Optimal reinsurance for variance related premium calculation principles

Topic 1 - Risk Management of an Insurance Enterprise: Risk Transfer

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Abstract: In this paper we compare stop loss reinsurance with the optimal form of reinsurance from the ceding company point of view, when the cedent seeks to maximize the adjustment coefficient of the retained risk and the reinsurance loading is an increasing function of the variance.

We arrive to the conclusion that the optimal arrangement can provide a significant improvement in the adjustment coefficient when compared to the best stop loss treaty.

Keywords: adjustment coefficient, optimal reinsurance, stop loss, standard deviation premium principle, variance premium principle

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1 Assumptions and Preliminaries

This paper deals with optimal reinsurance when the insurer seeks to maximize the adjustment coefficient of the retained risk and the reinsurer prices the reinsurance premium using as loading an increasing function of the variance, namely the variance principle or the standard deviation principle.

Let Y be a non-negative random variable, representing the annual aggregate claims and let us assume that aggregate claims over consecutive periods are i.i.d. random variables. We assume that Y is a continuous random variable, with density function f, and that $E[Y^2] < +\infty$. Let c, $c > E[Y]$, be the corresponding premium income, net of expenses. $Z : [0, +\infty) \longrightarrow [0, +\infty)$ identifies a reinsurance policy. The set of all possible reinsurance programmes is:

$$\mathcal{Z} = \{Z : [0, +\infty[\mapsto \mathbb{R} |\ Z \text{ is measurable and } 0 \leq Z(y) \leq y,\ \forall y \geq 0\}.$$

We do not distinguish between functions which differ only on a set of zero probability. i.e., two measurable functions, ϕ and ϕ' are considered to be the same whenever $\Pr\{\phi(Y) = \phi'(Y)\} = 1$. Similarly, a measurable function, Z, is an element of $\mathcal{Z}$ whenever $\Pr\{0 \leq Z(Y) \leq Y\} = 1$.

For a given reinsurance policy, $Z \in \mathcal{Z}$, the reinsurer charges a premium $P[Z]$ of the type

$$P[Z] = E[Z] + g(Var[Z]), \tag{1}$$

where $g : [0, +\infty[\mapsto [0, +\infty[$ is a function smooth in $]0, +\infty[$ such that $g(0) = 0$ and $g'(x) > 0,\ \forall x \in]0, +\infty[$. Further we assume that P is a convex functional. We call premium calculation principles of this type "variance related principles". The variance principle and the standard deviation principle are both under these conditions, with $g(x) = \beta x$ and $g(x) = \beta x^{1/2}$, $\beta > 0$, respectively (see Deprez and Gerber (1985) to check the convexity of these principles).

The net profit, after reinsurance, is

$$L_Z = c - P[Z] - (Y - Z(Y)).$$

We assume that P is such that

$$\Pr\{L_Z < 0\} > 0, \forall Z \in \mathcal{Z}, \tag{2}$$

otherwise there would exist a policy under which the risk of ruin would be zero. For the variance principle this requires that the inequality

$$\beta > (c - E[Y])/Var[Y] \tag{3}$$

holds. In the standard deviation principle case the required condition is

$$\beta > (c - E[Y])/\sqrt{Var[Y]}. \tag{4}$$

Consider the map $G : \mathbb{R} \times \mathcal{Z} \mapsto [0, +\infty]$, defined by

$$G(R, Z) = \int_0^{+\infty} e^{-RL_Z(y)} f(y)\ dy, \qquad R \in \mathbb{R},\ Z \in \mathcal{Z}. \tag{5}$$

Let R_Z denote the adjustment coefficient of the retained risk for a particular reinsurance policy, $Z \in \mathcal{Z}$. R_Z is defined as the strictly positive value of R which solves the equation

$$G(R, Z) = 1, \tag{6}$$

for that particular Z, when such a root exists. Equation (6) can not have more than one positive solution. This means the map $Z \mapsto R_Z$ is a well defined functional in the set

$$\mathcal{Z}^+ = \{Z \in \mathcal{Z} :\ (6)\ \text{admits a positive solution}\}.$$

