Measurement and Transfer of Catastrophic Risks. A Simulation Analysis

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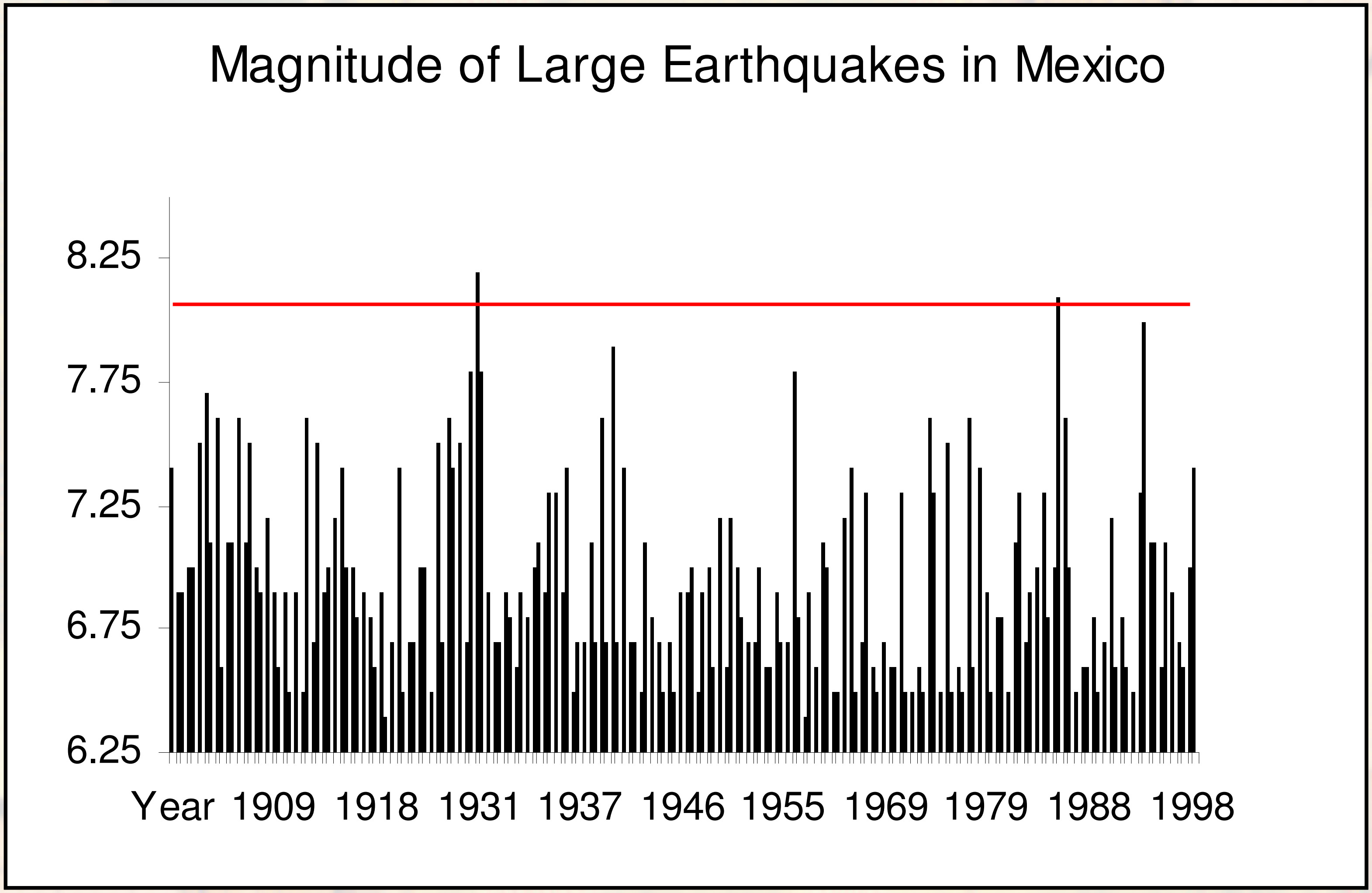

Magnitude of Large Earthquakes in Mexico
8.25
7.75
7.25
6.75
6.25
Year 1909 1918 1931 1937 1946 1955 1969 1979 1988 1998

Traditional measures for evaluating risk,

- probable maximum loss (PML),
- value at risk (VaR),
- Tail-VAR (CTE), and
- Probability of Ruin
- Risk Premium

