A model for numerical evaluation of continuous time ruin probabilities with a variable premium rate

Lourdes B. Afonso*
CMA and FCT, New University of Lisbon
lbafonso@fct.unl.pt

Alfredo D. Egídio dos Reis*
CEMAPRE and ISEG, Technical University of Lisbon
alfredo@iseg.utl.pt

Howard R. Waters
Department of Actuarial Mathematics and Statistics, Heriot-Watt University
H.R.Waters@ma.hw.ac.uk

February 1, 2007

Abstract

In this paper we present a method for the numerical evaluation of the ruin probability in continuous, finite or infinite time for a classical risk process where the premium can change from year to year. Our method is based on the simulation of the annual aggregate claims and then on the calculation of the ruin probability for a given surplus at the start and at the end of each year. We calculate the within-year ruin probability assuming first a Brownian motion approximation and, secondly, a translated gamma distribution approximation for the aggregate claim amount.

Keywords: Probability of ruin; finite time ruin probability; credibility; translated Gamma approximation; Brownian motion approximation; simulation.

*The authors gratefully acknowledge the partial financial support from *Fundação para a Ciência e a Tecnologia* (programme FEDER/POCI 2010).

1 Introduction

This paper presents a method for calculating the probability of ruin in continuous time, either finite or infinite, for a compound Poisson risk process where the premium rate can change at the start of each period, say year, although remaining constant throughout the year. The idea of the present work is to build up a model that allows the premium to be dependent on the portfolio past experience on annual aggregate claims. In particular we can think of the premium being dependent of the surplus level at the start of some previous year. We can think of experience rating models for instance, where past aggregate claims are a basis for calculating annual premiums.

Our model involves the simulation of annual aggregate claims, calculating the premium to be charged at the start of each year given the past aggregate claims, and then calculating the probability of ruin in the year, which we call the within year ruin probability. We will do this using two different approximation methods, a Brownian motion and a translated gamma distribution approximation to the surplus process, for which we can compute formulae for the within year probability of ruin. The Brownian motion approximation can be seen in (Klugman, Panjer & Willmot 2004), Sections 8.6 and 8.7 and we will study it in our problem in the next section. The translated gamma approximation have been recently used by (Dickson & Waters 2006), it will be presented in Section 3.

Let's set out our model and general procedure for calculating the ruin probabilities in finite and continuous time. Without loss of generality consider the period measured in years.

Consider a risk process over an n-year period which is described by

$$U(t) = u + \sum_{j=1}^{i-1} P_j + (t - i + 1)P_i - S(t), \quad 0 \le t \le n \tag{1}$$

where i is such that $t \in [i-1, i)$, $i = 1, 2, ..., n$ and, $U(t)$ is the insurer's surplus at time t, $0 \le t \le n$, $u = U(0)$ is the insurer's initial surplus and is assumed to be known, P_i is the premium set at the begining of year i, $S(t)$ is the aggregate claims up to time t, Y_i is the aggregate claims in year i, i.e. $Y_i = S(i+1) - S(i)$. We assume that $\{Y_i\}_{i=1}^{n}$ is a sequence of *i.i.d.* random variables, with a compound Poisson distribution whose first three moments exist. We denote by λ the Poisson parameter and the expected number of claims each year, and by $f(., s)$ the probability density of $S(s)$, $0 < s \le 1$.

We also assume that premiums are received continuously at a constant rate throughout each year and that the initial premium, P_1, is known. For $i = 2, \ldots, n$, we assume that P_i is a function of $\{Y_j\}_{j=1}^{i-1}$, the aggregate claims in the preceding years and is updated at the beginning of the year.

For $i \ge 2$, the premium P_i and surplus level $U(i)$ are random variables since they both depend on the claims experience in previous years. Where we wish to refer to a particular realization of these variables, we will use the lower case letters p_i and $u(i)$.

The probability of ruin in continuous time within n years is denoted by $\psi(u, n)$ and defined as:

$$\psi(u, n) \stackrel{def}{=} \Pr(U(t) < 0 \text{ for some } t \in (0, n])$$

Let $\psi(u(i-1), u(i), 1)$ be the probability of ruin within the year i, given the surplus $u(i-1)$ at the start of the year and the surplus $u(i)$ at the end. As referred above we will use the Brownian motion and a translated gamma distribution methods to approximate this probability. Both the methods are moment based, the former by matching two moments and the latter matching three.

