C_l resulting from model (4) also may vary over time.

The characteristic of being financially-balanced means that the penalties should be tough enough for bad drivers. That is, the bonuses awarded to good drivers should be compensated for by maluses from bad drivers. As mentioned above, the optimal BMS is fair and financially-balanced, but in some markets, it is too tough for bad drivers to be applied in practice. The extension model gives a way to consider the fairness and the financial balance of a practical BMS for an open portfolio at the same time. In the experimentation that we have done with this model we have selected M as a design characteristic, rather than solving for M to satisfy some specified criteria, but this alternative approach to solving the equation also could be explored further.

Although the characteristic of being financially-balanced is very important for a BMS, almost all BMS's in practice don't have this characteristic. In practice, the correction of a BMS financial imbalance frequently is achieved implicitly by loading additional provisions into other elements of the rating scheme; we believe it is preferable to view the effects of financial imbalance explicitly within the design of the BMS. And if the tolerance level of a policyholder is low in an open portfolio, there is a situation discussed in section 5 that makes the model given by Coene and Doray (1996) have no solution. However, model (4) may be used in this situation.

The optimization problem (4) is an inextricable nonlinear programming. In order to get some useful solution, we add two assumptions. First, we assume that the number of new policyholders entering the system is 0. Second, all the premium scales is less than the tolerance level of policyholders. So the number of leaving policyholders is also equal to 0 and the system is simplified to be a closed one with given G. Under these assumptions, the stationary probability is existent. Therefore we can obtain a simplified model for an open portfolio, that is

$$\min\left\{\sum_{l=1}^{K}\sum_{(N,t)} f_l\left[C_l - C_{(N,t)}\right]^2 + M\times\left[\sum_{l=1}^{K} f_l C_l - 1\right]^2\right\} \tag{6}$$

$$\begin{cases} C_{l+1} - C_l \geq 0, \quad l = 1,\ldots,K-1 \\ C_k = 1 \\ C_K \leq G \end{cases} \tag{7}$$

5. A simulation and some simple examples using the extension model

In this section, we give a simulation to show that the financial balance is impossible by requiring that the premium difference between two adjacent classes be at least, say 5%. And we get some numerical illustrations by the extension model to compare with examples from Coene and Doray (1996).

5.1. A case of financial imbalance

We consider the BMS defined in Taylor (1997). There are 9 classes. Class 6 is the starting class. If no claim has been reported by the policyholder during period t, then he moves one class down in period t + 1. If a number of claims, $m_t > 0$, have been reported during the period t, then the policyholder moves $2m_t$ classes up. The transition rules are described in Table 1.

Table 1 Transition Rules For BMS -1/+2

Class Level in period t	Number of claims reported in period t				
	0	1	2	3	4+
	Resulting Class Level in period t + 1				
9	8	9	9	9	9
8	7	9	9	9	9
7	6	9	9	9	9
6	5	8	9	9	9
5	4	7	9	9	9
4	3	6	8	9	9
3	2	5	7	9	9
2	1	4	6	8	9
1	1	3	5	7	9

We assume that the distribution function of the risk parameter Θ is a three-point discrete distribution, that is, Pr(θ= 0.05461) = 0.56187, Pr(θ= 0.24600) = 0.41465, Pr(θ= 0.95619) = 0.02348, as proposed by Walhin (1999). The constraint that $C_{l+1} - C_l > (\, a\,\%\text{ of } C_k\,) > 0, \quad l = 1, \ldots, 8$, is added for getting a practical BMS, i.e., a reasonable degree of pricing spread between classes, which is common in practice. Here we set a = 5%. We have not included here the financial balancing of the extension model.

For a given value of tolerance level G, there are four cases for an open portfolio. Case 1: $C_9 \leqslant G$, i.e., all policyholders are staying in the system. Case 2: $C_9 > G \geqslant C_8$, i.e., a policyholder leaves the company when he is moved into class 9. Case 3: $C_8 > G \geqslant C_7$, i.e., a policyholder leaves the company when he is moved into classes 8 or 9. Case 4: $C_7 > G \geqslant C_6$, i.e., a policyholder leaves the company when the premium is higher than the initial premium. Here we let G =1.5. For simplicity, we only list the former three cases in Table 2. In each case, we are using the assumption that in the open system 100% of policyholders who are set a premium greater than G would leave the insurer, and thus that G is a hard upper bound on the premium C_l for any class, as in formula (2).