Guerra and Centeno (2007) have studied the problem of determining the optimal reinsurance policy in such a way that the adjustment coefficient is maximized. The results were given for general convex premium principles and particularized for variance related principles. The results were proved by relating the adjustment coefficient criterion with the expected utility of wealth, for an exponential utility function. It was proved that the type of reinsurance arrangement that maximizes the expected utility of wealth, for the exponential utility, is the same type that maximizes the adjustment coefficient and vice versa. Further, the optimal policies for both problems coincide when the risk aversion coefficient is equal to the to the adjustment coefficient of the retained risk. For example, if for a given premium functional P, stop loss maximizes the adjustment coefficient (which will be the case when P is calculated according to the expected value principle), then stop loss is also optimal for the expected utility problem, and vice-versa. The retention limit on the expected utility problem will depend of course on the risk aversion coefficient of the exponential utility function. When

the risk aversion coefficient equals the adjustment coefficient of the retained risk, then that particular adjustment coefficient is maximal and the same retention limit maximizes the expected utility and the adjustment coefficient. In the following section we summarize the results obtained for variance related premium principles with respect to the adjustment coefficient.

2 Optimal reinsurance policies for variance related problems

The following theorem, which proof can be seen in Guerra and Centeno (2007), provides the solution, under the assumptions made on Section 1, to the following problem:

Problem 1 *Find* $\left(\hat{R}, \hat{Z}\right) \in]0, +\infty[\times \mathcal{Z}^+$ *such that*

$$\hat{R} = R_{\hat{Z}} = \max\left\{R_Z : Z \in \mathcal{Z}^+\right\}.\square$$

In what follows $\nu \in [0, +\infty)$ denotes the number

$$\nu = \sup\{y : \Pr\{Y \leq y\} = 0\}.$$

Theorem 1 *A solution to Problem 1 always exists.*

a) When g' is a bounded function in a neighborhood of zero, the adjustment coefficient of the retained aggregate claims is maximized when $Z(y)$ satisfies

$$y = Z(y) + \frac{1}{R}\ln\frac{Z(y)+\alpha}{\alpha}, \tag{7}$$

where α is a positive solution to

$$h(\alpha) = 0, \tag{8}$$

with

$$h(\alpha) = \alpha + E[Z] - \frac{1}{2g'(Var[Z])}. \tag{9}$$

and R is the only root to equation (6).

When g' is unbounded in any neighborhood of zero, if no contract of the type (7) is optimal then $Z(y) = 0$, $\forall y$ (no reinsurance at all) is optimal.

b) If $\nu = 0$, the solution is unique. If $\nu > 0$ the optimal solution to the problem is not unique, but they are all of the form $Z(y) + x$, where $Z(y)$ is the solution described in a) and x is any constant such that $-Z(\nu) \leq x \leq \nu - Z(\nu)$. $\square$

Note that when $\nu > 0$ the optimal solutions differ by a constant, which implies that they will provide the same profit (L_Z will be constant, since $P[Z + x] = P[Z] + x$), and hence indifferent from the economic point of view.

Let us define the functions

$$\Phi_k(R, \alpha) = \int_0^{+\infty} (1 + R(Z(y) + \alpha))^k f(y) \; dy, \qquad k \in \mathbb{Z}, \tag{10}$$

where $Z(y)$ is such that (7) holds for the particular (R, α) indicated. These functions are useful for the proofs of the properties that follow. We have assumed that $E[Y^2] < +\infty$, which implies that Φ_k is finite for all $k \leq 2$, $\alpha > 0$, $R > 0$.