2 The Brownian motion process approximation

Let $\{W(s); s \geq 0\}$ be a Brownian motion process with drift and variance per unit time denoted μ and σ^2, respectively. For the given values $u(i-1)$ and p_i, we approximate the surplus process, $\{U(t); i-1 \leq t \leq i\}$ by the Brownian motion process $\{u(i-1)+W(s); 0 \leq s \leq 1\}$ with $s = t-i+1$ and

$$\mu = p_i - \mathrm{E}[Y_i] \quad \text{and} \quad \sigma^2 = \mathrm{Var}[Y_i]$$

so that for $0 \leq s \leq 1$

$$\begin{aligned} \mathrm{E}[u(i-1)+W(s)] &= u(i-1)+s(p_i - \mathrm{E}[Y_i]) = \mathrm{E}[U(t)|U(i-1)=u(i-1)] \\ \mathrm{Var}[W(s)] &= s.\sigma^2 = \mathrm{Var}[U(t)|U(i-1)=u(i-1)] \end{aligned}$$

Let T denote the time until ruin for this process, so that

$$T = \inf(s > 0 : u(i-1)+W(s) < 0)$$

with the convention that $T = \infty$ if ruin never occurs. (Klugman et al. 2004), Corollaries 8.25 and 8.27, show that the probability that ruin ever occurs, denoted $\psi_{BM}(u(i-1))$, is

$$\psi_{BM}(u(i-1)) = \exp\left(-\frac{2\mu u(i-1)}{\sigma^2}\right), \tag{2}$$

and the conditional probability density of the time to ruin, given that ruin occurs, denoted $f_T(s)$, is given by

$$f_T(s) = \frac{u(i-1)}{\sqrt{2\pi\sigma^2}} s^{-3/2} \exp\left\{-\frac{(u(i-1)-\mu s)^2}{2\sigma^2 s}\right\}, \quad s > 0 \tag{3}$$

Hence, the (unconditional) probability density of the time to ruin, for finite s, without conditioning on whether ruin occurs, is the product of (2) and (3), that is

$$f_T(s)\psi_{BM}(u(i-1)) = \frac{u(i-1)}{\sqrt{2\pi\sigma^2}} s^{-3/2} \exp\left\{-\frac{(u(i-1)-\mu s)^2 + 4\mu s u(i-1)}{2\sigma^2 s}\right\}. \tag{4}$$

(Klugman et al. 2004) also show (p. 259) that, for $0 < s < 1$, the conditional probability density of $u(i-1)+W(1)$ at y, given that $T = s$, denoted $f(y|T=s)$, is:

$$f(y|T=s) = \exp\left\{\frac{y\mu}{\sigma^2}\right\} \frac{\exp\left(-\frac{y^2+\mu^2(1-s)^2}{2\sigma^2(1-s)}\right)}{\sqrt{2\pi\sigma^2(1-s)}} \tag{5}$$

Hence, for $0 < T < 1$, the joint probability density of $u(i-1)+W(1)$ and T is given by the product of (4) and (5) and the conditional probability density of T, given that $u(i-1)+W(1) = u(i)$, denoted $f_W(s)$, is the product of (4) and (5) divided by the (marginal) density of $u(i-1)+W(1)$ at the point $u(i)$. Since $W(1)$ follows a Normal distribution, we have:

$$f_W(s) = \frac{\exp\left(\frac{u(i)\mu}{\sigma^2}\right)\frac{u(i-1)s^{-3/2}}{2\pi\sigma^2}\frac{1}{\sqrt{1-s}}\exp\left(-\frac{u(i)^2+\mu^2(1-s)^2}{2\sigma^2(1-s)} - \frac{(u(i-1)-\mu s)^2}{2\sigma^2 s} - \frac{2\mu u(i-1)}{\sigma^2}\right)}{n(u(i)-u(i-1),\mu,\sigma^2)}$$

