Optimal strategies for ruin probabilities and expected gains

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Abstract

This paper studies ruin probabilities based on the classical discrete time surplus process. The individual claim size random variables come from one of nine combinations of tail types (heavy, neutral and light) and frequency/severity (low/high, mid/mid and high/low) distributions.

We consider strategies to reduce the ruin probabilities and enhance the expected profits or gains. First we analyze a pricing method where the renewal premiums are based on Buhlmann's credibility theory. Then we add two policy provisions, a deductible and a policy limit.

We also propose two criteria, an index and a value at risk measure, which can be used to select optimal strategies.

Keywords: Surplus process; Time of ruin; Probability of ruin; Buhlmann credibility theory;

Deductible; Policy limit, Value at risk.

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1. Introduction

Consider the surplus process of the insurer at time n, $n = 0, 1, 2, \ldots$, in the classical discrete time risk model,

$$U_n = u + n \times c - S_n, \qquad (1)$$

where $u = U_0$ is the initial surplus, c is the amount of premiums received each period, and S_n is the total claims in the first n periods. We also assume that

$$S_n = W_1 + W_2 + \cdots + W_n, \qquad (2)$$

where W_i is the aggregate claims in period i, and $W_1, W_2, \ldots, W_n$ are non-negative independent and identically distributed random variables with the same distribution as W which can be expressed as

$$W = X_1 + X_2 + \cdots + X_N. \qquad (3)$$

In equation (3), N is a random variable representing the number of claims in one period, and the individual claim sizes $X_1, X_2, \ldots$, are non-negative independent, identically distributed random variables identical to X. Moreover, $c = (1+\theta) \times \mathrm{E}[W] = (1+\theta) \times \mathrm{E}[N] \times \mathrm{E}[X]$ where θ is the relative security loading. Then $\{U_n : n = 0, 1, 2, \ldots\}$ is called the discrete time surplus process, and the time of ruin (the first time that the surplus becomes negative) is defined as $T = \min\{n\colon U_n < 0\}$ ($T = \infty$ if $U_n \geq 0$ for all n). We denote $\Psi(u) = \Pr\{T < \infty \mid U_0 = u\}$ as the probability of ultimate ruin in this context; the probability of ruin before or at time n (that is, the distribution function of T) is denoted by $\Psi(u, n) = \Pr\{T \leq n \mid U_0 = u\}$ which is more intractable mathematically than $\Psi(u)$. Obviously, both $\Psi(u)$ and $\Psi(u, n)$ are decreasing in u, and $\Psi(u, n)$ is non-decreasing in n with $\lim_{n\to\infty} \Psi(u,n) = \Psi(u) < 1$, implying that the distribution function of T, $\Psi(u,n)$, is defective.

The probability of ruin $\Psi(u)$ is one of many questions of interest in classical ruin theory. There have been many papers discussing the probability of ruin $\Psi(u)$, for example, Dufresne and Gerber (1988), DeVylder and Goovaerts (1988), Dickson and Waters (1991, 1992), and Cardoso and Egídio dos Reis (2002). For the continuous time surplus process, an explicit expression can be obtained if the individual claim size random variable X comes from some distribution families. However, there is no explicit expression for the probability of ruin for the discrete time surplus process.

Credibility is a form of insurance pricing that is widely used, particularly in property and casualty insurance. It is a type of experience rating that employs a weighted average of claims experience and a previously developed price to determine a new price for each class under consideration. Credibility theory has been studied in the actuarial literature; see Frees (2003), Herzog (1999), Hickman and Heacox (1999), Klugman, Panjer and Willmot (2004), and Water (1993) for more details.

Ruin is a very important issue for insurance regulators, policyholders and shareholders. From the insurance regulator’s and policyholders’ viewpoints, ruin is a major concern; sufficient fund is required to keep the ruin probability at a low and acceptable level. The shareholders are not only concerned with the ruin probability, but also with the gain or profit over a period of time. Tha gain is defined as the difference between the surplus at the end of a study period and the initial surplus. In this paper, in addition to studying ruin probabilities based on the classical discrete time surplus process (1), in which the premium received in each period is assumed to be a constant, we will apply the Buhlmann credibility theory to calculate the so called Buhlmann credibility premium as the renewal net premium received in each period. Unlike level premiums in life insurance, renewal premiums charged in property and casualty insurance are usually adjusted based on past experience. Intuitively, when the insurer has unfavorable past experience (larger actual claims than expected), then he charges policyholders higher premiums for the next period. This tends to lower the ruin probability compared to charging constant premiums. On the other hand, if the insurer gets a favorable claim experience, then he reduces the renewal premium as an experience refund, which tends to increase the ruin probability compared to charging constant premiums.. In addition to the dynamic premium scheme (Buhlmann credibility premium), we also impose a deductible and/or a policy limit on the individual claim size random variables. Here we are interested in dynamic credibility premium schemes, deductibles and policy limits that can significantly reduce the probability of ruin. The probabilities of ruin are calculated by Monte Carlo simulations.

The remainder of the paper is organized as follows. In section 2, the traditional and modified Buhlmann's credibility methods are applied to the surplus process for the purpose of reducing the probability of ruin. Section 3 analyzes the simulation results and discusses strategies that can significantly reduce the ultimate ruin probability. Since minimizing the ruin probability can result in small expected gains, we propose an index for selecting the optimal strategy in section 4. The index for a strategy combines its associated ultimate ruin probability and expected gain. Section 5 uses a different criteria to determine the optimal strategy. First, value at risk is used to determine the initial surplus given a specific confidence level. Then an annualized rate of return from the gains and the initial surplus is calculated. The best strategy is the one which yields the largest annualized rate of return. Finally, the conclusion in Section 6 summarizes our main findings.

2. Credibility premiums

In this section, we apply Buhlmann's credibility theory to the surplus process to allow the premium received in each period to vary depending on the past experience. The constant premium of the classical model is replaced by two types of premiums, the traditional credibility premium and a modified credibility premium, both with a security loading included. We study three mixtures of frequency and severity, and each severity is associated with three distributions of heavy, neutral and light tailed, respectively. The deductible and policy limit imposed on the individual claim amount are also introduced in this section.

First from equations (1) and (2), we have the following recursive formula:

$$U_{n+1} = U_n + c - W_{n+1}, \quad n = 0, 1, 2, \ldots, \tag{4}$$

with the initial value $U_0 = u$. Note that in the classical model, the amount of premiums received at each period is a constant c. If we let the amount of premiums received in period $(n+1)$ be c_{n+1}, then equation (4) becomes

$$U_{n+1} = U_n + c_{n+1} - W_{n+1}, \quad n = 0, 1, 2, \ldots \tag{5}$$

where c_{n+1} is to be determined based on the Buhlmann's credibility theory. First, we want to estimate the expected value of the random variable W_{n+1} for period $n+1$, given the realizations $W_1 = w_1$, $W_2 = w_2$, …, $W_n = w_n$. From Klugman, Panjer and Willmot (2004),

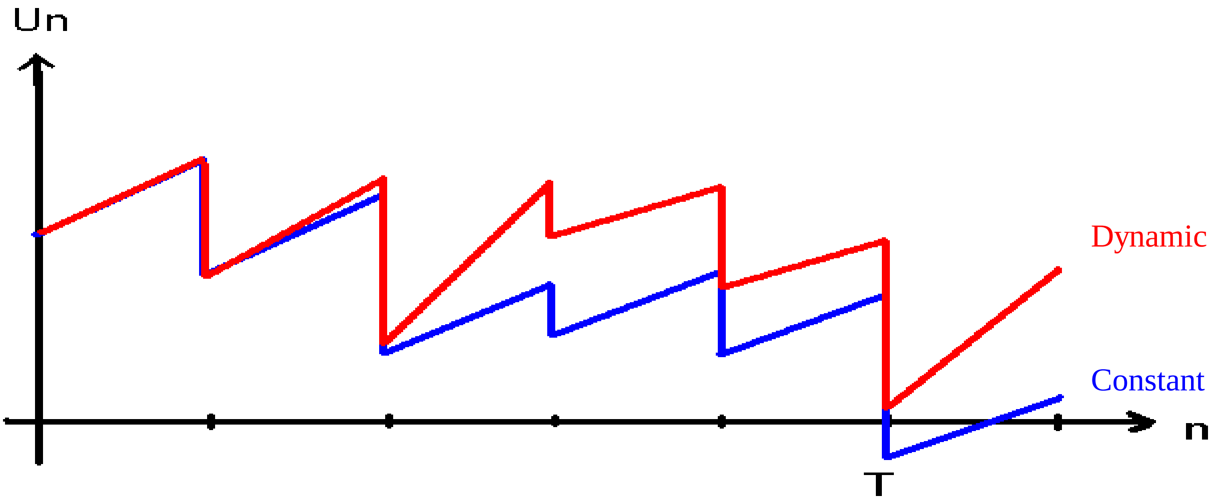
$$\omega_{n+1} \equiv E[W_{n+1} \mid W_1 = w_1, W_2 = w_2, \ldots, W_n = w_n] = Z_n \times \overline{w}_n + (1 - Z_n) \times \mu, \quad n = 0, 1, 2, \ldots, \tag{6}$$

where $\overline{w}_n = \frac{1}{n}\sum_{i=1}^{n} w_i$ is the mean of the past observations, $\mu = E[W]$ is the overall hypothetical mean, $Z_n = n / (n + v / a)$ is the Buhlmann's credibility factor, a is the variance of the hypothetical means, and v is the expected process variance. Thus, $c_{n+1} = (1 + \theta) \times \omega_{n+1}$. Let

$$\hat{\omega}_{n+1} = E[W_{n+1} \mid W_1, W_2, \cdots, W_n] = Z_n \times \overline{W}_n + (1 - Z_n) \times \mu, \; n = 0, 1, 2, \ldots,$$

where $\overline{W}_n = \frac{1}{n}\sum_{i=1}^{n} W_i$. Since $\mu = \mathrm{E}[W]$, we have $E[\hat{\omega}_{n+1}] = Z_n \times \mu + (1 - Z_n) \times \mu = \mu$, that is, $\hat{\omega}_{n+1}$ is an unbiased estimator of μ. Figure 1 illustrates a sample path for the surplus processes based on constant and credibility premiums, respectively.

Figure 1: A sample path for the surplus processes based on constant and dynamic premiums



Because $Var[W_j] = v + a$ and $Cov[W_i, W_j] = a$ for $i \neq j$ (see Klugman, Panjer and Willmot (2004)), we have

$$Var[\hat{\omega}_{n+1}] = \frac{Z_n^{\,2}}{n^2} Var[\sum_{i=1}^{n} W_i] = \frac{Z_n^{\,2}}{n^2}[\sum_{i=1}^{n} Var(W_i) + 2\sum_{i \neq j} Cov(W_i, W_j)] = \frac{Z_n^{\,2}}{n^2}[n(v+a) + n(n-1)a] = aZ_n$$

.