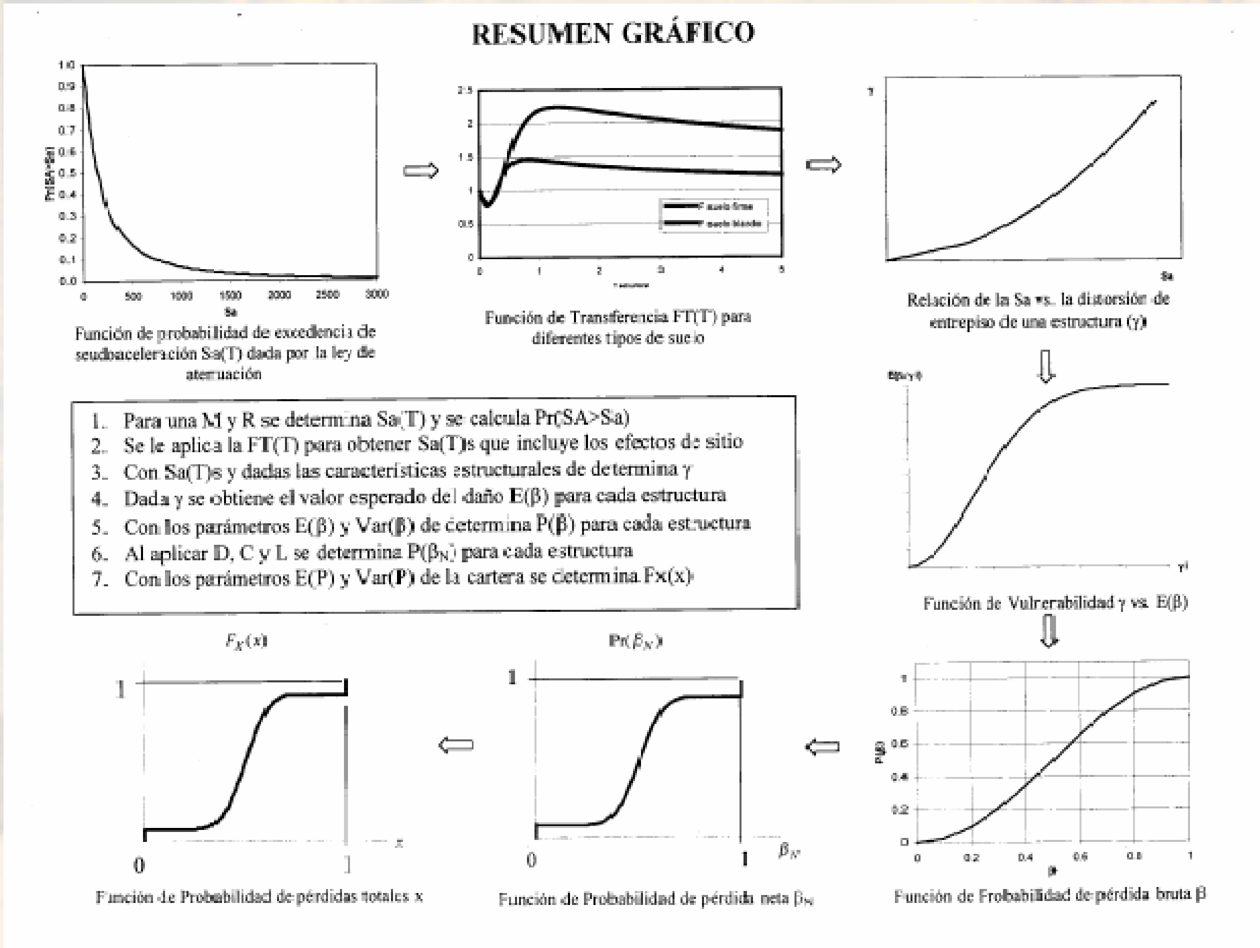
Practically impossible to obtain analytically in certain types of insurance, such as earthquake (earthquake generation, shock wave diffusion, etc.), e.g. if want to evaluate reinsurance

Normally losses are due to a main event and a replica. Of key importance for the industry, for a whole year of coverage, specially with multiple areas of exposure

PML defined as the losses y_0 that will be observed with a probability less than or equal to some previously agreed value, say .002. = 1/500, so that Pr{losses>= PML) = 1/500, the excedance probability.

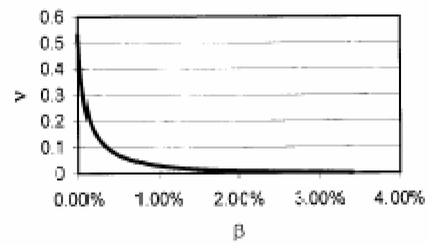
PML is usually expressed in terms of a return period, the time between events of a given magnitude,

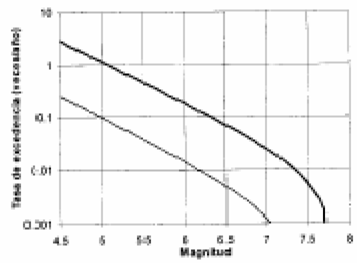
Return period = 1/Pr{losses>= PML) = 500. Usually very small and more difficult to compute.