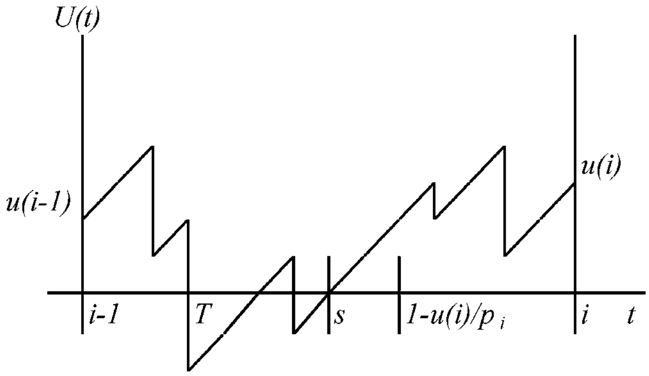

Figure 1: A sample path of the surplus in one period.

where $n(\cdot,\mu,\sigma^2)$ is the density function of the normal distribution.

Finally, the probability of ruin in the year, $\psi_{BM}(u(i-1),u(i),1)$, is the integral of this last conditional density from $s=0$ to $s=1$, i.e.

$$\psi_{BM}(u(i-1),u(i),1) = \int_{s=0}^{1} f_W(s)\,ds \tag{6}$$

We will use $\psi_{BM}(u(i-1),u(i),1)$ as an approximation to $\psi(u(i-1),u(i),1)$.

3 The Translated gamma distribution approximation

Consider the surplus process, $\{U(t)\}$ described by equation (1). Consider the time interval $[i-1,i]$ and assume we know the history of the process up to time $i-1$. Hence, the premium income in the year, p_i, is known.

Again, we are interested in $\psi(u(i-1),u(i),1)$, the probability of ruin within the year given the starting and final values for the surplus $U(t)$. A typical sample path of interest is illustrated in Figure 1. Here, T indicates the instant of ruin and s is the time of the last ruin recovery in the year before end. For convenience, we will measure T and s from the start of the year, so that $s=t-i+1$. Note that, given that $U(i)=u(i)$, the maximum value for s is $1-u(i)/p_i$, since in a time interval of length $(1-s)$, the maximum increase in the surplus is $p_i(1-s)$.

Let $f(\cdot,s)$ and $F(\cdot,s)$ denote respectively the density and the distribution function of the aggregate claims in a time interval of length s.

For sample paths as illustrated in Figure 1 to occur, two independent events (i) and (ii) must occur and we must allow a third event (iii) to occur:

(i) the aggregate claims in $[i-1,s]$ must be $u(i-1)+p_is$; the probability density for this is

$$p_i f\left(u(i-1)+p_is,s\right);$$

(ii) starting from an initial surplus of 0 at time $(i-s)$, the surplus process must reach $u(i)$ at time i with no ruin occurring in this interval; the probability density for this event is:

$$\frac{u(i)}{p_i(1-s)} f(p_i(1-s)-u(i),1-s);$$

(iii) ruin occurs for the final time at instant $s=1-u(i)/p_i$ and there are then no claims in the remaining time $u(i)/p_i$; the probability density for this event is:

$$f(u(i-1)+p_i-u(i),1-u(i)/p_i)\exp(-\lambda u(i)/p_i).$$

To get $\psi(u(i-1),u(i),1)$ we are going to use Section 3.2 of (Dickson & Waters 2006) with a change of notation. Hence, using their formulae (3.10) and (3.11) we have the survival probability $\delta(u(i-1),u(i),1)=1-\psi(u(i-1),u(i),1)$:

$$\begin{aligned}\delta(u(i-1),u(i),1) &= 1-f(u(i-1)+p_i-u(i),1) \\ &\quad -f(u(i-1)+p_i-u(i),1-u(i)/p_i)\exp(-\lambda u(i)/p_i) \\ &\quad -p_i\int_0^{1-u(i)/p_i} f(u(i-1)+cs,s)\delta(0,u(i),1-s)\,ds\end{aligned} \tag{7}$$

with

$$\delta(0,u(i),1-s)=\frac{u(i)}{p_i(1-s)} f(p_i(1-s)-u(i),1-s).$$

We divide (7) by $f(u(i-1)+p_i-u(i),1)$ to get the density conditional on the surplus at the end of the year being between $u(i)$ and $u(i)+du(i)$, and subtract from 1, since we want the density of the probability of ruin, rather than non-ruin, we arrive to formula (8).