Proposition 1 *For any $R > 0$, $\alpha > 0$, the expected value and the variance of Z, when Z is such that (7) holds, can be calculated by*

$$E[Z] = \frac{1}{R}(\Phi_1 - (1 + R\alpha)), \tag{11}$$

$$Var[Z] = \frac{1}{R^2}\left(\Phi_2 - \Phi_1^2\right). \; \square \tag{12}$$

Proof. Let $\phi = 1 + R(Z + \alpha)$. Then,

$$Z = \frac{1}{R}(1 + R(Z + \alpha) - R\alpha - 1) = \frac{1}{R}(\phi - (1 + R\alpha)).$$

Therefore,

$$\begin{aligned} E[Z] &= \int_0^{+\infty} Z(y) f(y) \; dy = \\ &= \frac{1}{R}\int_0^{+\infty} (\phi - (1 + R\alpha)) f(y) \; dy = \frac{1}{R}(\Phi_1 - (1 + R\alpha)) \end{aligned}$$

and

$$\begin{aligned} E\left[Z^2\right] &= \int_0^{+\infty} Z\left(y\right)^2 f\left(y\right)\ dy = \\ &= \int_0^{+\infty} \left(\frac{\phi - (1+R\alpha)}{R}\right)^2 f\left(y\right)\ dy = \\ &= \frac{1}{R^2}\left(\Phi_2 - 2\left(1+R\alpha\right)\Phi_1 + \left(1+R\alpha\right)^2\right). \end{aligned}$$

Hence

$$Var[Z] = E\left[Z^2\right] - E^2\left[Z\right] = \frac{1}{R^2}\left(\Phi_2 - \Phi_1^2\right).$$

■

Proposition 2

$$\frac{\partial \Phi_k}{\partial \alpha} = k\left(\frac{1}{\alpha} + R\right)\left(\Phi_{k-1} - \Phi_{k-2}\right).\square \tag{13}$$

Proof. From (7) it follows that

$$\frac{\partial Z(y)}{\partial \alpha} = \frac{Z(y)}{\alpha(1+R(Z(y)+\alpha)} = \frac{1}{\alpha R} - \frac{1+\alpha R}{\alpha R}\frac{1}{1+R(Z(y)+\alpha)}.$$

Then,

$$\begin{aligned} \frac{\partial \Phi_k}{\partial \alpha} &= \int_0^{+\infty} k(1+R(Z(y)+\alpha)^{k-1} R\left(\frac{\partial Z}{\partial \alpha} + 1\right) f(y)dy = \\ &= \frac{k(1+\alpha R)}{\alpha}\int_0^{+\infty}\left((1+R(Z(y)+\alpha))^{k-1} - (1+R(Z(y)+\alpha))^{k-2}\right) f(y)dy, \end{aligned}$$

from where follows (13). ■

Propositions 1 and 2 allow us to state the following proposition:

Proposition 3

$$\frac{\partial E\left[Z\right]}{\partial \alpha} = \frac{1}{R\alpha} - \frac{1+R\alpha}{R\alpha}\Phi_{-1}, \tag{14}$$

$$\frac{\partial Var\left[Z\right]}{\partial \alpha} = \frac{2(1+R\alpha)}{R^2\alpha}\left(\Phi_1\Phi_{-1} - 1\right).\ \square \tag{15}$$

Theorem 1 leaves some ambiguity about the number of roots of equation (8). We will show next that this equation has at most one solution. For the proof we will use the following Property, which follows easily from Theorems 5 and 6 in Deprez and Gerber (1985).

Proposition 4 *Assume that g is twice differentiable. $P[Z] = E[Z] + g(Var[Z])$ is a convex functional if and only if*

$$\frac{g''(x)}{g'(x)} \geq -\frac{1}{2x}, \ \forall x > 0. \square \tag{16}$$

Note that (16) holds as an equality for the standard deviation principle and that the left hand side is zero for the variance principle.

Proposition 5 *For any $R > 0$, consider $h(\alpha)$ given by (9). Then $\lim_{\alpha \to +\infty} h(\alpha) = +\infty$ and equation (8) has at most one positive solution. Let $\widehat{\alpha}$ be the root of (8) (assuming it exists). Then $h(\alpha) < 0$, $\forall\ \alpha \in]0, \widehat{\alpha}[$ and $h(\alpha) > 0$, $\forall\ \alpha \in]\widehat{\alpha}, +\infty[$. $\square$*