We start with a hypothetical portfolio of 50,000 policyholders initially in class 6 and the number of new policyholders entering the system each year is equal to 50000r, where r is a constant proportion. $N(t)$ is the total number of policyholders in year t. Table 2 contains the simulated distribution of drivers in each class after 40 and 100 years. An upper bound of the average premium, denoted by UBAP, is obtained by recognizing that the premium in any class will not exceed G.

$$\sum_{j=1}^{9} f_j(C,G)C(j) \le \sum_{j=1}^{6}(0.7+0.05\times j)f_j(C,G)+\sum_{j=7}^{9} f_j(C,G)G = \text{UBAP}$$

Table 2 Distribution of Drivers at time *t* for BMS -1/+2 and UBAP

	Case 1			Case 2			Case 3		
Class	r=0 t=41	r=0 t=101	r=0.1 t=101	r=0 t=41	r=0 t=101	r=0.1 t=101	r=0 t=41	r=0 t=101	r=0.1 t=101
1	0.666	0.666	0.628	0.774	0.822	0.732	0.802	0.849	0.758
2	0.075	0.075	0.077	0.071	0.062	0.072	0.067	0.057	0.070
3	0.090	0.090	0.092	0.082	0.069	0.084	0.076	0.063	0.080
4	0.040	0.040	0.047	0.028	0.019	0.037	0.023	0.014	0.033
5	0.037	0.037	0.046	0.024	0.016	0.035	0.019	0.011	0.031
6	0.025	0.025	0.036	0.012	0.007	0.026	0.008	0.004	0.023
7	0.022	0.022	0.025	0.008	0.005	0.010	0.004	0.002	0.005
8	0.020	0.020	0.023	0.003	0.002	0.005	0.000	0.000	0.000
9	0.026	0.026	0.027	0.000	0.000	0.000	0.000	0.000	0.000
UBAP	0.834	0.833	0.844	0.782	0.772	0.792	0.774	0.766	0.783
$N(t)$	49999	49999	549995	38073	32739	409946	33441	28803	361812

From Table 2, we can see that the upper bounds of the average premium are always lower than 1, which means that the financial balance is unsatisfied for the BMS. Intuitively, the insurer could increase the maluses for bad drivers or decrease the bonuses for good drivers in order to increase the average premium. But the former way is not necessarily effective for an open portfolio when the leaving behavior is like the assumption defined by formula (2). If the tolerance level G is particularly low, the tough BMS scales may push both good drivers and bad drivers out of the company.

A closer examination of the policyholder population after a number of years, in the assumed situations where the stream of incoming policyholders is smaller than the exodus (i.e., r = 0 or small), will indicate that in fact the financial imbalance is not quite as bad as suggested in Table 2 because the proportion of bad drivers who moved into a high class and therefore have left the company in response to $C_l > G$ is greater than the proportion of good drivers who have left the company (in particular, good drivers who happen to have had sufficient numbers of claims to push their premium above G in the short run). Thus, the required average premium may drift below 1.0 over time in this situation. In situations where the volume of incoming policyholders is greater than or equal to the volume of exiting policyholders, we would need to make additional assumptions about how much of the new business for our company is truly fresh drivers, versus a mix of drivers leaving competitor companies because of premium intolerance, and therefore biased towards a mix concentrated with bad drivers similar to the mix of exiting policyholders. To model this effect fully would be a useful extension of the current model, and could include some interesting considerations of alternative assumptions about competitor pricing behavior. For this paper, we have made the simplifying assumption that our target average premium remains at 1.0, which would be the case if the mix of good and bad drivers in the portfolio does not change significantly over time.

Related to this same dynamic, if the incoming policyholders are biased towards a mix concentrated with bad drivers, it would be worth considering establishing the entry class for new business at a class higher than C_k = 1.0, with the degree of shift depending on assumptions or model results regarding the particular mix of incoming policyholders. More practically, since C_{k+1} is considerably greater than 1.0 in many BMS, it may be better to leave k as the entry class, but to set C_k somewhat above 1.0, perhaps even in the optimal BMS against which practical BMS is being measured, reflecting the mix of old and new, bad and good drivers that are expected to be in class k. These are areas for further exploration.