Note that $Z_n = n / (n + v / a) \uparrow 1$ as $n \rightarrow \infty$, implying $Var[\hat{\omega}_{n+1}] \uparrow a$ and $\hat{\omega}_{n+1} \approx \overline{W}_n$ (the sample mean) as $n \rightarrow \infty$. Therefore, if n is large, there is very little change from $\hat{\omega}_n$ to $\hat{\omega}_{n+1}$ when one more aggregate claim W_n is observed (in fact, $\hat{\omega}_{n+1} - \hat{\omega}_n = \frac{W_n - \hat{\omega}_n}{n + v / a}$), which implies that the renewal premium c_{n+1} is very stable and the credibility impact disappears for large n. So, we would like to adopt a more dynamic and volatile premium approach which is also based on the concept of Buhlmann's credibility theory which can reflect recent observations more quickly for renewal premiums. The idea comes from property and casualty insurers that only consider the most recent k periods of claim experiences when setting renewal premiums. We may apply the approach in Klugman, Panjer and Willmot (2004) to obtain

$$\omega_{n+1,m} \equiv E[W_{n+1} \mid W_h = w_h, \cdots, W_n = w_n] = Z_{n,m} \times \overline{w}_{n,m} + (1 - Z_{n,m}) \times \mu \text{, } n = 0, 1, 2, \ldots, \qquad (7)$$

where $m = min\ (n, k)$, $h = max\ (n - k, 0) + 1 = n - m + 1$, $Z_{n,m} = m / (m + v / a)$ (called a modified credibility factor) and $\overline{w}_{n,m} = \frac{1}{m}\sum_{i=h}^{n} w_i$. If we denote

$$\hat{\omega}_{n+1,m} = E[W_{n+1} \mid W_h, W_{h+1}, \cdots, W_n] = Z_{n,m} \times \overline{W}_m + (1 - Z_{n,m}) \times \mu \text{, } n = 0, 1, 2, \ldots,$$

where $\overline{W}_{n,m} = \frac{1}{m}\sum_{i=h}^{n} W_i$, then $\hat{\omega}_{n+1,m}$ is also an unbiased estimator of μ with $Var[\hat{\omega}_{n+1,m}] = aZ_{n,m} \leq aZ_n = Var[\hat{\omega}_{n+1}]$ ("=" holds when $n \leq k$); also $Var[\hat{\omega}_{n+1,m}] = aZ_{n,m} = k\,a^2 / (k\,a + v)$ is independent of n when $n \geq k$. Note that equations (6) and (7) are identical for $n \leq k$. Also when $k = 0$ then $m = 0$ and $Z_{n,m} = 0$, and the modified credibility premium becomes constant; when $k = \infty$ then $m = n$ and $Z_{n,m} = Z_n$, and the modified credibility premium reduces to the Buhlmann credibility one.

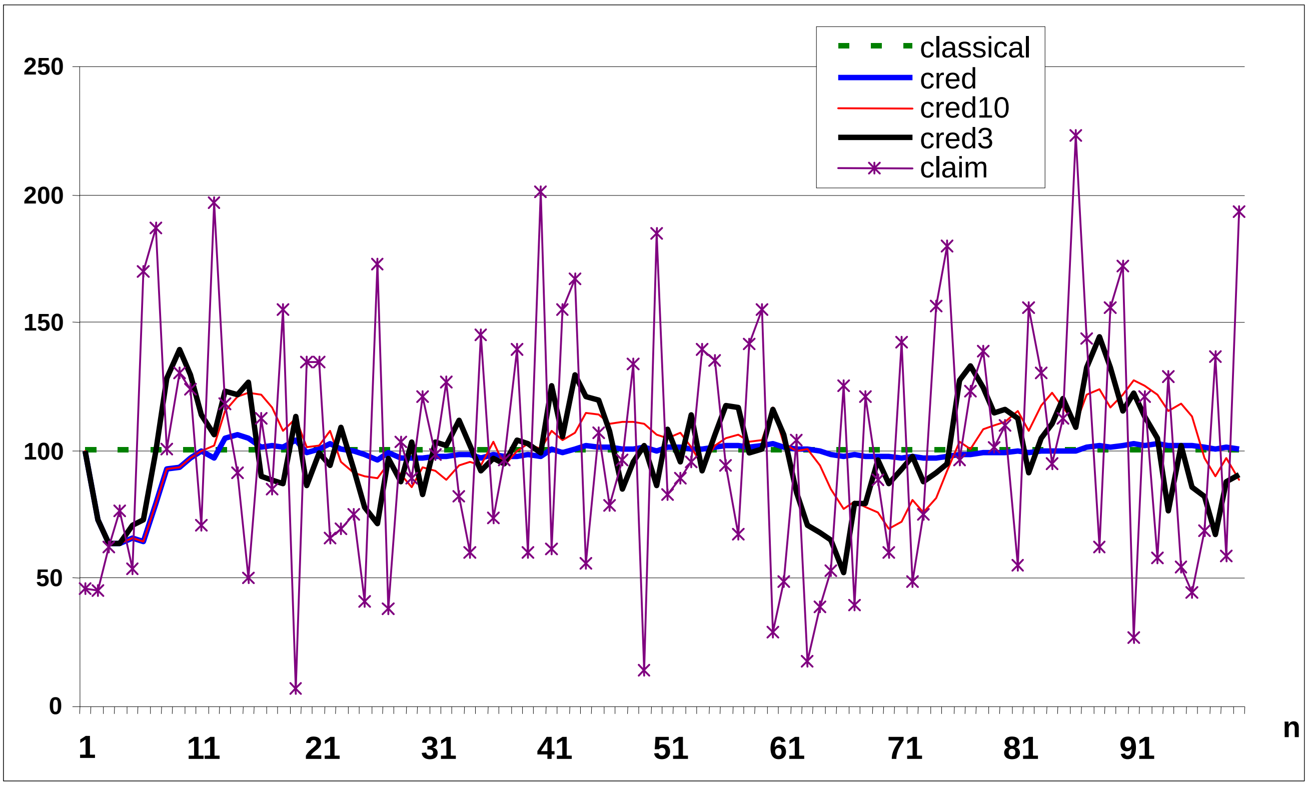
Unlike the situations in the classical Buhlmann's credibility theory, the modified credibility factor $Z_{n,m} = m / (m + v / a) \uparrow k / (k + v / a) < 1$ (also $Z_{n,m}$ is increasing in k) and $Var[\hat{\omega}_{n+1,m}] = aZ_{n,m} \uparrow a\,k / (k + v/a) < a = \lim_{n \to \infty} Var[\hat{\omega}_{n+1}]$ as $n \rightarrow \infty$. Therefore, $\hat{\omega}_{n+1} \approx \overline{W}_n$ (the sample mean) as $n \rightarrow \infty$ in the case of the classical Buhlmann's credibility theory does not hold any more. In fact,

$$\hat{\omega}_{n+1,m} - \hat{\omega}_{n,m} = \begin{cases} \dfrac{W_n - \hat{\omega}_n}{n + v/a}, n = 1,2,\ldots,k, \\ \dfrac{W_n - W_{n-k}}{k + v/a}, n = k+1, k+2,\ldots \end{cases}$$

Thus, $\hat{\omega}_{n+1,m} - \hat{\omega}_{n,m}$ depends largely on $W_n - W_{n-k}$ for $n > k$.

Here we assume $k = 3$ (cred3) and $k = 10$ (cred10) to represent the short- and long-term past experiences, respectively. Figure 2, illustrates a quite volatile claims process and corresponding renewal premiums. We observe that method cred3 quickly reflects claims' fluctuation over time while premiums $\omega_{n,m}$ based on the classical Buhlmann's credibility method (cred) quickly converge to 100 (the constant net premium for the classical surplus process). Since method cred3 ($k = 3$) can provide a quicker response to premium adjustment than the classical Buhlmann's credibility method, can it produce lower ruin probability as well? To investigate the question, we perform Monte Carlo method simulations based on a variety of assumptions and strategies.

Figure 2: A sample path for claims and $\omega_{n,m}$ for m = 0(classical), 3(cred3), 10(cred10) and n(cred)



First, we study three types of insurance business with equal expected aggregate claim ($E[W] = E[N] \times E[X]$) in each period. The first is a business with claims of low frequency (E[N] = 1) and high severity (E[X] = 100). The second has mid frequency (E[N] = 10) and mid severity (E[X] = 10) claims. The third one has claims of high frequency (E[N] = 100) and low severity (E[X] = 1). These three types of business are abbreviated as LF/HS, MF/MS and HF/LS, respectively. The number of claims, *N*, follows a Poisson distribution with parameter λ.

To observe whether tail behavior of a distribution will produce different results, we investigate three kinds of tail distributions based on the criteria of the hazard rate function (see Klugman, Panjer and Willmot (2004)): light-tailed (Weibull(α, θ) with $\alpha > 1$), neutral-tailed (Exponential(β)) and heavy-tailed (Pareto(τ, θ) with $\tau > 1$) distributions (abbreviated as LT, NT and HT, respectively) with the same mean $E[X]$ for the individual claim size random variable *X*. Table 1 lists the underlying severity distributions and associated parameters.

In practice, the insurer may manage higher risks by imposing a policy limit or by transferring claims in excess of a retention limit to a reinsurer. To lower administrative cost and consequently insurance premiums, the insurer may also set up a deductible below which claims are assumed by the policyholder. We study whether imposing a deductible or a policy limit, or both, will reduce the probability of ruin as well. Let the deductible $D = \mathrm{E}[X] / M$ and the policy limit $L = M \times \mathrm{E}[X]$ where $M > 0$ is called DL size modifier. Then $Y = [(X \wedge L) - D]_+ = \max(\min(X, L) - D, 0) = (X \wedge L) - (X \wedge D)$ is a per-loss random variable. In this case, equation (3) becomes $W = Y_1 + Y_2 + \cdots + Y_N$ and $E[W] = E[N] \times E[Y]$. Note that the policy limit *L* increases to infinity (no policy limit) and the deductible *D* decreases to zero (no deductible) as *M* goes to infinity, which reduces to the case $Y = X$, i.e. all claims are covered. From the point of view of the policyholder, the claims coverage should be broad enough to make the insurance product worthwhile. Since the insurer wants to sell as many policies as possible, it seems reasonable to require that $\mathrm{P}(D < X < L)$ is not too small. From Table 2, we observe that $\mathrm{P}(D < X < L) < 0.5$ for $M = 2$ when *X* has an Exponential or a Pareto distribution. Therefore, we adopt $M = 3$, 4 and 5 for our study.

Table 1: underlying severity distributions

Severity distribution for X	Light-tailed	Neutral-tailed	Heavy-tailed	E[X]
Low severity	Weibull (2, 1/Γ(1.5))	Exponential (1)	Pareto (3, 2)	1
Mid severity	Weibull (2, 10/Γ(1.5))	Exponential (10)	Pareto (3, 20)	10
High severity	Weibull (2, 100/Γ(1.5))	Exponential (100)	Pareto (3, 200)	100

Table 2: probabilities below the deductible, above the policy limit, and within these two quantities

Probabilities	M=2			M=3			M=4		
Distribution X	P(X<D)	P(D<X<L)	P(X>L)	P(X<D)	P(D<X<L)	P(X>L)	P(X<D)	P(D<X<L)	P(X>L)
Weibull	0.178	0.779	0.043	0.083	0.916	0.001	0.048	0.952	0.000
Exponential	0.393	0.471	0.135	0.283	0.667	0.050	0.222	0.760	0.018
Pareto	0.488	0.387	0.125	0.370	0.566	0.064	0.298	0.665	0.037

3. Ultimate ruin probabilities

In this section, we will show by simulation that the traditional and modified credibility methods can reduce the ruin probability except for small initial surpluses. The ruin probability further decreases if a deductible and/or policy limit is imposed. We also propose an optimal strategy which can significantly reduce the average ultimate ruin probability.