$$\begin{aligned}\psi(u(i-1),u(i),1) &= \frac{\int_{s=0}^{1-u(i)/p_i} f(u(i-1)+p_is,s)\frac{u(i)}{(1-s)} f(p_i(1-s)-u(i),1-s)ds}{f(u(i-1)+p_i-u(i),1)} \\ &\quad +\frac{f(u(i-1)+p_i-u(i),1-u(i)/p_i)\exp(-\lambda u(i)/p_i)}{f(u(i-1)+p_i-u(i),1)}\end{aligned} \tag{8}$$

Formula (8) is an exact expression for $\psi(u(i-1),u(i),1)$, however it is not easy to evaluate since it requires the probability density of the aggregate claims over a time interval of length s to be known, for all $s\in(0,1]$. Although these values can be calculated using well known recursive formulae, the number of values can be prohibitively large, particularly if λ is large, and so some practical approximate method is required. So, to evaluate formula (8) we assume that the probability densities can be approximated by the densities of translated gamma random variables, matched by moments.

This idea goes back at least to (Seal 1978*a*). In this approximation there are involved the first three moments of the distribution of individual claim. More recently, (Dickson & Waters 2006) also use this idea to obtain ruin probabilities in finite and continuous time under reinsurance strategies.

Let $H(s)$ be a random variable with a gamma distribution with parameters αs and β (so that its mean is $\alpha s/\beta$). Let κ be a constant. Then for $0<s\leq 1$ the random variable $H(s)+\kappa$ has

a translated gamma distribution, and we will choose the parameters α, β and κ so that the first three moments match those of $S(s)$. Let $F_G(.,s)$ and $f_G(.,s)$ denote the cumulative distribution function and pdf of $H(s)$, respectively. α, β and κ will be such that:

$$\begin{aligned}
\frac{2}{\sqrt{\alpha}} &= \frac{\mathrm{E}[(Y_i - \mathrm{E}[Y_i])^3]}{\mathrm{Var}[Y_i]^{3/2}}, \\
\frac{\alpha}{\beta^2} &= \mathrm{Var}[Y_i] \\
\frac{\alpha}{\beta} + \kappa &= \mathrm{E}[Y_i]
\end{aligned}$$

In formula (8) we replace $f(x,s)$ by $f_G(x-\kappa s, s)$ and $\exp(-\lambda t)$ by $F_G(\kappa t, t)$. Note that for the compound Poisson process $\exp(-\lambda t)$ is the probability of no claims in a time interval of length t. We approximate this by the probability that $H(t)+\kappa t$ is negative. Then our approximation for $\psi(u(i-1), u(i), 1)$ comes

$$\begin{aligned}
&\psi_{TG}(u(i-1), u(i), 1) = \\
&= \frac{\int_{s=0}^{1-u(i)/p_i} f_G\left(u(i-1)+p_i s - \kappa s; s\right) \frac{u(i)}{(1-s)} f_G\left(p_i(1-s) - u(i) - \kappa(1-s); 1-s\right) ds}{f_G\left(u(i-1)+p_i - u(i) - \kappa; 1\right)} \\
&+ \frac{f_G\left(u(i-1)+p_i - u(i) - \kappa(1-u(i)/p_i); 1-u(i)/p_i\right) F_G(-\kappa u(i)/p_i; u(i)/p_i)}{f_G\left(u(i-1)+p_i - u(i) - \kappa; 1\right)} \qquad (9)
\end{aligned}$$

The advantage of using this approximation is that there are well established and fast algorithms for calculating gamma densities so that the approximation can be calculated far more quickly.

4 Simulation procedure

Our goal is to estimate $\psi(u,n)$. To achieve this we will simulate N paths of the surplus process (1). Each path starts at u $(=U(0))$. The premium in the first year, $P_1 = p_1$, is given. We simulate the aggregate claims in each year and calculate the respective premium in order to calculate the probability of ruin given the surplus at the start and at the end of each year. If the surplus at the end of the year is negative, ruin has occurred, we stop this run, set as an *estimate* for the probability of ruin the value 1 and we start running another path.

The ruin probabilities within the year are calculated using either (6) or (9).

Let $\psi_j(u,n)$, $j = 1, 2, ..., N$, denote the estimate of $\psi(u,n)$ from the $j-th$ simulation. Our procedure is as follows:

(i) Simulate the values of $\{Y_i\}_{i=1}^n$ by assuming that each Y_i is approximately distributed as $H(1)+\kappa$, as defined in the previous section, so that Y_i follows a translated gamma distribution with parameters α, β and κ.