5.2. Some simplified examples using the extension model

In this section, we get numerical illustrations by the extension model described in formulas (6) and (7), compared with some examples from Coene and Doray (1996).

Coene and Doray (1996) have considered two BMS's. BMS1 is a system with 18 classes and a new driver starts from class 10. A driver with no claim during a year goes down one class, while a driver goes up by two classes for the first claim in a year and three classes for each subsequent claim in that year. BMS2 is a system with 24 classes and a new driver starts from class 10. A driver with no claim during a year goes down one class, while a driver goes up by 3 classes for the first claim in a year and 4 classes for each subsequent claim in that year. Coene and Doray simulated multiple years using the Negative Binomial distribution with parameters (α =1.0923183, β=7.70077). For that analysis, they limited themselves to a period of t = 9 years, and estimated the stationary probabilities and the optimal premium scales of being in class l, denoted by f_l and C_l in our Tables 3 and 4.

Here we consider the formula:

$$\min\left\{\sum_{l=1}^{K}\sum_{(N,t)} f_l\left[C_l - C_{(N,t)}\right]^2 + M\left[\sum_{l=1}^{K} f_l C_l - 1\right]^2\right\}$$

$$\begin{cases} C_{l+1} - C_l \geq 0 \\ C_{l+1} - C_l \geq a\%, \quad l = 1,\dots,9 \\ C_{10} = 1 \\ C_K \leq G \end{cases} \qquad (8)$$

The premium scales of BMS1 for an open portfolio are displayed in Table 3, with a = 0 and various tolerance levels G and financial-balancing weights M. The average premium (AP) is increasing with the tolerance level G and financial balance factor M. It is natural that when G is infinite and M is large enough, say M =100 in Table 3, the BMS scales are approaching the ones proposed by Coene and Doray (1996), which is displayed in column 3 of Table 3. Also shown in Tables 3 and 4 are the average premiums ("AP") generated in each scenario.

In Table 4, when penalty factor G is chosen to be 2, the premium scales from 15 to 24 are equal. It means that the number of the classes should be reduced to 15. It is interesting to note that, when G =2, M =1, a = 5, each claim-free year produces a 5% saving, just like the old Belgian BMS, although the transition rules are somewhat different. Furthermore, when M is large enough and G = ∞, the BMS scale from the extension model is approaching the financially-balanced BMS designed by Coene and Doray (1996), which is displayed in the third column of Table 4. In addition, all cases suggest that the penalty for the first claim should be more severe than the old Belgian BMS.

Table 3 Premium Scales of BMS1 for an Open Portfolio

	f_l	Balanced BMS	G=∞ M=100 a=0	G=∞ M=1 a=0	G=2 M=1 a=0	G=3 M=1 a=0
1	0.66	79.21	78.84	61.44	64.95	62.03
2	0.06	82.15	81.78	64.38	67.89	64.97
3	0.08	85.49	85.11	67.71	71.23	68.31
4	0.02	88.82	88.62	79.93	81.68	80.23
5	0.02	93.84	93.65	84.96	86.70	85.25
6	0.02	99.63	99.44	90.75	92.50	91.04

7	0.01	100.00	100.00	100.00	100.00	100.00
8	0.01	100.00	100.00	100.00	100.00	100.00
9	0.02	100.00	100.00	100.00	100.00	100.00
10	0.01	100.00	100.00	100.00	100.00	100.00
11	0.01	180.19	180.05	174.26	175.43	174.46
12	0.01	195.07	194.94	189.14	190.31	189.34
13	0.01	220.79	220.67	214.87	200.00	215.07
14	0.01	237.88	237.76	231.95	200.00	232.15
15	0.01	258.15	258.03	252.23	200.00	252.42
16	0.01	282.38	282.18	273.49	200.00	273.79
17	0.01	306.59	306.39	297.70	200.00	298.00
18	0.02	357.94	357.86	354.39	200.00	300.00
100×AP		100	99.67	84.67	81.20	84.10