First for each of nine combinations of tail type (HT, NT and LT) and frequency/severity (LF/HS, MF/MS and HF/LS) risks, we investigate forty strategies specified by three factors. The first factor is the premium scheme where 1 is used for a constant premium, 2 for regular credibility premium, 3 and 4 for modified credibility premiums for $k = 3$ and 10, respectively. The second factor is the DL indicator where 1 is for no deductible (D) and policy limit (L), 2 for deductible only, 3 for policy limit only, and 4 for both provisions. The third factor is the DL size modifier M (a value of 1 is assigned to this factor when there is no deductible or policy limit, or equivalently when M equals infinity). Table 3 summarizes these definitions and the codes that are used or the strategies studied in this paper. For example, (1, 3, 5) corresponds to the strategy using constant premium with a policy limit $L = 5\ \mathrm{E}[X]$, and (4, 1, 1) corresponds to the strategy using modified credibility premium with $k = 10$ but without the deductible and policy limit imposed.

Table 3: definitions of factors

Factors	Codes			
1- premium scheme	1: constant	2: cred	3: cred10	4: cred3
2- DL indicator	1: no D or L	2: D only	3: L only	4: D and L
3- DL size modifier	1: M=∞ (no D or L)	3: M=3	4: M=4	5: M=5

Next for each strategy, we study eleven initial surpluses: 0, 2,…, 20 for HF/LS risk; 0, 20,…, 200 for MF/MS risk; and 0, 200,…, 2000 for LF/HS risk. For each initial surplus, we create 1000 paths by Monte-Carlo simulation, and the numerical ruin probability by time n is defined as $\Psi(u, n) =$ number of $\{U_k < 0 \text{ for some } k \leq n \mid U_0 = u\}$ / 1000. Also, let the numerical ultimate ruin probability $\Psi(u) = \Psi(u, 100)$. The reason for setting different scales for the initial surplus for these three mixtures of frequency/severity of risks is that, for a given ruin probability, a larger initial surplus is needed for lower frequency and higher severity risks.

From Table 4, we have the following findings for the NT risk by comparing $\Psi_{(1,1,1)}(u)$, $\Psi_{(2,1,1)}(u)$, $\Psi_{(3,1,1)}(u)$ and $\Psi_{(4,1,1)}(u)$ (the same holds for HT and LT risks):

(A) The ruin probability for LF/HS risks is larger than the one for HF/LS risks. That is, for a fixed strategy and initial surplus u, $\Psi(u)_{(LF,HS)} > \Psi(u)_{(MF,MS)} > \Psi(u)_{(HF,LS)}$ eventhough they have the same mean $\mathrm{E}[W]$. Equivalently, to maintain a low ruin probability, the insurer needs a larger initial surplus for LF/HS risks than for HF/LS risks.

(B) A credibility premium scheme cannot reduce the ruin probability unless u is large enough. When the initial surplus u is small, $\Psi_{(k, 1, 1)}(u) > \Psi_{(1, 1, 1)}(u)$ where $k = 2$, 3 and 4. However, $\Psi_{(k, 1, 1)}(u)$ decreases in u faster than $\Psi_{(1, 1, 1)}(u)$ does , and $\Psi_{(k, 1, 1)}(u)$ eventually becomes smaller than $\Psi_{(1, 1, 1)}(u)$. That is, there exists a value u^* such that $\Psi_{(k, 1, 1)}(u) < \Psi_{(1, 1, 1)}(u)$ for all $u > u^*$ and $\Psi_{(k, 1, 1)}(u) > \Psi_{(1, 1, 1)}(u)$ for all $u < u^*$ and $k = 2, 3, 4$. Also, we need a larger u^* for the LF/HS risk.

(C) For the same initial surplus u, credibility-based ruin probability with longer period of past observations is bigger than the one with shorter period, that is, $\Psi_{(2, 1, 1)}(u) \geq \Psi_{(3, 1, 1)}(u) \geq \Psi_{(4, 1, 1)}(u)$ for all three mixtures of frequency and severity and in fact all three tail types of risks except for some larger initial surplus u for HT and LF/HS risk, $\Psi_{(2, 1, 1)}(u) \geq \Psi_{(4, 1, 1)}(u) \geq \Psi_{(3, 1, 1)}(u)$.

Since cred3 method can reduce the ruin probability the most, we now focus on this method, and study the impact of deductible and policy limit on the ruin probability. For the case that cred3 is adopted and the DL size modifier $M = 3$, we compare $\Psi_{(4, 1, 1)}(u)$, $\Psi_{(4, 2, 3)}(u)$, $\Psi_{(4, 3, 3)}(u)$ and $\Psi_{(4, 4, 3)}(u)$.

(D) A deductible does not significantly reduce the ruin probability, and even increases it for most initial surplus for the HF/LS risk. However, policy limit can significantly lower the ruin probability further. We note that the strategy with both deductible and policy limit produces the smallest ruin probability, $\Psi_{(4, 4, 3)}(u)$, for most initial surpluses for the mid frequency and severity risk and the low frequency and high severity risk .

Next we study the impact of the DL size modifier M on the ruin probability by comparing $\Psi_{(4, 4, 3)}(u)$, $\Psi_{(4, 4, 4)}(u)$ and $\Psi_{(4, 4, 5)}(u)$.

(E) The strategy with both policy limit and deductible imposed associated with the smallest DL size modifier M ($M = 3$) produces the lowest ruin probability, $\Psi_{(4, 4, 3)}(u)$, except for very few low initial surpluses.

Let the initial surplus equal to the mean of the aggregate loss in one period, that is, $u = E[W] = 100$; then for the MF/MS risk, the ruin probability reduces to 8.9% from 29.5% if cred3 method is adopted, and it is lowered further to 2.5% if both deductible and policy limit with modifier M=3 are imposed. See Table 4 and Figures 3, 4 and 5 for more details.

Table 4: ruin probabilities for neutral tailed case

ruin probabilities $\Psi(u)$ for the HF/LS risk											
u	0	2	4	6	8	10	12	14	16	18	20
$\boldsymbol{\Psi_{(1,1,1)}(u)}$	**33.2%**	**29.5%**	**25.7%**	**22.4%**	**19.5%**	**16.6%**	**14.3%**	**12.0%**	**9.8%**	**7.8%**	**6.6%**
$\Psi_{(2,1,1)}(u)$	30.8%	27.0%	22.6%	18.2%	15.1%	11.9%	9.1%	7.3%	5.6%	4.6%	2.9%
$\Psi_{(2,1,1)}(u)$	30.8%	27.0%	22.6%	18.2%	15.1%	11.9%	9.1%	7.3%	5.6%	4.6%	2.9%
$\boldsymbol{\Psi_{(4,1,1)}(u)}$	**30.1%**	**26.5%**	**22.2%**	**17.8%**	**14.8%**	**11.7%**	**8.8%**	**7.1%**	**5.5%**	**4.5%**	**2.8%**
$\Psi_{(4,2,3)}(u)$	36.5%	30.5%	25.7%	21.7%	17.1%	13.4%	10.6%	8.0%	5.9%	4.1%	2.9%
$\Psi_{(4,3,3)}(u)$	29.8%	24.0%	19.3%	14.1%	10.8%	8.8%	7.0%	5.6%	3.2%	2.3%	1.6%
$\boldsymbol{\Psi_{(4,4,3)}(u)}$	**33.8%**	**27.5%**	**22.6%**	**16.8%**	**12.9%**	**9.3%**	**7.3%**	**4.3%**	**2.6%**	**1.7%**	**1.1%**
$\Psi_{(4,4,4)}(u)$	34.6%	27.6%	23.6%	18.9%	15.8%	11.4%	9.1%	6.3%	4.3%	3.2%	1.6%
$\Psi_{(4,4,5)}(u)$	33.1%	27.7%	23.6%	19.1%	15.9%	12.3%	9.3%	6.9%	4.9%	3.7%	2.6%
ruin probabilities $\Psi(u)$ for the MF/MS risk											
u	0	20	40	60	80	100	120	140	160	180	200
$\boldsymbol{\Psi_{(1,1,1)}(u)}$	**70.2%**	**60.2%**	**50.3%**	**42.6%**	**34.8%**	**29.5%**	**23.8%**	**20.1%**	**16.6%**	**13.7%**	**11.1%**
$\Psi_{(2,1,1)}(u)$	95.1%	83.3%	66.1%	48.7%	33.2%	23.6%	16.3%	11.3%	8.5%	6.3%	5.1%
$\Psi_{(2,1,1)}(u)$	91.3%	76.1%	57.7%	40.1%	25.2%	16.1%	9.3%	5.6%	3.7%	1.7%	1.1%
$\boldsymbol{\Psi_{(4,1,1)}(u)}$	**81.8%**	**62.9%**	**43.8%**	**27.9%**	**16.5%**	**8.9%**	**4.6%**	**1.8%**	**0.8%**	**0.3%**	**0.1%**
$\Psi_{(4,2,3)}(u)$	80.9%	62.8%	44.4%	26.7%	15.3%	8.6%	4.1%	1.6%	0.7%	0.3%	0.0%
$\Psi_{(4,3,3)}(u)$	82.3%	61.3%	38.6%	19.8%	10.2%	4.1%	1.4%	0.3%	0.1%	0.1%	0.1%
$\boldsymbol{\Psi_{(4,4,3)}(u)}$	**81.1%**	**60.3%**	**33.0%**	**16.8%**	**6.9%**	**2.5%**	**0.7%**	**0.1%**	**0.1%**	**0.1%**	**0.0%**
$\Psi_{(4,4,4)}(u)$	81.9%	61.6%	39.8%	21.8%	11.3%	4.8%	2.0%	0.4%	0.1%	0.1%	0.0%
$\Psi_{(4,4,5)}(u)$	81.2%	62.8%	42.2%	24.6%	13.7%	6.4%	2.5%	0.8%	0.3%	0.1%	0.0%
ruin probabilities $\Psi(u)$ for the LF/HS risk											
u	0	200	400	600	800	1000	1200	1400	1600	1800	2000
$\boldsymbol{\Psi_{(1,1,1)}(u)}$	**83.6%**	**66.0%**	**52.0%**	**40.1%**	**31.8%**	**25.2%**	**20.3%**	**16.2%**	**12.5%**	**8.5%**	**6.2%**
$\Psi_{(2,1,1)}(u)$	100.0%	94.4%	71.2%	49.6%	34.9%	24.4%	15.6%	10.4%	7.0%	4.5%	2.3%
$\Psi_{(2,1,1)}(u)$	99.8%	85.5%	51.8%	23.2%	8.9%	3.3%	0.9%	0.3%	0.0%	0.0%	0.0%
$\boldsymbol{\Psi_{(4,1,1)}(u)}$	**97.1%**	**69.1%**	**32.1%**	**11.3%**	**3.6%**	**0.8%**	**0.1%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
$\Psi_{(4,2,3)}(u)$	94.6%	65.3%	32.3%	13.2%	4.7%	1.4%	0.3%	0.1%	0.0%	0.0%	0.0%
$\Psi_{(4,3,3)}(u)$	98.5%	62.3%	20.2%	2.5%	0.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
$\boldsymbol{\Psi_{(4,4,3)}(u)}$	**95.6%**	**54.0%**	**16.5%**	**2.7%**	**0.3%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
$\Psi_{(4,4,4)}(u)$	95.5%	62.5%	25.0%	6.5%	1.6%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%
$\Psi_{(4,4,5)}(u)$	95.6%	65.4%	28.6%	8.7%	2.4%	0.6%	0.1%	0.0%	0.0%	0.0%	0.0%