Proof. Throughout this proof we consider $Z(y)$ defined by (7). Notice that $\lim_{\alpha \to +\infty} Z(y) = y$, $\forall y > 0$. Therefore $\lim_{\alpha \to +\infty} h(\alpha) = +\infty$.
Differentiating $h(\alpha)$, for $\alpha > 0$, we get

$$h'(\alpha) = 1 + \frac{\partial E[Z]}{\partial \alpha} + \frac{1}{2} \frac{g''(Var[Z])}{(g'(Var[Z]))^2} \frac{\partial Var[Z]}{\partial \alpha}. \tag{17}$$

At a point where $h(\alpha) = 0$, we must have

$$\frac{1}{2} = (E[Z] + \alpha) g'(Var[Z]) \tag{18}$$

and hence

$$h'(\alpha)|_{h(\alpha)=0} = 1 + \frac{\partial E[Z]}{\partial \alpha} + (E[Z] + \alpha) \frac{g''(Var[Z]}{g'(Var[Z])} \frac{\partial Var[Z]}{\partial \alpha}. \tag{19}$$

Noticing that $E[Z] + \alpha$ and $\partial Var[Z]/\partial \alpha$ (given by (15)) are positive and using Proposition 4 we have that

$$h'(\alpha)|_{h(\alpha)=0} \geq 1 + \frac{\partial E[Z]}{\partial \alpha} - \frac{(E[Z] + \alpha)}{2Var[Z]} \frac{\partial Var[Z]}{\partial \alpha}. \tag{20}$$

Now, using (11), (12), (14) and (15) we get

$$\begin{aligned}
h'(\alpha)|_{h(\alpha)=0} &\geq \left(1+\frac{1}{R\alpha}\right)(1-\Phi_{-1}) - \frac{E[Z]+\alpha}{\Phi_2-\Phi_1^2}\left(\frac{1}{\alpha}+R\right)(\Phi_1\Phi_{-1}-1) = \\
&= \left(1+\frac{1}{R\alpha}\right)(1-\Phi_{-1}) - \frac{\frac{1}{R}(\Phi_1-1)}{\Phi_2-\Phi_1^2}\left(\frac{1}{\alpha}+R\right)(\Phi_1\Phi_{-1}-1) = \\
&= \left(1+\frac{1}{R\alpha}\right)\left(1-\Phi_{-1}-\frac{(\Phi_1-1)(\Phi_1\Phi_{-1}-1)}{\Phi_2-\Phi_1^2}\right) = \\
&= \frac{1+R\alpha}{R\alpha(\Phi_2-\Phi_1^2)}\left((1-\Phi_{-1})(\Phi_2-\Phi_1^2)-(\Phi_1-1)(\Phi_1\Phi_{-1}-1)\right) = \\
&= \frac{1+R\alpha}{R\alpha(\Phi_2-\Phi_1^2)}\left((\Phi_2-\Phi_1)(1-\Phi_{-1})-(\Phi_1-1)^2\right).
\end{aligned}$$

Noticing that

$$\begin{aligned}
\Phi_2-\Phi_1 &= \int_0^{+\infty} R(Z(y)+\alpha)(1+R(Z(y)+\alpha))f(y)\ dy, \\
1-\Phi_{-1} &= \int_0^{+\infty} \frac{R(Z(y)+\alpha)}{1+R(Z(y)+\alpha)}f(y)dy, \\
\Phi_1-1 &= \int_0^{+\infty} R(Z(y)+\alpha)f(y)\ dy = \\
&= \int_0^{+\infty} \sqrt{R(Z(y)+\alpha)(1+R(Z(y)+\alpha))}\sqrt{\frac{R(Z(y)+\alpha)}{(1+R(Z(y)+\alpha))}}\ f(y)\,dy,
\end{aligned}$$

and recalling that the Cauchy–Schwarz inequality states that

$$E^2[X_1X_2] \leq E[X_1^2]E[X_2^2],$$

holds for any random variables such that $E[X_1^2] < +\infty$ and $E[X_2^2] < +\infty$, with strict inequality when X_1, X_2 are linearly independent, we conclude that $h'(\alpha)|_{h(\alpha)=0} > 0$. Hence there exists at most a positive solution to equation (8). Let it be $\widehat{\alpha}$, when it exists. Since $h'(\hat{\alpha}) > 0$,we must have $h(\alpha) < 0$, $\forall\ \alpha \in]0, \widehat{\alpha}[$ and $h(\alpha) > 0$, $\forall\ \alpha \in]\widehat{\alpha}, +\infty[$. ■

3 Numerical Calculation of the Optimal Solution

The following propositions allow us to compute all the necessary functions to solve the problem. First we show that Φ_k can be given an explicit form thought Z is given only in implicit form (7).