Table 4 Premium Scales of BMS2 for an Open Portfolio

	f_l	Balanced BMS	G=∞ M=100 a =0	G=∞ M=100 a =5	G=∞ M=1 a=0	G=2 M=1 a=0	G=2 M=1 a=5	Belgian BMS
1	0.5602	54.10	54.01	53.75	49.80	58.38	55.00	54
2	0.0514	57.04	56.95	58.75	52.73	61.32	60.00	54
3	0.0449	60.38	60.29	63.75	56.07	64.65	65.00	54
4	0.05	64.20	64.11	68.75	59.89	68.48	70.00	57
5	0.0175	78.47	78.43	75.00	76.32	80.61	75.00	60
6	0.0354	83.88	83.83	80.00	81.72	86.02	80.00	63
7	0.015	90.15	90.10	85.00	88.00	92.28	85.00	66
8	0.0108	97.52	97.47	90.00	95.37	99.66	90.00	69
9	0.0073	100.00	100.00	95.00	100.00	100.00	95.00	73
10	0.0277	100.00	100.00	100.00	100.00	100.00	100.00	77
11	0.006	147.08	147.02	147.25	144.92	149.21	150.40	81
12	0.0054	159.60	159.55	159.78	157.44	161.72	162.91	85
13	0.0049	173.97	173.95	174.10	172.53	175.40	176.20	90
14	0.0226	189.02	188.98	189.22	186.88	191.17	192.36	95
15	0.005	204.00	203.95	204.18	201.84	200.00	200.00	100
16	0.0049	221.64	221.60	221.84	219.49	200.00	200.00	105
17	0.0059	233.55	233.52	233.68	232.12	200.00	200.00	111
18	0.0197	241.61	241.57	241.80	239.45	200.00	200.00	117
19	0.0078	260.91	260.86	261.09	258.75	200.00	200.00	123
20	0.0103	283.72	283.67	283.91	281.56	200.00	200.00	130
21	0.013	311.12	311.08	311.31	308.97	200.00	200.00	140
22	0.0224	314.81	314.72	315.19	310.50	200.00	200.00	160
23	0.0214	343.49	343.40	343.87	339.18	200.00	200.00	200
24	0.0305	395.34	395.30	395.53	393.19	200.00	200.00	
100×AP		100	99.92	99.92	96.31	87.73	85.35	

6. Conclusion

Due to lack of information disclosure among insurers in Chinese market, the cost to "evade" the malus and obtain a new start with another company is lower. It implies that the penalties cannot be tough enough for bad drivers, since a tough BMS may push both good and bad drivers out of the company. We considered a special case that the leaving probabilities depend on the premium scales for an open portfolio. We proposed a model to calculate the BMS scales, and the model is intended to reduce the approximation error to optimal BMS scales and also to reduce the financial imbalance. The result of the model is closer to the BMS scales that we see in practice, i.e., both bonus and malus scales are reduced to some milder levels than would be indicated in an optimal BMS system. The model can be applied to subsets of the insured population (e.g., a different BMS scale for inexperienced drivers than for experienced drivers), which would parallel some of the evolution of insurance rating schemes in mature markets.

The extension model may also be applied to the situation that the highest premium is regulated by the government (rather than by policyholder behavior) to be at most G times the average premium. In this case, the model proposed by Coene and Doray (1996) may not have a solution.

One of the shortcomings of the extension model is the assumption that the insured will leave the company with the probability one when the malus is beyond his tolerance level. In fact, a better way to describe the behavior of a policyholder's evasion is a probability distribution. Obviously this is worthy of further discussion. In particular, it would be useful to incorporate a policyholder response function that reflects not only the absolute level of the premium but also the relative level of the premium as compared to the optimal premium, so that the model could explore the effects of adverse selection if good drivers are somewhat overpriced and bad drivers are somewhat under-priced, as we believe would be fairly common when the bonuses and maluses are tempered – and as almost certainly would be the case for many of the worst drivers if the maximum premium is severely capped. Year-to-year price changes also could be an explanatory variable in policyholder evasion. An even more complex extension of the model would consider competitor behavior, since policyholders in fact can react to the availability of alternative prices in the marketplace rather than necessarily being able to react on the basis of (or even know) theoretically correct prices. We are confident that a simulation model approach will be more tractable than a closed-form solution in all of these further explorations. Some very interesting market research could help illuminate and quantify the key variables and parameters of policyholder price elasticity.

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