Table 5 compares the two best strategies, strategy (4, 1, 1) (cred3 method only) and strategy (4, 4, 3) (cred3 plus deductible and policy limit with M = 3), with the original strategy (1, 1, 1) for the nine combinations of tail type and frequency / severity risks. Here, $\overline{\Psi}_{(i,j,k)}(u)$ denotes the average ruin probability for strategy (i, j, k) over the ten positive initial surpluses with equal weight for u. Among these three tail types, HT risks always cause the largest average ruin probability which can be reduced ($\overline{\Psi}_{(1,1,1)}(u)$ - $\overline{\Psi}_{(4,4,3)}(u)$) most significantly, and LT risks always cause the smallest average ruin probability which can be reduced ($\overline{\Psi}_{(1,1,1)}(u)$ - $\overline{\Psi}_{(4,4,3)}(u)$) least significantly for all three combinations of frequency and severity.

Also, we note that credibility premium is a more effective way of reducing the ruin probability for NT and LT risks, i.e. $\overline{\Psi}_{(1,1,1)}(u)$ - $\overline{\Psi}_{(4,1,1)}(u)$) > ($\overline{\Psi}_{(4,1,1)}(u)$ - $\overline{\Psi}_{(4,4,3)}(u)$. Deductibles and policy limits are more effective for HT risks, i.e. $\overline{\Psi}_{(1,1,1)}(u)$ - $\overline{\Psi}_{(4,1,1)}(u)$) < ($\overline{\Psi}_{(4,1,1)}(u)$ - $\overline{\Psi}_{(4,4,3)}(u)$.

Figure 3: ruin probability for the NT and MF/MS risk without D/L case

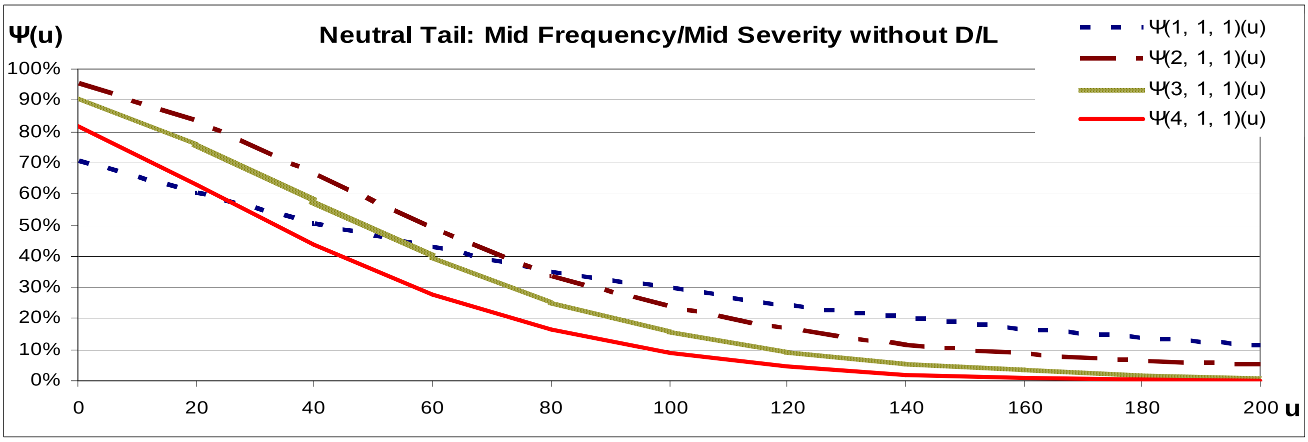


Figure 4: ruin probability for the NT and MF/MS risk with cred3 and M = 3

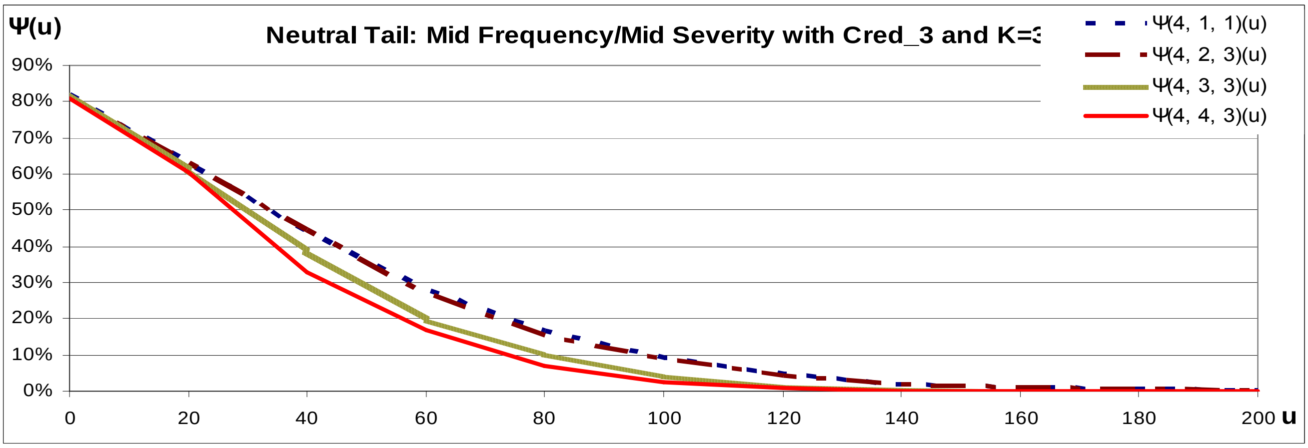


Figure 5: ruin probability for the NT and MF/MS risk with cred3 and D/L

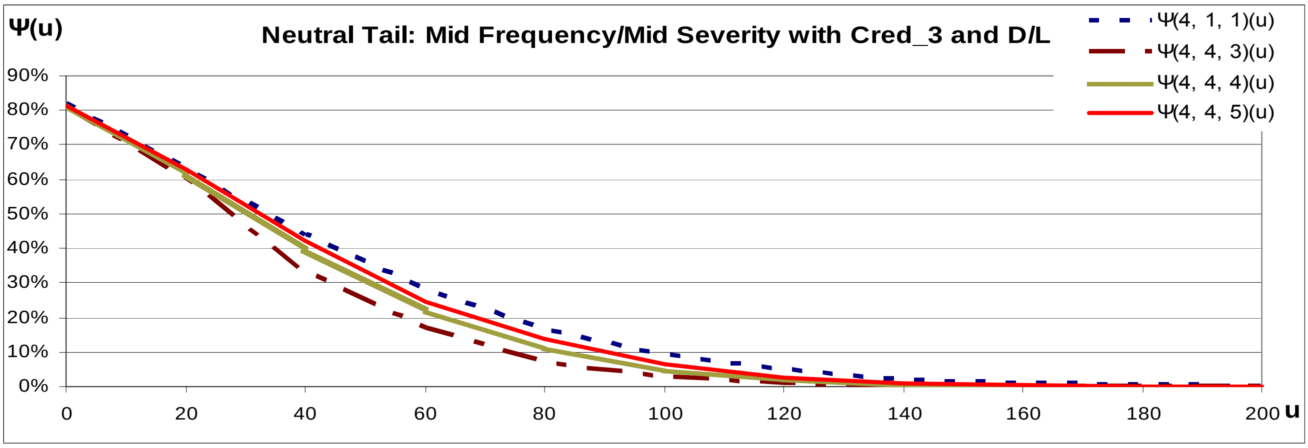


Table 5: Strategies and corresponding average ruin probabilities

Frequency Severity	Tail type / Strategy	$\overline{\Psi}_{(1,1,1)}(u)$	$\overline{\Psi}_{(4,1,1)}(u)$	$\overline{\Psi}_{(4,4,3)}(u)$	$\overline{\Psi}_{(1,1,1)}(u)$ - $\overline{\Psi}_{(4,1,1)}(u)$	$\overline{\Psi}_{(4,1,1)}(u)$ - $\overline{\Psi}_{(4,4,3)}(u)$
LF / HS	Heavy	0.3439	0.2620	0.0892	0.0819	0.1728
	Neutral	0.2788	0.1170	0.0735	0.1618	0.0435
	Light	0.2012	0.0616	0.0379	0.1396	0.0237
MF / MS	Heavy	0.4597	0.3675	0.1255	0.0922	0.2420
	Neutral	0.3027	0.1676	0.1205	0.1351	0.0471
	Light	0.2017	0.1032	0.0730	0.0985	0.0302
HF / LS	Heavy	0.2988	0.2640	0.1123	0.0348	0.1517
	Neutral	0.1642	0.1217	0.1061	0.0425	0.0156
	Light	0.0862	0.0628	0.0563	0.0234	0.0065

4. Ruin ratio, gain ratio and index

In Section 3, we proposed an optimal one among forty strategies for each combination of frequency, severity and tail type based on the purpose of reducing the average ultimate ruin probabilities. Although the insurer and its shareholders prefer a lower ruin probability, they also prefer strategies that produce larger expected gains. In this section, we would like to seek strategies which have relatively small ruin probabilities and large expected gains. Since the strategies studied tend to affect these objectives in opposite directions, we propose an index which is the product of a ruin ratio and a gain ratio that appear so be a reasonable comprise to identifies optimal strategies.

First, we introduce some notation. Let

S: the set of forty strategies $\{(i, j, k) \mid i, j = 1, 2, 3, 4$ and $k = 1, 3, 4, 5\}$(see Table 3);

$\overline{U}_s(u,n)$: the average surplus at time n over 1000 simulations for the initial surplus u and strategy $s \in S$;

$\overline{G}_s(n) = \overline{U}_s(u,n) - u$: the average gain at time n over 1000 simulations for strategy $s \in S$;

$\overline{G}(n) = \max\{\overline{G}_s(n) : s \in S\}$: the largest average gain at time n over 1000 simulations among $s \in S$;

$\overline{\Psi}_s(u,n)$: the average ruin probability by or at time n over 1000 simulations for the initial positive surplus u and strategy $s \in S$;

$\overline{\Psi}_s(n) = \frac{1}{10}\sum_u \overline{\Psi}_s(u,n)$: the average ruin probability by or at time n over 10 positive initial surpluses for strategy $s \in S$;

$\overline{\Psi}(n) = \min\{\overline{\Psi}_s(n) : s \in S\}$: the smallest average ruin probability by or at time n over 10 positive initial surpluses among $s \in S$;

$GR_s(n) = \dfrac{\overline{G}_s(n)}{\overline{G}(n)} \leqq 1$: a gain ratio for the study period n and strategy $s \in S$; and

$RR_s(n) = \dfrac{\overline{\Psi}(n)}{\overline{\Psi}_s(n)} \leqq 1$: a ruin ratio for the study period n and strategy $s \in S$.