Proposition 6 *Φ_k can be represented as the integral :*

$$\Phi_k(R,\alpha) = \frac{1}{R}\int_0^{+\infty} \frac{(1+R(\zeta+\alpha))^{k+1}}{\zeta+\alpha} f\left(\zeta + \frac{1}{R}\ln\frac{\zeta+\alpha}{\alpha}\right)\, d\zeta. \ \square \tag{21}$$

Proof. Using the change of variable $y = \zeta + \frac{1}{R}\ln\frac{\zeta+\alpha}{\alpha}$, $\zeta \in [0,+\infty[$, we obtain

$$\begin{aligned}
\Phi_k &= \int_0^{+\infty} (1+R(Z(y)+\alpha))^k f(y)\ dy = \\
&= \int_0^{+\infty} (1+R(\zeta+\alpha))^k f\left(\zeta+\frac{1}{R}\ln\frac{\zeta+\alpha}{\alpha}\right)\left(1+\frac{1}{R(\zeta+\alpha)}\right)\ d\zeta = \\
&= \frac{1}{R}\int_0^{+\infty} \frac{(1+R(\zeta+\alpha))^{k+1}}{\zeta+\alpha} f\left(\zeta+\frac{1}{R}\ln\frac{\zeta+\alpha}{\alpha}\right)\ d\zeta.
\end{aligned}$$

■

Let $G(R,\alpha)$ be defined as $G(R,Z)$ with Z satisfying (7) for that particular (R,α). We can also calculate $G(R,\alpha)$ easily.

Proposition 7 *$G(R,\alpha)$ can be computed by*

$$G(R,\alpha) = \frac{1}{\alpha}(E[Z]+\alpha)\, e^{R(P[Z]-c)}. \ \square$$

Proof.

$$\begin{aligned}
G(R,\alpha) &= e^{R(P[Z]-c)} \int_0^{+\infty} e^{R(y-Z(y))} f(y) dy = \\
&= e^{R(P[Z]-c)} \int_0^{+\infty} \frac{Z(y)+\alpha}{\alpha} dy = \\
&= \frac{1}{\alpha}(E[Z]+\alpha)\, e^{R(P[Z]-c)}.
\end{aligned}$$

■

Propositions 3, 6 and 7 together with Propositions 5, 6 and 7 can be used to calculate the optimal solution when the premium follows the variance principle or the standard deviation principle. Summarizing:

1. When the premium is the variance premium principle, i.e. when $P[Z] = E[Z] + \beta Var[Z]$, the adjustment coefficient of the retained aggregate claims is maximized when $(\widehat{Z}(y), \widehat{R}, \widehat{\alpha}) = (Z(y), R, \alpha)$ is the only solution to

$$\begin{cases} y = Z(y) + \frac{1}{R} \ln \frac{Z(y)+\alpha}{\alpha}, \forall y > 0, \\ \alpha = \frac{1}{2\beta} e^{R(P[Z]-c)}, \\ \alpha = \frac{1}{2\beta} - E[Z], \end{cases}$$

with $E[Z]$, $Var[Z]$ computed by (11), (12) respectively.

2. When the premium follows the standard deviation principle, i.e. when $P[Z] = E[Z] + \beta\sqrt{Var[Z]}$,

   (a) if $\exists \alpha > 0 : h(\alpha) < 0$, with

   $$h(\alpha) = \alpha + E[Z] - \frac{\sqrt{Var[Z]}}{\beta}, \tag{22}$$

   the adjustment coefficient of the retained aggregate claims is maximized when $(\widehat{Z}(y), \widehat{R}, \widehat{\alpha}) = (Z(y), R, \alpha)$ is the only solution to

   $$\begin{cases} y = Z(y) + \frac{1}{R} \ln \frac{Z(y)+\alpha}{\alpha}, \forall y > 0, \\ \alpha = (E[Z] + \alpha)\, e^{R(P[Z]-c)}, \\ \alpha = \frac{\sqrt{Var[Z]}}{\beta} - E[Z], \end{cases}$$

   with $E[Z]$, $Var[Z]$ computed by (11), (12) respectively;