* Provided by “**ERN, Ingenieros Consultores**”

$$\frac{1}{Tr} = \nu(\beta) = \sum_{i=1}^{Nfuentes} \int_{Moi}^{Mui} -\frac{d\lambda i(M)}{dM} \Pr(B > \beta / M, Ri)\, dM \quad \Rightarrow$$

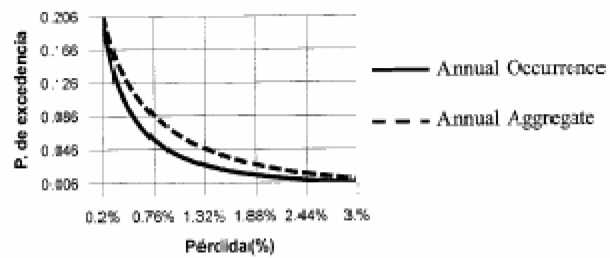




Tasa de excedencia de magnitudes, λ(M) para dos fuentes

8. Para determinar el riesgo sísmico se utiliza λ(M) aplicado a Fx(x) para todas las magnitudes y para todas las fuentes
9. Así se obtiene la tasa de excedencia de pérdida ν(β)
10. También aplicando λ(M) a E(P) para todas las magnitudes y para todas las fuentes se obtiene la Prima pura (pp)
11. Con esta se puede determinar las curvas de Probabilidad de quiebra (AO y AA)

$$pp = \sum_{i=1}^{Nfuentes} \int_{Moi}^{Mui} -\frac{d\lambda i}{dM} E[\beta / M, Ri]\, dM \quad \Rightarrow$$



$$\nu_i(y) = -\int_{M_0}^{M_{ui}} P(Y > y \mid M, f_i)\, d\lambda_i(M)$$

$\lambda_i(M)$ is the number of earthquakes of magnitude greater than M at source i.

$\nu_i(y)$ = average number of events, by unit time, that produce losses larger than y at seismic source i

Total exceedance rate for the whole portfolio

$$\nu(y) = \sum_{i=1}^{Nf} \int_{Mo}^{Mui} -\frac{d\lambda_i(M)}{dM} \Pr(Y > y \mid M, f_i) \Pr(f_i)\, dM$$

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Probability distribution for the losses of the portfolio

$$F(y) = 1 - \frac{\nu(y)}{\nu(0)}$$

$$\Pr(Y > y \mid M, f_i) = \Pr(S_{\exp}\beta > y \mid M, f_i) = \Pr(\beta > y / S_{\exp} \mid M, f_i)$$

$$\beta = \frac{Y}{S_{\exp}}$$

$$f_B(\beta) = P_0\delta(\beta) + (1 - P_0 - P_1)B(\beta; a, b \mid M, f_i) + P_1\delta(\beta - 1)$$

$$0 \le \beta \le 1$$

(1) Excess-loss treaty: the reinsurer pays that part of each claim that exceeds the retention level (the priority) $M \geq 0$

$$X_n^R = \{ X_n - M \}+, \quad n = 1,2...,$$

where $\{x\}+ = \max\{x,0\}$, $x \in$ R.

The ceding insurer covers claims that do not exceed the retention M and pays amount M when $X_n > M$;

$$X_n^I = min\{ X_n, M \}, \quad n = 1,2....$$

Usually, excess-loss treaties have provisions for coverage reinstatement, after they are exhausted in an event or a series of them

(2) Quota-share treaty: reinsurer accepts a certain proportion *a,* $0 < a < 1$, of each individual claim.

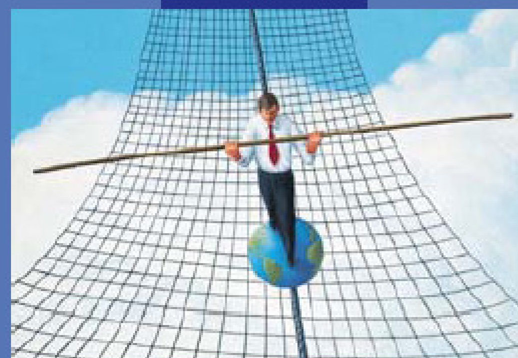
$$X_n^R = a \cdot X_n,$$

cedent retains the remaining $1 - a$,

$$X_n^I = (1 - a) \cdot X_n.$$

$1 - a$ = retention

A Global Framework for Insurer Solvency Assessment



AAI
IAA

Research Report of the Insurer Solvency Assessment Working Party

INTERNATIONAL ACTUARIAL ASSOCIATION
ASSOCIATION ACTUARIELLE INTERNATIONALE

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"While proper treatments and recognition of reinsurance arrangements are necessary to assess the impact of the of a ceding company's risk profile, this is a difficult task for a number of reasons.