Note that since $\overline{U}_s(u,n)$ is an increasing function of u, and $\overline{G}_s(n)$ is independent of u, using $\overline{G}_s(n)$ as a measure can remove the effect of the initial surplus, and thus $\overline{G}_s(n)$ is better than $\overline{U}_s(u,n)$. Also, the gain ratios, $GR_s(n)$, preserve the order of $\overline{G}_s(n)$, while the ruin ratios, $RR_s(n)$, reverse the order of $\overline{\Psi}_s(n)$. Recall that strategies with lower average ruin probability (bigger $RR(n)$) and larger average gain (higher $GR(n)$) are preferred. Therefore,

$$Index_s(n) = GR_s(n) \times RR_s(n) \leqq 1,\ s \in S, \tag{8}$$

is a reasonable and appropriate measure for ranking the forty strategies. The best strategy is the one with the largest index value.

Tables 6, 7, and 8 list the top four gain ratios, ruin ratios and indices, respectively, for $n = 5$, 20 and 100 representing short, mid and long terms, respectively.

(A) From Table 6 we conclude that to maximize the average gain $\overline{G}_s(n)$ (or to maximize the gain ratio $GR_s(n)$) for $n = 5$, 20 and 100, strategies without deductible or policy limit imposed (that is, (i, 1, 1) for $i = 1, 2, 3$, and 4) should be adopted for all three mixtures of frequency and severity risks. Further, the classical credibility strategy (2, 1, 1) is the overall best. In fact, strategies without deductible and policy limit, (i, 1, 1), or strategies with policy limit imposed only, (i, 3, k), produce higher gains than strategies with deductible imposed only,
(i, 2, k), and strategies with both deductible and policy limit, (i, 4, k), for $i = 1, 2, 3, 4$ and $k = 3, 4, 5$.

(B) From Table 7, minimizing the average ruin probability $\overline{\Psi}_s(n)$ (or maximizing the ruin ratio $RR_s(n)$) for $n = 5$, 20 and 100 can be done by adopting a modified credibility premium (cred3). The best DL indicator and DL size modifier depends on the type of risk and tail distribution.
Overall, strategy (4, 4, 3) is the best one for minimizing the average ruin probability. Note that none of strategies (k, 1, 1) for $k = 1, 2, 3$, and 4 (that is, without deductible or policy limit imposed) producing the highest expected gains (see Table 6) ranks in the top four for minimizing the ruin probabilities.

(C) By Table 8, from the viewpoint of maximizing $Index_s(n)$ (that is, achieving a balance between ta high average gain $\overline{G}_s(n)$ and a low average ruin probability $\overline{\Psi}_s(n)$), strategy (4, 3, 3) is the overall best choice since it has the largest $Index_s(n)$ values for most cases. We note some exceptions however, for example strategy (4, 3, 3) ranks fourth for LT and MF/MS risks and for n = 20, 100, and is not even in the top four for LT and LF/HS risks. Note that among the strategies (k, 1, 1) for k = 1, 2, 3, 4, strategy (4, 1, 1) is the only one in the top four.

Table 6: Top four Gain Ratio(n) for n = 5, 20 and 100

Tail and Years		**High Frequency / Low Severity**				**Mid Frequency / Mid Severity**				**Low Frequency / High Severity**			
HT	Strategy	(1, 1, 1)	**(2, 1, 1)**	(3, 1, 1)	(4, 1, 1)	(1, 1, 1)	(4, 1, 1)	**(2, 1, 1)**	(3, 1, 1)	(2, 3, 5)	(3, 3, 5)	(4, 3, 5)	(2, 3, 4)
5 yrs	Gain R.	1.0000	0.9929		0.9924	1.0000	0.9904	0.9781		1.0000		0.9888	0.9780
HT	Strategy	(1, 1, 1)	(4, 1, 1)	(3, 1, 1)	**(2, 1, 1)**	**(2, 1, 1)**	(3, 1, 1)	(4, 1, 1)	(1, 1, 1)	**(2, 1, 1)**	(3, 1, 1)	(1, 1, 1)	(4, 1, 1)
20 yrs	Gain R.	1.0000	0.9957	0.9918	0.9913	1.0000	0.9997	0.9847	0.9751	1.0000	0.9586	0.9307	0.9260
HT	Strategy	(1, 1, 1)	(4, 1, 1)	(3, 1, 1)	**(2, 1, 1)**	**(2, 1, 1)**	(3, 1, 1)	(4, 1, 1)	(1, 1, 1)	(1, 1, 1)	**(2, 1, 1)**	(4, 1, 1)	(3, 1, 1)
100 yrs	Gain R.	1.0000	0.9959	0.9937	0.9913	1.0000	0.9990	0.9975	0.9915	1.0000	0.9526	0.9478	0.9353
NT	Strategy	**(2, 1, 1)**	(3, 1, 1)	(4, 1, 1)	(2, 3, 5)	(1, 1, 1)	(1, 3, 5)	(4, 1, 1)	(4, 3, 5)	**(2, 1, 1)**	(3, 1, 1)	(1, 1, 1)	(1, 3, 4)
5 yrs	Gain R.	1.0000		0.9984	0.9923	1.0000	0.9936	0.9935	0.9882	1.0000		0.9955	0.9949
NT	Strategy	**(2, 1, 1)**	(3, 1, 1)	(4, 1, 1)	(2, 3, 5)	**(2, 1, 1)**	(3, 1, 1)	(1, 1, 1)	(4, 1, 1)	(1, 3, 4)	(1, 3, 3)	(4, 1, 1)	(1, 3, 5)
20 yrs	Gain R.	1.0000	0.9979	0.9952	0.9929	1.0000	0.9999	0.9990	0.9968	1.0000	0.9985	0.9979	0.9971
NT	Strategy	(1, 1, 1)	(4, 1, 1)	(3, 1, 1)	**(2, 1, 1)**	**(2, 1, 1)**	(1, 1, 1)	(2, 3, 5)	(4, 1, 1)	**(2, 1, 1)**	(2, 3, 5)	(3, 1, 1)	(1, 1, 1)
100 yrs	Gain R.	1.0000	0.9959	0.9937	0.9913	1.0000	0.9956	0.9937	0.9933	1.0000	0.9924	0.9843	
LT	Strategy	**(2, 1, 1)**	(3, 1, 1)	(2, 3, 4)	(3, 3, 4)	**(2, 1, 1)**	(3, 1, 1)	(2, 3, 4)	(3, 3, 4)	(4, 1, 1)	(4, 3, 4)	(4, 3, 5)	(4, 3, 3)
5 yrs	Gain R.	1.0000				1.0000				1.0000			0.9995
LT	Strategy	**(2, 1, 1)**	(2, 3, 5)	(2, 3, 4)	(2, 3, 3)	(3, 1, 1)	(3, 3, 5)	(3, 3, 4)	(3, 3, 3)	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	(4, 3, 3)
20 yrs	Gain R.	1.0000			0.9999	1.0000			0.9999	1.0000			0.9999
LT	Strategy	(1, 1, 1)	(1, 3, 5)	(1, 3, 4)	(1, 3, 3)	**(2, 1, 1)**	(2, 3, 5)	(2, 3, 4)	(2, 3, 3)	**(2, 1, 1)**	(2, 3, 5)	(2, 3, 4)	(2, 3, 3)
100 yrs	Gain R.	1.0000			0.9999	1.0000			0.9999	1.0000			

Table 7: Top four Ruin Ratio(n) for n = 5, 20 and 100

Tail and Years		**High Frequency / Low Severity**				**Mid Frequency / Mid Severity**				**Low Frequency / High Severity**			
HT	Strategy	(4, 3, 3)	(3, 3, 3)	(2, 3, 3)	**(4, 4, 3)**	**(4, 4, 3)**	(2, 4, 3)	(3, 4, 3)	(4, 3, 3)	**(4, 4, 3)**	(2, 4, 3)	(3, 4, 3)	(1, 4, 3)
5 yrs	Ruin R.	1.0000	0.9961		0.9079	1.0000	0.9869		0.8652	1.0000	0.9862		0.8166
HT	Strategy	(4, 3, 3)	(3, 3, 3)	(2, 3, 3)	**(4, 4, 3)**	**(4, 4, 3)**	(4, 3, 3)	(4, 3, 4)	(4, 4, 4)	**(4, 4, 3)**	(4, 3, 3)	(3, 4, 3)	(4, 4, 4)
20 yrs	Ruin R.	1.0000	0.9845		0.9038	1.0000	0.9507	0.7836	0.7797	1.0000	0.8217	0.7924	0.7485
HT	Strategy	(4, 3, 3)	(3, 3, 3)	(2, 3, 3)	**(4, 4, 3)**	**(4, 4, 3)**	(4, 3, 3)	(4, 3, 4)	(4, 4, 4)	**(4, 4, 3)**	(4, 3, 3)	(4, 3, 4)	(3, 4, 3)
100 yrs	Ruin R.	1.0000	0.9845		0.9038	1.0000	0.9654	0.7953	0.7785	1.0000	0.9867	0.7866	0.7763
NT	Strategy	(4, 3, 3)	(3, 3, 3)	(2, 3, 3)	**(4, 4, 3)**	**(4, 4, 3)**	(2, 4, 3)	(3, 4, 3)	(4, 4, 4)	**(4, 4, 3)**	(2, 4, 3)	(3, 4, 3)	(1, 4, 3)
5 yrs	Ruin R.	1.0000	0.9949		0.9175	1.0000	0.9818		0.8232	1.0000	0.9879		0.7625

NT	Strategy	(4, 3, 3)	(3, 3, 3)	(2, 3, 3)	**(4, 4, 3)**	**(4, 4, 3)**	(4, 3, 3)	(4, 4, 4)	(4, 3, 4)	**(4, 4, 3)**	(4, 3, 3)	(4, 4, 4)	(3, 4, 3)
20 yrs	Ruin R.	1.0000	0.9877		0.9114	1.0000	0.8794	0.8494	0.7926	1.0000	0.7641	0.7600	0.7206
NT	Strategy	(4, 2, 3)	(3, 3, 3)	(2, 3, 3)	**(4, 4, 3)**	(4, 3, 3)	(3, 3, 3)	(2, 3, 3)	**(4, 4, 3)**	**(4, 4, 3)**	(4, 3, 3)	(4, 4, 4)	(4, 3, 4)
100 yrs	Ruin R.	1.0000	0.9845		0.9038	1.0000	0.9877		0.9114	1.0000	0.8607	0.7664	0.7372
LT	Strategy	(4, 2, 3)	**(4, 4, 3)**	(2, 2, 3)	(3, 2, 3)	**(4, 4, 3)**	(4, 2, 3)	(2, 4, 3)	(3, 4, 3)	(4, 2, 3)	**(4, 4, 3)**	(2, 2, 3)	(3, 2, 3)
5 yrs	Ruin R.	1.0000	0.9964	0.9929		1.0000		0.9703		1.0000	0.9930	0.9658	
LT	Strategy	(4, 2, 3)	**(4, 4, 3)**	(4, 2, 4)	(4, 4, 4)	**(4, 4, 3)**	(4, 2, 3)	(4, 2, 4)	(4, 4, 4)	**(4, 4, 3)**	(4, 2, 3)	(4, 2, 4)	(4, 4, 4)
20 yrs	Ruin R.	1.0000	0.9964	0.9912		1.0000	0.9986	0.9068		1.0000		0.8549	
LT	Strategy	(4, 2, 3)	**(4, 4, 3)**	(4, 2, 4)	(4, 4, 4)	**(4, 4, 3)**	(4, 2, 3)	(4, 2, 4)	(4, 4, 4)	**(4, 4, 3)**	(4, 2, 3)	(4, 2, 4)	(4, 4, 4)
100 yrs	Ruin R.	1.0000	0.9964	0.9912		1.0000	0.9986	0.9068		1.0000		0.8713	