(ii) From the simulated values of $\{Y_i\}_{i=1}^{n}$, say $\{y_i\}_{i=1}^{n}$, calculate succesively $u(1) = u + p_1 - y_1$, then p_2 (as a function of u and $u(1)$), $u(2) = u(1) + p_2 - y_2$ and so on until the surplus at the end of each year, $\{u(i)\}_{i=0}^{n}$, has been calculated.

(iii) If $u(i) < 0$ for any $i, i = 1, 2, \ldots, n$, then we set $\psi_j(u,n) = 1$ and we start simulation $j+1$.

(iv) If $u(i) \geq 0$ for all $i, i = 1, 2, \ldots, n$, we calculate $\psi_j(u(i-1), u(i), 1)$, using either a Brownian motion or translated gamma approximation, as follows.

$$\begin{aligned}\psi_j(u,n) &= 1 - \Pi_{i=1}^{n}(1 - \psi_{BM}(u(i-1), u(i), 1)) \\ \psi_j(u,n) &= 1 - \Pi_{i=1}^{n}(1 - \psi_{TG}(u(i-1), u(i), 1))\end{aligned}$$

The mean of our N estimates, $\{\psi_j(u,n)\}_{j=1}^{N}$ is then our estimate of $\psi(u,n)$ and we can use the sample standard deviation of the N estimates to calculate approximate confidence intervals for the estimate. We will denote our estimates $\hat{\psi}_{BM}(u,n)$ and $\hat{\psi}_{TG}(u,n)$.

5 Accuracy of the procedure

We consider the two approximation methods described above and we expect both to be reasonable approximations if the expected number of claims each year, λ, is large and the individual claim size distribution does not have a too fat tail. In the case $u = 0$ we cannot use the Brownian motion approximation to the within year probability of ruin since each simulation will give $\psi_j(0, u(1), 1) = 1$.

(Wikstad 1971) and (Seal 1978*b*), provide values of ruin probabilities in finite and continuous time for some compound Poisson risk process, in all cases with a fixed premium rate. (Gerber 1979) and (Bowers, Gerber, Hickman, Jones & Nesbitt 1997), for instance, provide values for ultimate ruin probabilitites. We can test the accuracy of our method by applying it to their examples. We restrict our comparisons to cases where the probability of ruin lie between 0.001 and 0.05, which we consider covers all values of practical interest.

Example 1:

(Wikstad 1971) in his case IA considered exponentially distributed individual claims with mean 1 and with one claim expected each year ($\lambda = 1$). Table 1 shows his values for selected cases and our estimates, $\hat{\psi}_{BM}(u,n)$ and $\hat{\psi}_{TG}(u,n)$, of these values together with the standard errors of these estimates.

In Table 1, ζ denotes the premium loading factor, so that the premium rate each year is $(1+\zeta)$, and $\psi(u,n)$ denotes Wikstad's value.

u	n	ζ	$\psi(u,n)$	$\hat{\psi}_{BM}(u,n)$	SD$[\hat{\psi}_{BM}(u,n)]$	$\hat{\psi}_{TG}(u,n)$	SD$[\hat{\psi}_{TG}(u,n)]$
10	10	0.05	0.03670	0.03621	0.0008	0.03487	0.0008
10	10	0.15	0.02770	0.02808	0.0007	0.02832	0.0007
10	10	0.25	0.02090	0.01897	0.0006	0.02011	0.0006

Table 1: Values and estimates of $\psi(u,n)$. Wikstad (1971, Case IA).