   (b) if $h(\alpha) \geq 0$, $\forall \alpha > 0$, with $h(\alpha)$ given by (22), then the adjustment coefficient of the retained aggregate claims is maximized when $\widehat{Z}(y) = 0$, $\forall y$ (in this case $\widehat{R}$ is the adjustment coefficient associated to the gross claim amount.

3. If $\nu = 0$, the solution to the problem is unique. If $\nu > 0$ the optimal solution to the problem is not unique, but they are all of the form $\widehat{Z}(y) + x$, with x constant such that $-\widehat{Z}(\nu) \leq x \leq \nu - \widehat{Z}(\nu)$.

4 Examples

In this section we give two examples for the standard deviation principle. In the first example we consider that Y follows a Pareto distribution. In the second example we consider a

generalized gamma distribution. The parameters of these distributions where chosen such that $E[Y] = 1$ and both distributions have the same variance (which was set to $Var[Y] = \frac{16}{5}$, for convenience of the choice of parameters). Notice that thought they have the same mean and variance, the tails of the two distributions are rather different. However, none of them has a moment generating function defined in any neighbourhood of the origin. Hence the optimal solution must always be different than no reinsurance.

In both examples we consider the same premium income $c = 1.2$ and the same loading coefficient $\beta = 0.25$.

Example 1 *We consider that Y follows the Pareto distribution*

$$f(y) = \frac{32 \times 21^{32/11}}{(21 + 11y)^{43/11}}, \qquad y > 0.$$

The first column of Table 1 shows the optimal value of α and the corresponding values of R, $E[Z]$, $Var[Z]$, $P[Z]$, and $E[L_Z]$, while the second column shows the corresponding values for the best (in terms of the adjustment coefficient) stop loss treaty. The optimal policy improves the adjustment coefficient by 16.1% with respect to the best stop loss treaty, at the cost of an increase of 111% in the reinsurance premium. However, notice that the relative contribution of the loading to the total reinsurance premium is much smaller in the optimal policy, compared with the best stop loss. Hence, thought a larger premium is ceded under the optimal treaty than under the best stop loss, this is made mainly through the pure premium, rather than the premium loading, so the expected profits are not very different.

Table 1: Y - Pareto random variable

	Optimal Treaty	Best Stop Loss
	$\alpha = 1.74411$	$M = 67.4436$
R	0.055406	0.047703
$E[Z]$	0.098018	0.001050
$Var[Z]$	0.212089	0.160269
$P[Z]$	0.213151	0.101134
$E[L_Z]$	0.084867	0.099916

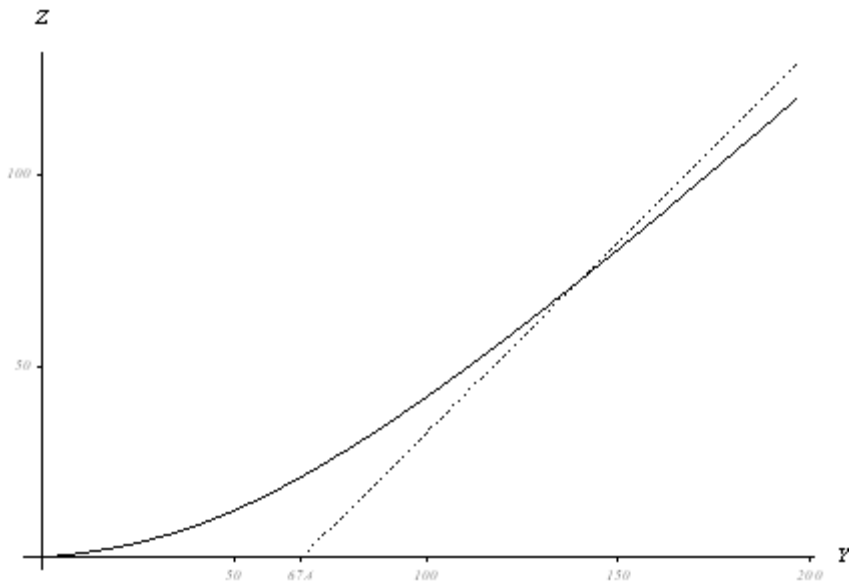

Figure 1: Optimal policy (full line) versus best stop loss (dashed line): the Pareto case