Table 8: Top four *Index*(n) for n = 5, 20 and 100

Tail and Years		**High Frequency / Low Severity**				**Mid Frequency / Mid Severity**				**Low Frequency / High Severity**			
HT	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)
5 yrs	Index	0.8298	0.8285		0.7376	0.7290	0.7185		0.6542	0.7341	0.7215		0.6238
HT	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(3, 3, 3)	**(4, 3, 3)**	(4, 3, 4)	(4, 4, 3)	(3, 3, 3)
20 yrs	Index	0.8360	0.8194	0.8191	0.7416	0.7879	0.6877	0.6223	0.5707	0.6276	0.5385	0.5044	0.4843
HT	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 4, 3)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 4, 3)
100 yrs	Index	0.8344	0.8202	0.8180	0.7403	0.8115	0.7075	0.6373	0.5754	0.7514	0.6366	0.5497	0.5235
NT	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	(4, 4, 3)	(3, 4, 3)	(2, 4, 3)	**(4, 3, 3)**
5 yrs	Index	0.9468	0.9428		0.8580	0.7695	0.7492		0.7182	0.6846	0.6786		0.6457
NT	Strategy	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)	**(4, 3, 3)**	(4, 4, 3)	(4, 3, 4)	(4, 3, 5)
20 yrs	Index	0.9450	0.9375	0.9355	0.8561	0.8299	0.7738	0.7480	0.7118	0.7342	0.6796	0.6553	0.6162
NT	Strategy	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 4, 3)
100 yrs	Index	0.9463	0.9367	0.9351	0.8572	0.8358	0.7780	0.7514	0.7142	0.8358	0.7100	0.6607	0.6570
LT	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 1, 1)	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	(4, 2, 3)	(4, 4, 3)	(4, 2, 4)	(4, 4, 4)
5 yrs	Index	0.9001	0.8974		0.8902	0.6907	0.6900			0.6824	0.6770	0.6268	
LT	Strategy	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(4, 1, 1)	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**	(4, 2, 3)	(4, 4, 3)	(4, 2, 4)	(4, 4, 4)
20 yrs	Index	0.8980	0.8946	0.8907	0.8881	0.7061			0.7060	0.6746	0.6744	0.6436	
LT	Strategy	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(4, 1, 1)	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**	(4, 2, 3)	(4, 4, 3)	(4, 2, 4)	(4, 4, 4)
100 yrs	Index	0.9013	0.8940	0.8932	0.8914	0.6997			0.6996	0.6506	0.6504	0.6318	

5. Value at risk

In this section, we propose another measure for selecting an optimal strategy. First, we apply value at risk to determine the initial surplus U_0; then we calculate the rate of return, and select an optimal strategy which results in the highest rate of return.

Value at risk (VaR) has been widely used in finance and insurance as a risk measure. The value at risk of a random variable X at a confidence level $1-\alpha$, $0 < \alpha < 1$, is defined as $VaR(\alpha) = \inf\{t\text{: } S_X(t) \leqq \alpha\}$ = $100(1-\alpha)^{\text{th}}$ percentile of X where S_X is the survival function of X. Obviously, $VaR(\alpha)$ is non-increasing in α. In the continuous time surplus process, the ultimate ruin probability $\Psi(u) = S_Z(u)$, the survival function of the maximal aggregate loss Z. In our discrete time case, we similarly define

$$VaR_{\text{s}}(\alpha, n) = \inf\{u\text{: } \Psi_{\text{s}}(u, n) \leqq \alpha\}$$

for the confidence level $1-\alpha$, study period n and strategy $s \in S$. Given a specific level confidence $1-\alpha$ = 90% or 95% and a study period n = 5, 20 or 100 years, we can find a $VaR_{\text{s}}(\alpha, n)$ (initial surplus) for each strategy $s \in S$ such that the corresponding ruin probability $\Psi_{\text{s}}(u, n)$ is kept at 10% or 5%. Also, denote the total and annualized rates of return for initial surplus u and n years as

$$TRR_s(u,n) = \frac{\overline{G}_s(n)}{u} \tag{9}$$

and

$$ARR_s(u,n) = \sqrt[n]{\frac{\overline{U}(u,n)}{u}} - 1 = \sqrt[n]{TRR_s(u,n)+1} - 1, \tag{10}$$

respectively. Note that both $TRR_{\text{s}}(VaR_{\text{s}}(\alpha, n), n)$ and $ARR_{\text{s}}(VaR_{\text{s}}(\alpha, n), n)$ are increasing functions of α since $\overline{G}_s(n)$ is independent of u and $VaR_{\text{s}}(\alpha, n)$ is decreasing in α Then the optimal strategy is the one which yields the largest $TRR_{\text{s}}(VaR_{\text{s}}(\alpha, n), n)$ or $ARR_{\text{s}}(VaR_{\text{s}}(\alpha, n), n)$. That is, given α and n, we want $\underset{s \in S}{Max}\{\frac{U(\text{VaR}_{\text{s}}(\alpha,n),n) - \text{VaR}_{\text{s}}(\alpha,n)}{\text{VaR}_{\text{s}}(\alpha,n)}\}$ for each of nine combinations of tail, frequency and severity types.

From the simulations, we found that for α = 5%, 10%, n = 5, 20, 100, and each $s = (i, j, k) \in S$,

(A) $ARR_{\text{s}}(VaR_{\text{s}}(\alpha, n), n)$ is decreasing in n for most types of risks. Exceptions are that $ARR_{\text{s}}(VaR_{\text{s}}(5\%, 5), 5) < ARR_{\text{s}}(VaR_{\text{s}}(5\%, 20), 20)$ for seven strategies and HT/LF/HS risks.

(B) HF/LS and LF/HS risks produce the largest and smallest annualized rates of return (that is, ARR_{s}(LF/HS) < ARR_{s}(MF/MS) < ARR_{s}(HF/LS)), respectively, for all three tail types.

(B) LT and HT risks yield the biggest and smallest annualized rates of return (that is, ARR_{s}(HT) < ARR_{s}(NT) < ARR_{s}(LT)), respectively, for all three mixtures of frequency and severity except that ARR_{s}(NT) < ARR_{s}(HT) < ARR_{s}(LT) for s = (2, 3, 3), n = 100, α = 5%, 10% and LF/HS risks, and for s = (2, 4, 3), n = 100, α = 10% and LF/HS risks.

(D) summarizing statements (B) and (C) above, we have the following ARR ordering relationships for different risk attributes except the three cases mentioned in (C):

$$\begin{array}{ccccc} \text{HT / LF / HS} & < & \text{HT / MF / MS} & < & \text{HT / HF / LS} \\ \wedge & & \wedge & & \wedge \\ \text{NT / LF / HS} & < & \text{NT / MF / MS} & < & \text{NT / HF / LS} \\ \wedge & & \wedge & & \wedge \\ \text{LT / LF / HS} & < & \text{LT / MF / MS} & < & \text{LT / HF / LS} \end{array}$$

By the relationships above, we conclude that LT/HF/LS and HT/LF/HS risks lead to the largest and smallest annualized rates of return, respectively, among these nine risk attributes.

Tables 9 and 10 list the top four strategies based on the ARR measure for $\alpha = 5\%$ and 10%, respectively; and goves the corresponding $\overline{G}_s(n)$, U_0. No doubt, strategy (4, 3, 3) is the overall best for all cases. The rankings between Tables 8 and 10 are quite similar, which shows that Index measure is consistent with the ARR one for $\alpha = 10\%$. Note that we cannot compare strategies among these three mixtures of frequency/ severity risks due to different initial surplus scales. However, for ARR measure, we may conclude that strategies (2, 3, 3) and (4, 3, 3) yields the largest annualized rates of return for all cases (all strategies for n = 5, 20, 100, and for all tail types and all mixtures of frequency/severity risks) for $\alpha = 5\%$ and 10%, respectively. Surprisingly, the rates of return of the top four strategies for $n = 5$ and HF/LS risks are very high, ranging from 29.716% to 44.860% for $\alpha = 5\%$, and from 37.749% to 56.722% for $\alpha = 10\%$. Also, none of the top four strategies adopts a deductible (that is, there is no code 2 or 4 appearing in the second entry of all top four strategies) for HF/LS risks on which the insurers usually impose a deductible to eliminate small claims and reduce administration work. This situation occurs for MF/MS and LF/HS risks as well. In fact, strategies imposing a deductible tend to reduce the expected gains (see statement (A) of section 4), and therefore, do not rank very high.