Example 2:

(Wikstad 1971) in his case IIA considered a compound Poisson surplus model where individual claim amounts have the following distribution:

$$P(x) = 1 - 0.0039793\exp(-0.014631x) - 0.1078392\exp(-0.19206x) - 0.8881815\exp(-5.514588x)$$

This is described by Wikstad as an attempt to model Swedish non-industrial fire insurance data from 1948-1951. He also describes the distribution as 'extremely skew'. Table 2 shows values of the probability of ruin, with $\lambda = 1$, in the same format as Tables 1 and 3.

u	n	ζ	$\psi(u,n)$	$\hat{\psi}_{BM}(u,n)$	SD$[\hat{\psi}_{BM}(u,n)]$	$\hat{\psi}_{TG}(u,n)$	SD$[\hat{\psi}_{TG}(u,n)]$
100	10	0.05	0.00940	0.01044	0.0005	0.00970	0.0004
10	1	0.15	0.01880	0.01813	0.0004	0.00963	0.0004
100	10	0.15	0.00930	0.01088	0.0005	0.01010	0.0005
10	1	0.25	0.01870	0.01763	0.0004	0.00935	0.0004
100	10	0.25	0.00920	0.00999	0.0004	0.00914	0.0004

Table 2: Values and estimates of $\psi(u,n)$. Wikstad (1971, Case IIA).

Example 3:

(Seal 1978*b*) also considered exponentially distributed individual claims with mean 1 and with one claim expected each year ($\lambda = 1$). Table 3 shows his values for selected cases and our estimates using the same format as in Table 1.

u	n	ζ	$\psi(u,n)$	$\hat{\psi}_{BM}(u,n)$	SD$[\hat{\psi}_{BM}(u,n)]$	$\hat{\psi}_{TG}(u,n)$	SD$[\hat{\psi}_{TG}(u,n)]$
10	10	0.10	0.03190	0.03491	0.0008	0.03105	0.0008
22	50	0.10	0.01562	0.01628	0.0005	0.01539	0.0005
44	600	0.10	0.01348	0.01338	0.0005	0.01361	0.0005
66	600	0.10	0.00135	0.00130	0.0001	0.00133	0.0001

Table 3: Values and estimates of $\psi(u,n)$. Seal (1978).

It can be seen that in our three examples above that $\psi_{TG}(u,n)$ is generally closer to $\psi(u,n)$ than is $\psi_{BM}(u,n)$. This is as expected since the former approximation is based on matching three moments and the latter is based on matching only two. The notable exceptions are values for $u = 10,\ n = 1$ in Table 2. These are extreme cases since one claim is expected each year and the probability that this claim on its own exceeds the initial surplus is 0.0192, which is almost the same as the probability of ruin over 10 years in each case.

The standard errors of our estimates are almost identical for the two approximations to the within year probability of ruin. This is not surprising since the major source of randomness comes from the simulation of the aggregate annual claims and the same simulations are used for the two approximations.

Example 4:

If the single claim amount distribution is exponential with parameter $\nu > 0$ the probability of ruin is an exponential function of the initial surplus measured in mean claim amounts, see formula (3.14) Chapter 8 of (Gerber 1979) .

$$\psi(u) = \frac{1}{1+\zeta} exp\left(-\frac{\nu\zeta}{1+\zeta}u\right), u \geq 0 \tag{10}$$

Tables 4 shows the values of the probability of ruin in continuous and infinite time for selected cases and our estimates $\hat{\psi}_{BM}(u, 1000)$ and $\hat{\psi}_{TG}(u, 1000)$, for the Compound Poisson risk process with exponentially distributed individual claims. The premium is set constant and equals $c = (1+\zeta)\lambda/\nu = (1+\zeta)20000$, with $\lambda = 1000$ and $\nu = 0.05$. We show also the standard errors of these estimates. $\psi(u)$ denote Gerber's values.

The drift and variance of the Brownian motion process are:

$$\mu = \frac{\zeta\lambda}{\nu} \quad \text{and} \quad \sigma^2 = \frac{2\lambda}{\nu^2}$$

The parameters for the translated gamma approximation α, β and κ are given by:

$$\alpha = \tfrac{2^3}{3^2}\lambda \qquad \beta = \tfrac{2\nu}{3} \qquad \kappa = -\tfrac{\lambda}{3\nu}$$

	u	$\psi(u)$	$\psi_{TG}(u,n)$	SD$[\hat{\psi}_{TG}(u,n)]$	$\psi_{BM}(u,n)$	SD$[\hat{\psi}_{BM}(u,n)]$
$\zeta = 0.05$	1100	0.06940	0.07096	0.00089	0.06767	0.00088
	1300	0.04311	0.04427	0.00073	0.04206	0.00072
$\zeta = 0.15$	500	0.03335	0.03345	0.00022	0.02390	0.00018
	700	0.00905	0.00912	0.00011	0.00551	8.66E-05
$\zeta = 0.25$	300	0.03983	0.03998	0.00013	0.02357	0.00008
	500	0.00539	0.00546	0.00003	0.00195	1.48E-05

Table 4: Values and estimates of $\psi(u)$: exponentially distributed claim amounts. (Gerber 1979).