- *Typical reinsurance arrangement comprise both proportional and non-proportional covers*
- *Some contracts have variable rating terms, ... for a proportional reinsurance treaty, and reinstatements or contingent commissions for an excess-of-loss treaty*
- *Some contracts cover just one line of business, others cover multiple lines of business ...*
- *Some contracts are on an aggregate basis, with aggregate deductibles and aggregate limits*
- *Some financial type reinsurance contracts cover a hybrid of underwriting and financial risks.*
- *Many reinsurance contracts do not bear a linear relationship with the underlying risks."*

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Proper evaluation of the risk reducing impact of non-proportional reinsurance contracts is still not possible without either relatively complex mathematical transformations, which are typically beyond the of supervisory control mechanisms, or the use of simulations, which are standard routines for more complex risk modelling in internal models.”

ISAWG: if applied properly to evaluate the solvency of a direct insurer, reinsurance is a very efficient means of reducing risk (particularly if measured by TVaR).

It becomes very complicated, if not impossible, to evaluate their mitigation or transfer effects of reinsurance schemes analytically.

It may be necessary to use alternative approaches, such as Monte Carlo simulation methods.

- How to include the effect of excess of loss reinsurance contracts?
- How to include reinstatements?
- How are reinstatement premiums taken into account?
- How to measure the effect of aggregate limits?

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The Model

The insurance regulatory body in Mexico (Comision Nacional de Seguros y Fianzas) commissioned the construction of an earthquake loss model that must be used to compute the pure risk premium as well as the PML, ERN (2002).

The algorithm includes the possibility of simulating

a) the occurrence of earthquakes
b) their impact on the insured portfolio
c) the risk transfer effect of reinsurance programs that mix different types or reinsurance
d) descriptive statistics for, gross losses, and losses net of reinsurance
e) the proportion of times the reinsurance is exhausted
f) average cost per year of reinstatements
g) distribution of loss by reinsurance layer according to their magnitude
h) percentage of years it was necessary to contract additional reinstatements

Sitio	magnitud	Probabilidad de ocurrencia anual	P0	P1	a	b
329 Costero en San Marcos M>7	7	1.55E-04	0	0	10.89002044	3088.203697
329 Costero en San Marcos M>7	7.48	6.78E-04	0	0	12.41154765	1762.965162
329 Costero en San Marcos M>7	7.8	3.38E-04	0	0	12.84697736	1068.781339
329 Costero en San Marcos M>7	8.05	1.13E-04	0	0	13.58280005	733.1621638
329 Costero en San Marcos M>7	8.22	4.01E-05	0	0	14.26929591	546.7891018
329 Costero en San Marcos M>7	8.4	6.60E-06	0	0	16.82657019	444.6389496
336 Costero en San Marcos M>7	7	1.54E-04	0	0	7.457165067	3128.50799
336 Costero en San Marcos M>7	7.48	6.75E-04	0	0	8.348849803	1517.767939
336 Costero en San Marcos M>7	7.8	3.36E-04	0	0	9.24219238	870.2394296
336 Costero en San Marcos M>7	8.05	1.13E-04	0	0	10.88808887	608.6445064
336 Costero en San Marcos M>7	8.22	3.99E-05	0	0	13.14780003	477.3056236
336 Costero en San Marcos M>7	8.4	6.58E-06	0	0	17.35110288	410.5728332
335 Costero en San Marcos M>7	7	1.54E-04	0	0	8.229884326	3475.708135
335 Costero en San Marcos M>7	7.48	6.75E-04	0	0	9.231863779	1798.39439
335 Costero en San Marcos M>7	7.8	3.36E-04	0	0	9.864955143	1039.022446
335 Costero en San Marcos M>7	8.05	1.13E-04	0	0	11.08425307	708.0447135
335 Costero en San Marcos M>7	8.22	3.99E-05	0	0	12.80889625	541.0697722
335 Costero en San Marcos M>7	8.4	6.57E-06	0	0	16.01680773	446.2955057
340 Costero en Guerrero M>7	7	3.53E-04	0	0	6.726432535	3027.937959
340 Costero en Guerrero M>7	7.48	1.55E-03	0	0	7.496139761	1559.603733
340 Costero en Guerrero M>7	7.8	7.81E-04	0	0	8.484783019	929.3486296
340 Costero en Guerrero M>7	8.04	2.68E-04	0	0	9.905775103	650.3443772
340 Costero en Guerrero M>7	8.22	9.53E-05	0	0	11.86444037	507.8211019
340 Costero en Guerrero M>7	8.4	1.54E-05	0	0	15.33513999	427.0501233
334 Costero en San Marcos M>7	7	1.54E-04	0	0	5.574821489	3436.1093
334 Costero en San Marcos M>7	7.48	6.75E-04	0	0	6.618722768	1589.073048
334 Costero en San Marcos M>7	7.8	3.36E-04	0	0	7.655479477	885.477713
334 Costero en San Marcos M>7	8.05	1.13E-04	0	0	9.301085166	610.7060624
334 Costero en San Marcos M>7	8.22	3.99E-05	0	0	11.53050592	476.7256471
334 Costero en San Marcos M>7	8.4	6.58E-06	0	0	15.50250525	408.3611405
331 Costero en San Marcos M>7	7	1.55E-04	0	0	10.1962632	3426.483496
331 Costero en San Marcos M>7	7.48	6.78E-04	0	0	12.56316414	2182.140523
331 Costero en San Marcos M>7	7.8	3.38E-04	0	0	13.48131973	1407.369141
331 Costero en San Marcos M>7	8.05	1.13E-04	0	0	13.83638387	951.9643486
331 Costero en San Marcos M>7	8.22	4.01E-05	0	0	13.94485997	693.7577321
331 Costero en San Marcos M>7	8.4	6.60E-06	0	0	15.38012792	535.4167295