Figure 1 shows the optimal reinsurance arrangement versus the best stop loss treaty $Z_M(Y) = \max\{0, Y - M\}$. It shows that the improved performance of the optimal policy is achieved partly by compensating a lower level of reinsurance against very high losses (which occur rarely) by reinsuring a substantial part of moderate losses, which occur more frequently but are inadequately covered or not covered at all by the stop-loss treaty.

Table 2: Y - Generalized gamma random variable

	Optimal Treaty	Best Stop Loss
	α =0.813383	$M = 47.8468$
R	0.084709	0.078571
$E[Z]$	0.076969	0.000204
$Var[Z]$	0.049546	0.004950
$P[Z]$	0.132616	0.017794
$E[L_Z]$	0.144353	0.182410

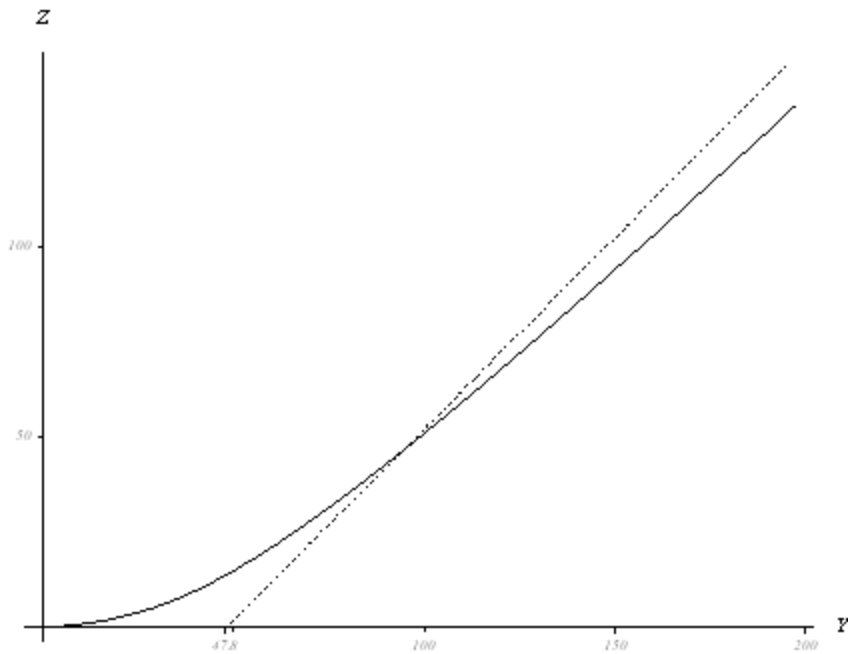

Figure 2: Optimal policy (full line) versus best stop loss (dashed line): the generalized gamma case

Example 2 *In this example, Y follows the generalized gamma distribution with density*

$$f(y) = \frac{b}{\Gamma(k)\theta}\left(\frac{y}{\theta}\right)^{kb-1} e^{-\left(\frac{y}{\theta}\right)^b}, \qquad y > 0,$$

with $b = 1/3$, $k = 4$ and $\theta = 3!/6!$. Table 2 shows the results for this example. The general features are similar to Example 1 but the improvement with respect to the best stop loss is smaller (the optimal policy increases the adjustment coefficient by about 7.8% with respect to the best stop loss). The optimal policy presents a larger increase in the sharing of risk and profits and a sharp increase in the reinsurance premium (more than seven-fold) with respect to the best stop loss. However, in both cases the amount of the risk and of the profits which is ceded under the reinsurance treaty is substantially smaller than in the Pareto case.

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