Example 5:

We consider here that the claim amounts distribution is a mixture of exponentials with pdf $g(z) = 3/2e^{-3z} + 7/2e^{-7z}$, $z > 0$, and ultimate ruin probability

$$\psi(u) = \frac{24}{35}e^{-u} + \frac{1}{35}e^{-6u}, u \geq 0 \ ,$$

please see Example 3.2 p. 116 of (Gerber 1979) or Example 13.6.2 of (Bowers et al. 1997).

Table 5 shows the values of the probability of ruin in continuous and infinite time for selected cases and our estimates $\hat{\psi}_{BM}(u, 1000)$ and $\hat{\psi}_{TG}(u, 1000)$, for a premium loading factor $\zeta = 2/5$.

The premium is set constant and equals $c = (1+\zeta)\lambda E[Z] = 5 \times 10^6$, with $\lambda = 1000$. We show also the standard errors of these estimates. $\psi(u)$ denote Gerber's values. The drift and variance of the Brownian motion process are:

$$\mu = \frac{5\zeta\lambda}{21} \quad \text{and} \quad \sigma^2 = \frac{58\lambda}{441}$$

The parameters for the translated gamma approximation α, β and κ are given by:

$$\alpha = \frac{195112}{308025}\lambda \qquad \beta = \frac{406}{185} \qquad \kappa = -\frac{589}{11655}\lambda$$

u	$\psi(u)$	$\psi_{TG}(u,n)$	SD$[\hat{\psi}_{TG}(u,n)]$	$\psi_{BM}(u,n)$	SD$[\hat{\psi}_{BM}(u,n)]$
3	0.03414	0.03428	7.92E-05	0.01298	3.30E-05
4	0.01256	0.01272	3.94E-05	0.00305	1.10E-05
5	0.00462	0.00472	1.88E-05	0.00072	3.54E-06
6	0.00170	0.00175	8.74E-06	0.00017	1.12E-06

Table 5: Values and estimates of $\psi(u)$: individual claim amounts being a mixture of exponentials. (Gerber 1979).

Again, in these two last examples we can see that the translated gamma approximation is better than the Brownian motion approximation, especially in the mixture of exponentials case.

References

Bowers, N., Gerber, H., Hickman, J., Jones, D. & Nesbitt, C. (1997), *Actuarial Mathematics*, 2nd edn, The Society of Actuaries.

Dickson, D. & Waters, H. (2006), 'Optimal dynamic reinsurance', *ASTIN Bulletin* **36**(2), 415–432.

Gerber, H. U. (1979), *An Introduction to Mathematical Risk Theory*, Huebner Foundation for Insurance Education University of Pennsylvania, Philadelphia, Pa 19104 USA.

Klugman, S., Panjer, H. & Willmot, G. (2004), *Loss models: from data to decisions*, 2nd edn, Jonh Wiley and Sons, Inc.

Seal, H. (1978*a*), 'From aggregate claims distribution to probability of ruin', *ASTIN Bulletin* **10**(1), 47–53.

Seal, H. L. (1978*b*), *Survival probabilities, the goal of risk theory*, Wiley, New York.

Wikstad, N. (1971), 'Exemplification of ruin probabilities', *ASTIN Bulletin* **6**(2), 147–152.

Lourdes B. Afonso
Departamento de Matemática
Faculdade Ciências e Tecnologia
Universidade Nova de Lisboa
Monte da Caparica
2829-516 Caparica
Portugal
email: mla@fct.unl.pt

Alfredo D. Egídio dos Reis
Departamento de Matemática
Instituto Superior de Economia e Gestão
Rua do Quelhas 6 (G. 201 Q)
1200-781 Lisboa
Portugal
email: alfredo@iseg.utl.pt

Howard R. Waters
Department of Actuarial Mathematics and Statistics
Heriot-Watt University
Edinburgh EH14 4AS
Great Britain
email: H.R.Waters@ma.hw.ac.uk