In very broad terms the simulation algorithm is as follows:

a) Choose an earthquake site at random
b) Given the site, generate a magnitude at random from the corresponding distribution
c) Use the distribution of proportion of losses for the site-magnitude combination to generate a random proportion
d) Multiply the proportion resulting in c) by the total value insured for the portfolio and obtain a loss amount.
e) Apply any reinsurance that is in effect.

This process is applied as many times as there are earthquakes in a year to derive a figure of total losses for a year. As many replications are generated according to the required precision.

An Application

Real Mexican insurance company. The portfolio consists of 25,000 buildings. The reinsurance scheme is as follows:

Layers	Priority	Cover	Reinstatement Premium	RoI	Reins
1	$ 7,500	$ 7,500	$ 1,586	21.15%	2
2	$ 15,000	$ 15,000	$ 1,890	12.60%	2
3	$ 30,000	$ 30,000	$ 2,268	7.56%	1
4	$ 60,000	$ 40,000	$ 1,548	3.87%	1
5	$ 100,000	$ 130,000	$ 2,574	1.98%	1
Superior	$ 230,000	None	NA	NA	NA

Includes a quota-share reinsurance program in priority, with 10% retention

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Gross Losses	Net Losses	Net Losses without Reinstatement Premiums	% Reduction	Fn	Return Period
$ 300,710	$ 14,278	$ 9,094	95.25%	0.999333333	1500
$ 231,938	$ 10,968	$ 8,448	95.27%	0.999	1000
$ 143,763	$ 9,313	$ 7,827	93.52%	0.998	500
$ 90,040	$ 8,038	$ 7,587	91.07%	0.995	200
$ 78,011	$ 7,684	$ 7,537	90.15%	0.99	100

	MEAN	ST. D.	MINIMUM	Q1	MEDIAN	Q3	MAXIMUM
GROSS LOSS	$ 4,873	$ 18,881	$ 6	$ 754	$ 1,565	$ 3,682	$ 1,213,000
NET LOSS	$ 526	$ 5,970	$ 1	$ 75	$ 157	$ 368	$ 762,500
NET LOSS W.O. REINS	$ 516	$ 5,815	$ 1	$ 75	$ 157	$ 368	$ 752,600
RETENTION	10.80%	31.62%	10.00%	10.00%	10.00%	10.00%	62.86%

Layers	% years all reinstatements were used	% years only one reinstatement was used	% years the second reinstatement was used	Distribution of events by layer
(1)	(2)	(3)	(4)	(5)
Priority	NA	NA	NA	99.9937%
1	0.00%	0.49%	0.00%	0.0040%
2	0.18%	0.18%	0.00%	0.0012%
3	0.09%	0.09%	NA	0.0002%
4	0.07%	0.07%	NA	0.0002%
5	0.05%	0.05%	NA	0.0002%
Sup	NA	NA	NA	0.0005%

Reinstatements in all layers were exhausted in 840 years (0.56%)

Conditional Tail Expectation (tail-VaR)

Tail-VaR										
			Gross Losses				Net Losses			
alpha	Return	k	CTE(alpha)	FSE(CTE)	LL	UL	CTE(alpha)	FSE(CTE)	LL	UL
0.01	100	1500	124,320	3423	117,610	131,030	16,339	1478	13,443	19,236
0.005	200	750	185,873	6051	174,012	197,733	26,399	2909	20,697	32,102
0.002	500	300	309,852	11877	286,573	333,130	53,237	6991	39,535	66,940
0.001	1000	150	442,367	17975	407,136	477,599	96,297	13059	70,703	121,892
0.000667	1500	100	530,843	22049	487,626	574,059	138,066	18178	102,436	173,695