Table 9: Top four ARR(u,n) for $\alpha = 5\%$ and n = 5, 20, 100

Tail and Years		**High Frequency / Low Severity**				**Mid Frequency / Mid Severity**				**Low Frequency / High Severity**			
HT 5 yrs	Strategy	(3, 3, 3)	(2, 3, 3)	**(4, 3, 3)**	(4, 3, 4)	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)
	Gains	42.24		42.14	44.68	41.55	41.57		43.84	37.39	37.35		38.82
	U_0	13.9			16.7	90.1	92.0		101.2	447.0			504.0
	ARR	32.234%		32.186%	29.716%	7.879%	7.744%		7.464%	1.620%	1.618%		1.495%
HT 20 yrs	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(3, 3, 3)	**(4, 3, 3)**	(4, 3, 4)	(3, 3, 3)	(4, 3, 5)
	Gains	168.02	167.29	167.22	177.84	168.57	178.51	184.70	170.42	163.22	172.43	171.04	177.75
	U_0	13.9			16.8	96.0	109.0	120.0	120.7	517.0	629.0	655.4	722.0
	ARR	13.722%	13.699%	13.697%	13.031%	5.200%	4.969%	4.769%	4.500%	1.381%	1.219%	1.166%	1.107%
HT 100 yrs	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(3, 3, 3)	**(4, 3, 3)**	(4, 3, 4)	(3, 3, 3)	(4, 3, 5)
	Gains	840.29	839.01	836.77	889.50	840.92	889.82	919.43	842.34	844.81	897.74	859.43	929.75
	U_0	13.9			16.8	96.0	110.4	120.1	121.1	554.0	682.0	725.3	806.0
	ARR	4.204%	4.203%	4.200%	4.069%	2.304%	2.228%	2.182%	2.096%	0.931%	0.844%	0.785%	0.770%
NT 5 yrs	Strategy	(2, 3, 3)	(3, 3, 3)	**(4, 3, 3)**	(2, 3, 4)[1]	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(2, 3, 4)[1]
	Gains	47.56		47.52	49.18	46.97	46.71		48.50	48.21	48.12		49.83
	U_0	14.4			16.1	92.3	94.6		98.3	415.9	417.0		450.7
	ARR	33.849%		33.830%	32.333%	8.576%	8.355%		8.351%	2.218%	2.208%		2.119%
NT 20 yrs	Strategy	(2, 3, 3)	(3, 3, 3)	**(4, 3, 3)**	(2, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)
	Gains	190.51	190.10	189.69	196.89	190.52	197.08	199.61	201.23	190.13	194.96	196.58	197.45
	U_0	14.4			16.1	97.0	103.3	109.5	115.6	523.0	583.0	624.0	678.0
	ARR	14.187%	14.176%	14.164%	13.789%	5.583%	5.482%	5.326%	5.170%	1.562%	1.453%	1.379%	1.286%
NT 100 yrs	Strategy	(2, 3, 3)	(3, 3, 3)	**(4, 3, 3)**	(2, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)
	Gains	952.79	951.18	950.76	984.07	950.17	981.96	993.69	1000.55	954.26	984.24	994.83	1001.15
	U_0	14.4			16.1	96.8	103.3	109.5	115.2	545.0	623.1	663.0	726.0
	ARR	4.295%	4.293%	4.293%	4.217%	2.410%	2.380%	2.337%	2.297%	1.017%	0.952%	0.921%	0.870%
LT 5 yrs	Strategy	(2, 3, 3)	(3, 3, 3)	(2, 1, 1)	(3, 1, 1)	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	(4, 2, 4)	(4, 4, 4)	(2, 2, 4)	(3, 2, 4)
	Gains	49.97		49.98		49.40	49.41	49.41	49.41	37.48	37.48	35.36	35.36
	U_0	9.3		9.3		72.2	77.3			244.0		247.0	
	ARR	44.860%		44.840%		10.395%	10.389%	10.389%	10.389%	2.899%	2.899%	2.712%	2.712%
LT 20 yrs	Strategy	(2, 3, 3)	(2, 1, 1)	(2, 3, 5)	(2, 3, 4)	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**
	Gains	200.95	200.98	200.98	200.98	200.62	200.62	200.62	200.59	200.23	200.23	200.23	200.21
	U_0	9.3		9.3		80.5			80.5	437.7			437.7
	ARR	16.879%	16.874%	16.874%	16.874%	6.454%	6.454%	6.454%	6.451%	1.901%	1.901%	1.901%	1.901%
LT 100 yrs	Strategy	(2, 3, 3)	(3, 3, 3)	(2, 3, 4)	(2, 1, 1)	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**
	Gains	1000.75	999.86	1000.91	1000.91	1000.41	1000.41	1000.41	1000.24	1001.64	1001.64	1001.64	1001.45
	U_0	9.3		9.3		80.5			80.6	438.7			438.8
	ARR	4.800%	4.800%	4.800%	4.800%	2.632%	2.632%	2.632%	2.630%	1.196%	1.196%	1.196%	1.196%

[1] tied with (3, 3, 4)

Table 10: Top four ARR(u,n) for α = 10% and n = 5, 20, 100

Years and Tail		**High Frequency / Low Severity**				**Mid Frequency / Mid Severity**				**Low Frequency / High Severity**			
HT 5 yrs	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(2, 3, 4)[1]	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)
	Gains	42.14	42.24		44.78	41.55	41.57		43.84	37.39	37.35		38.82
	U_0	9.4	9.5		11.3	72.9	74.8		85.3	346.0	352.0		396.0
	ARR	40.539%	40.351%		37.749%	9.440%	9.242%		8.648%	2.073%	2.037%		1.888%
HT 20 yrs	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(3, 3, 3)	**(4, 3, 3)**	(4, 3, 4)	(3, 3, 3)	(4, 3, 5)
	Gains	168.02	167.29	167.22	177.84	168.57	178.51	184.70	170.42	163.22	172.43	171.04	177.75
	U_0	9.4	9.5		11.3	80.1	93.7	104.7	98.6	469.5	538.0	555.6	607.0
	ARR	15.823%	15.735%	15.733%	15.124%	5.828%	5.477%	5.215%	5.147%	1.503%	1.400%	1.351%	1.292%
HT 100 yrs	Strategy	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(3, 3, 3)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(3, 3, 3)
	Gains	840.29	839.01	836.77	889.50	840.92	889.82	919.43	842.34	844.81	897.74	929.75	859.43
	U_0	9.4	9.5		11.3	80.6	94.0	104.8	99.9	491.0	585.0	674.0	648.3
	ARR	4.607%	4.594%	4.591%	4.475%	2.466%	2.376%	2.306%	2.269%	1.006%	0.934%	0.871%	0.848%
NT 5 yrs	Strategy	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(3, 3, 3)	(2, 3, 3)	(2, 3, 4)[1]
	Gains	47.52	47.56		49.12	46.97	46.71		48.50	48.21	48.12		49.83
	U_0	8.8	8.9		10.6	72.9	75.7		80.2	333.6	333.7		373.0
	ARR	44.955%	44.593%		41.284%	10.455%	10.088%	10.088%	9.921%	2.737%	2.731%		2.540%
NT 20 yrs	Strategy	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)
	Gains	189.69	190.51	190.10	196.00	190.52	197.08	199.61	201.23	190.13	194.96	196.58	197.45
	U_0	8.8	9.0		10.6	81.0	88.8	90.5	96.5	451.0	510.0	541.2	568.1
	ARR	16.859%	16.758%	16.746%	16.004%	6.235%	6.020%	5.998%	5.795%	1.774%	1.632%	1.561%	1.503%
NT 100 yrs	Strategy	**(4, 3, 3)**	(2, 3, 3)	(3, 3, 3)	(4, 3, 4)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)	**(4, 3, 3)**	(4, 3, 4)	(4, 3, 5)	(4, 1, 1)
	Gains	950.76	952.79	951.18	982.37	950.17	981.96	993.69	1000.55	954.26	984.24	994.83	1001.15
	U_0	8.8	9.0		10.6	80.4	88.7	90.4	96.5	471.0	540.0	571.0	626.0
	ARR	4.804%	4.782%	4.781%	4.644%	2.584%	2.522%	2.515%	2.461%	1.113%	1.043%	1.014%	0.960%
LT 5 yrs	Strategy	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**
	Gains	49.80	49.81	49.81	49.81	49.40	49.41	49.41	49.41	49.30	49.30	49.30	49.27
	U_0	5.9	6.0			63.3	63.5			293.0			
	ARR	56.722%	56.118%	56.118%	56.118%	12.230%	12.200%	12.200%	12.200%	3.159%	3.159%	3.159%	3.158%
LT 20 yrs	Strategy	**(4, 3, 3)**	(2, 3, 3)	(4, 1, 1)	(4, 3, 5)	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**
	Gains	199.78	200.95	199.81	199.81	200.62	200.62	200.62	200.59	200.23	200.23	200.23	200.21
	U_0	6.1	6.0	6.0		67.4			67.5	370.7			370.8
	ARR	19.343%	19.319%	19.315%	19.315%	7.143%	7.143%	7.143%	7.142%	2.183%	2.183%	2.183%	2.182%
LT 100 yrs	Strategy	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**	(4, 1, 1)	(4, 3, 5)	(4, 3, 4)	**(4, 3, 3)**
	Gains	999.32	999.49	999.49	999.49	1000.41	1000.41	1000.41	1000.24	1001.64	1001.64	1001.64	1001.45
	U_0	6.0	6.0			67.4			67.5	378.9			
	ARR	5.256%	5.251%	5.251%	5.251%	2.801%	2.801%	2.801%	2.800%	1.301%	1.301%	1.301%	1.301%

[1] tied with (3, 3, 4)

6. Conclusions

In this paper we apply the traditional/modified credibility methods and deductible/policy limit to the discrete time surplus process (single business line) for alternative premium schemes compared with constant premium. We mentioned in section 3 that strategy (4, 1, 1) (that is, cred3 method only) is better at reducing the ruin probability of strategy (1, 1, 1) than strategies (2, 1, 1) and (3, 1, 1). When deductible and/or policy limit are/is imposed, strategy (4, 4, 3) can further reduce the ruin probability significantly for most risk attributes. In section 4, we state that strategies with deductible imposed (that is, (*i*, *j*, *k*) for $j = 2, 4$) yield smaller gains than those without deductible imposed (that is, (*i*, 1, 1) and (*i*, 3, *k*)). Moreover, strategy (4, 3, 3) is overall the best based on an index measure. The overall good performance of strategy (4, 3, 3) is also verified by the ARR measure in section 5.

Table 11 gives the average ruins, $U_0(\alpha = 10\%)$ and associated ranks for strategies (1, 1, 1), (4, 1, 1), (4, 4, 3) and (4, 3, 3). The original strategy (1, 1, 1) gets very bad ranks for all risks, and strategy (4, 1, 1) can improve some but not much for HT, NT/LF/HS (5 years), LT/HS/LS and LT (5 years) risks. For all MF/MS and LF/HS risks, strategy (4, 4, 3) ranks the best for almost all cases; while for HT/HF/LS and NT/HF/LS risks, strategy (4, 3, 3) is the top one except in one case. However, from the viewpoint of maximizing gains, strategies (1, 1, 1) and (4, 1, 1) are much better than (4, 4, 3) and (4, 3, 3). Strategy (4, 4, 3) ranks best for minimizing the initial surplus or the ruin probability for almost all cases but does poorly with the index and the ARR measure. Strategy (1, 1, 1) got very bad rankings for producing small ruin probability for all risks but is the best in some cases. See Table 12 for more details. To consider maximization of gain and minimization of ruin probability (or initial surplus), we introduce an index and an ARR measure in sections 4 and 5, respectively. Table 13 illustrates Indices, ARRs ($\alpha = 10\%$) and associated ranks for these four strategies. Strategy (1, 1, 1) cannot get good rankings because it tends to produce relatively high ruin probabilities. Strategy (4, 1, 1) reaches better ranks for NT and LT risks. Strategy (4, 4, 3) ranks in the top four for only few cases due to its small gains for all cases. Finally, strategy (4, 3, 3) catches the first place for almost all cases because it does relatively well on average gains and average ruin probabilities (or initial surpluses).

The schemes we have proposed can be applied by property and casualty insurers in a variety of business lines with individual claims following specific loss distributions. The insurer first identifies the risk attributes of the nine combinations of tail types, frequency and severity that best corresponds to its line of business; then decides which strategy should be adopted based on the maximization of gain, the minimization of ruin probability or both. The resulting surplus process should help the insurer achieve its long-term goals.

Table 11: Average ruins, U_0 (α = 10%) and associated ranks for four strategies

Years and Tail		**High Frequency / Low Severity**				**Mid Frequency / Mid Severity**				**Low Frequency / High Severity**			
Strategy		(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)	(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)	(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)
HT 5 yrs	Avg ruin	26.30%	24.92%	11.18%	10.15%	21.53%	20.37%	9.05%	10.46%	7.22%	6.56%	2.85%	3.65%
	and rank	28	25	4	1	40	34	1	4	40	36	1	5
	U_0 for 10%	30.0	26.8	10.1	9.4	167.0	146.0	66.6	72.9	545.0	509.0	306.1	346.0
	and rank	34	25	4	1	40	34	1	4	40	38	3	4
HT 20 yrs	Avg ruin	29.78%	26.40%	11.23%	10.15%	37.25%	32.86%	12.35%	12.99%	16.93%	14.62%	6.22%	7.57%
	and rank	28	25	4	1	29	25	1	2	39	28	1	2
	U_0 for 10%	37.6	28.0	10.4	9.4	307.8	213.0	76.7	80.1	1133.0	882.0	429.0	469.5
	and rank	37	25	4	1	37	25	1	2	40	27	1	2
HT 100 yrs	Avg ruin	29.88%	26.40%	11.23%	10.15%	45.97%	36.75%	12.55%	13.00%	34.39%	26.20%	8.92%	9.04%
	and rank	28	25	4	1	33	25	1	2	36	18	1	2
	U_0 for 10%	38.1	28.0	10.1	9.4	452.0	232.6	76.9	80.6	2429.0	1494	509.1	491.0
	and rank	37	25	4	1	37	24	1	2	40	23	2	1
NT 5 yrs	Avg ruin	15.29%	12.15%	10.54%	9.67%	15.60%	13.51%	9.17%	11.28%	5.16%	4.50%	2.44%	3.56%
	and rank	35	14	4	1	40	30	1	5	40	35	1	11
	U_0 for 10%	15.3	11.2	9.6	8.8	112.3	86.4	64.2	72.9	478.0	401.0	290.3	333.6
	and rank	40	16	4	1	40	29	1	4	40	32	1	7
NT 20 yrs	Avg ruin	16.42%	12.17%	10.61%	9.67%	25.09%	16.75%	11.96%	13.60%	13.47%	9.58%	5.70%	7.46%
	and rank	35	13	4	1	39	11	1	2	38	13	1	2
	U_0 for 10%	15.9	11.2	8.8	9.7	169.6	96.5	72.2	81.0	887.0	568.1	411.4	451.0
	and rank	36	16	1	4	40	12	1	2	40	13	1	2
NT 100 yrs	Avg ruin	16.42%	12.17%	10.61%	9.67%	30.27%	16.76%	12.05%	13.60%	27.88%	11.70%	7.35%	8.54%
	and rank	35	13	4	1	34	10	1	2	34	10	1	2
	U_0 for 10%	15.9	11.2	9.7	8.8	210.1	96.5	72.2	80.4	1703.0	626.0	452.1	471.0
	and rank	36	14	4	1	40	11	1	2	40	8	1	2
LT 5 yrs	Avg ruin	8.33%	6.28%	5.63%	6.21%	10.81%	8.98%	6.21%	8.97%	3.30%	2.74%	1.42%	2.74%
	and rank	38	22	2	19	38	22	1	21	37	25	2	25
	U_0 for 10%	7.8	6.0	5.6	5.9	82.2	63.5	50.3	63.3	350.0	293.0	226.2	293.0
	and rank	37	20	7	19	37	20	1	19	37	23	1	23
LT 20 yrs	Avg ruin	8.62%	6.28%	5.63%	6.21%	17.80%	10.32%	7.30%	10.32%	9.71%	5.53%	3.24%	5.52%
	and rank	38	22	2	19	38	7	1	10	33	10	1	9
	U_0 for 10%	8.0	6.0	5.6	6.0	126.3	67.4	53.0	67.5	681.8	370.7	290.0	370.8
	and rank	37	20	7	19	37	9	2	12	37	7	1	10
LT 100 yrs	Avg ruin	8.62%	6.28%	5.63%	6.21%	20.17%	10.32%	7.30%	10.32%	20.12%	6.16%	3.79%	6.14%
	and rank	38	22	2	19	38	7	1	7	33	8	1	7
	U_0 for 10%	8.0	6.0	5.6	6.0	141.0	67.4	53.0	67.5	1177.0	378.9	305.3	378.9
	and rank	38	20	7	19	37	9	2	12	37	7	1	7

Table 12: Average gains and associated ranks for four strategies

Years and Tail		**High Frequency / Low Severity**				**Mid Frequency / Mid Severity**				**Low Frequency / High Severity**			
Strategy		(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)	(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)	(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)
HT 5 yrs	Avg gain	50.78	50.39	29.02	42.14	49.31	48.83	28.18	41.55	29.33	34.06	23.23	37.39
	and rank	1	4	40	16	1	2	39	15	19	10	29	7
HT 20 yrs	Avg gain	200.99	200.12	115.22	168.02	198.34	200.30	115.56	168.57	198.88	197.88	107.80	163.22
	and rank	1	2	38	14	4	3	40	16	3	4	39	19
HT 100 yrs	Avg gain	1007.08	1002.95	575.31	840.29	991.77	997.75	575.63	840.92	1109.33	1051.44	580.69	844.81
	and rank	1	2	38	14	4	3	39	15	1	3	40	23
NT 5 yrs	Avg gain	49.56	50.11	33.24	47.52	49.62	49.30	32.99	46.97	50.95	50.48	35.04	48.21
	and rank	7	3	39	15	1	3	38	14	3	9	40	14
NT 20 yrs	Avg gain	199.10	199.75	132.88	189.69	201.67	201.23	133.78	190.52	197.19	197.45	134.46	190.13
	and rank	5	3	39	15	3	4	39	14	5	3	38	14
NT 100 yrs	Avg gain	1004.73	1000.83	667.60	950.76	1002.82	1000.55	667.00	950.17	1005.89	1001.15	671.36	954.26
	and rank	1	4	40	16	2	4	39	14	3	5	38	16
LT 5 yrs	Avg gain	49.51	49.81	33.56	49.80	48.55	49.41	33.29	49.40	47.38	49.30	33.61	49.27
	and rank	13	9	38	12	14	9	38	12	5	1	34	4
LT 20 yrs	Avg gain	199.30	199.81	134.99	199.78	194.21	200.62	135.20	200.59	192.41	200.23	135.04	200.21
	and rank	13	9	38	12	14	5	38	8	13	1	34	4
LT 100 yrs	Avg gain	1001.63	999.49	675.57	999.32	987.60	1000.41	675.41	1000.24	1014.90	1001.64	678.08	1001.45
	and rank	1	13	40	16	14	9	38	12	5	13	40	16

Table 13: Indices, ARR (α = 10%) and associated ranks for four strategies

Years and Tail		**High Frequency / Low Severity**				**Mid Frequency / Mid Severity**				**Low Frequency / High Severity**			
Strategy		(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)	(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)	(1, 1, 1)	(4, 1, 1)	(4, 4, 3)	(4, 3, 3)
HT 5 yrs	Index	0.3859	0.4042	0.5189	0.8298	0.4203	0.4400	0.5714	0.7290	0.2911	0.3721	0.5841	0.7341
	and rank	28	25	13	1	27	23	10	1	30	22	9	1
	APP for 10%	21.903%	23.562%	31.106%	40.539%	5.310%	5.940%	7.311%	9.440%	1.054%	1.304%	1.474%	2.073%
	and rank	28	27	13	1	28	22	10	1	26	22	15	1
HT 20 yrs	Index	0.3408	0.3828	0.5182	0.8360	0.3233	0.3701	0.5681	0.7879	0.3419	0.3939	0.5044	0.6276
	and rank	28	22	13	1	25	21	5	1	24	14	3	1
	APP for 10%	9.679%	11.058%	13.419%	15.823%	2.518%	3.370%	4.702%	5.828%	0.812%	1.017%	1.127%	1.503%
	and rank	28	25	12	1	34	22	8	1	31	19	10	1
HT 100 yrs	Index	0.3397	0.3829	0.5163	0.8344	0.2707	0.3406	0.5754	0.8115	0.2594	0.3227	0.5235	0.7514
	and rank	28	22	13	1	23	19	4	1	20	13	4	1
	APP for 10%	3.367%	3.672%	4.143%	4.607%	1.168%	1.680%	2.161%	2.466%	0.377%	0.534%	0.764%	1.006%
	and rank	28	25	12	1	36	21	6	1	35	16	7	1
NT 5 yrs	Index	0.6246	0.7946	0.6077	0.9468	0.5878	0.6743	0.6647	0.7695	0.4707	0.5348	0.6846	0.6457
	and rank	17	10	20	1	25	10	11	1	36	20	1	4

	APP for 10%	33.495%	40.495%	34.872%	44.955%	7.593%	9.449%	8.646%	10.455%	2.046%	2.400%	2.305%	2.737%
	and rank	28	12	21	1	34	10	13	1	34	12	16	1
NT 20 yrs	Index	0.5842	0.7908	0.6034	0.9450	0.4762	0.7118	0.6627	0.8299	0.4217	0.5937	0.6796	0.7342
	and rank	25	10	18	1	23	4	5	1	24	5	2	1
	APP for 10%	13.911%	15.811%	14.395%	16.859%	3.995%	5.795%	5.381%	6.235%	1.009%	1.503%	1.424%	1.774%
	and rank	28	10	18	1	34	4	6	1	34	4	5	1
NT 100 yrs	Index	0.5889	0.7915	0.6056	0.9463	0.3963	0.7142	0.6622	0.8358	0.2595	0.6154	0.6570	0.8037
	and rank	22	10	17	1	27	4	5	1	24	5	4	1
	APP for 10%	4.250%	4.607%	4.337%	4.804%	1.769%	2.461%	2.353%	2.584%	0.465%	0.960%	0.914%	1.113%
	and rank	28	10	18	1	34	4	6	1	34	4	6	1
LT 5 yrs	Index	0.6671	0.8902	0.6690	0.9001	0.5631	0.6900	0.6723	0.6907	0.4107	0.5146	0.6770	0.5143
	and rank	30	4	27	1	33	2	10	1	37	19	2	22
	APP for 10%	49.012%	56.118%	47.500%	56.722%	9.726%	12.200%	10.693%	12.230%	2.572%	3.159%	2.810%	3.158%
	and rank	25	2	31	1	32	2	26	1	25	1	15	4
LT 20 yrs	Index	0.6454	0.8881	0.6693	0.8980	0.3963	0.7061	0.6727	0.7060	0.3206	0.5859	0.6744	0.5869
	and rank	32	4	26	1	32	1	9	4	27	8	2	7
	APP for 10%	17.673%	19.315%	17.477%	19.343%	4.766%	7.143%	6.541%	7.142%	1.251%	2.183%	1.930%	2.182%
	and rank	25	3	31	1	32	1	10	4	31	1	10	4
LT 100 yrs	Index	0.6508	0.8914	0.6721	0.9013	0.3534	0.6997	0.6679	0.6996	0.1834	0.5911	0.6504	0.5929
	and rank	32	4	26	1	26	1	9	4	21	8	2	7
	APP for 10%	4.957%	5.251%	4.916%	5.256%	2.102%	2.801%	2.656%	2.800%	0.624%	1.301%	1.177%	1.301%
	and rank	26	2	30	1	32	1	10	4	25	1	9	4

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