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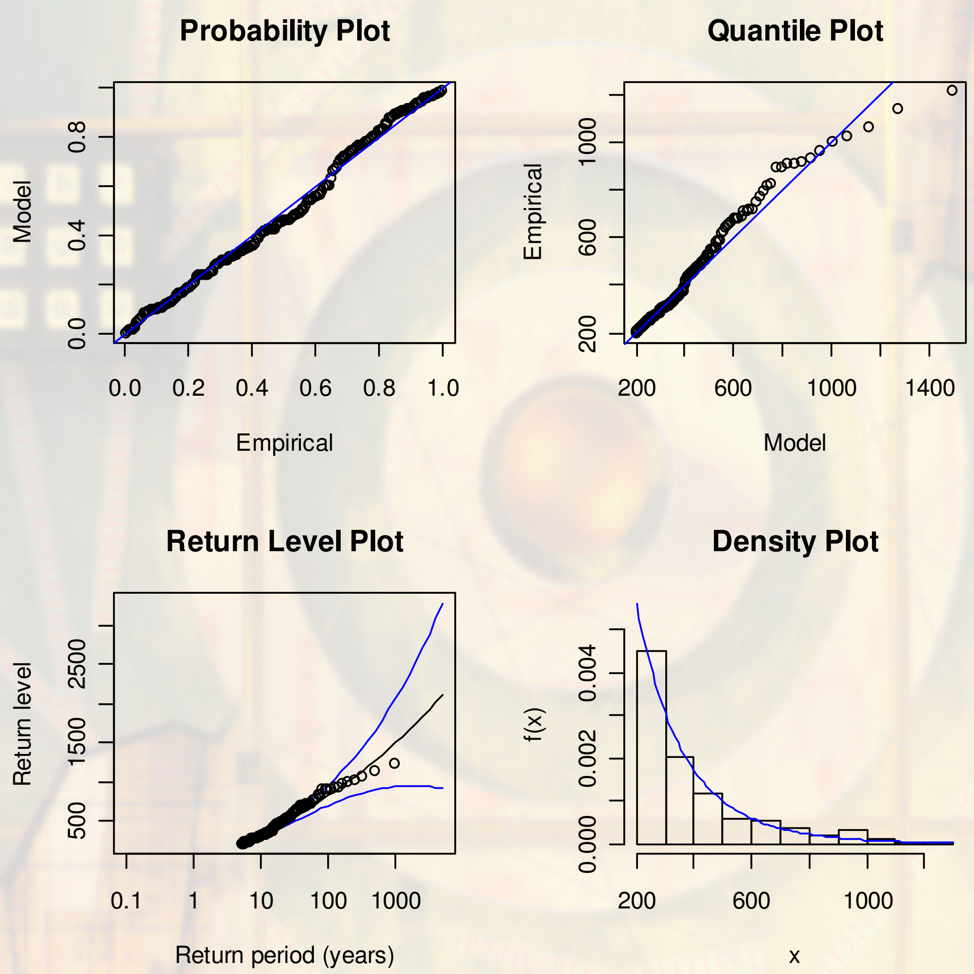
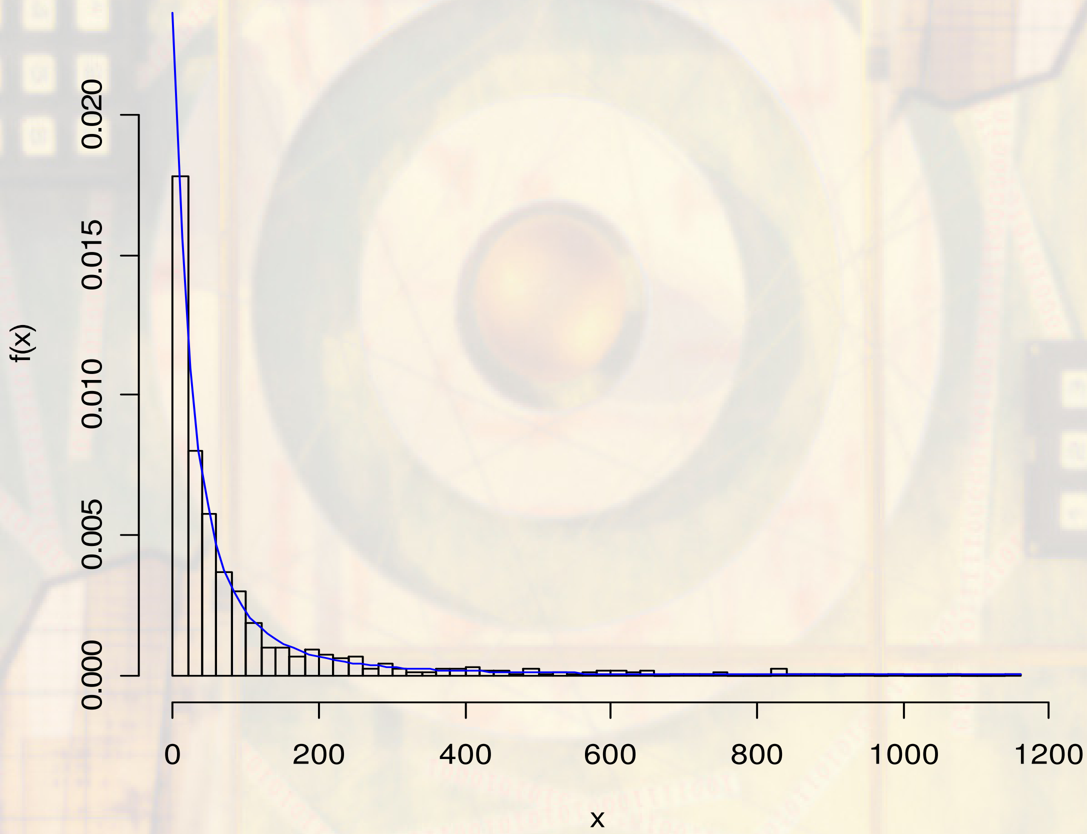
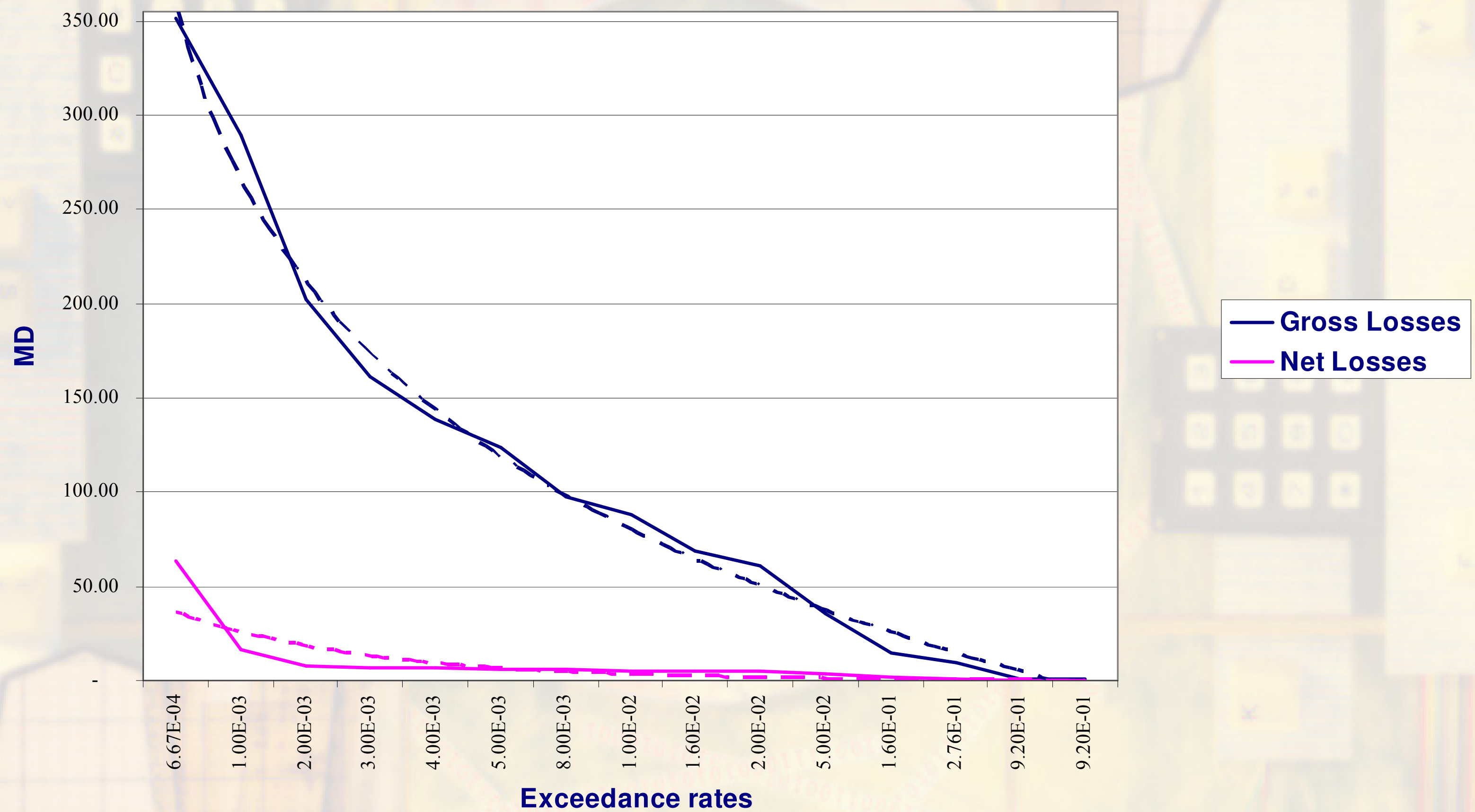

Probability Plot
Model
0.0
0.4
0.8
0.0
0.2
0.4
0.6
0.8
1.0
Empirical
Quantile Plot
Empirical
200
600
1000
200
600
1000
1400
Model
Return Level Plot
Return level
500
1500
2500
0.1
1
10
100
1000
Return period (years)
Density Plot
f(x)
0.000
0.002
0.004
200
600
1000
x

Density Plot

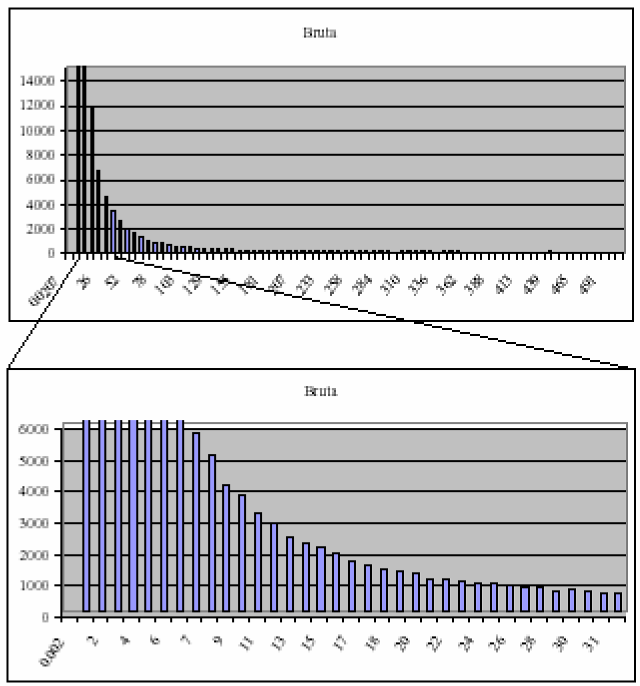


RETURN PERIODS	SIMULATION	EXTREME VALUE	LOWER LIMIT	UPPER LIMIT	ERN
1500	$ 300,710	$ 280,076	$ 256,524	$ 309,758	$ 284,718
1000	$ 231,938	$ 217,937	$ 203,170	$ 235,519	$ 224,775
500	$ 143,763	$ 141,439	$ 134,747	$ 148,824	$ 144,087

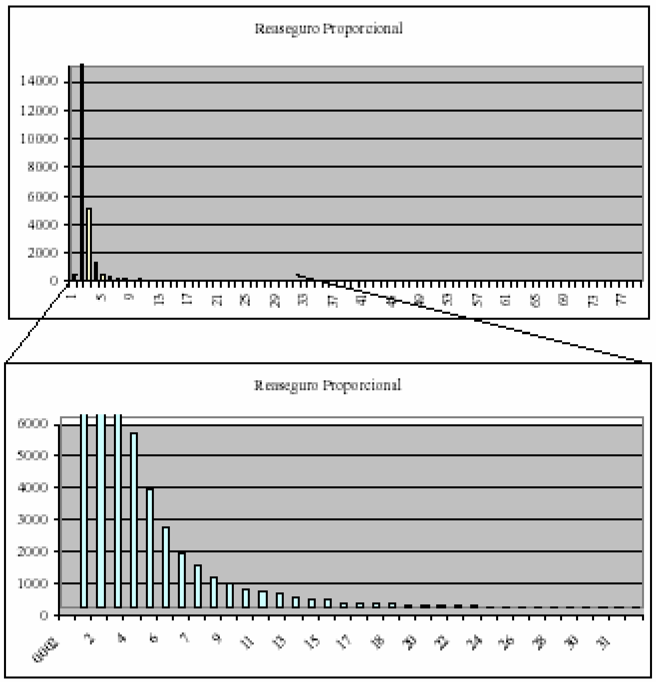
Comparison of Loss Curves



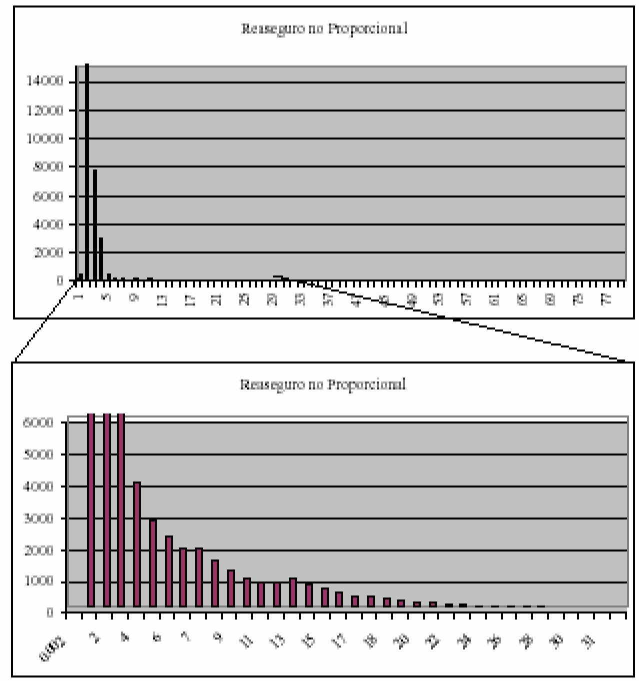
Distribución de Pérdidas Brutas



Distribución de Pérdida Neta de Reaseguro Proporcional



Distribución de Pérdida Neta de Reaseguro No Proporcional



Distribución de Pérdidas